Modelling Stochastic Processes in Geodetic VLBI Analysis

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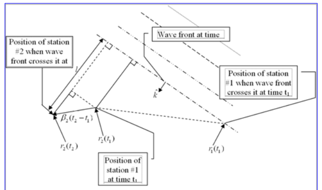
Abstract

Based on the recommendations (common standards in modelling, parameterization and analysis strategies) by the Global Geodetic Observing System (GGOS), analysis and modelling will be homogenized and improved across the various space geodetic techniques, such as Very Long Baseline Interferometry (VLBI), Global Navigation Satellite Systems (GNSS), Satellite Laser Ranging (SLR) and Doppler Orbitography by Radiopositioning Integrated on Satellite (DORIS). Furthermore, there is the project VLBI2010 of the International VLBI Service for Geodesy and Astrometry (IVS) which is asking for major updates of the VLBI software packages because of the significant increase of the number of observations with the new system. The analysis-related enhancements are planned to be implemented within two years in the VLBI analysis software Occam (or a Matlab-based equivalent) which is widely used all over the world by several institutes for VLBI parameter estimation. At the end of the study, the Occam software will fulfill all future requirements of GGOS with regard to modelling, parameterization and analysis. The work done up to now is the modelling of stochastic processes (VLBI clock errors and tropospheric wet delays). These parameters are estimated as 'piecewise linear offsets', i.e. only offsets are estimated at integer hours or integer fractions of it (every 20, 10, 5 minutes). In order to avoid numerical problems (as e.g. rank deficiencies) and to stabilize the parameter estimation process, constraints (pseudo observations) have to be included. Usually this type of pseudo-observations constrains the particular rate segment to zero and allows for a certain variation by assigning an appropriate formal error. In this paper, the VLBI delay model, the clock error model due to the inconsistencies of the hydrogen masers and atomic clocks at each antenna and the tropospheric wet delay model will be discussed.

Introduction

• VLBI is the only technique which connects the sky-fixed reference system (CRF) directly to the Earth-fixed system (TRF) via the Earth orientation parameters (EOP).
• In a VLBI experiment the parameters which can be estimated are,
- station locations in a *Terrestrial Reference Frame*,
- offsets of zenith wet delays (ZWD) and clocks,
- and troposphere gradients.
• In addition to these station specific parameters, a number of "global" parameters can also be estimated. These are common to the entire network and describe
- the orientation of the Earth (5 EOP parameters)
- and the positions of the sources (two angular parameters per source) in a *Celestial Reference Frame*.



Geometry for calculating the transmit time of a plane wave front (Sovers and Jacobs, 1994)

• The basic functional model of VLBI is:

$$\tau_{obs} = -\frac{1}{c}\cdot\begin{pmatrix} X_A - X_B \\ Y_A - Y_B \\ Z_A - Z_B \end{pmatrix}\cdot R(x_p, y_p, dUT1, d\Psi, d\varepsilon, z, \xi_A, \Theta_A)\cdot\begin{pmatrix} \cos\delta\cdot\cos h(t) \\ \cos\delta\cdot\sin h(t) \\ \sin\delta \end{pmatrix}$$
$$+\tau_{j-abb.}+\tau_{t-abb.}+\tau_{Rel.}+\tau_{Tid.}+\tau_{Load.}$$
$$+\tau_{Instr.}+\tau_{Clock}+\tau_{Ion.}+\tau_{Atm_h}+\tau_{Atm_w}$$

where the following terms are used:
$\tau_{j-abb.}$: annual abberation because of the motion of the Earth around the Solar System barycentre
$\tau_{t-abb.}$: diurnal abberation because of the rotation of the Earth
$\tau_{Rel.}$: relativistic effects
$\tau_{Tid.}$: deformation of Earth because of tides and because of changes of the angular momentum due to ocean tides
$\tau_{Load.}$: deformation of the Earth because of loading effects e.g. due to ocean tides and atmospheric pressure changes.
$\tau_{Ion.}$: Ionospheric correction
$\tau_{Instr.}$: Instrumental corrections
τ_{Atm_h} : Hydrostatic delay through the troposphere
τ_{Atm_w} : Wet delay through the troposphere
τ_{Clock} : Relative clock error

Modelling VLBI clock behaviour and zenith wet delays

- One of the largest constituents of the signal delays is caused by the differences in the behaviour of the station frequency standards. After choosing an arbitrary clock as the reference clock for the entire observing network the remaining clocks show both a constant difference (=clock offset) and a linear (=clock trend) or even higher rate of change relative to the reference clock.
- In order to describe the clock behaviour in a mathematical way, usually a simple polynomial approach is chosen,

$$\tau_{clk_1} = \beta_0^{clk1} + \beta_1^{clk1}(t_1^{clk1} - t_0) + \beta_2^{clk1}(t_1^{clk1} - t_0)^2$$
$$\tau_{clk_2} = \beta_0^{clk2} + \beta_1^{clk2}(t_1^{clk2} - t_0) + \beta_2^{clk2}(t_1^{clk2} - t_0)^2$$
$$\vdots$$
$$\tau_{clk_s} = \beta_0^{clk(s)} + \beta_1^{clk(s)}(t_1^{clk(s)} - t_0) + \beta_2^{clk(s)}(t_1^{clk(s)} - t_0)^2$$

where s is the notation assigned for the VLBI clocks included in the session. Observation equations of time delays are,

$$\Delta\tau_{clk(1,2)} = \tau_{clk_1} - \tau_{clk_2}$$
$$\Delta\tau_{clk(1,3)} = \tau_{clk_1} - \tau_{clk_3}$$
$$\vdots$$
$$\Delta\tau_{clk(s-1,s)} = \tau_{clk_{s-1}} - \tau_{clk_s}$$

The partial derivatives of the observation equations according to the unknowns of the clocks are, ⇨

$$\frac{\partial\tau_1}{\partial\beta_0} = 1d\beta_0^{clk1} - 1d\beta_0^{clk2}$$
$$\frac{\partial\tau_1}{\partial\beta_1} = (t_1^{clk1} - t_0)\,d\beta_1^{clk1} - (t_1^{clk2} - t_0)\,d\beta_1^{clk2}$$
$$\frac{\partial\tau_1}{\partial\beta_2} = (t_1^{clk1} - t_0)^2\,d\beta_2^{clk1} - (t_1^{clk2} - t_0)^2\,d\beta_2^{clk2}$$

The design matrix for the clocks can be formed as:

$$A_{clk.offset} = \begin{array}{c} \\ \tau_1 \\ \tau_2 \\ \vdots \\ \tau_{m-1} \\ \tau_m \end{array}\begin{bmatrix} d\beta_0^{clk1} & d\beta_0^{clk2} & \cdots & d\beta_0^{clk(s-1)} & d\beta_0^{clk(s)} \\ 1 & -1 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & -1 \\ 0 & 0 & \cdots & 1 & -1 \end{bmatrix}$$

$$A_{clk.rate} = \begin{array}{c} \\ \tau_1 \\ \tau_2 \\ \vdots \\ \tau_{m-1} \\ \tau_m \end{array}\begin{bmatrix} d\beta_1^{clk1} & d\beta_1^{clk2} & \cdots & d\beta_1^{clk(s-1)} & d\beta_1^{clk(s)} \\ t_{clk1}-t_0 & -(t_{clk2}-t_0) & \cdots & 0 & 0 \\ t_{clk1}-t_0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & -(t_{clk(s)}-t_0) \\ 0 & 0 & \cdots & t_{clk(s-1)}-t_0 & -(t_{clk(s)}-t_0) \end{bmatrix}$$

$$A_{clk.quadr.term} = \begin{array}{c} \\ \tau_1 \\ \tau_2 \\ \vdots \\ \tau_{m-1} \\ \tau_m \end{array}\begin{bmatrix} d\beta_2^{clk1} & d\beta_2^{clk2} & \cdots & d\beta_2^{clk(s-1)} & d\beta_2^{clk(s)} \\ (t_{clk1}-t_0)^2 & -(t_{clk2}-t_0)^2 & \cdots & 0 & 0 \\ (t_{clk1}-t_0)^2 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & -(t_{clk(s)}-t_0)^2 \\ 0 & 0 & \cdots & (t_{clk(s-1)}-t_0)^2 & -(t_{clk(s)}-t_0)^2 \end{bmatrix}$$

$$A_{clock} = \begin{bmatrix} A_{clk.offset} & A_{clk.rate} & A_{clk.quadr.term} \end{bmatrix}$$

- In addition to a simple polynomial further so-called piecewise linear parameters are used to account for higher variations of the frequency standards. For piecewise linear modelling a linear behaviour of the effect to be modelled is assumed for certain intervals.

$$\Delta\tau_{clk(i)}(t) = \Delta\tau_{clkoffset} + \Delta\tau_{clkrate1}(t_1 - t_0) + \Delta\tau_{clkrate2}(t_2 - t_1) + \cdots + \Delta\tau_{clkrate(n)}(t_n - t_{(n-1)})$$
$$\Delta\tau_{clk(i)}(t) = a_0 + \frac{a_1 - a_0}{t_1 - t_0}(t_1 - t_0) + \frac{a_2 - a_1}{t_2 - t_1}(t_2 - t_1) + \frac{a_3 - a_2}{t_3 - t_2}(t - t_2) + \cdots + \frac{a_n - a_{n-1}}{t_n - t_{n-1}}(t_n - t_{n-1})$$

Piecewise linear modelling of zenith wet delays

- The path delay due to the neutral atmosphere for microwave signals emitted by radio sources which is called as tropospheric delay.
- Tropospheric delay is one of the major error sources in the analyses of VLBI observations.
- The tropospheric delay correction is applied to the baseline as follows:

$$\Delta\tau_i^e(t) = -\frac{1}{c}[\Delta L_2^e(t) - \Delta L_1^e(t)]$$
$$\Delta L_i(e,t) = \Delta L_h^z(t)\cdot mf_h(e) + \Delta L_w^z(t)\cdot mf_w(e)$$
$$\Delta L_h^z = 0.0022768\cdot\frac{p}{(1 - 0.00266\cdot\cos(2\varphi) - 0.28\cdot 10^{-6}\cdot h)}$$

$$mf_{h,w}(e) = \frac{1 + \dfrac{a_i}{1 + \dfrac{b_i}{1 + c_i}}}{\sin(e) + \dfrac{a_i}{\sin(e) + \dfrac{b_i}{\sin(e) + c_i}}}$$

Piecewise linear modelling of zenith wet delays

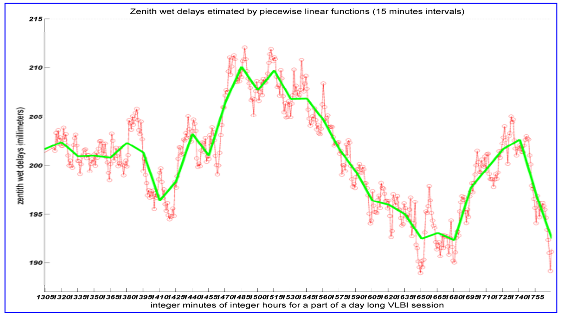
The ZWDs can be gathered by estimating the offsets of the continuous piecewise linear functions with Least Squares Method in the stage of analysis of VLBI session. The offsets are chosen as the parameters of the continuous piecewise linear function

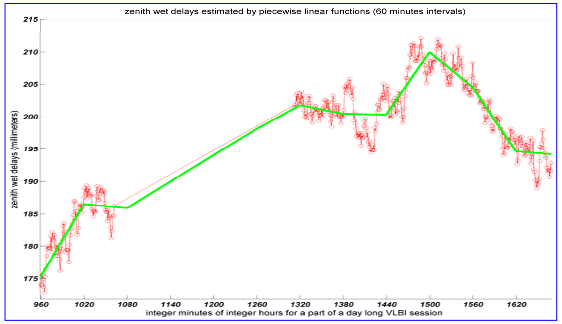
$$\Delta L_w^z(t) = a_0 + \frac{a_1 - a_0}{t_1 - t_0}(t_1 - t_0) + \frac{a_2 - a_1}{t_2 - t_1}(t_2 - t_1) + \frac{a_3 - a_2}{t_3 - t_2}(t - t_2) + \cdots + \frac{a_n - a_{n-1}}{t_n - t_{n-1}}(t_n - t_{n-1})$$

$$y = a_{n-1} + \frac{a_n - a_{n-1}}{t_n - t_{n-1}}(t - t_{n-1})$$

The partial derivatives of the observation equations according to the unknowns of the ZWD's piecewise linear offsets are, ⇨

$$\frac{\partial y}{\partial a_n} = \left(\frac{t - t_{n-1}}{t_n - t_{n-1}}\right)da_n$$
$$\frac{\partial y}{\partial a_{n-1}} = \left(1 - \frac{t - t_{n-1}}{t_n - t_{n-1}}\right)da_{n-1}$$





Conclusion

From the investigations carried out within this study the following conclusions can be drawn:
• *Piecewise linear offsets* should be supported for all parameters, i.e. ZWDs, troposphere gradients, clocks, Earth orientation parameters, and station coordinates.
• The offsets should be determined at *integer days, integer hours, or integer fractions of integer hours*, respectively.
•In order to avoid numerical problems (as e.g. rank deficiencies) and to stabilize the parameter estimation process, *loose constraints as pseudo observations should be included in the parameter estimation stage* of the VLBI analysis.
• *The compatibility of the weights of constraints (pseudo-observations) and of the real observations has utmost importance* because of their effect on the estimated parameters.
• The application of scans instead of observations as primary unit of the structure array has shown several advantages. One of those certainly is that the concept is *appropriate for Kalman Filter solutions* with updates every scan. Also, station-related quantities are only stored once per epoch, which *decreases the memory allocation significantly. The major advantage of these structure arrays is that they can be extended at any place in the programs very easily without updating every other call to them. Furthermore, it is very easy to read certain properties of these arrays with Matlab and to e.g. plot them.*

Least squares adjustment introduced with loose constraints

VLBI parameters can be estimated according to the least-squares (LS) adjustment based on the Gauss-Markov model. The goal of LS adjustment is to minimize the square sum of residuals. LS where loose constraints are introduced as pseudo-observations with linearized observation equations can be applied for the estimation of VLBI parameters like, e.g., the coefficients of the continuous piecewise linear functions for the clocks and the tropospheric zenith delays.

$$\begin{bmatrix} v \\ v_c \end{bmatrix} = \begin{bmatrix} A \\ H \end{bmatrix}dx - \begin{bmatrix} dl \\ h \end{bmatrix} \quad K_{ll} = \begin{bmatrix} K & 0 \\ 0 & K_c \end{bmatrix} = S_{0c}\begin{bmatrix} P^- & 0 \\ 0 & p_c^- \end{bmatrix} \quad ; \quad (A'PA + H'P_cH)\,dx = A'Pl + H'P_ch \quad ; \quad \begin{array}{l} N_{total} = A'PA + H'P_cH \\ b_{total} = A'Pl + H'P_ch \end{array} \quad ; \quad x_c = x_0 + N_{total}^- b_{total} \quad ; \quad \begin{array}{l} S_{0c} = (v'Pv + v_c'P_cv_c)/(n_{obs} + n_{constr} - n_{unk}) \\ K_{xx} = S_{0c}(N_{total})^- \end{array}$$

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