STRATEGY FOR DESIGNING GEODETIC GPS NETWORKS WITH HIGH RELIABILITY AND ACCURACY

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ABSTRACT

Global Positioning System (GPS) is increasingly coming into use to establish geodetic networks aiming especially forecasting earthquakes, tracking crustal movements, planning large engineering projects, implementing cadastral works and designing urbanization activities. In order to meet the establishment aim of a geodetic network, it has to be optimized, depending on a design criterion in designing stage. GPS networks can be optimized according to the chosen reliability and precision criteria. Reliability, the success of detecting outliers in a geodetic network and accuracy, the quality of a network in terms of random errors, largely depend on the configuration of the measurements. This study concentrates on the solution of designing the most effective GPS vector configuration and planning sessions for survey schedules so as to reach the demanded accuracy and reliability criteria. In this context, to design a GPS network survey schedule, a near optimal solution depending on the chosen reliability and accuracy criteria was illustrated, the results were analyzed and applicable solutions for designing GPS networks were suggested.

1. INTRODUCTION

Geodetic networks have to satisfy demanded accuracy and reliability criteria to ensure the functions of their usage. Geodetic networks should be optimized to reach the demanded accuracy and reliability criteria. The main optimization purpose of geodetic networks is to establish them with high reliability, good accuracy and low cost by reaching the best configuration and survey plan (Grafarend 1974; Schaffrin, 1985).

Nowadays, with developments in technology, GPS networks have taken place of terrestrial networks. Optimization of a GPS network can be carried out by selecting baseline vectors from the all-probable baseline vectors that can be measured in a GPS network. The aim of this method is to reach the demanded accuracy and reliability with minimum cost (Grafarend, and Sanso, 1985; Kuang, 1996; Konak, 1994; Even-Tzur, 2001). The general optimal design problems of GPS networks have been discussed in, among others, the works of Lindlohr and Wells (1985), Wells et al. (1986, 1987) and Kuang (1996).

In this study, an A-optimal solution, which leads to a homogenous network and an E-optimal solution, which leads an isotropic network, were applied for a GPS network. So as to minimize the objective function of the two classes of optimality mentioned above, first order design (FOD), which is the determination of the network configuration, was carried out. FOD can be characterized by fixed and free parameters. The weight matrix of observations (P) and the cofactor matrix (Q_{xx}) of the unknown parameters are assumed as fixed parameters and the configuration or design matrix (A) is assumed as free parameter in FOD (Grafarend, 1974). Sequential least squares method was used as a solution method to apply in FOD.

2. METHODOLOGY

The basic idea of network optimization is to recognize that it is possible to estimate the quality of a network before any observations are made, provided its configuration is known. This idea opens the possibility to make a detailed analysis of the design concerning its accuracy, reliability and costs, and to improve it in parts if necessary. For engineering applications, the locations of the ground network stations are selected to serve the purpose of the network and the

precision of GPS observations dictated by the chosen instruments (GPS receivers). Therefore, the only variable left for a surveyor to decide with freedom is the total number and the distribution of the baselines to be measured in the field (Kuang, 1996).

2.1. Geodetic GPS Network Accuracy Optimization

The three main classes of optimality, well known in statistics, are A-, D- and E- optimality. In the designing stage of a GPS network, an objective function is selected from local or global accuracy criteria and minimized by optimizing observation plan, which is known as configuration problem. Determined objective function has to be suitable for the network usage. Generally geodetic networks are desired to be homogenous and isotropic. An A-optimal solution minimizes the mean variance tr(Q_{xx}), computed as the sum of the eigenvalues (λ_i) of cofactor matrix, leads to an homogenous network. E-optimal solution provides a minimum value for the maximal eigenvalue (λ_n) of cofactor matrix. This minimum value is reached in the case that all eigenvalues are equal. Then the hyper-ellipsoid gets a hyper-sphere called as property isotropy (Wolf and Ghilani, 1997).

All the accuracy information of a network can be derived from cofactor matrix (Q_{xx}) of the unknown parameters, which are mainly coordinates (Baarda, 1977). Scalar precision criteria are made of global or local scalar precision measures that serve as an overall representation of the precision of a network (Wolf and Ghilani, 1997). Different scalar precision criteria were proposed for geodetic networks by Grafarend (1974), which can be considered as accuracy objective functions in network optimization. In Table 1, certain scalar accuracy objective functions (Z), which are minimizing in the course of geodetic network optimization, are given.

Accuracy Objective Functions					
Local	Helmert point error	$Z = m_{P_i} = \sqrt{m_{X_i}^2 + m_{Y_i}^2 + m_{Z_i}^2}$			
	Werkmeister point error	$Z = w_{p_1} = m_X m_Y m_Z$			
	Helmert point error ellipsoid semi-axes	$Z = A_{\rm H} = m_0 \sqrt{\lambda_1}; B_{\rm H} = m_0 \sqrt{\lambda_2}; C_{\rm H} = m_0 \sqrt{\lambda_3}$			
Global	Mean coordinate error	$Z = m_x, m_y, m_z = m_0 \sqrt{\frac{\text{tr}(Q_{xx})}{3p}}$			
	Objective Function for A-Optimal Network	$Z = tr(Q_{xx}) = \lambda_1 + \lambda_2 + \ldots + \lambda_{3p} = \sum_{i=1}^{3p} \lambda_i$			
	Objective Function for D-Optimal Network	$Z = det(Q_{xx}) = \lambda_1 \lambda_2 \lambda_{3p} = \prod_{i=1}^{3p} \lambda_i$			
	Objective Function for E-Optimal Network	$Z = \lambda_{max.}$			
	Objective Function for S-Optimal Network	$Z = \lambda_{max.} - \overline{\lambda_{min.}}$			
	Objective Function for I-Optimal Network	$Z = 1 - \lambda_{\min} / \lambda_{\min}$			

Table 1. Accuracy objective functions for the geodetic network optimization

In the equations above, m_x, m_y, m_z are standard deviations of coordinate unknowns, λ_i are the eigenvalues of the cofactor matrix.

A GPS network can be optimized with the objective functions selected from Table 1. The procedure is explanined as the follow steps.

• The network points are marked on the map of the study area. In that procedure, square-shaped geometry should be formed to reach the best accuracy and reliability for the network.

• The baselines are designed in maximum and minimum number. After this preliminary design, measurement weights and priory variance are determined. Then, network is adjusted with free network adjustment method.

• The selected objective function from accuracy or reliability criteria for the optimization of the network is calculated.

• Local accuracy criteria (coordinate errors, point errors, point error ellipsoid semi-axes) of the network are calculated. According to the simulation method, baselines are added to or removed from the observation plan iteratively so as to reach the demanded value of the selected objective function. In this method, when the first observation plan is constituted with a maximum number of baselines, the baseline which has the minimum effect on objective function is removed from the observation plan or when the first observation plan is constituted with a minimum number of baselines, the baseline which has the maximum effect on objective function is added to the observation plan. The baselines having the maximum or minimum effects on the objective functions are selected by investigating local accuracy criteria of the network (Dare, 1995; Dare, and Saleh, 2000; Even-Tzur, 2001).

• New points are planned on the area where the accuracy cannot be improved adequately by the new baselines.

• When the changes of the accuracy objective function values lessen and the values of local accuracy criteria reach a homogenous form for the whole network, iteration is ended. After that, construction and measurement works are begun.

2.2. Geodetic GPS Network Reliability Optimization

In order to increase the capability of detecting model errors and outliers in a geodetic network, it has to be optimized. Baarda (1968) distinguishes "*internal reliability*" and "*external reliability*". While internal reliability of a control network measures the marginal undetectable error in the measurements, external reliability measures the effect of an undetectable gross error on the network coordinates and on quantities computed from them. The aim of geodetic networks adjustment is to calculate coordinates so external reliability is of more practical value than internal reliability. There is a strong correlation between internal and external reliability; high internal reliability leads to high external reliability (Even-Tzur and Papo, 1996).

The reliability of a network is considered high when the network can identify even small gross errors. Gross errors in the measurements affect the adjustment parameters; therefore, the reliability of the network is useful as a design criterion (Biacs et al. 1990).

So as to reach global relative redundancy number of GPS network at the value of 0.5, first observation plan is constituted with adequate baselines. When optimizing GPS networks in respect to reliability redundancy numbers of the baselines, internal and external reliability of the network are taken into consideration as objective functions. New baselines are planned perpendicular to the baselines of which redundancy numbers are under the value of 0.5. Also new baselines are planned perpendicular to the baselines of which internal and external reliability values are higher than the critical values (Dare and Saleh, 2000). In Table 2, certain scalar objective functions, which are to ensure the limit of the critical values, are given.

Reliability Ob	Critical Values	
Individual Redundancy	$\mathbf{Z} = \mathbf{r}_{j} = (\mathbf{Q}_{vv})_{j} \mathbf{P}_{j}$	$Z = r_j > 0.5 \text{ or } r_j > 0.3$
Internal Reliability	$Z = \left \Delta_{0j} \right = m_0 \sqrt{\frac{w_0}{P_j r_j}}$	$Z = \Delta_{0j} \cong (6 \text{ or } 8) \text{ m}_j$
External Reliability	$Z = \delta_{0j}^2 = \frac{1 - r_j}{r_j} w_0$	$Z = \delta_{0j} \cong 6 \text{ or } 10$

Table 2. Reliability objective functions for the geodetic network optimization

In the equations above, $Q_{\nu\nu}$ is the cofactor matrix of the residuals, *P* is the weight matrix of the observations, m₀ is the standard deviation of unit weight and w₀ is the lower bound for non-centrality parameter in dependency of the significance level (α_0) and the required minimum power of the test (1- β_0).

A geodetic network can be optimized depending on the reliability objective functions selected from Table 2 as the following steps;

• The global relative redundancy number (r_0) of the network is and individual redundancy numbers of the baselines are (r_j) calculated from the first observation plan.

• If some of the individual redundancy numbers are lower than the global relative redundancy it is decided that the concerned baselines cannot be checked adequately by the other baselines. For that reason, new baselines are planned perpendicular to the relevant baselines.

• Internal reliability criteria (Δ_{0j}) and external reliability criteria (δ_{0j}) are calculated for the network.

• The baselines of which internal reliability values are above the critical values $(6m_j)$ are decided that they cannot be checked adequately by the other baselines. For that reason, new baselines are planned perpendicular to the relevant baselines.

• When the cost of the network is taken into consideration, the baselines of which redundancy numbers are $r_i >> r_0$ and $\delta_{0i} \le 6$ would be removed from the network.

• The improved session plan is reviewed lastly if it ensures the selected objective function. Then the design is applied to the field (Baarda, 1977; Gazdzicki, 1976).

3. GPS NETWORK DESIGN (APPLICATION)

In this study, Karadeniz Technical University campus and a part of Trabzon airport were selected as the study area to design the GPS network. The size of network is approximately 1x1.5 km, and the network consists of 11 points. Optimization of the network was realized by planning sessions. As a result, optimum observation scheme was designed, which provides minimum values of objective functions. At first, the points of the network were marked in the map of the study area. In this procedure, there is no need for station intervisibilities for GPS networks but sky visibility is essential. To provide the best accuracy and reliability, the ground network stations were selected on the locations that ensures square-shaped network (Figure 1).



Figure 1. Designed GPS Network

In order to determine the baseline vector components for planning sessions to constitute observation scheme, calculation of the points' coordinates in WGS84 was needed. Therefore, the baseline vectors were calculated before the optimization of the network by subtracting points' coordinates from each other. For convenience, the steps in the determination of baselines' vectors in WGS84 are summarized below.

Step 1: Network points were marked in the map of the study area. The Universal Transverse Mercator (UTM) projection coordinates of the marked points were read from the map. Then, UTM coordinates of the points were converted into Gauss-Kruger projection coordinates (x, y).

Step 2: The Gauss-Kruger (x, y) coordinates of the points were converted into the geodetic coordinates (B, L) in ED50.

Step 3: Elevations referred to the geoid of the network points (orthometric heights) were read from the map of the study area. To convert orthometric heights (H) to ellipsoid heights (h), geoid heights (vertical distances between the ellipsoid and geoid) were added to orthometric heights.

Step 4: Earth-related geocentric coordinates (X, Y, Z) in ED50 (European Datum 1950) of the GPS network points were converted from their geodetic coordinates (B, L, h)_{ED50}. The equations for making these conversations are as follows:

$$e^{2} = \frac{a^{2} - b^{2}}{a^{2}}, N = \frac{a}{\sqrt{1 - e^{2} \sin^{2} B}},$$

$$X_{ED50} = (N + h) \cos B \cos L$$

$$Y_{ED50} = (N + h) \cos B \cos L$$

$$Z_{ED50} = (\frac{b^{2}}{a^{2}}N + h) \sin B$$
(2)

In the equations above, h is the ellipsoid height of the point, B is the geodetic latitude of the point, L is the geodetic longitude of the point, e is the eccentricity, a and b are the semi-major and semi-minor axis of the Hayford ellipsoid respectively and N is the normal to ellipsoid at the points.

Step 5: Geocentric coordinates (X, Y, Z) in ED50 of the network points were transferred to WGS-84 geocentric coordinates by using seven-parameter similarity transformation with 7 transformation parameters which are three rotations, three translations and one scale factor. In order to calculate these seven unknowns (ε_X , ε_Y , ε_Z , t_X , t_Y , t_Z , k) for a unique solution, seven equations must be written. However, in the study area, geocentric coordinates (X, Y, Z) of three points (1, 6, 9) were known both in ED50 and in WGS84. For that reason, a least square solution was used to calculate the transformation parameters. All the ED50 geocentric coordinates of the network were transformed into WGS84 geocentric coordinates by the equation below:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{WGS84} = \begin{bmatrix} t_X \\ t_Y \\ t_Z \end{bmatrix} + k \begin{bmatrix} 1 & \varepsilon_Z & -\varepsilon_Y \\ -\varepsilon_Z & 1 & \varepsilon_X \\ \varepsilon_Y & -\varepsilon_X & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{ED50}$$
(3)

In the equations above, ε_X , ε_Y , ε_Z are rotations to make the reference axes of the two systems parallel, t_X , t_Y , t_Z are translations to create a common origin for the two coordinate systems and k is the scale factor to create equal dimensions in the two coordinate systems (Wolf and Ghilani, 1997; Özbenli, 2001).

Step 6: The designed baseline vector components $(\Delta X, \Delta Y, \Delta Z)_{WGS84}$ were calculated from geocentric coordinates $(X, Y, Z)_{WGS84}$ of the network points by subtracting from each other.

3.1. Accuracy Optimization of Observation Scheme

In the application, it was aimed to constitute a homogenous and isotropic network. For that reason, an A-optimal solution ($Z = tr(Q_{xx}) \Rightarrow min$.) that leads to a homogenous network and an E-optimal solution ($Z = \lambda_{max} \Rightarrow min$.) that leads to an isotropic network were selected as objective functions. First order design (optimization of observation scheme) was selected as an appropriate design order to minimize the selected objective functions. Optimum observation scheme was formed by using sequential least square method which is a method of geodetic network simulation. When the first observation scheme was constituted with minimum baselines, the minimum number of the sessions (s) was calculated with the following equation:

$$s = (n-m)/(r-m) \tag{1}$$

In the equation above, r is the number of receivers, n is the number of points, m is the number of points that are measured in two different sessions. When the first observation scheme was constituted with maximum baselines, all the probable baselines were planned taking the sessions into consideration.

<u>A- and E- Optimal network design beginning from minimum baselines</u>: In the application, the number of receivers was 3, the number of points was 11 and the number of points, which are measured in two different sessions, was 2. Solving the equation (1) with the above-mentioned variables, the value of 9 was calculated as the minimum number of the sessions. It was seen that the network could be measured at least 9 sessions with 3 receivers (Figure 2).

The determined baselines of the first observation scheme were considered as the measurements of the network and the network was adjusted with the free adjustment method. Then the values of objective functions were calculated (Table 3). As shown in Table 3, the maximum eigenvalue was approximately half of the total eigenvalues. This shows that the accuracy distribution was not homogenous for the first observation scheme. Therefore, new baselines were planned to the network depended on the Helmert point errors and semi-axes of the points' error ellipsoids (Table 4).



Figure 2. The first observation plan based on minimum sessions

Table 3. The objective function values calculated from the first observation scheme

Objective Function for A-Optimal Network	Objective Function for E-Optimal Network
$\sum_{i=1}^{11} \lambda_i = iz(Q_{xx}) = 1.524$	$\lambda_{\rm max} = 0.613$

Point	Helmert Point Errors	Semi-Axes of Error Ellipsoids			
Number	(cm)	A _H (cm)	B _H (cm)	C _H (cm)	
1	0.263	0.310	0.111	0.149	
2	0.385	0.454	0.162	0.218	
3	0.461	0.544	0.194	0.261	
4	0.402	0.474	0.169	0.227	
5	0.360	0.425	0.152	0.204	
6	0.290	0.342	0.122	0.164	
7	0.402	0.474	0.169	0.227	
8	0.385	0.454	0.162	0.218	
9	0.263	0.310	0.111	0.149	
10	0.360	0.425	0.152	0.204	
11	0.461	0.544	0.194	0.261	

Table 4. Helmert point errors and semi-axes of point error ellipsoids

As shown in Table 4, accuracy of the points 11 and 3 is insufficient. Therefore, the second observation scheme was constituted by adding the baselines 3-11 and 1-11 to the first observation scheme (Figure 3).



Figure 3: The network constituted with the second observation scheme

Each observation scheme was formed iteratively, depending on the values of the objective functions and the local precision criteria. The number of baselines, mean coordinate errors and the values of the objective functions are summarized in Table 5 for each observation scheme. In Figure 4, the effects of the added baselines on the values of the objective functions are given.



Table 5. A- and E- optimal network design from minimum baselines to maximum

Figure 4. The effects of added baselines on objective functions when designing E- and A- optimal networks

Observation

As shown in Figure 4, after the fifth observation scheme the objective function value of Eoptimality and after the sixth observation scheme the objective function value of A-optimality cannot be reduced significantly by the added baselines. When the effects of added baselines on objective functions were very little, it was decided that E-optimal network was reached with the fifth observation scheme and A-optimal network was reached with the sixth observation scheme (Figure 5).



Figure 5. E- and A- Optimal networks beginning from minimum baselines

<u>A- and E- Optimal network design beginning from maximum baselines:</u> The maximum observation scheme was designed assuming three receivers are used and was constituted with all probable baselines depended on the sessions. The baselines of which effects on the objective functions were minimum were removed from the observation scheme. When deciding the baselines which had to be removed from the network, the Helmert point errors and semi-axes of the point error ellipsoids were taken into consideration. The optimal networks were reached iteratively. In Figure 6, the effects of the removed baselines on the values of the objective functions are given.



Figure 6. The effects on the objective functions of removed baselines in designing E- and A-optimal networks

As seen in Figure 6, after the fourth observation scheme the objective function value of Eoptimality and after the sixth observation scheme the objective function value of A-optimality are increased significantly by the removed baselines. When the effects of removed baselines on objective functions were very big, it was decided that E-optimal network was reached with the fourth observation scheme and A-optimal network was reached with the sixth observation scheme (Figure 7).



Figure 7. E- and A- Optimal networks beginning from maximum baselines

3.2. Reliability Optimization of Observation Scheme

In the application, it was aimed that global redundancy number should be greater than the value of 0.5, individual redundancy numbers should be close to the global relative redundancy, internal reliability criterion values should be under the "6m_j" critical values and external reliability criterion values should be under the "6" critical value. Assuming three receivers are used, the first observation scheme of the network was designed with 20 baselines so that the global relative redundancy would be the value of 0.5 (Figure 8).



Figure 8. The network formed according to the first observation scheme in reliability optimization

The calculated redundancy numbers of the baseline vector components and the selected critical value (0.5) are shown in Figure 9.



Figure 9. Individual redundancy numbers of the first observation scheme

When Figure 9 is examined, it can be seen that redundancy numbers of the baselines 10-2, 4-2, 4-3, 11-3, 6-5, 10-6, 8-7, 5-8 and 7-11 are under the critical value. The internal reliability criterion values of the baselines related to the first observation scheme and " $6m_j$ " critical value are given in Figure 10.



Figure 10. Internal reliability values of the network related to the first observation scheme

When Figure 10 is examined, internal reliability values of the baselines 4-3, 11-3, 7-11 can be seen that they exceed the critical value and internal reliability values of the baselines 10-2, 4-2, 6-5, 10-6, 8-7, 5-8 can be seen that they are close to the critical value. The external reliability values of the baselines related to the first observation scheme and critical value "6" are given in Figure 11.



Figure 11. External reliability values of the network related to the first observation scheme

As shown in Figure 11, all the external reliability values of the network were found under the "6" critical value. As shown in Figure 12, reliability values of the baselines 4-3, 11-3 and 7-11 were improved by planning the new baselines 11-2 and 3-2 perpendicular to them. In this way, second observation scheme was formed (Figure 12).



Figure 12. The network formed according to the second observation scheme in reliability optimization

The same procedure as mentioned above was performed iteratively and with the third observation scheme all the individual reliability criteria were achieved to ensure the critical values. The third observation scheme was decided to be reliable to detect the model errors and outliers (Figure 13).



Figure 13. The network formed according to the third observation scheme in reliability optimization

4. CONCLUSIONS

In designing A and E optimal networks, homogeneous network structure could be obtained regarding to accuracy by beginning from observation schemes with minimum baselines. Beginning from observation schemes with maximum baselines homogeneous structure of the network has been lost. Objective function values of A- and E-optimal networks obtained approximately the same with equal iteration created with same amount of baselines but different network configuration. Therefore, it was seen that it is possible to reach A- and E-optimal networks by using equal iteration from beginning maximum observation scheme and changing to minimum one or vice-versa. It was seen that point error ellipsoids were small at the points having adequate observations and large at the outer network points not having adequate observations. It was seen that values of accuracy objective functions were inversely proportional with the number of baselines and not change in linear form.

In reliability optimization of the network, it was seen that with the first observation scheme the external reliability criterion values of the network ensured the related critical value, the internal reliability criterion values of the network ensured the related critical values with the second observation scheme, and the redundancy numbers of all baselines ensured the related critical value with the third observation scheme. Reliable network geometry was reached by the third observation scheme, with another expression; capability of the network geometry in detecting model errors and outliers was increased.

Consequently, when designing a GPS network,

- forming baselines with loops,
- planning new baselines in directions collinear with the existing ones in order to strengthen the point positions,
- due to the fact that having the value of a good global accuracy criterion does not mean that the network is homogeneous, improving the values of each local accuracy criteria of the network,
- in reliability optimization, having a high global relative redundancy does not mean that the network is reliable, ensuring the values of each reliability criteria of the network for critical values so as to reach a homogeneous structure for the entire network,
- planning new baselines in directions perpendicular to the existing baselines in order to strengthen the reliability of the network,

are proposed.

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