Different Tropospheric Mapping Functions and cut off Angles Investigated by Processing VLBI CONT05 Sessions

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Abstract. The speed of microwave radio signals that radiate from a quasar is altered when passing through the troposphere due to the particular atmospheric conditions. This effect is called tropospheric delay and is included in the mathematical model of Very Long Baseline Interferometry (VLBI) measurements, which are based on the time delay of signal arrivals between two ground stations. Various mapping functions have been developed to map the tropospheric delay onto zenith direction. The accuracy of VLBI results significantly depends on the reduction of tropospheric delay to zenith direction. Among various outcomes of VLBI, baseline length repeatabilities can be considered as important accuracy criteria since baseline lengths are independent of rotations of the polyhedron formed by several VLBI stations.

In this study, baseline length repeatabilities of 15 sessions of the continuous VLBI CONT05 campaign were investigated by comparing the Vienna Mapping Function (VMF1), the Global Mapping Function (GMF) and the Niell Mapping Function (NMF) for various cut off elevation angles (5°, 6°, 7°, 8°, 9°, 10°, 12°, 15°, 20° and 30°). From the analysis with the VLBI software OCCAM 6.1, the following conclusions can be drawn: All three mapping functions yield about similar baseline length repeatabilities for cut off angles 5° to 10°, but significantly worse repeatabilities for 12° to 30°. In spite of the small differences, the mapping function VMF1 gives always the best baseline length repeatabilities for all cut off angles. For cut off angle 7°, the best results are obtained for all mapping functions. There is no need to observe radio sources below a cut off angle of 7° unless the wet zenith delay parameters will be measured more accurately and the related mapping function models will be improved.

Keywords. VLBI, CONT05, tropospheric mapping function, cut off angle, baseline length repeatability.

1 Introduction

The term "mapping function" is used to describe the relation between a signal delay at zenith direction and an arbitrary angle above the horizon. Modelling the path delays due to the neutral atmosphere for microwave signals emitted by satellites or radio sources is one of the major error sources in the analyses of GPS and VLBI observations. Troposphere mapping functions are used in the analyses of Global Positioning System (GPS) and Very Long Baseline Interferometry (VLBI) observations to map a priori zenith hydrostatic delay to any elevation and to estimate the wet zenith delays, respectively (Boehm et al., 2006b). Throughout the history of VLBI, extensive attention has been paid to tropospheric mapping functions, in view of the dominance of tropospheric delay mismodelling in the error budget. The concept is based on the separation of the path delays into a hydrostatic and a wet part, see Marini (1972), Chao (1974), Niell (1996), Davis et al. (1985), and Boehm et al. (2006a).

The accuracy of VLBI results significantly depends on the tropospheric delay reduction to zenith direction. Among various outcomes of VLBI, baseline length repeatabilities can be considered important accuracy criteria since baseline lengths are independent of rotations of the polyhedron formed by several VLBI stations (Niell, 2006).

For each baseline, the repeatability σ can be determined as the standard deviation of the n estimates L_i with regard to the corresponding value L_0 on a regression polynomial of first order as e.g. given by Boehm et al. (2006a) (Equation 1).

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(L_i - L_0)^2}{n-2}} \qquad (1)$$

To describe the increase of the baseline length repeatability with increasing baseline length, Equation (2) can be used,

$$y = a^2 + b^2\, ppb^2 L^2 \qquad (2)$$

where a and b are the parameters to be estimated by least-squares method (LSM), y are the repeatability values (σ) w.r.t. baseline lengths (L) (Niell, 2006). In the first part of this study, based on Equation (1) and (2), baseline length repeatabilities of 15 sessions of the continuous VLBI CONT05 campaign were investigated for the different mapping functions VMF1 (Boehm et al., 2006a), GMF (Boehm et al. 2006b), and NMF (Niell 1996), and different cut off elevation angles (5°, 6°, 7°, 8°, 9°, 10°, 12°, 15°, 20° and 30°).In the second part of this study, the observations of the VLBI sessions were simulated. Then the output of the analysis of observed and simulated CONT05 observations were compared w.r.t. baseline length repeatabilities. The plan for the CONT05 campaign was to acquire state of the art VLBI data over a two-week period to demonstrate the highest accuracy of which VLBI is capable. CONT05 was a two-week campaign of continuous VLBI sessions, scheduled for observing during September 2005. The CONT05 sessions with an observing network consists of 11 stations were the follow-on to the spectacularly successful CONT94 (January 1994) CONT95 (August 1995), CONT96 (fall 1996), and CONT02 (October 2002) ([URL 1]).

2 Baseline Length Repeatabilities Derived from Different Mapping Functions and cut off Angles

The CONT05 sessions were processed with the software OCCAM 6.1 using different mapping functions (VM1, GMF, NMF) and for different cut off angles. The parameters of the regression function given in Equation (2) were obtained by LSM adjustment as follows:

$$x = (A^T W A)^{-} A^T W\, y \qquad (3)$$

where the measurement vector (y) and the vector of unknown parameters (x) were formed as shown in Equation (4).

$$y = \begin{bmatrix} rms_1^2 \\ rms_2^2 \\ \vdots \\ rms_n^2 \end{bmatrix};\; x = \begin{bmatrix} a^2 \\ b^2 \end{bmatrix} \qquad (4)$$

The design matrix (A) and the weight matrix of the adjusted baselines (W) were set up according to Equation (5)

$$A = \begin{bmatrix} 1 & ppb^2 L_1^2 \\ 1 & ppb^2 L_2^2 \\ \vdots & \vdots \\ 1 & ppb^2 L_n^2 \end{bmatrix};\; W = \begin{bmatrix} 1/s_1^2 & & & \\ & 1/s_2^2 & & \underline{0} \\ & & \ddots & \\ & \underline{0} & & \\ & & & 1/s_n^2 \end{bmatrix} \qquad (5)$$

where s is the mean value of the standard deviations of the adjusted baselines estimated from the 15 CONT05 sessions. The parameters of the regression function for different mapping functions and cut off angles are shown in Table 1. Also the number of the observations that were included for each cut off angle is added. From Table 1 it can be seen that the parameters of the regression function computed from the data of GMF and NMF are nearly the same for all cut off angles. However VMF1 yields better results for cut off angle 5° to 10°. From 10° to 30° all mapping functions approximately produce the same outcomes when comparing baseline length repeatabilities.

Table 1. Parameters of the regression function for each mapping function and cut off angle. The numbers of observables available for each solution are given in parentheses.

Mapping Functions	*Parameters of the Equation (2) for different cut off angles*									
	5° (6156)		*6°(6028)*		*7°(5907)*		*8°(5818)*		*9°(5646)*	
	a *(cm)*	b	a *(cm)*	b	a *(cm)*	b	a *(cm)*	b	a *(cm)*	b
VM1	0,505	0,853	0,515	0,817	0,517	0,801	0,523	0,796	0,510	0,836
GMF	0,524	0,879	0,521	0,844	0,521	0,823	0,522	0,806	0,512	0,844
NMF	0,528	0,879	0,520	0,844	0,521	0,826	0,522	0,808	0,512	0,845
Mapping Functions	*10°(5502)*		*12°(5207)*		*15°(4730)*		*20°(3906)*		*30°(2491)*	
	a *(cm)*	b	a *(cm)*	b	a *(cm)*	b	a *(cm)*	b	a *(cm)*	b
VM1	0,501	0,859	0,489	0,927	0,428	1,078	0,404	1,229	0,657	1,542
GMF	0,500	0,866	0,488	0,931	0,426	1,081	0,403	1,229	0,656	1,542
IMF	0,500	0,867	0,489	0,931	0,428	1,081	0,404	1,228	0,655	1,543

When Table 1 is investigated an unambiguous comparison cannot be achieved. For that reason the value of the initial parameter a of the regression function is fixed to 0.5 cm in the adjustment stage. So parameter b can be used to find the optimal mapping function and cut off angle w.r.t. baseline length repeatability.

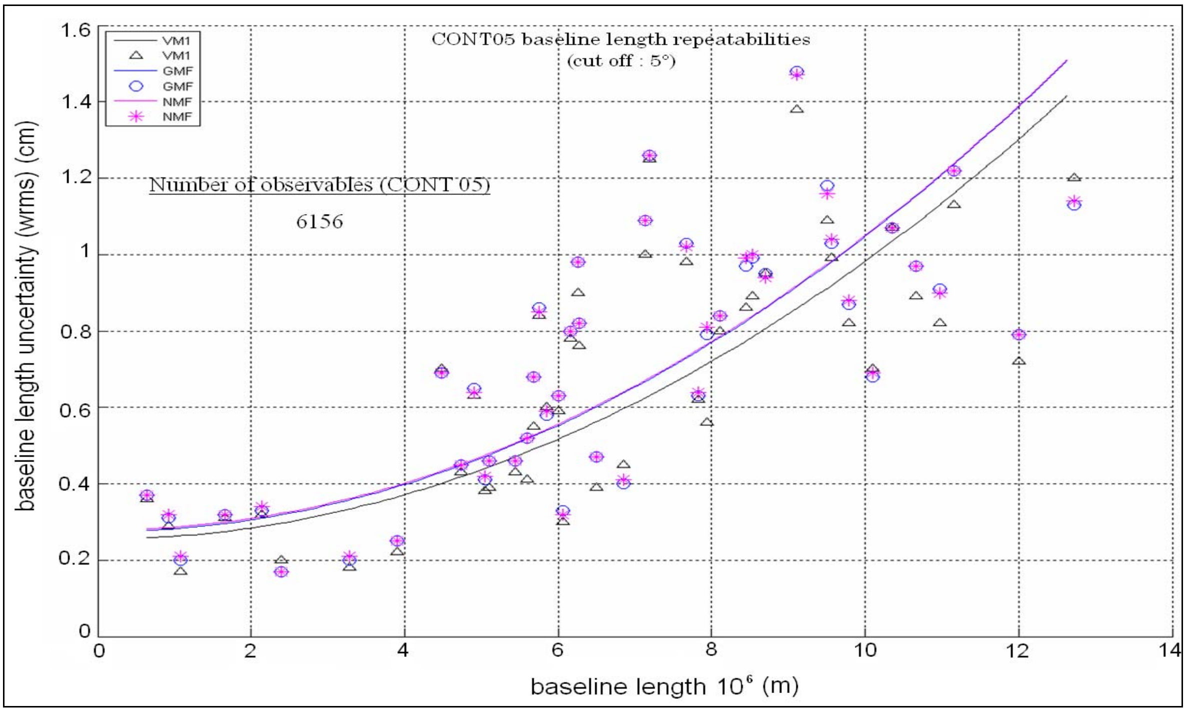
Table 2. Parameters of the regression function for each mapping function and cut off angle (parameter of the regression function fixed to 0.5 cm in order to ensure an unambiguous comparability between mapping functions and cut off angles). The numbers of observables available for each solution are given in parentheses.

Mapping Functions	*Parameters of the Equation (2) for different cut off angles*									
	5°(6156)		*6°(6028)*		*7°(5907)*		*8°(5818)*		*9°(5646)*	
	a (cm)	*b*	*a (cm)*	*b*	*a (cm)*	*b*	*a (cm)*	*b*	*a (cm)*	*b*
VM1	0,5	0,597	0,5	0,559	0,5	0,537	0,5	0,540	0,5	0,582
GMF	0,5	0,657	0,5	0,605	0,5	0,577	0,5	0,554	0,5	0,595
NMF	0,5	0,660	0,5	0,605	0,5	0,580	0,5	0,558	0,5	0,597
Mapping Functions	*10°(5502)*		*12°(5207)*		*15°(4730)*		*20°(3906)*		*30°(2491)*	
	a (cm)	*b*	*a (cm)*	*b*	*a (cm)*	*b*	*a (cm)*	*b*	*a (cm)*	*b*
VM1	0,5	0,600	0,5	0,680	0,5	0,823	0,5	0,994	0,5	1,506
GMF	0,5	0,610	0,5	0,685	0,5	0,824	0,5	0,993	0,5	1,506
IMF	0,5	0,611	0,5	0,686	0,5	0,826	0,5	0,992	0,5	1,506

From Table 2 the following can be concluded:

- The Vienna Mapping Function VMF1 gives the best baseline length repeatabilities for all cut off angles.
- Comparing the different cut off angles cut off angle 7° gives the best baseline length repeatabilities for all mapping functions w.r.t. baseline length repeatabilities. Thus, from the investigations of CONT05 sessions the optimal tropospheric mapping function was found to be VMF1 with the optimal cut off angle at 7°. On the other hand it must be highlighted that this conclusion can only be drawn for CONT05 sessions and if the baseline length repeatabilities are chosen as accuracy criterion.

The scattered data of the baseline length repeatabilities for different mapping functions and cut off angles with their fitted curves are given in the graphs of Figure 1.



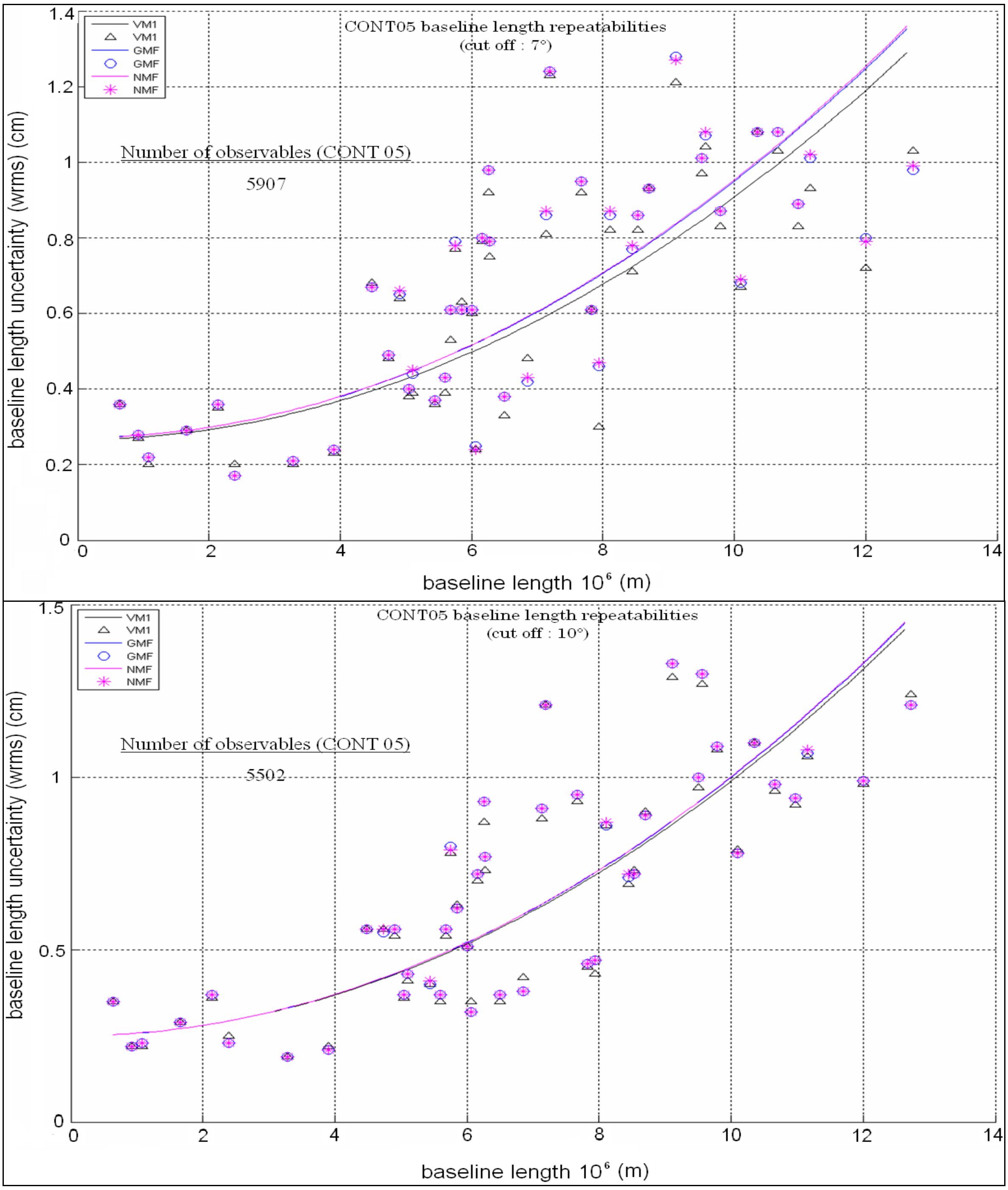


Fig. 1 Baseline length repeatability values provided by the tropospheric mapping functions VMF1, GMF, and NMF for cut off angles 5°, 7°, and 10°.

As it can be seen in Figure 1 VMF1 produces better results w.r.t. baseline length repeatabilities than NMF and GMF for the cut off angle 5°. From cut off angle 5° to 20° the differences decrease between VMF1 and the other mapping functions. For cut off angles 20° and higher all mapping functions yield approximately the same baseline length repeatabilities.

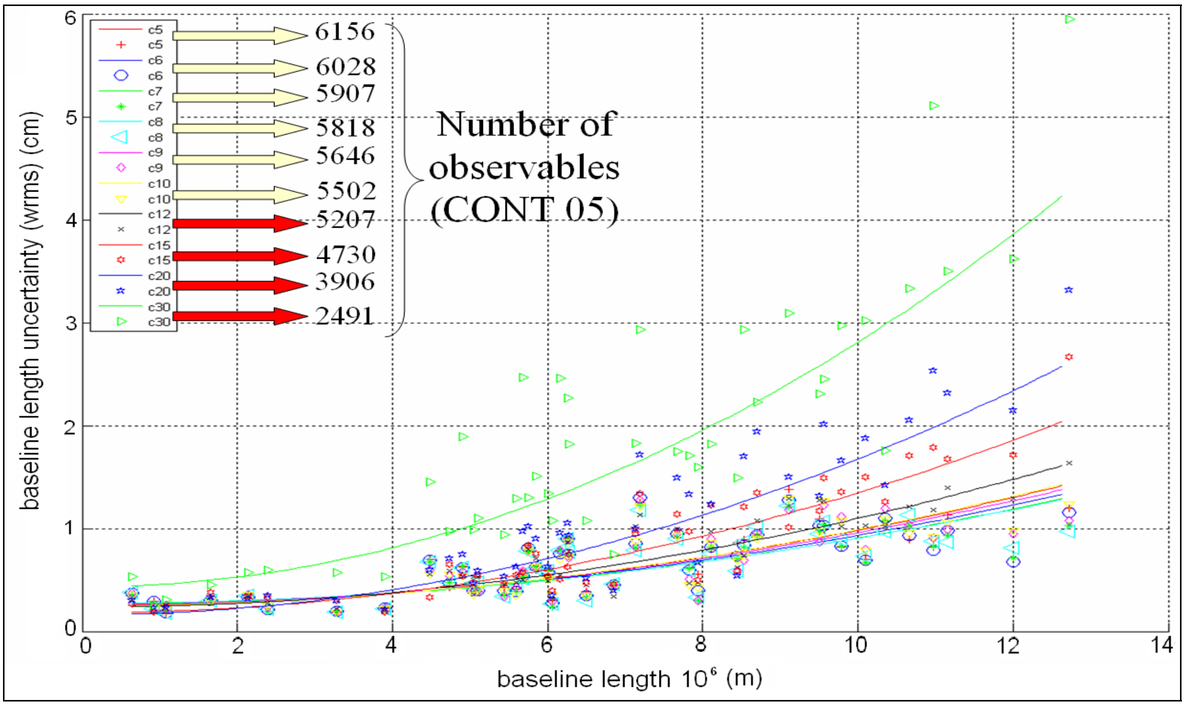


Fig. 2 Baseline length repeatabilities obtained with Vienna Mapping Function (VMF1) for different cut off angles also added the number of observables in CONT05 is.

Figure 2 shows that the number of observations is significantly decreasing with increasing elevation cutoff angle. This is mainly due to the fact that the CONT05 sessions were scheduled for a cut off angle of 5°; with higher cut off angles several observations were simply discarded. Thus, this comparison is not fully objective because the schedule should have been determined for each cut off angle separately.

In Section 3 schedules were created for each cut off angle and the observations were filled with simulated values.

3 Comparison of Simulated and Observed CONT05 Sessions derived from Different Mapping Functions and cut off Angles

The main idea of simulation methods in an optimization procedure is to catch the maximum or minimum value of a mathematical function by trial and error. In geodesy, most of the statistical functions are used to find out the accuracy of the measurements and of the unknowns. The values obtained from these statistical accuracy functions are desired to be a minimum. Simulation methods such as Monte Carlo and Sequential Least Squares are effective methods as to reach the accuracy objective functions. Simulation methods are not as rigid as analytical optimization methods and more suitable for computer programming. The simulated and observed files of CONT05 sessions are compared w.r.t. baseline length repeatabilities.

The group delay ($\Delta\tau$) is simulated according to Equation (6),

$$\begin{aligned}\Delta\tau = \Delta\tau_{comp} &+ (WZD_2\ mfw_2(e) + cl_2) \\ &- (WZD_1\ mfw_1(e) + cl_1) + wn_{bsl(1-2)}\end{aligned} \qquad (6)$$

where WZD denotes to wet zenith delay, mfw is the wet mapping function (NMF was used here), cl is the clock value, and wn is the white noise added to the baselines. The simulated files have been processed by the OCCAM 6.1 software for the cut off angles 5°, 7°, 10°, 15°, and 20°. The objective function for the optimization is shown in Equation (7) where m is the number of baselines.

$$\sum_{j=1}^{m}(rep_{observed} - rep_{simulated})^2 \Rightarrow \min \qquad (7)$$

After varying the driving parameters for the wet zenith delays, the clocks and the white noise, the best agreement with real observations was found with a power spectrum density (PSD) of 0.5 psec^2/sec for the wet zenith delay (except for Kokee Park with 0.8, HartRAO with 0.1, and Tsukuba with 0.6 psec^2/sec), an Allan standard deviation of 10^{-15}@15 min for all clocks and a white noise of 12 psec for all baselines.

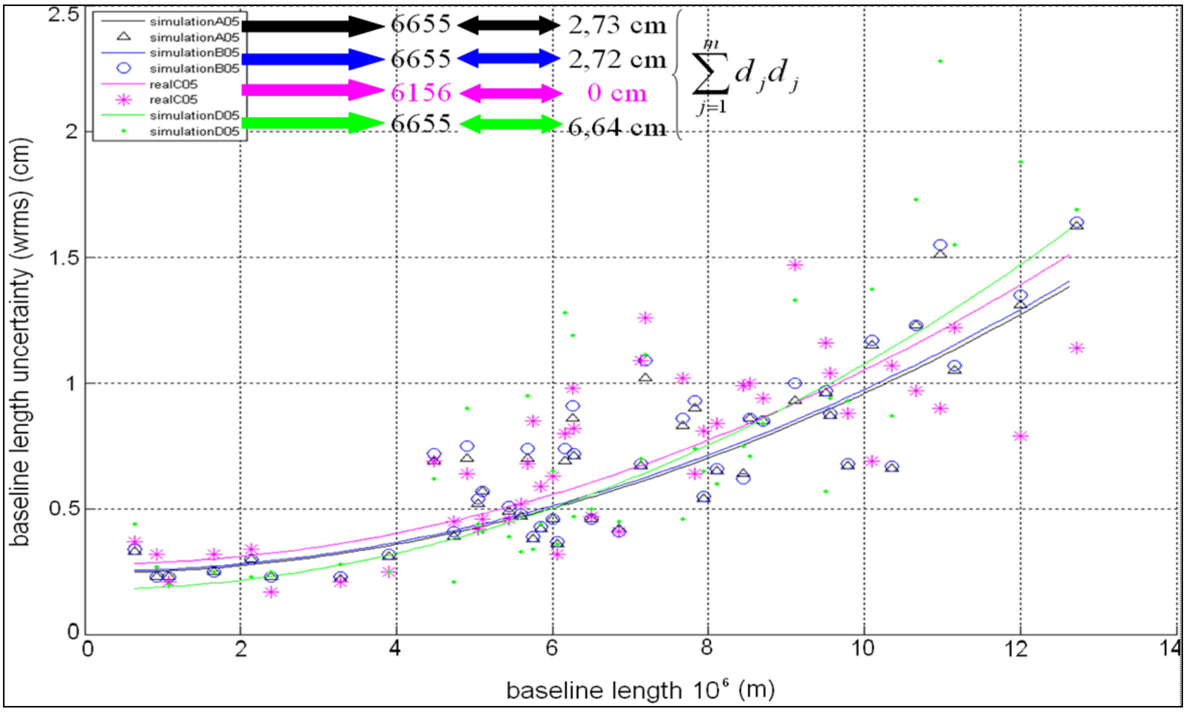


Fig. 3 Comparison of baseline length repeatabilities derived from simulated CONT05 NGS files with real values and a cut off angle of 5°.

Results of three simulations in terms of baseline length repeatabilities and their fitted curves are shown in Figure 3. In Table 3 the parameters that were changed for the simulations are given. The WZD are simulated as random walk process, although a turbulence model should be used in future investigations to create more realistic values. The most compatible outcomes are found out in the 2nd simulation (B05) since this curve in Figure 3 agrees best with the estimates of the real measurements.

Table 3. The parameters changed for the last three simulations.

Parameters	Simulations					
	1st Simulation (A05)		2nd Simulation (B05)		3rd Simulation (D05)	
white noise	8psec (2.4 mm)		12psec (3.6mm)		8psec (2.4mm)	
predicted clock	1e-15@15min		1e-15@15min		1e-15@15min	
predicted wet zenith delay	Station	PSD (psec^2/sec)	Station	PSD (psec^2/sec)	Station	PSD (psec^2/sec)
	HARTRAO	0.1	HARTRAO	0.1	All stations	0.5
	KOKEE	0.8	KOKEE	0.8		
	TSUKUB32	0.6	TSUKUB32	0.6		
	The rest of all stations	0.5	The rest of all stations	0.5		

4 Conclusions and Outlook

From the investigations of CONT05 baseline repeatabilities for different mapping functions (VMF1, GMF, NMF) and cut off angles (5°, 6°, 7°, 8°, 9°, 10°, 12°, 15°, 20° and 30°) the following conclusions can be drawn:

- All mapping functions produced rather similar baseline uncertainty values for cut off angles 5° to 10° but not for 12° to 30°. This difference occurred because of the various numbers of observables and their distribution on the sky.

- In spite of the small differences, the mapping function VMF1 gives always the best baseline length repeatabilities for all cut off angles.

From the comparison of simulated and observed CONT05 sessions the following conclusions can be drawn,

- For the cut off angle 7° the simulated observations for CONT05 yielded approximately the same baseline length repeatabilities as the real observations.
- No need to observe radio sources below a cut off angle 7° unless the wet zenith delay parameters will be measured more accurately and the related mapping function models will be improved.
- Future simulations should use the turbulence model for the wet zenith delays, and also down-weighting of low elevation observations should be tested.

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