FULL NAME ______________________________

ID NUMBER ______________________________

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Midterm Exam
November 28, 2013

Instructions

1. This exam's contribution to your final grade is 50%.
2. There are three questions in this exam. You should answer all of these questions.
3. You do not need a calculator.
4. Mobile phones and laptop computers should be turned off.
5. You are not allowed to leave the room for the first 20 minutes of the exam time.
6. This is a closed-books and closed-notes exam.
7. You are not allowed to talk to each other during the exam.
8. *Student Discipline Regulations of the Institutions of Higher Education* are in effect. According to the 9th article, cheating in this exam may have severe consequences for you—including a temporary suspension of your studies up to two semesters.
9. You have exactly 80 minutes to complete the exam.

Questions

1. **[10 points]** Consider an economy that produces cookies using labor. The price of a cookie is p, and the price of a unit of labor is w. In the cookie market, the excess demand is defined by

$$c \equiv \frac{\alpha w}{p} - \beta$$

where $\alpha > 0$ and $\beta > 0$ are fixed parameters. In the labor market, the excess demand is defined by

$$\ell \equiv \gamma - \frac{1}{(w/p)}$$

where $\gamma > 0$ is a fixed parameter. Under what condition does there exist a Walrasian equilibrium?

Answer:

In a Walrasian equilibrium, we must have $c = \ell = 0$. Hence, $(p^\star, w^\star)$ is a Walrasian equilibrium price vector if the following hold:

$$\frac{\alpha w^\star}{p^\star} = \beta \qquad \text{and} \qquad \gamma = \frac{1}{(w^\star/p^\star)}.$$

These imply

$$\frac{w^\star}{p^\star} = \frac{\beta}{\alpha} \qquad \text{and} \qquad \frac{w^\star}{p^\star} = \frac{1}{\gamma}.$$

Thus, if the parameters satisfy

$$\frac{\beta}{\alpha} = \frac{1}{\gamma},$$

a Walrasian equilibrium exists.

2. **[25 points]** The Gale-Nikaido mapping, defined as in

$$\varphi_i(p) \equiv \frac{p_i + \max[0, \zeta_i(p)]}{1 + \sum_{j=1}^N \max[0, \zeta_j(p)]},$$

satisfies three key properties to be a legitimate price-updating function under the normalization

$$\sum_{i=1}^N p_i = 1.$$

Two of these properties are

$$\varphi_i(p^\star) = p_i^\star \qquad \zeta_i(p) > 0 \Rightarrow \varphi_i(p) > p_i$$

where $p^\star$ is an equilibrium price vector.

Consider an alternative price-updating function defined as in

$$f_i(p) \equiv p_i + \eta\zeta_i(p) \qquad \text{for all } i$$

under the normalization

$$\prod_{i=1}^N p_i = 1$$

where $\eta > 0$ is a fixed parameter and $\zeta_i(\bullet)$ is the excess demand function for commodity i. The question is whether this alternative price-updating function is a legitimate one.

Provide, firstly, a yes-or-no answer. Then, support your answer formally.

Answer:

No! To be a legitimate price-updating function, f should satisfy all of the three properties stated below for any p:

$$f_i(p^\star) = p_i^\star \qquad \prod_{i=1}^{N} f_i(p) = 1 \qquad \zeta_i(p) > 0 \Rightarrow f_i(p) > p_i.$$

It satisfies the first property because

$$\zeta_i(p^\star) = 0.$$

It satisfies the third property because

$$\eta > 0.$$

However, there is no guarantee that the second property is satisfied for any p:

$$\prod_{i=1}^{N} f_i(p) = \prod_{i=1}^{N} [p_i + \eta\zeta_i(p)] \lesseqgtr 1.$$

3. [**15 points**] Consider an economy that produces lemonade. There are H households and one dictator that consume lemonade. After the production occurs, the dictator gets one half of the total production, and the other half is distributed equally across all H households. There is no technological progress or capital accumulation.

- Does the described allocation sound like a Pareto-efficient one? Discuss.
- Suppose that an earthquake destroys one half of the identical factories that produce lemonade but the dictator now gets only the 5% of the total production before the rest of it is distributed equally across all H households. Does this allocation sound like a Pareto-efficient one? Discuss.

Answer:

Both allocations are Pareto-efficient. The first one is Pareto-efficient because the total level of production does not change with the distributional rules. No individual can be made better off without making at least one individual worse off. Nothing essentially is different to make the allocation in the second case not Pareto-efficient. Specifically, there are two changes: a decrease in the total level of production due to the earthquake and a decrease in the dictator's allocation in percentage terms. Yet, after the production occurs and the dictator and the rest get their allocations, we still have Pareto-efficiency.