Comparison and Cut off Angle Tests for Observed and Simulated CONT05 Sessions

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Abstract. Baseline length repeatabilities can be taken as accuracy criteria of the VLBI network, because they are independent of rotations of the polyhedron formed by several VLBI stations.

In the first part of this study, baseline length repeatabilities of 15 sessions of the VLBI CONT05 campaign were investigated for certain mapping functions (VMF1, GMF, NMF) and cut off elevation angles (5°, 6°, 7°, 8°, 9°, 10°, 12°, 15°, 20° and 30°). From the analysis with the VLBI software OCCAM 6.1, the following conclusions can be drawn: All three mapping functions yield about similar baseline length repeatabilities for cut off angles 5° to 10°, but significantly larger repeatabilities for 12° to 30°. The cut off angle of 7° gives the best results for all mapping functions, and baseline length repeatabilities obtained with VMF1 are slightly better than those with NMF and GMF.

In the second part of this study, the observations of the VLBI sessions (NGS files) were simulated and compared with the real observations in terms of baseline length repeatabilities. For the cut off angle 7° the simulated observations for CONT05 yielded approximately the same baseline length repeatabilities as the real observations. One of the main conclusions from the simulation study is that there is no need to observe radio sources below the cut off angle 7° unless the modelling of wet delay parameters is improved.

Keywords. VLBI, CONT05, mapping function, cut off angle, baseline length repeatability, simulation.

1 Introduction

The term “mapping function” is used to describe the relation between the tropospheric delay at zenith direction and an arbitrary angle above the horizon. Throughout the history of VLBI, extensive attention has been paid to tropospheric mapping functions, in view of the dominance of tropospheric delay mismodelling in the error budget.

The baseline length is independent of rotations of the polyhedron formed by several VLBI stations. Thus, baseline length repeatability is a good measure of the accuracy achieved for geodetic VLBI (Niell, 2006). For each baseline, the repeatability σ can be determined as the standard deviation of the n estimates L_i with regard to the corresponding value L_0 on a regression polynomial of first order as e.g. given by (Boehm et al., 2006a),

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(L_i - L_0)^2}{n-2}} \qquad (1)$$

To describe the increase of the baseline length repeatability with increasing baseline length, Equation (2) can be used,

$$y = a^2 + b^2 ppb^2 L^2 \qquad (2)$$

where a and b are the parameters to be estimated by least-squares method (LSM), y are the repeatability values (σ) w.r.t. baseline lengths (L) (Niell, 2006).

Baseline length repeatability can be taken as criterion for evaluating the accuracy of the mapping functions which changes with cut off angle.

In the first part of this study, based on Eq. (1) and (2), baseline length repeatabilities of 15 sessions of the VLBI CONT05 campaign were investigated for the mapping functions VMF1 (Boehm et al., 2006a), GMF (Boehm et al. 2006b), and NMF (Niell 1996) and the cut off elevation angles 5°, 6°, 7°, 8°, 9°, 10°, 12°, 15°, 20° and 30°.

In the second part of this study, the observations of the VLBI sessions were simulated. Then observed and simulated CONT05 NGS files were compared based on baseline length repeatabilities.

2 Baseline Length Repeatabilities Derived from Different Mapping Functions and Cut off Angles

The CONT05 sessions were processed with the software OCCAM 6.1 for different mapping functions (VM1, GMF, NMF) and for the different cut off angles. The parameters a and b of the regression function given in Eq. (2) were obtained by Least Squares (LS) adjustment as follows:

$$x = (A^T W A)^{-} A^T W\, y \tag{3}$$

where the measurement vector (y) and the vector of unknown parameters (x) were formed as shown in Eq. (4).

$$y = \begin{bmatrix} rms_1^2 \\ rms_2^2 \\ \vdots \\ rms_n^2 \end{bmatrix} ; \; x = \begin{bmatrix} a^2 \\ b^2 \end{bmatrix} \tag{4}$$

The design matrix (A) and the weight matrix of the adjusted baselines (W) were set up according to Eq. (5)

$$A = \begin{bmatrix} 1 & ppb^2 L_1^2 \\ 1 & ppb^2 L_2^2 \\ \vdots & \vdots \\ 1 & ppb^2 L_n^2 \end{bmatrix} ; W = \begin{bmatrix} 1/s_1^2 & & & \\ & 1/s_2^2 & & \underline{0} \\ & & \ddots & \\ & \underline{0} & & \\ & & & 1/s_n^2 \end{bmatrix} \tag{5}$$

where s is the mean value of the standard deviations of the adjusted baselines estimated from the 15 CONT05 sessions. The parameters of the regression function for different mapping functions and cut off angles are shown in Table 1. Also the number of the observations that were included for each cut off angle are added. From Table 1 it can be seen that the parameters of the regression function computed from the data of GMF and NMF are nearly the same for all cut off angles. However VMF1 yielded better results for cut off angle 5° to 10°. From 10° to 30° all mapping functions approximately produce the same outcomes w.r.t. baseline length repeatabilities.

Table 1. The parameters of the regression function for each mapping function and cut off angle. The numbers of observables available for each solution are given in parentheses.

Mapping Functions	*Parameters of the function for different cut off angles*									
	5° (6156)		*6°(6028)*		*7°(5907)*		*8°(5818)*		*9°(5646)*	
	a (cm)	*b*	*a (cm)*	*b*	*a (cm)*	*b*	*a (cm)*	*b*	*a (cm)*	*b*
VM1	0,505	0,853	0,515	0,817	0,517	0,801	0,523	0,796	0,510	0,836
GMF	0,524	0,879	0,521	0,844	0,521	0,823	0,522	0,806	0,512	0,844
NMF	0,528	0,879	0,520	0,844	0,521	0,826	0,522	0,808	0,512	0,845
Mapping Functions	*10°(5502)*		*12°(5207)*		*15°(4730)*		*20°(3906)*		*30°(2491)*	
	a (cm)	*b*	*a (cm)*	*b*	*a (cm)*	*b*	*a (cm)*	*b*	*a (cm)*	*b*
VM1	0,501	0,859	0,489	0,927	0,428	1,078	0,404	1,229	0,657	1,542
GMF	0,500	0,866	0,488	0,931	0,426	1,081	0,403	1,229	0,656	1,542
IMF	0,500	0,867	0,489	0,931	0,428	1,081	0,404	1,228	0,655	1,543

When Table 1 is investigated an unambiguous comparison cannot be achieved. For that reason the value of the initial parameter a of the regression function is fixed to 0.5 cm in the adjustment stage. So parameter b can be used to find the optimal mapping function and cut off angle w.r.t. baseline length repeatability.

Table 2. The parameters of the regression function for each mapping function and cut off angle (parameter of the regression function fixed to 0.5 cm in order to ensure an unambiguous comparability between mapping functions and cut off angles). The numbers of observables available for each solution are given in parentheses.

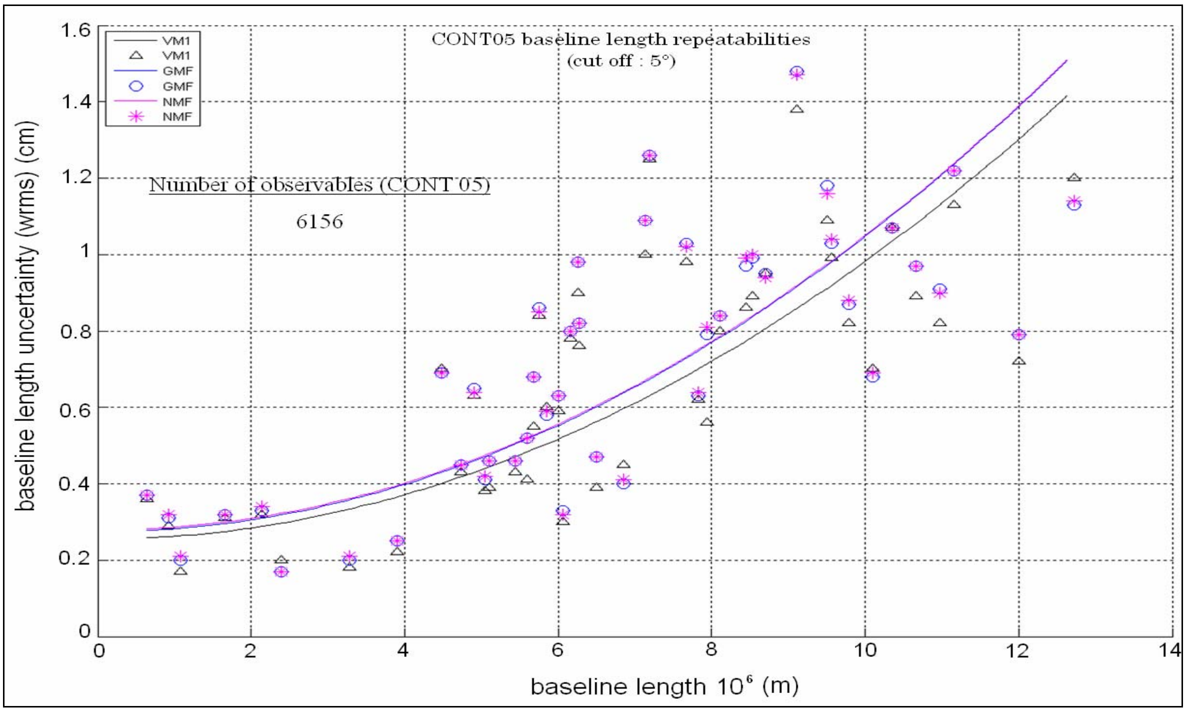
Mapping Functions	*Parameters of the function for different cut off angles*				
	5°(6156)	*6°(6028)*	*7°(5907)*	*8°(5818)*	*9°(5646)*

	a (cm)	b	a (cm)	b	a (cm)	b	a (cm)	b	a (cm)	b
VM1	0,5	0,597	0,5	0,559	0,5	0,537	0,5	0,540	0,5	0,582
GMF	0,5	0,657	0,5	0,605	0,5	0,577	0,5	0,554	0,5	0,595
NMF	0,5	0,660	0,5	0,605	0,5	0,580	0,5	0,558	0,5	0,597
Mapping Functions	*10°(5502)*		*12°(5207)*		*15°(4730)*		*20°(3906)*		*30°(2491)*	
	a (cm)	b	a (cm)	b	a (cm)	b	a (cm)	b	a (cm)	b
VM1	0,5	0,600	0,5	0,680	0,5	0,823	0,5	0,994	0,5	1,506
GMF	0,5	0,610	0,5	0,685	0,5	0,824	0,5	0,993	0,5	1,506
IMF	0,5	0,611	0,5	0,686	0,5	0,826	0,5	0,992	0,5	1,506

From Table 2 the following can be concluded:

- The Vienna Mapping Function VMF1 gives the best baseline length repeatabilities for all cut off angles.
- The cut off angle 7° gives the best baseline length repeatabilities for all mapping functions w.r.t. baseline length repeatabilities.
- From the investigations of CONT05 sessions the optimal tropospheric mapping function was found to be VMF1 with the optimal cut off angle at 7°.
- On the other hand it must be highlighted that this conclusion can only be drawn for CONT05 sessions and when the baseline length repeatabilities are chosen as accuracy criterion.

The scattered data of the baseline length repeatabilities for different mapping functions and cut off angles with their fitted curves are given in the graph below.



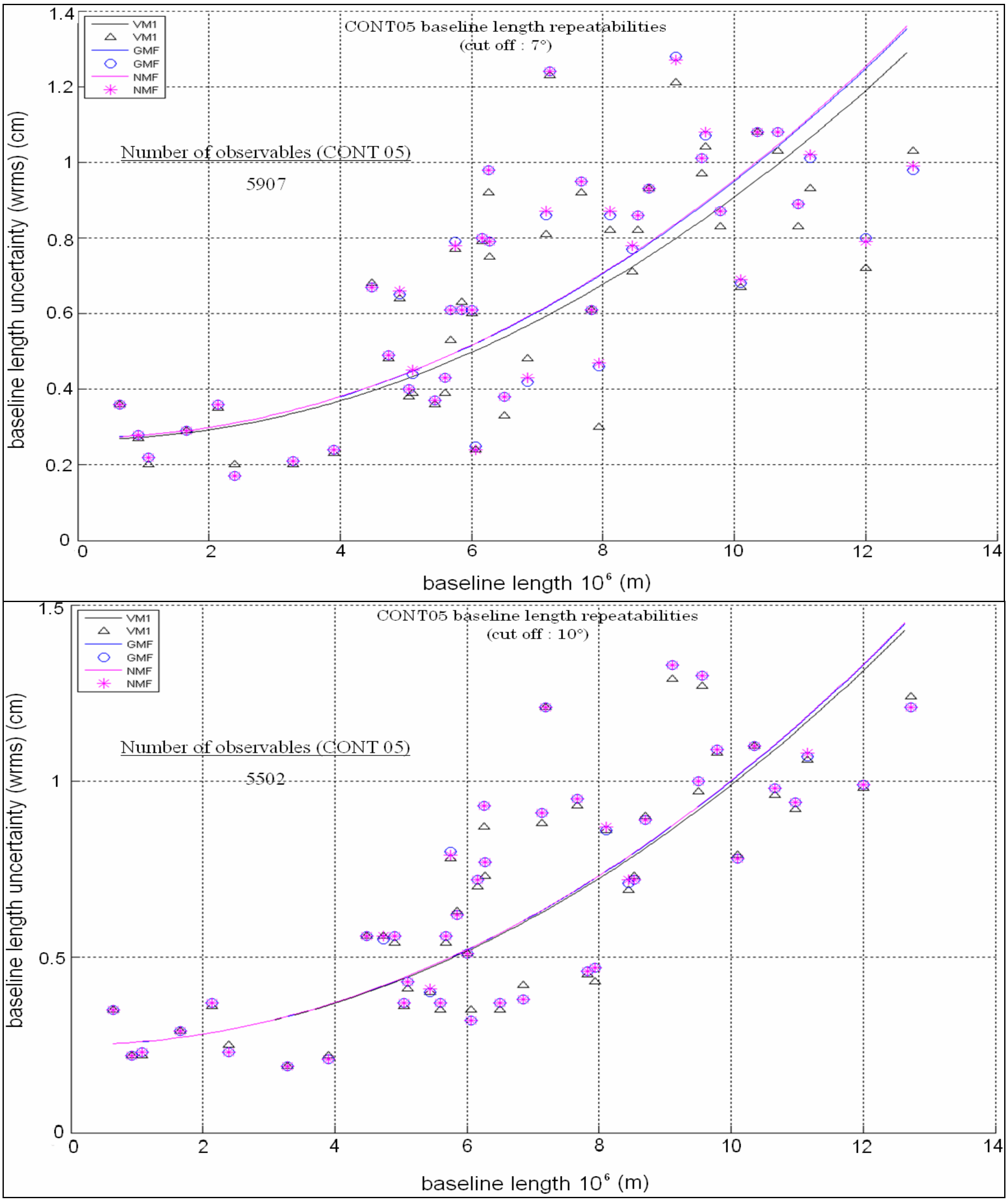


Fig. 1 Baseline length repeatability values provided by the tropospheric mapping functions VMF1, GMF, and NMF for cut off angles 5°, 7°, and 10°.

As it can be seen in Fig. 1 VMF1 produces better results w.r.t. baseline length repeatabilities than NMF and GMF for the cut off angle 5°. From cut off angle 5° to 20° the differences decrease between VMF1 and the other mapping functions. For cut off angles 20° and higher all mapping functions yield approximately the same baseline length repeatabilities.

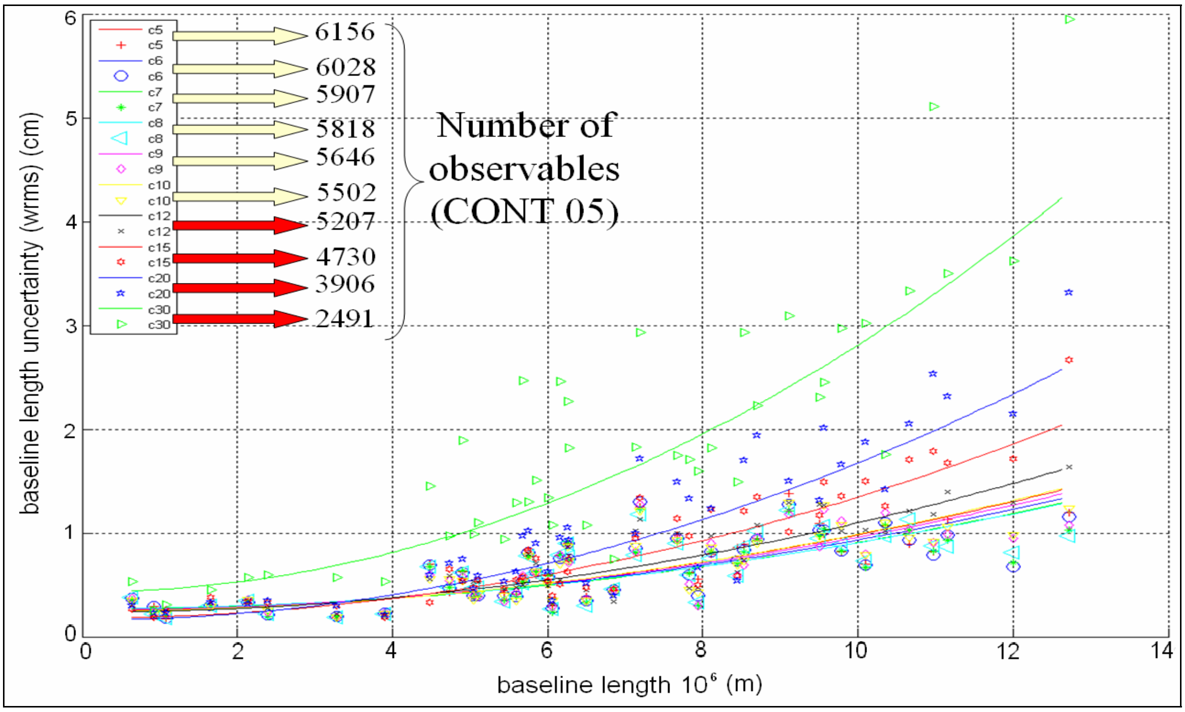


Fig. 2 Baseline length repeatabilities obtained with Vienna Mapping Function (VMF1) for different cut off angles. Also added is the number of observables in CONT05.

Figure 2 shows that the number of observations is significantly decreasing with increasing elevation cutoff angle. This is mainly due to the fact that the CONT05 sessions were scheduled for a cut off angle of 5°; with higher cut off angles several observations were simply discarded. Thus, this comparison is not fully objective because the schedule should have been determined for each cutoff angle separately.

In Section 3 schedules were created for each cut off angle and the observations were filled with simulated values.

3 Comparison of Simulated and Observed CONT05 Sessions derived from Different Mapping Functions and Cut off Angles

The main idea of simulation methods in an optimization procedure is to catch the maximum or minimum value of a mathematical function by trial and error. In geodesy, most of the statistical functions are used to find out the accuracy of the measurements and the unknowns. The values obtained from these statistical accuracy functions are desired to be a minimum. Simulation methods such as Monte Carlo and Sequential Least Squares are effective methods as to reach the accuracy objective functions. Simulation methods are not rigid as analytical optimization methods and more suitable for computer programming.

The simulated and observed NGS files of CONT05 sessions were compared w.r.t. baseline length repeatabilities. It is important to emphasize that these comparisons were based on baseline length repeatabilities.

The group delay ($\Delta\tau$) is simulated according to Eq. (6),

$$\begin{aligned}\Delta\tau = \Delta\tau_{comp} &+ (WZD_2\, mfw_2(e) + cl_2) \\ &- (WZD_1\, mfw_1(e) + cl_1) + wn_{bsl(1-2)}\end{aligned} \qquad (6)$$

where WZD denotes to wet zenith delay, mfw is the wet mapping function (NMF was used here), cl is the clock error, wn is the white noise added to the baselines. The simulated NGS files have been processed by the OCCAM 6.1 software for the cut off angles 5°, 7°, 10°, 15° and 20°. The objective function for the optimization is shown in Eq. (7).

$$\sum_{j=1}^{m} (rep_{observed} - rep_{simulated})^2 \Rightarrow \min \qquad (7)$$

After varying the driving parameters for the wet zenith delays, the clocks and the white noise, the best agreement with real observations was found with a power spectrum density of 0.5 $psec^2/sec$ for the wet zenith delay (except for Kokee Park with 0.8, HartRAO with 0.1, and Tsukuba with 0.6 $psec^2/sec$), an Allan standard deviation of $2 \cdot 10^{-15}$@15 min for all clocks and a white noise of 12 psec for all baselines.

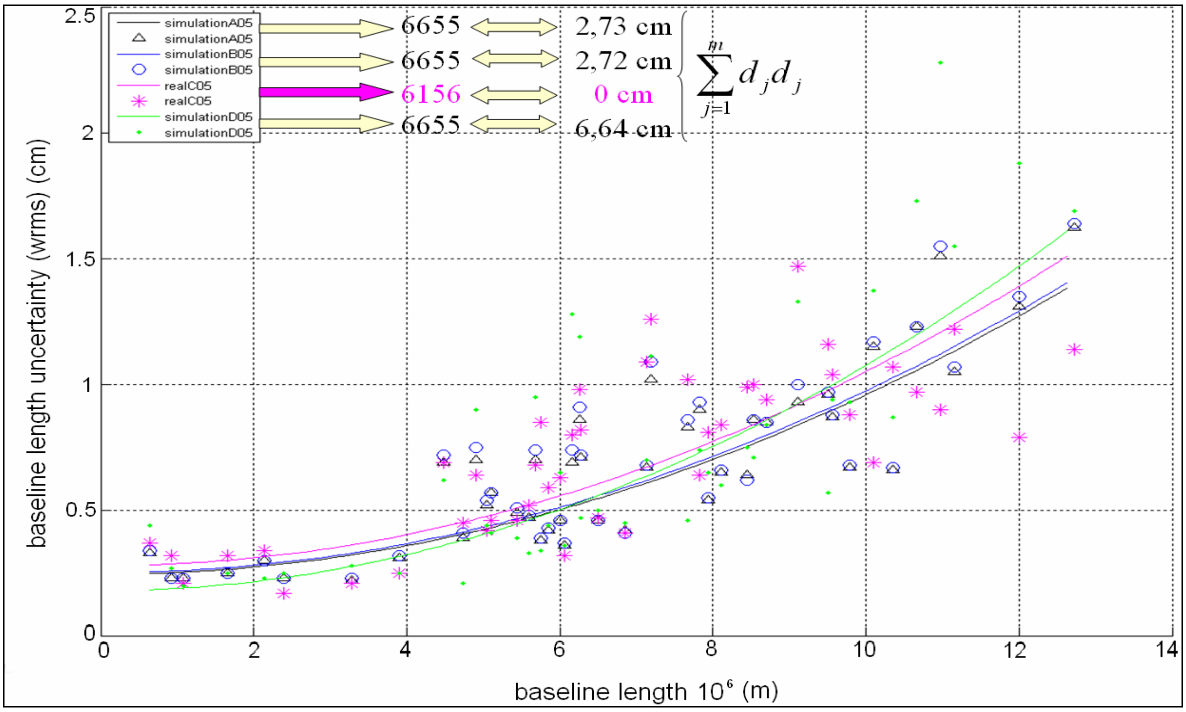


Fig. 3 Comparison of baseline length repeatabilities derived from simulated CONT05 NGS files with respect to real values and a cut off angle of 5°.

In Figure 3 the abbreviation “simulationA05” denotes the first attempt of the creation of simulated NGS files. Then the other attempts were carried out which were named as “simulationB05” and “simulationD05”. It can be seen that in Figure 3 the number of the observables in simulated sessions is nearly the same as in the observed sessions. After applying all the procedures which were done for cut off angle 5° also for the cut off angle 7° an excellent agreement was found with the observed NGS files (Figure 4). “Simulated C07” NGS file succeeded to fit to the “Real C07” NGS file w.r.t. baseline length repeatabilities.

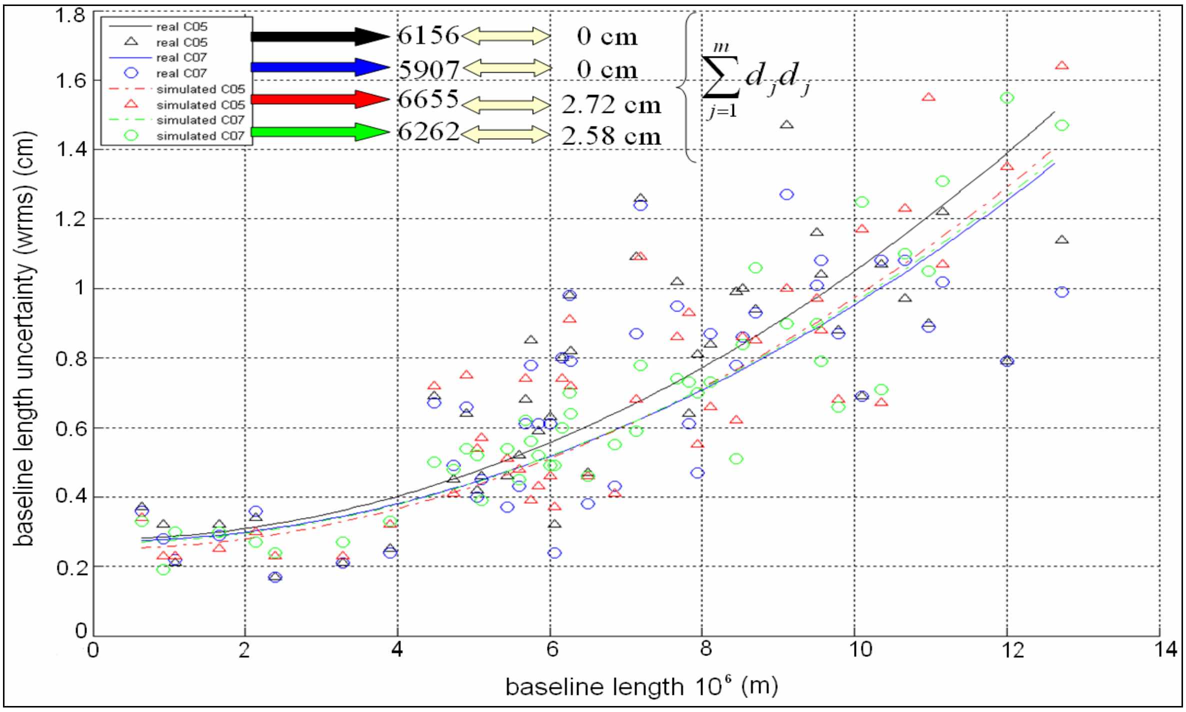


Fig. 4 Comparison of baseline length repeatabilities derived from simulated and observed CONT05 NGS files

4 Conclusions and Outlook

From the investigations of CONT05 baseline repeatabilities for different mapping functions (VMF1, GMF, NMF) and cut off angles (5°, 6°, 7°, 8°, 9°, 10°, 12°, 15°, 20° and 30°) the following conclusions can be drawn:

- The mapping functions produced rather similar baseline uncertainty values for cut off angles 5° to 10° but not for 12° to 30°. This difference occurred because of the various numbers of observables and their distribution on the sky.
- In spite of the small differences, the mapping function VMF1 gives always the best baseline length repeatabilities for all cut off angles.

From the comparison of simulated and observed CONT05 sessions the following conclusions can be drawn,

- For the cut off angle 7° the simulated observations for CONT05 yielded approximately the same baseline length repeatabilities as the real observations.
- No need to observe radio sources below a cut off angle 7° unless the wet zenith delay parameters will be measured more accurately and the related mapping function models will be improved.

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