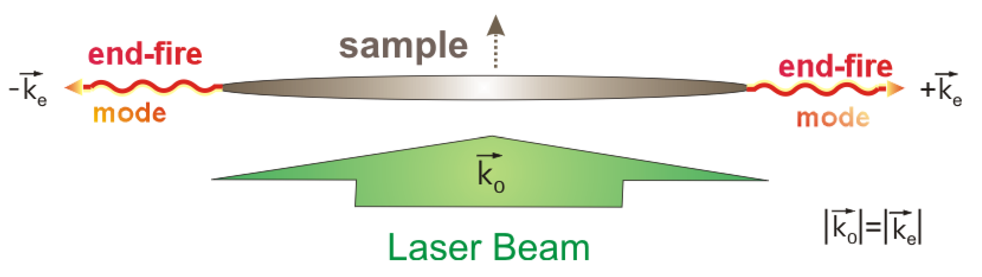
Continuous Variable Entanglement of Photons via Sequential Superradiant scattering from BEC

M. E. Taşgın[a], M. Ö. Oktel[a] , and Ö. E. Müstecaplıoğlu[b]

[a]Bilkent University, Department of Physics, 06800 Bilkent, Ankara, Turkey
[b]Departments of Physics, Koç University, Rumelifeneri Yolu, Sariyer 34450 Istanbul, Turkey

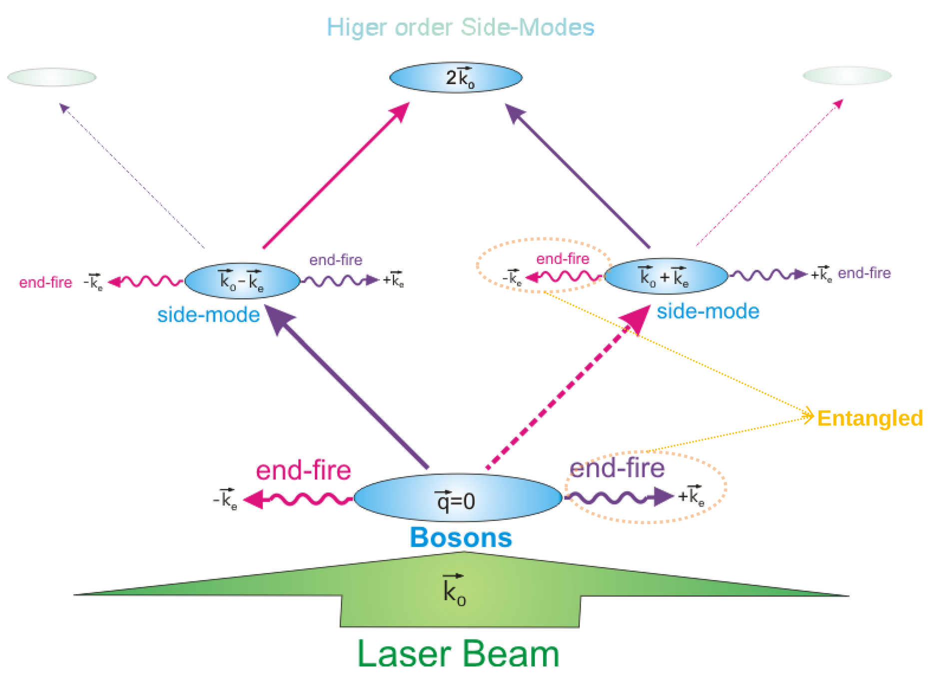
1. Superradiance (SR)

- Superradiance(SR) is the collective spontaneous emission of an ensemble of excited atoms.
- Atoms are exposed to a strong laser beam, and excited very quickly.
- Outgoing radiation($\mp\vec{k}_e$) is coherent and well aligned, perpendicular to the laser beam($\vec{k}_0$). Thus, they are called as end-fire modes.
- Alignment in SR is merely due to the elongated sample.



2. Sequential Superradiance

- If sample is a Bose-Einstein condensate(BEC), **recoiling of atoms are also collective.** These are called side-modes. $(\vec{k}_0 \mp \vec{k}_e)$
- For a rightward end-fire mode, a leftward side-mode is generated, vice versa (indicated with colors).
- If the laser pump is open for long enough time, the first order side-modes also superradiate to give higher order modes. $2(\vec{k}_0-\vec{k}_e)$ $2\vec{k}_0$ $2(\vec{k}_0+\vec{k}_e)$



- Entanglement is swapped via (k_0-k_e) side mode.
- We investigate the entanglement in the counter propagating end fire modes $+k_e$ and $-k_e$.

3. Hamiltonian

$$\hat{H} = \int d^3\boldsymbol{k}\,\hbar\omega(\boldsymbol{k})\hat{a}(\boldsymbol{k})^\dagger\hat{a}(\boldsymbol{k}) + \sum_{\boldsymbol{q}} \hbar\omega_{\boldsymbol{q}}\hat{c}_{\boldsymbol{q}}{}^\dagger\hat{c}_{\boldsymbol{q}}$$

$$-\frac{g(\boldsymbol{k}_0)}{\Delta}\sum_{\boldsymbol{q},\boldsymbol{q}'}\int d^3\boldsymbol{k}\rho_{\boldsymbol{q},\boldsymbol{q}'}(\boldsymbol{k})\hbar g^*(\boldsymbol{k})\hat{c}_{\boldsymbol{q}}{}^\dagger\hat{a}_{\boldsymbol{k}}{}^\dagger\hat{a}_{\boldsymbol{k}_0}\hat{c}_{\boldsymbol{q}'}$$

- We treat the laser pump quantum $\hat{a}_{k_0}$.
- Ignore the angular distribution about end-fire mode ($\pm\vec{k}_e$) $\Rightarrow$ single mode.
- Treat side mode as single mode.
- Ignore higher order side modes. $2(\vec{k}_0 \mp \vec{k}_e)$
- Move to rotating frame.

$$\hat{H} = -\hbar\frac{g^2}{\Delta}\left(\hat{c}_+^\dagger\hat{a}_-^\dagger\hat{a}_0\hat{c}_0 + \hat{c}_-^\dagger\hat{a}_+^\dagger\hat{a}_0\hat{c}_0 + \hat{c}_2^\dagger\hat{a}_-^\dagger\hat{a}_0\hat{c}_- + \hat{c}_2^\dagger\hat{a}_+^\dagger\hat{a}_0\hat{c}_+\right) + H.c.$$

$$\hat{a}_\pm \equiv \hat{a}_{\pm k_e}\ ,\ \ \hat{a}_0 \equiv \hat{a}_{\pm k_0}\ ,\ \ \hat{c}_\pm \equiv \hat{c}_{(k_0 \pm k_e)}\ ,\ \ \hat{c}_2 \equiv \hat{c}_{2k_0}$$

4. Continuous Variable Entanglement Parameter

- If density-matrix is <u>inseparable</u> $\Rightarrow$ $\langle\Delta\hat{u}^2\rangle + \langle\Delta\hat{v}^2\rangle \le \left(c^2 + \frac{1}{c^2}\right)$
- Heisenberg uncertainty $\Rightarrow$ $\langle\Delta\hat{u}^2\rangle + \langle\Delta\hat{v}^2\rangle \ge \left|c^2 - \frac{1}{c^2}\right|$
- **<u>Entanglement Parameter</u>**: $\lambda(t) = \langle\Delta\hat{u}^2\rangle + \langle\Delta\hat{v}^2\rangle - \left(c^2 + \frac{1}{c^2}\right)$
- Due to symmetry of end-fire modes ($\hat{a}_+ \leftrightarrow \hat{a}_-$) ➡ $c^2 = 1$
- If $\lambda(t) < 0$ end-fire modes are entangled.
- Minimum value of lambda can be **-2**, due to Heisenberg !

$$\hat{u} = |c|\hat{x}_1 + \hat{x}_2/c \quad \text{and} \quad \hat{v} = |c|\hat{p}_1 - \hat{p}_2/c$$

are Einstein-Podolsky-Rosen(EPR) operators with

$$\hat{x}_{1,2} = (\hat{a}_\pm + \hat{a}_\pm^\dagger)/\sqrt{2} \quad \text{and} \quad \hat{p}_{1,2} = (\hat{a}_\pm - \hat{a}_\pm^\dagger)/i\sqrt{2}$$

5. Entanglement Swap

Approximation

Initial Times

$\hat{a}_0 \to \sqrt{M}e^{i\theta_0}$, $\hat{c}_0 \to \sqrt{N}e^{i\phi_1}$

$$\hat{H}_1 = -\hbar\chi_1\left[e^{i\theta_1}(\hat{a}_+^\dagger\hat{c}_-^\dagger + \hat{a}_-^\dagger\hat{c}_+^\dagger) + \text{H.c.}\right]$$

couples $|\hat{c}_-\rangle \leftrightarrow |\hat{a}_+\rangle$

Later Times

$\hat{a}_0 \to \sqrt{M}e^{i\theta_0}$, $\hat{a}_2 \to \sqrt{N_2}e^{i\phi_2}$

$$\hat{H}_2 = -\hbar\chi_2\left[e^{i\theta_2}(\hat{a}_-^\dagger\hat{c}_- + \hat{a}_+^\dagger\hat{c}_+) + \text{H.c.}\right]$$

couples $|\hat{a}_-\rangle \leftrightarrow |\hat{c}_-\rangle$

couples $|\hat{a}_+\rangle \leftrightarrow |\hat{a}_-\rangle$

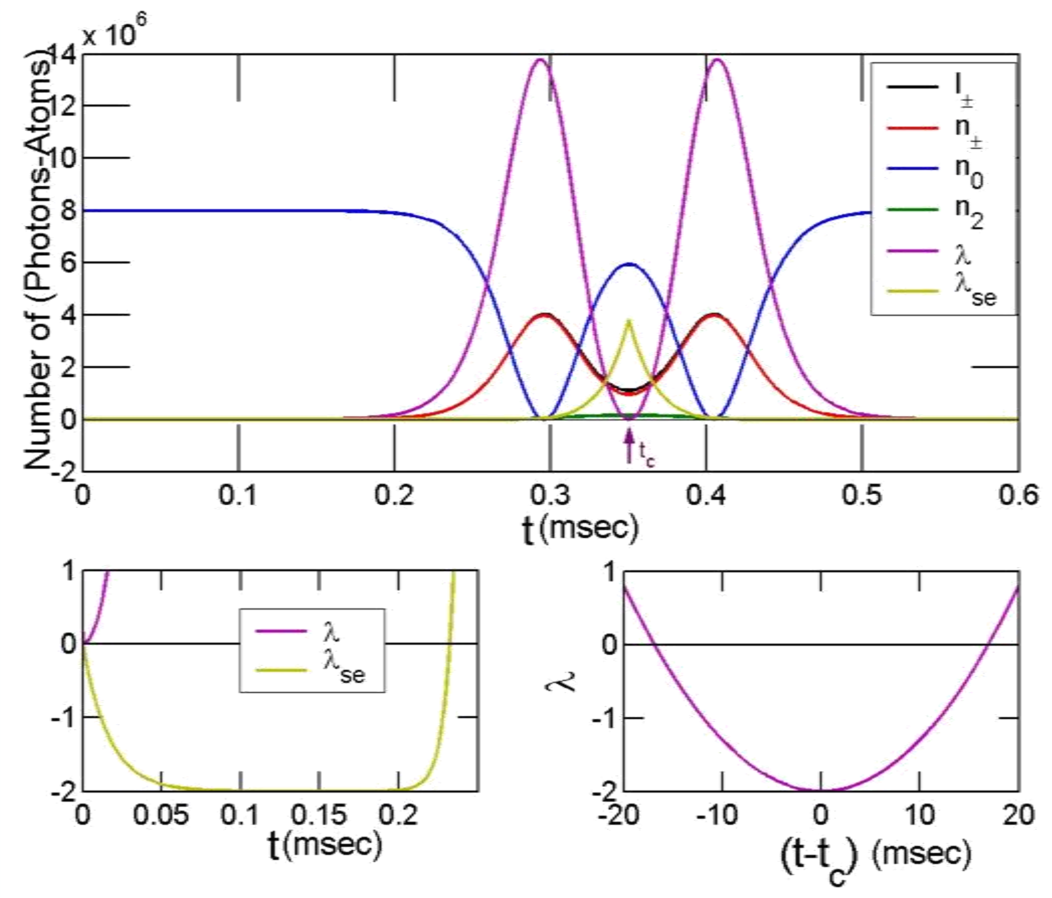
Analytical Result

$$\lambda(t) = 2[2\sinh^2(\chi_1 t_0) - \cos(\theta_1+\theta_2)\sinh(2\chi_1 t_0)\sin(2\chi_2(t-t_0))]$$

- When $\cos(\theta_1+\theta_2) \cong 1$ ➡ $\lambda < 0$
- λ starts at ~N~10⁶ and approaches **-2 !!**

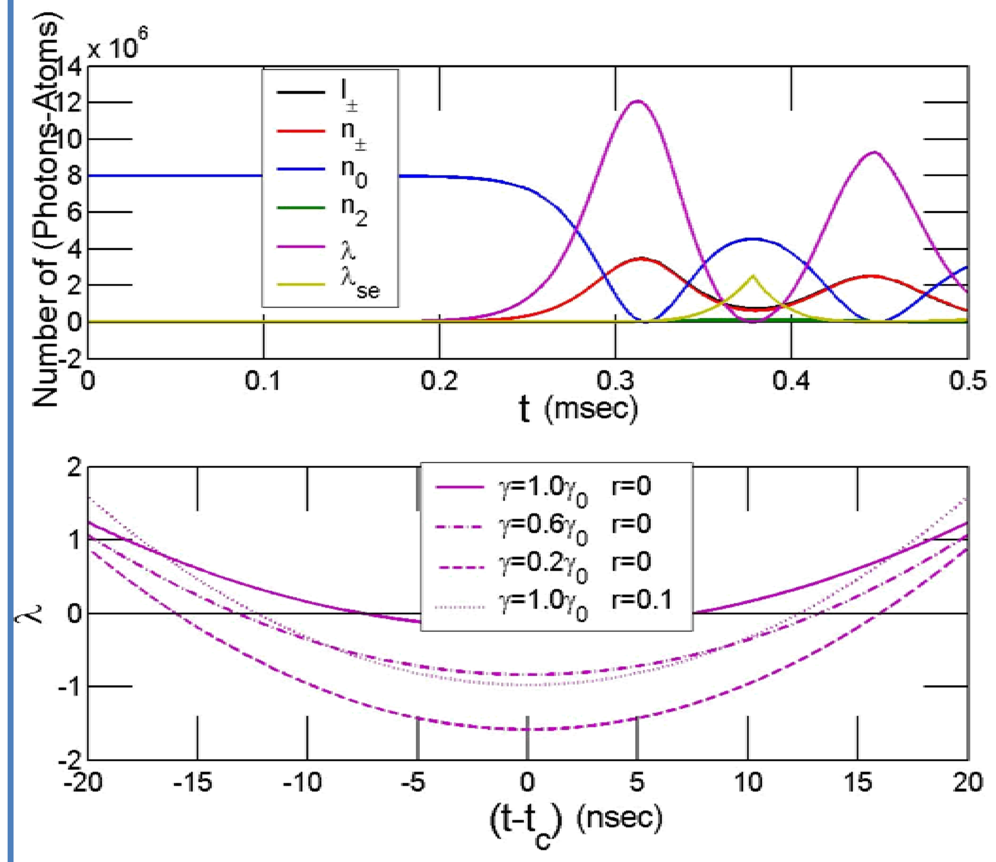
6. Numerical Results

A: No Damping



- The entanglement between (side mode)⇔(end-fire) : λ_{se} is swapped to the entanglement between (end-fire) ⇔(end-fire) : λ
- <u>λ approaches minimum possible value</u>, the Heisenberg limit, to -2.
- As expected; entanglement establishes after 2nd order SR occurs. (when n_2 is occupied)

B: with Experimental Parameters



γ : the decoherence rate.
r : squeezing parameter of vacuum(initial).

7. Conclusion

- EPR type correlation is derived for counter-propagating ($\pm k_e$) end-fire modes of a superradiant BEC.
- Entanglement is due to the swap mechanism in between sequences of superradiance.
- Entanglement can be enhanced by preparing the end-fire modes in the squeezed vacuum initial state.