FULL NAME ______________________________

ID NUMBER ______________________________

SIGNATURE ______________________________

Final Exam
January 13, 2014

Instructions

1. This exam's contribution to your final grade is 50%.
2. There are three questions in this exam. You should answer only two of these questions.
3. You do not need a calculator.
4. Mobile phones and laptop computers should be turned off.
5. You are not allowed to leave the room for the first 20 minutes of the exam time.
6. This is a closed-books and closed-notes exam.
7. You are not allowed to talk to each other during the exam.
8. *Student Discipline Regulations of the Institutions of Higher Education* are in effect. According to the 9th article, cheating in this exam may have severe consequences for you—including a temporary suspension of your studies up to two semesters.
9. You have exactly 90 minutes to complete the exam.

1. The Walrasian Equilibrium

- Imagine an economy with N produced goods—such as pen and paper—and K primary inputs—such as labor and capital. Let $n \in \{1, 2, ..., N\}$ and $k \in \{1, 2, ..., K\}$ be the corresponding index variables.
- Denote the total production of good n by $Y_n \geq 0$. In this economy, no produced good is used as an intermediate input, and the production of good n satisfies the technology

$$Y_n = F(Z_{n1}, Z_{n2}, ..., Z_{nK})$$

  where $Z_{nk} \geq 0$ denotes the total flow of input k used in the production of good n.
- The total endowment of input k is exogenously given and equal to $E_k > 0$. In this economy, primary inputs are supplied inelastically.
- The conditional demand Z_{nk} for good-input pair (n, k) satisfies

$$Z_{nk} = \frac{\theta_k p_n Y_n}{\omega_k} \qquad \theta_k \in (0,1), \sum_{k=1}^{K} \theta_k = 1$$

  where $p_n > 0$ denotes the price of a unit of good n and $\omega_k > 0$ denotes the price of a unit of input k.
- The consumption demand C_n for good n satisfies

$$C_n = \frac{\alpha_n \left(\sum_{k=1}^{K} \omega_k E_k\right)}{p_n} \qquad \alpha_n \in (0,1), \sum_{n=1}^{N} \alpha_n = 1$$

1.1. Excess Demand Functions

No points

Define the excess demand functions for a typical good market n and for a typical input market k.

1.2. The Walras Law

10 points

Show that the Walras Law holds.

1.3. The Characterization of the Equilibrium

15 points

Suppose that $N = K = 1$. The production function then reads

$$Y = F(Z)$$

Impose the price normalization $\omega = 1$. Characterize the Walrasian equilibrium (Hint: You have $\alpha = \theta = 1$ as well).

2. Intermediate Inputs

- Consider an economy with four intermediate inputs. There are four firms, and each firm produces one of the inputs. Denote the prices by

$$p_1, p_2, p_3 \text{ and } p_4.$$

- The four firms are tied together with the following input-output relations:

  i. Firm 1 sells 20% of its product to Firm 2, 50% of its product to Firm 3, and 30% of it to Firm 4.

  ii. Firm 2 sells 100% of its product to Firm 1.

  iii. Firm 3 sells 50% of its product to Firm 1 and 50% of its product to Firm 4.

  iv. Firm 4 sells 30% of its product to Firm 2 and 70% of its product to Firm 3.

2.1. The Input-Output Matrix

15 points

Write down the 4×4 input-output matrix A for this economy where a_{ij} denotes the units of input i required to produce 1 unit of input j.

2.2. The Social Accounting Matrix

10 points

Let $p_i^\star$ denote the Walrasian equilibrium price of input i. Suppose that each firm produces 100 units at this equilibrium. Write down the 4×4 social accounting matrix S.

3. Calibration

- The solution of a general equilibrium model with two markets is in the form of

$$p^\star = \alpha x + \beta \qquad \text{and} \qquad \omega^\star = 1 - p^\star$$

  where $(p^\star, \omega^\star)$ is the unique equilibrium price vector, $x > 0$ is the only exogenous variable of the model, and $\alpha > 0$ and $\beta \geq 0$ are structural parameters.

3.1. Unidentified Case

5 points

Suppose that $p^\star = 1/2$ and $x = 10$. Explain why you cannot calibrate both α and β.

3.2. Identified Case

5 points

Suppose that $\omega^\star = 1/2$ and $x = 10$. Let β be equal to 0. Calibrate α.

3.3. Identified or not?

15 points

Suppose that the excess demand in the market with price ω satisfies

$$\zeta(\omega) \equiv -\gamma\omega + x$$

such that

$$\zeta(\omega^\star) = 0$$

where $\gamma > 0$ is a structural parameter. Suppose, once again, that $p^\star = 1/2$ and $x = 10$. Let β still be equal to 0. Can you calibrate γ? Explain.