Modelling Stochastic Processes in Geodetic VLBI Analysis

Kamil TEKE[1,2], Johannes BOEHM[1], Harald SCHUH[1]

Abstract

Based on the recommendations (common standards in modelling, parameterization and analysis strategies) by the Global Geodetic Observing System (GGOS), analysis and modelling will be homogenized and improved across the various space geodetic techniques, such as Very Long Baseline Interferometry (VLBI), Global Navigation Satellite Systems (GNSS), Satellite Laser Ranging (SLR) and Doppler Orbitography by Radiopositioning Integrated on Satellite (DORIS). Furthermore, there is the project VLBI2010 of the International VLBI Service for Geodesy and Astrometry (IVS) which is asking for major updates of the VLBI software packages because of the significant increase of the number of observations with the new system. The analysis-related enhancements are planned to be implemented within two years in the VLBI analysis software Occam (or a Matlab-based equivalent) which is widely used all over the world by several institutes for VLBI parameter estimation. At the end of the study, the Occam software will fulfill all future requirements of GGOS with regard to modelling, parameterization and analysis. The work done up to now is the modelling of stochastic processes (VLBI clock errors and tropospheric wet delays). These parameters are estimated as 'piecewise linear offsets', i.e. only offsets are estimated at integer hours or integer fractions of it (every 20, 10, 5 minutes). In order to avoid numerical problems (as e.g. rank deficiencies) and to stabilize the parameter estimation process, constraints (pseudo observations) have to be included. Usually this type of pseudo-observations constrains the particular rate segment to zero and allows for a certain variation by assigning an appropriate formal error. In this paper, the VLBI delay model, the clock error model due to the inconsistencies of the hydrogen masers and atomic clocks at each antenna and the tropospheric wet delay model will be discussed.

Keywords
Global Geodetic Observing System (GGOS), Very Long Baseline Interferometry, VLBI geometric model, VLBI antenna atomic clock errors, zenith wet delays (ZWD).

Özet

Jeodezik VLBI analizinde stokastik süreclerin modellenmesi

Küresel Jeodezik Gözlem Sistemi'nin (Global Geodetic Observing System (GGOS)) önerileri temelinde (modelleme, parametrizasyon ve analiz stratejilerine iliskin genel standartlar), Very Long Baseline Interferometry (VLBI), Global Navigation Satellite Systems (GNSS), Satellite Laser Ranging (SLR) and Doppler Orbitography by Radiopositioning Integrated on Satellite (DORIS) gibi farkli uzay tabanli jeodezik tekniklerin analizleri ve modellemeleri homojenlestirilecek ve gelistirilecektir. Bu kapsamda Jeodezi ve Astrometri VLBI Servisi'nin (International VLBI Service for Geodesy and Astrometry (IVS)) VLBI2010 adli projesi uygulamaya konulmustur. Bu proje kapsaminda, yeni nesil VLBI sistemi ile elde edilecek önemli sayida ölcünün analizine olanak saglamak üzere VLBI yazilim paketlerinde büyük capta güncellemeler yapilacaktir. VLBI parametre kestirimi icin tüm dünya capinda bir cok enstitü tarafindan kullanilan Occam (veya bir Matlab-temelli dengi) yazilim paketine, VLBI2010 projesi kapsaminda, analiz-iliskili gelistirmelerin iki yil icerisinde uygulanmasi planlanmaktadir. Calisma sonucunda, VLBI modelleri, parametreleri ve analizlerine iliskin tüm gelecek GGOS gereksinimleri Occam yaziliminca karsilanacaktir. Simdiye kadar, VLBI stokastik süreclerinin (saat hatalari ve troposferik sinyal gecikmeleri) modellenmesi gerceklestirilmistir. Ilgili stokastik süreclere iliskin kestirimi yapilan parametreler ise 'parcali lineer ofsetler' dir (her 20, 10, 5 dakika ofset degerleridir). Sayisal problemlerden kurtulmak ve parametre kestirimini tutarlilastirmak icin kisitlayicilar (pseudo-gözlemler) kullanilmistir. Genellikle pseudo-gözlemler seklinde olusturulan kisitlayicilar parametreler arasindaki orani sifira esitlerken uygun degerde bir hata atayarak parametrelerin belli bir miktardaki degisimine izin verirler. Bu makalede, VLBI sinyal gecikme modeli, her antendeki hydrogen maser ve atomik saatlerin uyusum tutarsizliklarina iliskin saat modeli ve troposferik sinyal gecikme modeli ele alinacaktir.

Anahtar Sözcükler
Küresel Jeodezik Gözlem Sistemi (GGOS), Cok Uzun Baz Enterferometrisi, VLBI geometrik modeli, VLBI anteni atomik saat hatalari, islak basucu gecikmesi

1. Introduction

The principle of Very Long Baseline Interferometry (VLBI) has been developed in the 1970ies and was at first mainly used for the investigation of astronomical and astrophysical phenomena (e.g. COHEN and SHAFFER 1971). This principle is based on a classical interferometer in the visible spectrum which has been invented as early as 1890 by Michelson (MICHELSON 1890). The distances between the receivers can be up to 12000 km. At both stations, the signals of an extra-galactic radio source are received and provided with time marks generated by highly precise atomic clocks (usually hydrogen-masers) before they are stored digitally on tapes or discs (CAMPBELL 2004). The data is sent to specially designed computers (correlators) and brought to coherency. Within the correlation process the difference τ of the arrival times of the signal at both stations is determined and represents the primary geodetic observable. It is often called 'delay' and can nowadays be determined with an accuracy of approximately 20-30 picoseconds (= 6- 10 mm) (e.g. SOVERS et al. 1998). Soon after the first use for astronomical purposes the

Ars. Gr.,[1]Vienna University of Technoloy, Institute of Geodesy and Geophysics, WG on Advanced Geodesy, Vienna.
[2]Hacettepe Üniversitesi, Mühendislik Fakültesi, Jeodezi ve Fotogrametri Mühendisligi Bölümü, Ankara.
Assoc. Prof. Dr., [1]Vienna University of Technoloy, Institute of Geodesy and Geophysics, WG on Advanced Geodesy, Vienna.
Prof. Dr., [1]Vienna University of Technoloy, Institute of Geodesy and Geophysics, WG on Advanced Geodesy, Vienna.

geodetic use of the VLBI-principle was recognized (e.g. SHAPIRO 1974, MA 1978, CAMPBELL and WITTE 1978). In addition, besides the baseline vector other parameters such as e.g. the rotation of the Earth (i.e. polar motion x_p, y_p and ΔUT1 as well as nutation $d\psi$, $d\varepsilon$), atmospheric behaviour, tidal effects, etc. can be determined and are included in the functional model. By performing common observations of the same radio source by different stations, global observation networks can be formed. These networks can be used to connect regional geodetic reference systems (and can therefore be used for the generation of global reference systems) as well as for a more precise determination of Earth orientation parameters (compared to observations on single baselines). Compared to other space geodetic techniques (such as GPS, SLR/LLR or DORIS) VLBI has the advantage of having a direct connection to the quasi-inertial system of the radio sources which enables analysts to determine Earth orientation parameters with a long time stability and free of any hypothesis. Hence VLBI is the only technique which connects the sky-fixed reference system (CRF) directly to the Earth-fixed system (TRF) via the Earth orientation parameters (EOP). In a VLBI experiment the parameters which can be estimated are namely, station locations in a *Terrestrial Reference Frame*, offsets of zenith wet delays (ZWD) and clocks, and troposphere gradients. In addition to these station specific parameters, a number of "global" parameters can also be estimated. These are common to the entire network and describe the orientation of the Earth (5 EOP parameters) and the positions of the sources (two angular parameters per source) in a *Celestial Reference Frame* (SOVERS and FANSELOW 1998). The basic principle of VLBI has been described by many authors. For more details see e.g. CAMPBELL 1979, SCHUH 1987, NOTHENAGEL 1991, and TAKAHASHI et al. 1994.

The Global Geodetic Observing System (GGOS) of the International Association of Geodesy (IAG) is aiming at an accuracy of 1 mm and 0.1 mm/year for station coordinates and velocities, respectively, which requires major improvements of the space geodetic techniques (DREWES and REIGBER 2005). In this respect, the International VLBI Service for Geodesy and Astrometry (IVS) set up the VLBI2010 Committee that is considering developments of the Very Long Baseline Interferometry (VLBI) technique to fulfill these requirements. The Institute of Geodesy and Geophysics (IGG) of the Vienna University of Technology contributes to this effort with simulation studies. Within these studies it became evident that existing VLBI software packages are at the limits because of the larger numbers of observations and the shorter time intervals for the estimation of troposphere parameters that are planned for VLBI2010 (PETRACHENKO et al., 2008). To enhance modelling, parameterization and analysis of the VLBI software Occam, e.g. the classical Gauß-Markov model (TITOV et al., 2004) one part of the work done up to now will be summarized in this paper. It will be beneficial to introduce the basic functional model of VLBI before discussing the modeling of stochastic processes that are namely VLBI clock errors and tropospheric wet delays.

2. The Basic Functional Model of VLBI

The time delay of a non-rotating plane or spatial radio interferometer whose two stations are connected by the baseline vector $b = r_2 - r_1$ (with r_i being the geocenteric vectors of the observation sites, respectively) can be mathematically described by the scalar product,

$$\tau_{geom} = \tau_2 - \tau_1 = -\frac{1}{c} \cdot b \cdot k \tag{1}$$

where c denotes the velocity of the radio signal (i.e., the velocity of light), k denotes a unit vector in the direction of the radio source and τ_1 and τ_2 denote the arrival times of the radio signal at the two stations respectively (e.g. NOTHNAGEL 1991). Since the baseline vector b is defined in an Earth-fixed reference system while the vector k in direction of the radio source is defined in a sky-bound reference system, one of these reference systems has to be transformed into the other one. For a better physical interpretation, the three rotations necessary for this transformation are usually decomposed into five individual rotations which are represented by four rotation matrices W, S, N, and P. These matrices correspond to polar motion (<u>w</u>obble, x_p and y_p), Earth rotation (<u>s</u>pin, dUT1), <u>n</u>utation ($d\psi, d\varepsilon$) and precession (z, ξ_A, Θ_A) respectively (e.g. MA 1978, NOTHNAGEL 1991). Equation 1 thus becomes:

$$\begin{aligned}\tau_{geom} &= -\frac{1}{c} \cdot b \cdot R \cdot k \\ &= -\frac{1}{c} \cdot b \cdot W \cdot S \cdot N \cdot P \cdot k\end{aligned} \tag{2}$$

The matrices W, S, N, and P are usually expressed by means of Eulerian rotation angles around the respective rotation axes. A more detailed description can be found in e.g. NOTHNAGEL 1991, and SOVERS et al. 1998. Since Equation 2 only describes the geometrical delay, a more sophisticated model has to be used to model real VLBI observations which are affected by various effects on their way through interstellar space, the Solar System, and the delay caused by atmospheric influences, tidal or loading effects, etc. have to be added. Hence the basic functional model of VLBI has to be extended to

$$\begin{aligned}\tau_{obs} = &-\frac{1}{c} \cdot b \cdot W \cdot S \cdot N \cdot P \cdot k \\ &+ \tau_{j-abb.} + \tau_{t-abb.} + \tau_{Rel.} + \tau_{Tid.} + \tau_{Load.} \\ &+ \tau_{Instr.} + \tau_{Clock} + \tau_{Ion.} + \tau_{Atm_h} + \tau_{Atm_w}\end{aligned} \tag{3}$$

where the following terms are used:

$\tau_{j-abb.}$: annual abberation because of the motion of the Earth around the Solar System barycentre

$\tau_{t-abb.}$: diurnal abberation because of the rotation of the Earth

$\tau_{Rel.}$: relativistic effects

$\tau_{Tid.}$: deformation of Earth because of tides and because of changes of the angular momentum due to ocean tides

$\tau_{Load.}$: deformation of the Earth because of loading effects e.g. due to ocean tides and atmospheric pressure changes.

$\tau_{Ion.}$: ionospheric correction

$\tau_{Instr.}$: instrumental corrections

τ_{Atm_h} : hydrostatic delay through the troposphere

τ_{Atm_w} : wet delay through the troposphere

τ_{Clock} : relative clock error

A more explicit formulation of Equation 3 reads:

$$\begin{aligned}\tau_{obs} = -\frac{1}{c}\cdot\begin{pmatrix} X_A - X_B \\ Y_A - Y_B \\ Z_A - Z_B \end{pmatrix}\cdot R(x_p, y_p, dUT1, d\Psi, d\varepsilon, z, \xi_A, \Theta_A)\cdot\begin{pmatrix} \cos\delta\cdot\cos h(t) \\ \cos\delta\cdot\sin h(t) \\ \sin\delta \end{pmatrix} \\ + \tau_{j-abb.} + \tau_{t-abb.} + \tau_{Rel.} + \tau_{Tid.} + \tau_{Load.} \\ + \tau_{Instr.} + \tau_{Clock} + \tau_{Ion.} + \tau_{Atm_h} + \tau_{Atm_w}\end{aligned} \quad (4)$$

with X_i, Y_i, Z_i being the geocentric coordinates of the particular station, R being the rotation matrix between the celestial and the terrestrial reference system, $h(t)$ being Greenwich hour angle of the radio source, δ denotes the declination of the radio source. The way of parameterization usually depends on the target parameters to be investigated and depends on the number of stations participating as well as on the duration of the session. Although official recommendations exist (McCARTY and PETIT 2003), the choice of a particular model and the choice of a particular parameterization is quite arbitrary and may vary from analyst to analyst.

3. Modelling VLBI clock behaviour and zenith wet delays with Matlab

The analysis of VLBI measurements is carried out according to scanwise parameterization. For instance, lets assume scan1, scan2 and scan3 observations as seen in Figure 1 are read from an NGS observation file and stored in separate "structure arrays", which are data entities in Matlab.

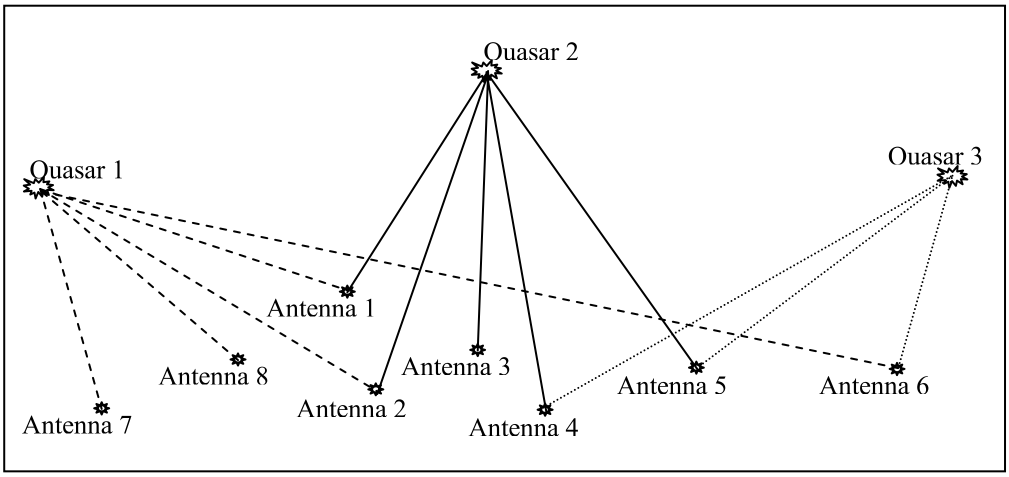

Figure 1: The geometric illustration of VLBI scans

Each entry in the structure array consists of sub-arrays. These structure arrays are based on scans, and each scan contains the structures arrays for the observations within the scan (*obs*) and the stations at the epoch of the scan (*st*) (see Table 1). The application of scans instead of observations as primary unit of the structure array has several advantages. One of those certainly is that the concept is appropriate for Kalman Filter solutions with updates every scan. Also, station-related quantities are only stored once per epoch, which decreases the memory allocation significantly. The major advantage of these structure arrays is that they can be extended at any place in the programs very easily without updating every other call to them. Furthermore, it is very easy to read certain properties of these arrays with Matlab and to e.g. plot them.

Table 1: Possible structure array for one VLBI session to be used as internal database in the Vienna VLBI software. The units given here in the last column are only examples and will be changed as appropriate.

scan.	**obs.**	*iso*	number of source in internal source list	1
		i1, i2	number of stations in internal station list	1
		obs	group delay observation	sec

		sig	sigma of group delay observation	sec
		com	theoretical group delay observation	sec
		pnut, ppol	partial derivatives for the Earth orientation parameters	sec/rad
		dra, dde	partial derivatives for the source coordinates	mas
		...		
	st.	*x, y, z*	cartesian coordinates	m
		dx, dy, dz	partial derivatives for cartesian coordinates	1
		az, zd	azimuth, zenith distance	rad
		zhd	a priori zenith delay	m
		gmfh, gmfw	global mapping functions	1
		...		
	nobs		number of observations in the scan	1
	mjd		time epoch of the scan	UTC

3.1. Modelling clock behaviour

In the parameter estimation stage of VLBI analysis the coordinates $(X, Y, Z)_{ITRF2000}$ and clock parameters (offset, rate, and quadratic term) of the first station which is considered as master station are taken as fixed. One of the largest constituents of the signal delays is caused by the differences in the behaviour of the station frequency standards. After choosing the reference clock for the entire observing network the remaining clocks show both a constant difference (=clock offset) and a linear (=clock trend) or even higher rate of change relative to the reference clock. Thus, an appropriate clock with presumably high frequency stability should be chosen as the reference standard for the entire network. From an algebraic point of view it is of no consequence to the least–squares solution which station clock is chosen as the reference one. Since the clock parameters also 'absorb' physical effects with a similar signature (as e.g. instrumental effects and relativistic effects of higher order) special attention should be paid to this type of parameter (SCHUH 1987). In order to describe the clock behaviour in a mathematical way, usually a simple polynomial approach is chosen,

$$\begin{aligned} \tau_{clk_1} &= \beta_0^{clk1} + \beta_1^{clk1}(t_1^{clk1} - t_0) + \beta_2^{clk1}(t_1^{clk1} - t_0)^2 \\ \tau_{clk_2} &= \beta_0^{clk2} + \beta_1^{clk2}(t_1^{clk2} - t_0) + \beta_2^{clk2}(t_1^{clk2} - t_0)^2 \\ &\vdots \\ \tau_{clk_s} &= \beta_0^{clk(s)} + \beta_1^{clk(s)}(t_1^{clk(s)} - t_0) + \beta_2^{clk(s)}(t_1^{clk(s)} - t_0)^2 \end{aligned} \tag{5}$$

where s is the notation assigned for the VLBI clocks included in the session. Observation equations of time delays are,

$$\begin{aligned} \Delta\tau_{clk(1,2)} &= \tau_{clk_1} - \tau_{clk_2} \\ \Delta\tau_{clk(1,3)} &= \tau_{clk_1} - \tau_{clk_3} \\ &\vdots \\ \Delta\tau_{clk(s-1,s)} &= \tau_{clk_{s-1}} - \tau_{clk_s} \end{aligned} \tag{6}$$

The partial derivatives of the observation equations according to the unknowns of the clocks are,

$$\begin{aligned} \frac{\partial\tau_1}{\partial\beta_0} &= 1d\beta_0^{clk1} - 1d\beta_0^{clk2} \\ \frac{\partial\tau_1}{\partial\beta_1} &= (t_1^{clk1} - t_0)\,d\beta_1^{clk1} - (t_1^{clk2} - t_0)\,d\beta_1^{clk2} \\ \frac{\partial\tau_1}{\partial\beta_2} &= (t_1^{clk1} - t_0)^2\,d\beta_2^{clk1} - (t_1^{clk2} - t_0)^2\,d\beta_2^{clk2} \end{aligned} \tag{7}$$

The design matrix of the clock offsets, rates, and quadratic terms will be,

$$A_{clk.offset} = \begin{array}{cc} & \overbrace{\begin{matrix} d\beta_0^{clk1} & d\beta_0^{clk2} & \cdots & d\beta_0^{clk(s-1)} & d\beta_0^{clk(s)} \end{matrix}} \\ \begin{matrix} \tau_1 \\ \tau_2 \\ \vdots \\ \tau_{m-1} \\ \tau_m \end{matrix} & \begin{bmatrix} 1 & -1 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & -1 \\ 0 & 0 & \cdots & 1 & -1 \end{bmatrix} \end{array}$$

$$A_{clk.rate} = \begin{array}{cc} & \overbrace{\begin{matrix} d\beta_1^{clk1} & d\beta_1^{clk2} & \cdots & d\beta_1^{clk(s-1)} & d\beta_1^{clk(s)} \end{matrix}} \\ \begin{matrix} \tau_1 \\ \tau_2 \\ \vdots \\ \tau_{m-1} \\ \tau_m \end{matrix} & \begin{bmatrix} t_{clk1} - t_0 & -(t_{clk2} - t_0) & \cdots & 0 & 0 \\ t_{clk1} - t_0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & -(t_{clk(s)} - t_0) \\ 0 & 0 & \ldots & t_{clk(s-1)} - t_0 & -(t_{clk(s)} - t_0) \end{bmatrix} \end{array} \tag{8}$$

$$A_{clk.quadr.term} = \begin{array}{c} \\ \tau_1 \\ \tau_2 \\ \vdots \\ \tau_{m-1} \\ \tau_m \end{array} \left[\begin{array}{ccccc} d\beta_2^{clk1} & d\beta_2^{clk2} & \cdots & d\beta_2^{clk(s-1)} & d\beta_2^{clk(s)} \\ (t_{clk1}-t_0)^2 & -(t_{clk2}-t_0)^2 & \cdots & 0 & 0 \\ (t_{clk1}-t_0)^2 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & -(t_{clk(s)}-t_0)^2 \\ 0 & 0 & \cdots & (t_{clk(s-1)}-t_0)^2 & -(t_{clk(s)}-t_0)^2 \end{array} \right]$$

$$A_{clock} = \lfloor A_{clk.offset} \quad A_{clk.rate} \quad A_{clk.quadr.term} \rfloor \tag{9}$$

In addition to the simple quadratic polynomial, further so-called piecewise linear parameters are used to account for higher variations of the frequency standards. For piecewise linear modelling a linear behaviour of the effect to be modelled is assumed for certain intervals. (e.g. SCHUH 1987, TITOV et al. 2004, TESMER 2004, HOBIGER et al. 2008). Previously, clock rates of continuous piecewise linear functions have been estimated as shown in Equation (10).

$$\begin{aligned} \Delta\tau_{clk(i)}(t) = \Delta\tau_{clkoffset} + \Delta\tau_{clkrate1}(t_1 - t_0) + \Delta\tau_{clkrate2}(t_2 - t_1) + \\ \cdots + \Delta\tau_{clkrate(n)}(t - t_{(n-1)}) \end{aligned} \tag{10}$$

However, it has been recommended by the IERS Analysis Coordinator (Rothacher 2008, personal communication) that so-called 'piecewise linear offsets as expressed by Equation (11) should be used for the representation of clocks and zenith delays.

$$\begin{aligned} \Delta\tau_{clk(i)}(t) = a_0 + \frac{a_1 - a_0}{t_1 - t_0}(t_1 - t_0) + \frac{a_2 - a_1}{t_2 - t_1}(t_2 - t_1) \\ + \frac{a_3 - a_2}{t_3 - t_2}(t - t_2) + \cdots + \frac{a_n - a_{n-1}}{t_n - t_{n-1}}(t - t_{n-1}) \end{aligned} \tag{11}$$

In order to avoid numerical problems (as e.g. rank deficiencies) and to stabilize the parameter estimation process, constraints (or pseudo observations) have to be included in intervals with only a small number of observations. Usually this type of pseudo-observations constraints the particular rate segment to zero and allows for a certain variation by assigning an appropriate formal error. Piece-wise linear modelling is also used when describing other effects such as e.g. atmospheric behaviour, atmosphere gradients or sub-daily Earth rotation variations.

3.2. Piecewise linear modelling of zenith wet delays

On their way to the radio telescopes, radio signals have to pass the atmosphere, i.e., the electrically charged part (ionosphere) and the electrically neutral part (troposphere) of the atmospheric layers. Depending on the state of these layers the signals are distorted. The impact of the ionosphere can be eliminated almost completely by performing dual frequency measurements. The signal path delay caused by the atmosphere (as well as the ionospheric delay) is called refraction and can change between approximately 2.3 m in zenith direction (approximately 8 ns) and almost 25m at elevations of 5°. The path delay due to the neutral atmosphere for microwave signals emitted by radio sources which is called tropospheric delay $\Delta\tau_{tro}$ is one of the major error sources in the analyses of VLBI observations. The tropospheric delay $\Delta\tau_{tro}$ correction is applied to the baseline

$$\Delta\tau_i^e(t) = -\frac{1}{c}[\Delta L_2^e(t) - \Delta L_1^e(t)] \tag{12}$$

where $\Delta L_i^e(t)$ is the tropospheric delay at station i, on time t, and at the elevation angle e. The concept is based on the separation of the total slant path delay at station i, $\Delta L_i^e(t)$, into a hydrostatic and wet part (e.g. DAVIS et al. 1985)

$$\Delta L_i(e,t) = \Delta L_h^z(t) \cdot mf_h(e) + \Delta L_w^z(t) \cdot mf_w(e) \tag{13}$$

where $mf_{h,w}$ are the hydrostatic and wet mapping functions, $\Delta L_{h,w}^z$ are the wet and hydrostatic delays at zenith direction. The term "mapping function" is used to describe the relation between the tropospheric delay at zenith direction and an arbitrary angle above the horizon. Throughout the history of VLBI, extensive attention has been paid to tropospheric mapping functions, in view of the dominance of tropospheric delay mismodeling in the error budget. Various mapping functions have been developed to map the tropospheric delay onto zenith direction. Most of the geodetic-quality mapping functions use the continued fraction form (e.g. NIELL 1996). All the parameters in the mapping function can be estimated by least-squares fitting with ray-tracing delay values at various elevation angles. The simplest method to relate the tropospheric delay for an oblique path to the delay of the received signal from directly overhead is to assume a flat Earth covered by an azimuthally symmetric troposphere layer. Since the delay is proportional to the path length through this layer, the mapping function is then simply equal to the cosecant of the elevation angle. Mapping functions, which are independent of the azimuth of the observation, have been determined for hydrostatic and wet separately by fitting the coefficients a, b, and c of a continued fraction form (MARINI 1972) (Equation 14) to standard atmospheres (e.g. CHAO 1974), to radisonde data (NIELL 1996), or recently to numerical weather models (NWMs) (BOEHM et al. 2006a).

$$mf_{h,w}(e)=\frac{1+\cfrac{a_i}{1+\cfrac{b_i}{1+c_i}}}{\sin(e)+\cfrac{a_i}{\sin(e)+\cfrac{b_i}{\sin(e)+c_i}}} \tag{14}$$

where also $a_i, b_i, c_i, \ldots$ can be derived from functions $f(\varphi, h, doy, t, P, \alpha, \ldots)$ of e.g. latitude φ, ellipsoidal height h, day of the year doy, surface temperature t, surface total pressure P, temperature lapse α. The zenith hydrostatic delays (ZHD), $\Delta L_h^z(m)$, can be determined from the total pressure p in hPa and the station coordinates (latitude φ and height h in m) at a site (SAASTAMOINEN 1973) as follows:

$$\Delta L_h^z = 0.0022768 \cdot \frac{p}{(1-0.00266\cdot\cos(2\varphi)-0.28\cdot 10^{-6}\cdot h)} \tag{15}$$

The ZWDs, ΔL_w^z, can be estimated by estimating the offsets of the continuous piecewise linear functions with Least Squares Method in the stage of analysis of VLBI session. When the offsets are chosen as the parameters of the continuous piecewise linear function

$$\Delta L_w^z(t) = a_0 + \frac{a_1-a_0}{t_1-t_0}(t_1-t_0) + \frac{a_2-a_1}{t_2-t_1}(t_2-t_1) + \frac{a_3-a_2}{t_3-t_2}(t-t_2) + \cdots + \frac{a_n-a_{n-1}}{t_n-t_{n-1}}(t_n-t_{n-1})$$
$$y = a_{n-1} + \frac{a_n - a_{n-1}}{t_n - t_{n-1}}(t-t_{n-1}) \tag{16}$$

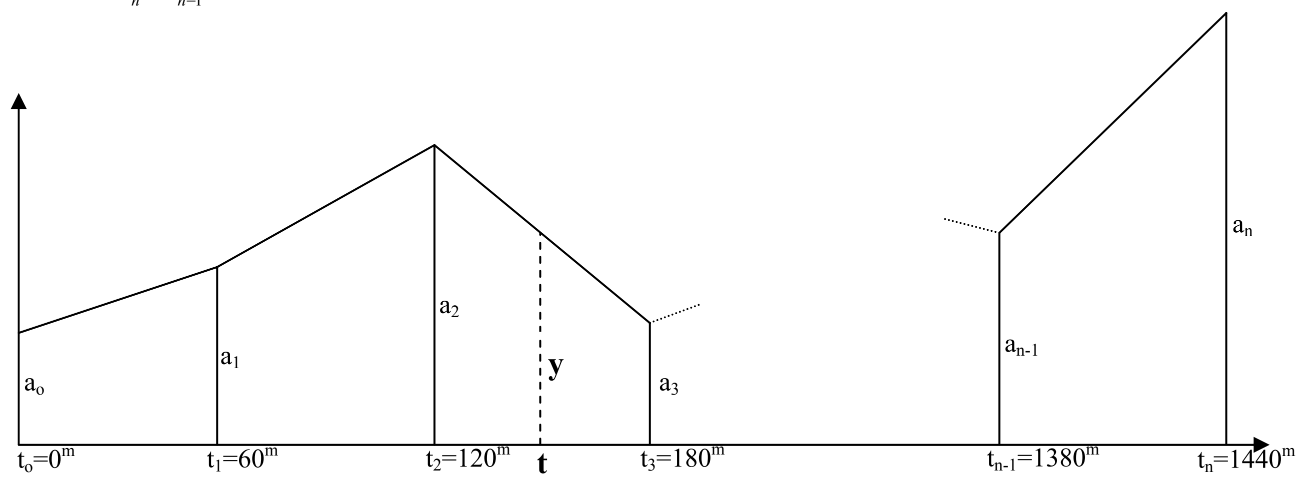

Figure 2: Offsets of continuous piecewise linear function for estimating the ZWDs (estimate interval is 60 min for 24 hour session)

for the case of the illustration in Figure 2 the function will be

$$\Delta L_w^z(t) = a_2 + \frac{a_3-a_2}{t_3-t_2}(t-t_2) \tag{17}$$

The partial derivatives of the continuous piecewise linear function are

$$\frac{\partial y}{\partial a_{n-1}} = (1-\frac{t-t_{n-1}}{t_n-t_{n-1}})da_{n-1} \tag{18}$$

$$\frac{\partial y}{\partial a_n} = (\frac{t-t_{n-1}}{t_n-t_{n-1}})da_n$$

where n is the number of unknown offsets of the continuous piecewise linear function. Depending on the analyst the interval length for piecewise linear modelling of the atmosphere is usually set to values between 2 hours to 5 minutes. In case, for the hourly estimation, in total 24 (n) ZWD linear offsets per station for one session is computed (Figure 3). Then, the design matrix will be

$$A_{wetdelay} = \begin{bmatrix} 1-\dfrac{t_1^{'}-t_0}{t_1-t_0} & \dfrac{t_1^{'}-t_0}{t_1-t_0} & 0 & 0 & \cdots & 0 \\ 1-\dfrac{t_2^{'}-t_0}{t_1-t_0} & \dfrac{t_2^{'}-t_0}{t_1-t_0} & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1-\dfrac{t_{m/n}^{'}-t_0}{t_1-t_0} & \dfrac{t_{m/n}^{'}-t_0}{t_1-t_0} & 0 & 0 & \cdots & 0 \\ 0 & 1-\dfrac{t_1^{''}-t_1}{t_2-t_1} & \dfrac{t_1^{''}-t_1}{t_2-t_1} & 0 & \cdots & 0 \\ 0 & 1-\dfrac{t_2^{''}-t_1}{t_2-t_1} & \dfrac{t_2^{''}-t_1}{t_2-t_1} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 1-\dfrac{t_{m/n}^{''}-t_1}{t_2-t_1} & \dfrac{t_{m/n}^{''}-t_1}{t_2-t_1} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & 1-\dfrac{t_1^{(n)}-t_{n-1}}{t_n-t_{n-1}} & \dfrac{t_1^{(n)}-t_{n-1}}{t_n-t_{n-1}} \\ 0 & \ddots & 0 & 0 & 1-\dfrac{t_2^{(n)}-t_{n-1}}{t_n-t_{n-1}} & \dfrac{t_2^{(n)}-t_{n-1}}{t_n-t_{n-1}} \\ \vdots & \cdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & 0 & 1-\dfrac{t_{m/n}^{(n)}-t_{n-1}}{t_n-t_{n-1}} & \dfrac{t_{m/n}^{(n)}-t_{n-1}}{t_n-t_{n-1}} \end{bmatrix}_{mxn} \qquad (19)$$

where m is the number of observations carried out in a 24 hour VLBI session. The weight matrix can be chosen as unit matrix,

$$P = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}_{mxm} \qquad (20)$$

Let's assume that the difference between the hourly estimated offsets of the piecewise linear functions of ZWDs should be equal with a formal error of $\pm$ 10 mm. Then, the pseudo-observations for the constraints will be

$$a_{i+1} - a_i = 0 \pm 10\,mm \qquad i = 0,1,2,\ldots,n-1 \qquad (21)$$

The Jacobian and weight matrix, the vector of observed minus computed for pseudo-observation equations of constraints are

$$H = \begin{array}{c} \overbrace{\begin{array}{cccccc} a_0 & a_1 & a_3 & \cdots & a_{n-1} & a_n \end{array}} \\ \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & -1 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & -1 \end{bmatrix}_{(n-1)xn} \end{array} \qquad (22)$$

$$p_c = \begin{bmatrix} 0.01 & 0 & \cdots & 0 \\ 0 & 0.01 & \cdots & 0 \\ \vdots & \vdots & \ddots & \\ 0 & 0 & 0 & 0.01 \end{bmatrix}_{(n-1)x(n-1)} \qquad h = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{(n-1)x1}$$

The weights of pseudo-observations as constraints are introduced looser than the weights of observations. This leads the normal equation matrix change in regular form but without any bias on the results. Also, these loose constraints get the normal equation matrix invertible. Their use allows to efficiently bridging time intervals with gaps in the data (Figure 4) without any re-parameterization (BOEHM et al. 2006b, KUTTERER 2003).

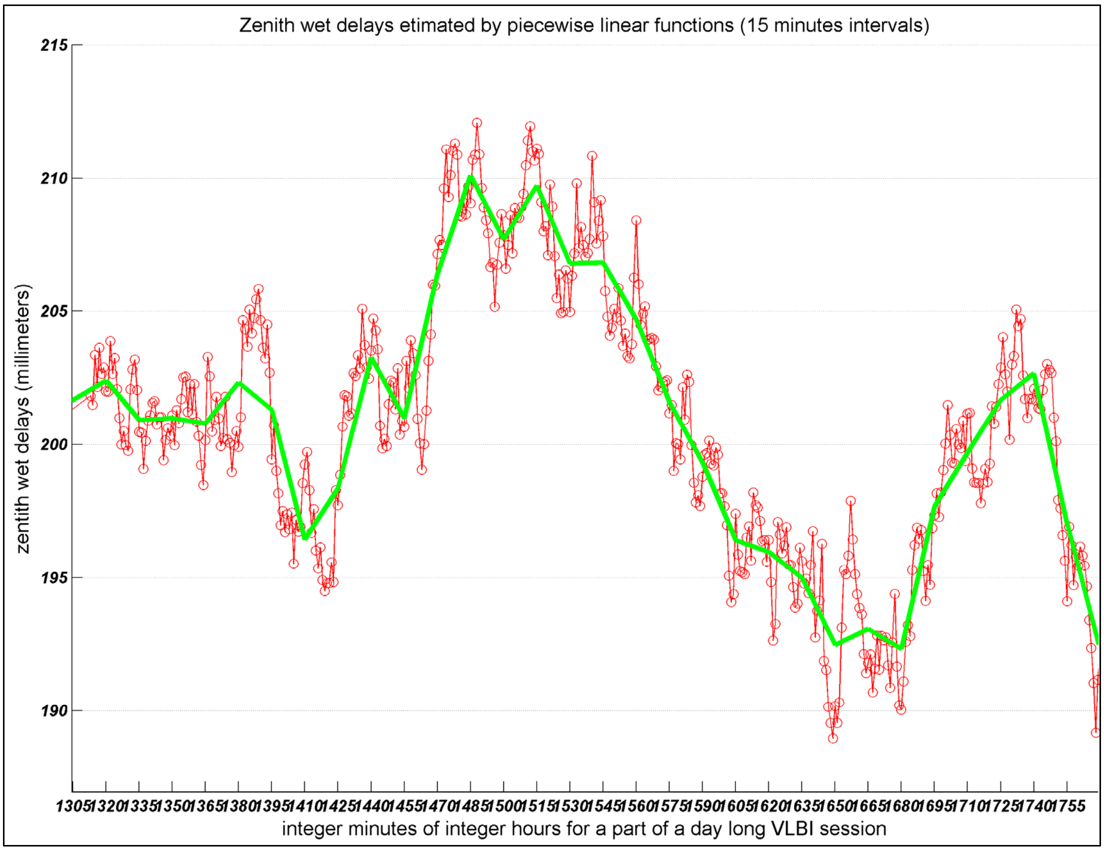

Figure 3: ZWDs estimated by piecewise linear functions within 15 minutes intervals

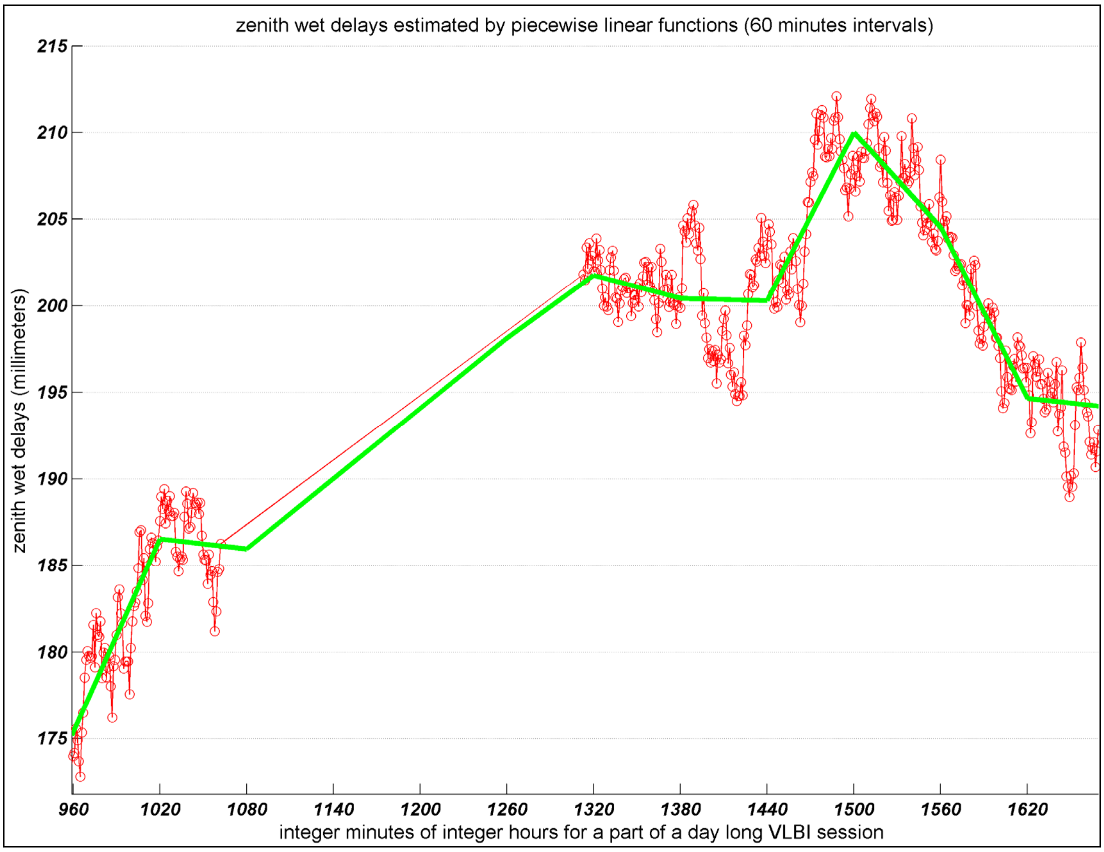

Figure 4: ZWDs estimated by piecewise linear functions in hourly intervals (Applying loose constraints as pseudo observations on ZWD results in consistent solutions (invertible normal equation matrix) when unobserved parts occurred).

4. Least squares adjustment introduced with constraints

VLBI parameters can be estimated according to the least-squares (LS) adjustment based on the Gauss-Markov model. The goal of LS adjustment is to minimize the square sum of residuals. LS where loose constraints are introduced as pseudo-observations with linearized observation equations can be applied for the estimation of VLBI parameters like, e.g., the offsets of the continuous piecewise linear functions for the clocks and the tropospheric zenith delays. Constraints as pseudo-observations with linearized observation equations are introduced

$$v_c = Hdx - h \tag{23}$$

where v_c is the residuals of the constraints, H is the Jacobian matrix for pseudo-observation equations. h is the vector of pseudo-observations introduced in order to stabilize the estimation of some parameter types such as those referring to piecewise continuous linear functions for clocks and for the tropospheric ZWDs. Nevertheless, only the original observations have physical meaning. There is a possibility to use 'arbitrary' values which just fulfill some best-fit criterion. Hence, in neither of these cases constraints need to be physically interpretable (KUTTERER 2003). The complete functional and stochastic model can be formed as follows

$$\begin{aligned} \begin{bmatrix} v \\ v_c \end{bmatrix} &= \begin{bmatrix} A \\ H \end{bmatrix} dx - \begin{bmatrix} dl \\ h \end{bmatrix} \\ K_{ll} = \begin{bmatrix} K & 0 \\ 0 & K_c \end{bmatrix} &= S_{0c} \begin{bmatrix} P^- & 0 \\ 0 & p_c^- \end{bmatrix} \end{aligned} \tag{24}$$

where dl is the vector of reduced observations (observed minus computed), K_{ll} denotes to the variance-covariance matrix for observations and pseudo-observations, P and P_c are the weight matrices for the observations and pseudo-observations, respectively. The complete normal equation system for the constrained solution can be computed as follows

$$(A'PA + H'P_cH)\,dx = A'Pl + H'P_ch \tag{25}$$

with the constrained normal equation matrix (N_{total}) and the vector of the right hand side of the constrained normal equation system (b_{total}) given below

$$N_{total} = A'PA + H'P_cH \qquad b_{total} = A'Pl + H'P_ch \tag{26}$$

The unknown parameters of the constrained solution can be computed with

$$x_c = x_0 + N_{total}^- b_{total} \tag{27}$$

The a posteriori variance-factor for the constrained normal equation system is

$$S_{0c} = (v'Pv + v_c'P_cv_c)/(n_{obs} + n_{constr} - n_{unk}) \tag{28}$$

where n_{obs}, n_{constr}, n_{unk} are denote to the number of observations, constraints as pseudo-observations, and unknowns, respectively. The variance-covariance matrix for the unknowns of this constrained normal equation system can be computed with

$$K_{xx} = S_{0c}(N_{total})^- \tag{29}$$

The resulted different elements belonging to the normal equations are stored in separate SINEX blocks.

5. Conclusion

From the investigations carried out with in this study the following conclusions can be drawn:

- *Piecewise linear offsets* should be supported for all parameters, i.e. ZWDs, troposphere gradients, clocks, Earth orientation parameters, and station coordinates.
- The offsets should be determined at integer days, integer hours, or integer fractions of integer hours, respectively.
- In order to avoid numerical problems (as e.g. rank deficiencies) and to stabilize the parameter estimation process, loose constraints as pseudo observations should be included in the parameter estimation stage of the VLBI analysis.
- The compatibility of the weights of constraints (pseudo-observations) and of the real observations has utmost importance because of their effect on the estimated parameters.
- The application of scans instead of observations as primary unit of the structure array has shown several advantages. One of those certainly is that the concept is appropriate for Kalman Filter solutions with updates every scan. Also, station-related quantities are only stored once per epoch, which decreases the memory allocation significantly. The major advantage of these structure arrays is that they can be extended at any place in the programs very easily without updating every other call to them. Furthermore, it is very easy to read certain properties of these arrays with Matlab and to e.g. plot them.

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