ECO 241 — Mathematical Economics I — Final Exam
Time Allowed: 60 minutes
Points Available: 75 pts (25 pts bonus)

Questions

1. [**10 points**] Consider a firm that uses two inputs (k and ℓ). The firm operates with the production target $y > 0$, and its cost function is $C(k, \ell) \equiv 3k + \ell$. The Lagrange function associated with the firm's cost minimization problem is

$$L(k, \ell, \lambda) \equiv -(3k + \ell) + \lambda(\sqrt{k\ell} - y).$$

What is the economic interpretation of $\lambda^\star$? (*Hint*: Notice that a unique solution exists.)

2. [**10 points**] Let $Ax = 0$ where A is a 2×2 matrix, and $x = (x_1, x_2)$ is a column vector. If $x_1/x_2 = 3$ is the general solution, what does this imply about the matrix A?

3. [**10 points**] Solve the following optimization problem:

$$\max_x \ f(x) \equiv 9 - x^2 \quad \text{where } x \in D \equiv \{..., -2, -1, 0, 1, 2, ...\} \setminus \{0\}$$

4. [**15 points**] You are in an exam. You can cheat or not. If you cheat, you may get caught and be punished. The probability that you get caught is $p \in (0, 1)$. Your utilities from different outcomes satisfy the following:

- If you don't cheat in the exam, your utility is 5.
- If you cheat, there are two possibilities:
  - If you get caught, your utility is -40.
  - If you don't get caught, your utility is 10.

Assuming that you are an expected utility maximizer, what is the minimum level of p that leads you to not to cheat in the exam?

5. [**15 points**] A firm that produces a good under perfect competition can use two different technologies. The first technology satisfies constant-returns-to-scale as in $Y_1 = AK_1$ where $A > 0$ is some measure of productivity, and K_1 is the flow of machine-hours allocated to the first technology. The second technology is of decreasing-returns-to-scale and reads $Y_2 = \sqrt{K_2}$. Suppose that the firm has a total flow $K > 0$ of machine-hours. Solve the problem of maximizing $Y_1 + Y_2$ subject to the resource constraint $K_1 + K_2 = K$.

6. [**15 points**] Consider a closed economy IS-LM model. Equilibrium-defining equations are

$$Y = C + I + G, \qquad C = C_0 + f(Y) - g(r), \qquad I = I_0 - h(r),$$
$$G = G_0, \qquad \text{and} \qquad M_0 = 3Y - i(r)$$

where $C_0, I_0, G_0, M_0 > 0$ are exogenously given, and f, g, h, i are continuously differentiable and strictly increasing functions satisfying

$$\begin{aligned} Y^\star &= C_0 + f(Y^\star) - g(r^\star) + I_0 - h(r^\star) + G_0 \\ M_0 &= 3Y^\star - i(r^\star). \end{aligned}$$

for the unique equilibrium $(r^\star, Y^\star)$. You should also suppose that $f'(Y^\star) \in (0, 1)$ (*Why*?). Find $\mathrm{d}r^\star/\mathrm{d}M_0$ and $\mathrm{d}Y^\star/\mathrm{d}G_0$, and interpret your results.