FULL NAME ______________________________

ID NUMBER ______________________________

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Midterm Exam
November 25, 2013

Instructions

1. This exam's contribution to your final grade is 50%.

2. There are four questions in this exam. You should answer only two of these questions.

3. You do not need a calculator.

4. Mobile phones and laptop computers should be turned off.

5. You are not allowed to leave the room for the first 20 minutes of the exam time.

6. This is a closed-books and closed-notes exam.

7. You are not allowed to talk to each other during the exam.

8. *Student Discipline Regulations of the Institutions of Higher Education* are in effect. According to the 9th article, cheating in this exam may have severe consequences for you—including a temporary suspension of your studies up to two semesters.

9. You have exactly 60 minutes to complete the exam.

Questions

1. [**5 points**] Consider the market for a commodity. The demand and the supply functions satisfy

$$q^d \equiv \alpha \qquad q^s \equiv \beta - \frac{1}{p-\gamma}$$

where $\alpha, \beta, \gamma > 0$ and $\beta > \alpha$. Let $q^\star$ and $p^\star$ respectively denote the equilibrium levels of quantity and price, and find $q^\star$ and $p^\star$.

Answer:

From $q^d = q^s = q^\star$, we have

$$\alpha = \beta - \frac{1}{p^\star - \gamma} \Rightarrow \frac{1}{p^\star - \gamma} = \beta - \alpha \Rightarrow p^\star - \gamma = \frac{1}{\beta - \alpha} \Rightarrow p^\star = \gamma + \frac{1}{\beta - \alpha}.$$

Then, from $q^d = \alpha$, we have $q^\star = \alpha$.

2. [**15 points**] Consider the model defined by

$$E \equiv C + I \qquad C \equiv f(Y, r) \qquad \text{and} \quad I \equiv g(r)$$

where Y is the national income, E is the aggregate expenditure, C is the consumption expenditure, I is the investment expenditure and r is the real interest rate. The functions $f(\bullet, \bullet)$ and $g(\bullet)$ satisfy

$$f_Y(Y, r) > 0 \quad f_r(Y, r) < 0 \quad \text{for all } (Y, r) \quad \text{and} \quad g_r(r) < 0 \quad \text{for all } r.$$

Let $Y^\star$ denote the equilibrium level of Y such that $E^\star = Y^\star$. The implicit function theorem holds locally around this unique equilibrium so that the function $Y^\star \equiv Y(r)$ exists. Find

$$\frac{\mathrm{d}Y^\star}{\mathrm{d}r}.$$

Answer:

By the definition of the equilibrium, we have

$$Y^\star = f(Y^\star, r) + g(r).$$

Taking the total derivative of both sides, we have

$$\mathrm{d}Y^\star = f_Y(Y^\star, r)\mathrm{d}Y^\star + f_r(Y^\star, r)\mathrm{d}r + g_r(r)\mathrm{d}r$$
$$\left(1 - f_Y(Y^\star, r)\right)\mathrm{d}Y^\star = \left(f_r(Y^\star, r) + g_r(r)\right)\mathrm{d}r$$
$$\frac{\mathrm{d}Y^\star}{\mathrm{d}r} = \frac{f_r(Y^\star, r) + g_r(r)}{1 - f_Y(Y^\star, r)}$$

3. [**30 points**] There are $n \in \mathbb{N}$ markets in an economy. In market $i \in \{1, 2, ..., n\}$, commodity i is traded at price p_i. Let $p \in \mathbb{R}^n$ denote the (column) vector of prices. The linear demand and supply functions for commodity i are such that

$$q_d^i \equiv \delta_i p \qquad q_s^i \equiv \sigma_i p$$

where q_d^i denotes the quantity demanded and8 q_s^i denotes the quantity supplied. Here, $\delta_i \equiv (\delta_{i1}, ..., \delta_{in}) \in \mathbb{R}^n$ and $\sigma_i \equiv (\sigma_{i1}, ..., \sigma_{in}) \in \mathbb{R}^n$ are (row) vectors of known constants. Notice that these markets are interconnected because q_d^i and q_s^i change not only with p_i but also with $(p_1, ..., p_{i-1}, p_{i+1}, ..., p_n)$. In a general equilibrium of this economy, all markets clear. Formally, in equilibrium, $q_d^i = q_s^i$ for all i.

i. [**5 points**] Define the appropriate system of linear equations in matrix form where $p^\star$ denotes the unknown vector of equilibrium prices.
Answer:
In general equilibrium, we have, for each i,

$$\delta_i p^\star = \sigma_i p^\star.$$

This can be written, for each i, as

$$\delta_i p^\star - \sigma_i p^\star = 0.$$

More explicitly, this reads, for each i,

$$\delta_{i1} p_1^\star + ... + \delta_{in} p_n^\star - (\sigma_{i1} p_1^\star + ... + \sigma_{in} p_n^\star) = 0.$$

Then, we have

$$Ap^\star = 0$$

where A is defined as the $n \times n$ matrix with the typical element

$$a_{ij} \equiv \delta_{ij} - \sigma_{ij}.$$

ii. [**5 points**] Is this system homogeneous? Explain.
Answer:
This system is homogeneous because the known coefficient vector on the right-hand side is equal to the null vector 0.

iii. [**5 points**] Set $n = 2$. Under what condition(s) on $(\delta_{11}, \delta_{12}, \delta_{21}, \delta_{22}, \sigma_{11}, \sigma_{12}, \sigma_{21}, \sigma_{22})$ the system has a nontrivial solution?
Answer:
With $n = 2$, the system $Ap^\star = 0$ reduces into

$$\begin{bmatrix} \delta_{11} - \sigma_{11} & \delta_{12} - \sigma_{12} \\ \delta_{21} - \sigma_{21} & \delta_{22} - \sigma_{22} \end{bmatrix} \begin{bmatrix} p_1^\star \\ p_2^\star \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

This system attains a nontrivial solution $p^\star \neq 0$ if $\det A = 0$. This condition implies

$$(\delta_{11} - \sigma_{11})(\delta_{22} - \sigma_{22}) = (\delta_{21} - \sigma_{21})(\delta_{12} - \sigma_{12}).$$

iv. [**15 points**] Find the general solution of the 2×2 system.
Answer:
The general solution of the system is a price ratio k such that

$$\frac{p_2^\star}{p_1^\star} = k.$$

Under the condition $(\delta_{11} - \sigma_{11})(\delta_{22} - \sigma_{22}) = (\delta_{21} - \sigma_{21})(\delta_{12} - \sigma_{12})$, we have

$$k = -\frac{\delta_{11} - \sigma_{11}}{\delta_{12} - \sigma_{12}} = -\frac{\delta_{21} - \sigma_{21}}{\delta_{22} - \sigma_{22}}$$