Quantum entanglement
and
light propagation
through Bose-Einstein condensate (BEC)

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Outline

- Superradiance and BEC Superradiance
- Motivation: Entanglement of scattered pulses.
- Our Model Hamiltonian
- Entanglement parameter
- Swap Mechanism
- Simulations
- Conclusions

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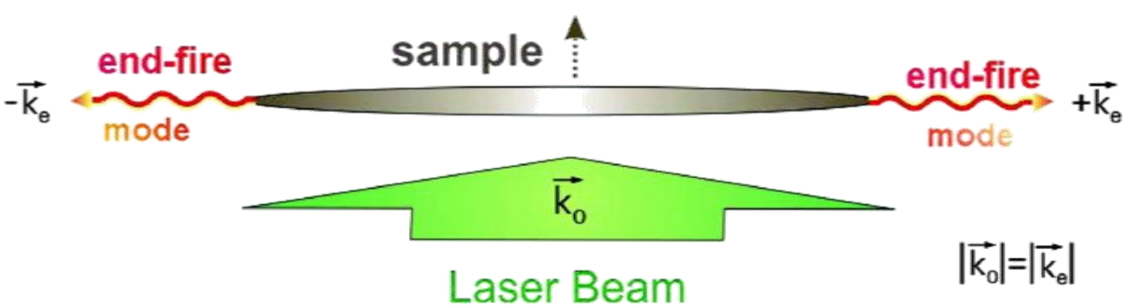
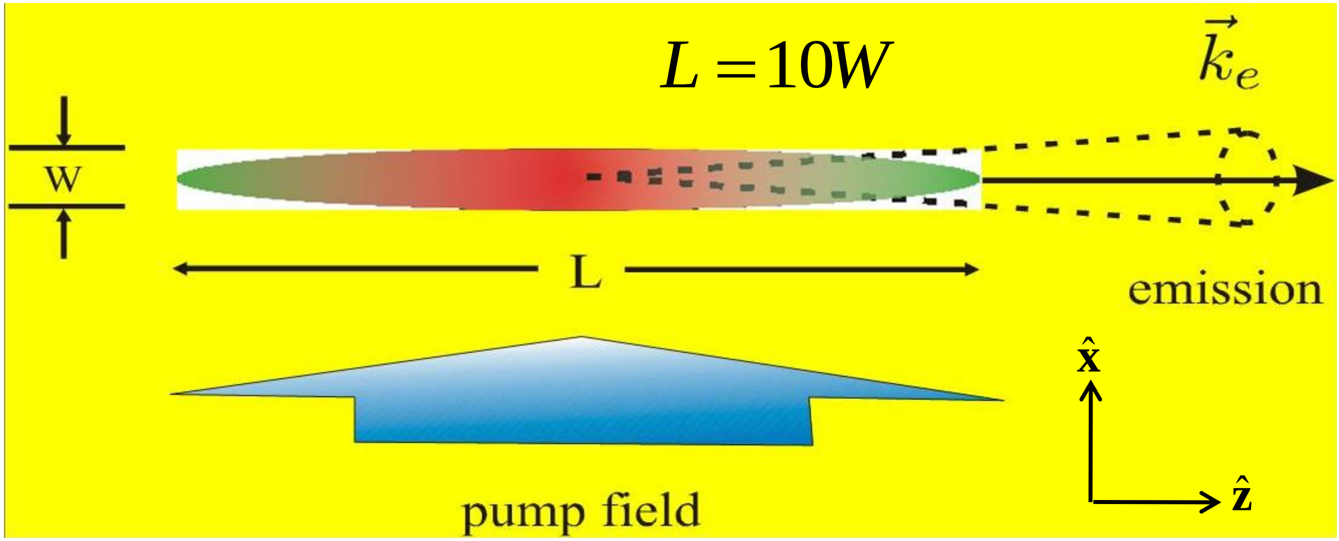
Superradiance (SR) 1

- SR: Collective spontaneous emission → $I_{Nor} \sim N$; $I_{SR} \sim N^2$

- Must $\longrightarrow$ excite very quickly $\longrightarrow$ strong pump

- Scattered radiation → Intense ; Coherent ; Directional





end-fire mode

- Elongated sample ⇨ SR is directional.

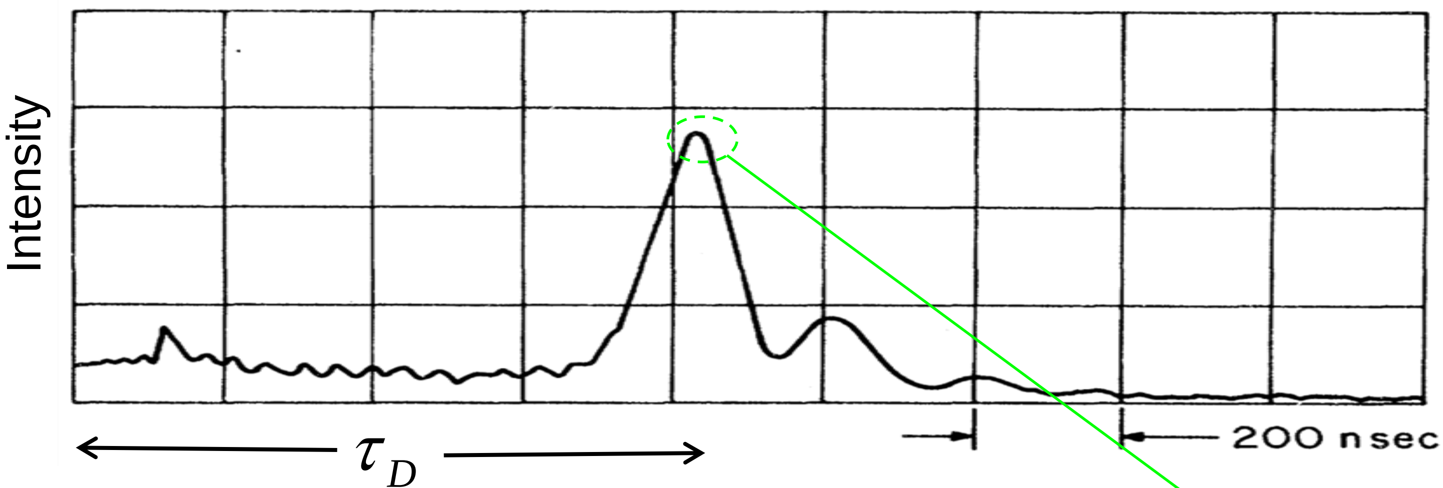
- Modes along the long-direction (**z**) is occupied by more atoms.

$$\frac{I_{\mathbf{x}}}{I_{\mathbf{z}}} = \left(\frac{N_{\mathbf{x}}}{N_{\mathbf{z}}}\right)^2 \sim \left(\frac{W}{L}\right)^2 = \frac{1}{100}$$

$N_{\mathbf{x},\mathbf{z}}$: # of atoms on $\hat{\mathbf{x}}, \hat{\mathbf{z}}$ line.

$$L = 10W$$

Superradiance (SR) (Pulse Shape) 3



Delay time

Establishment of atomic coherence.

Decay time $\sim \dfrac{T_1}{N}$

at peak

First experiment:

[N. Skribanowitz *et al.* , PRL **30**, 309 (1973).]

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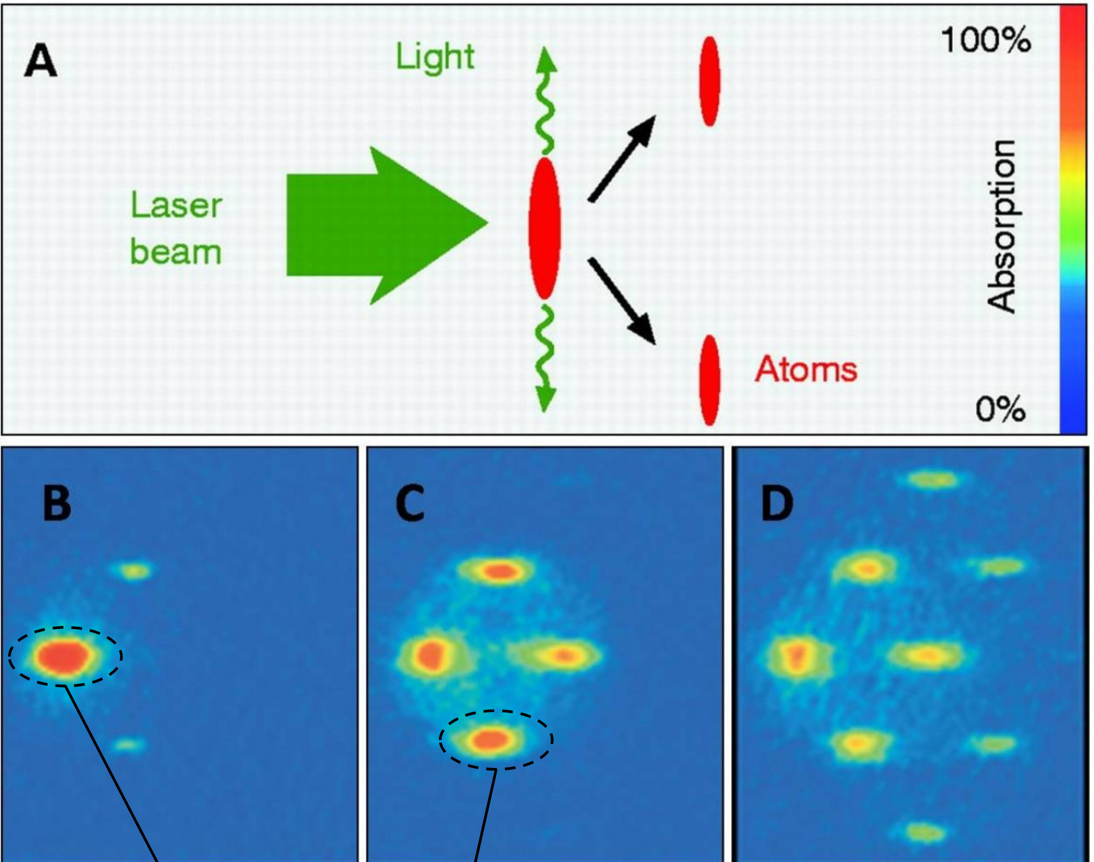
BEC Superradiance (SR) (experiment*) 1

Absorption Images: (in **p**-space)



*[S. Inouye *et al.*, Science **285**, 571 (1999).]

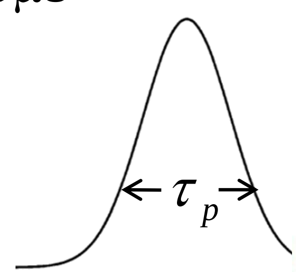
•fan-shaped pattern

Different pulse times:

B) $\tau_p =$ 35μs

C) $\tau_p =$ 75μs

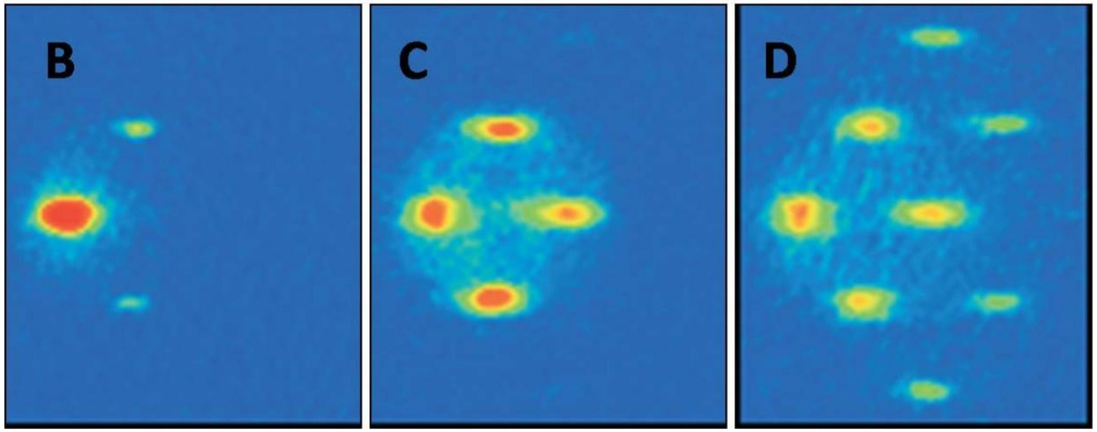
D) $\tau_p =$ 100μs



BEC **p**=0

Many-atoms in the same **p**-state

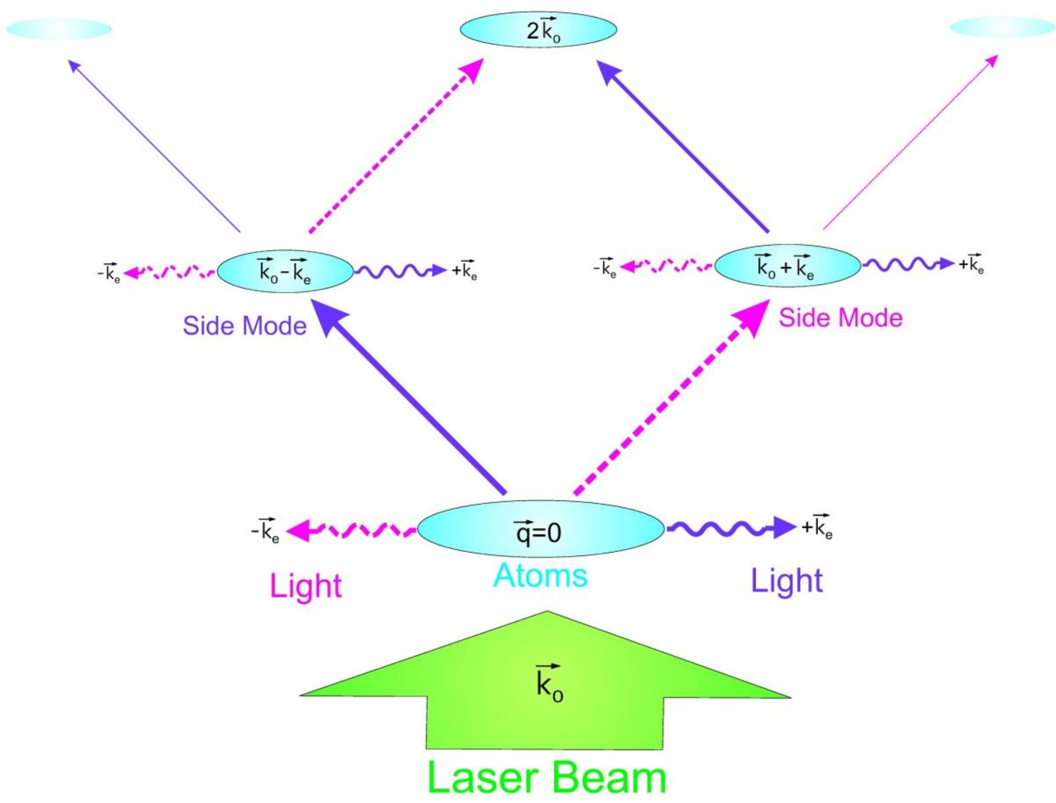
BEC Superradiance (SR)

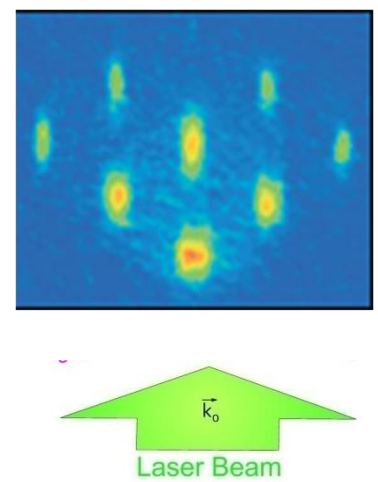


•SR emission:
- collective
- coherent
- directional (end-fire mode)

•Atom scattering:
- collective
- coherent
- same-momentum (side-mode)

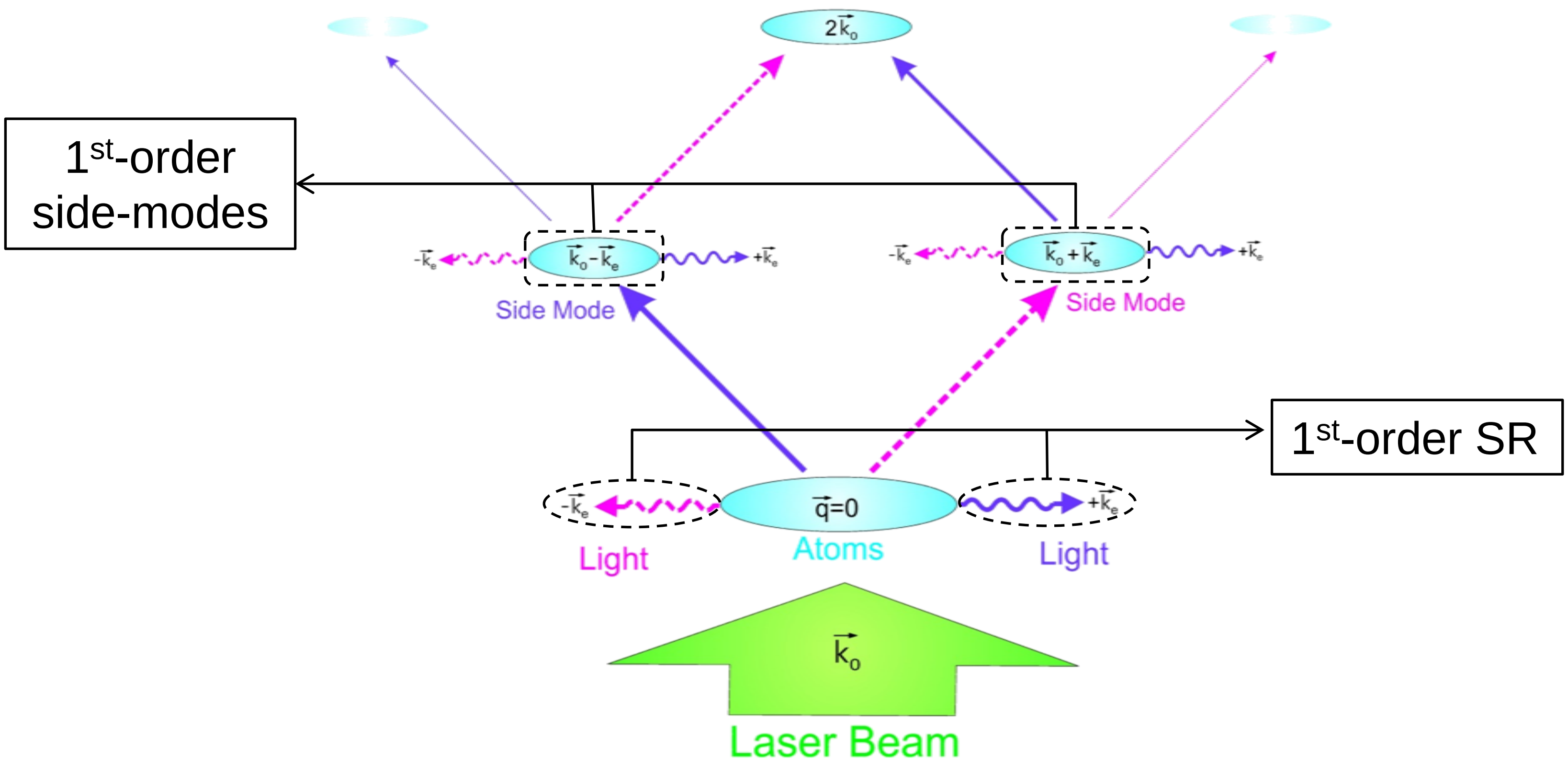
BEC Superradiance (sequential SR) 3





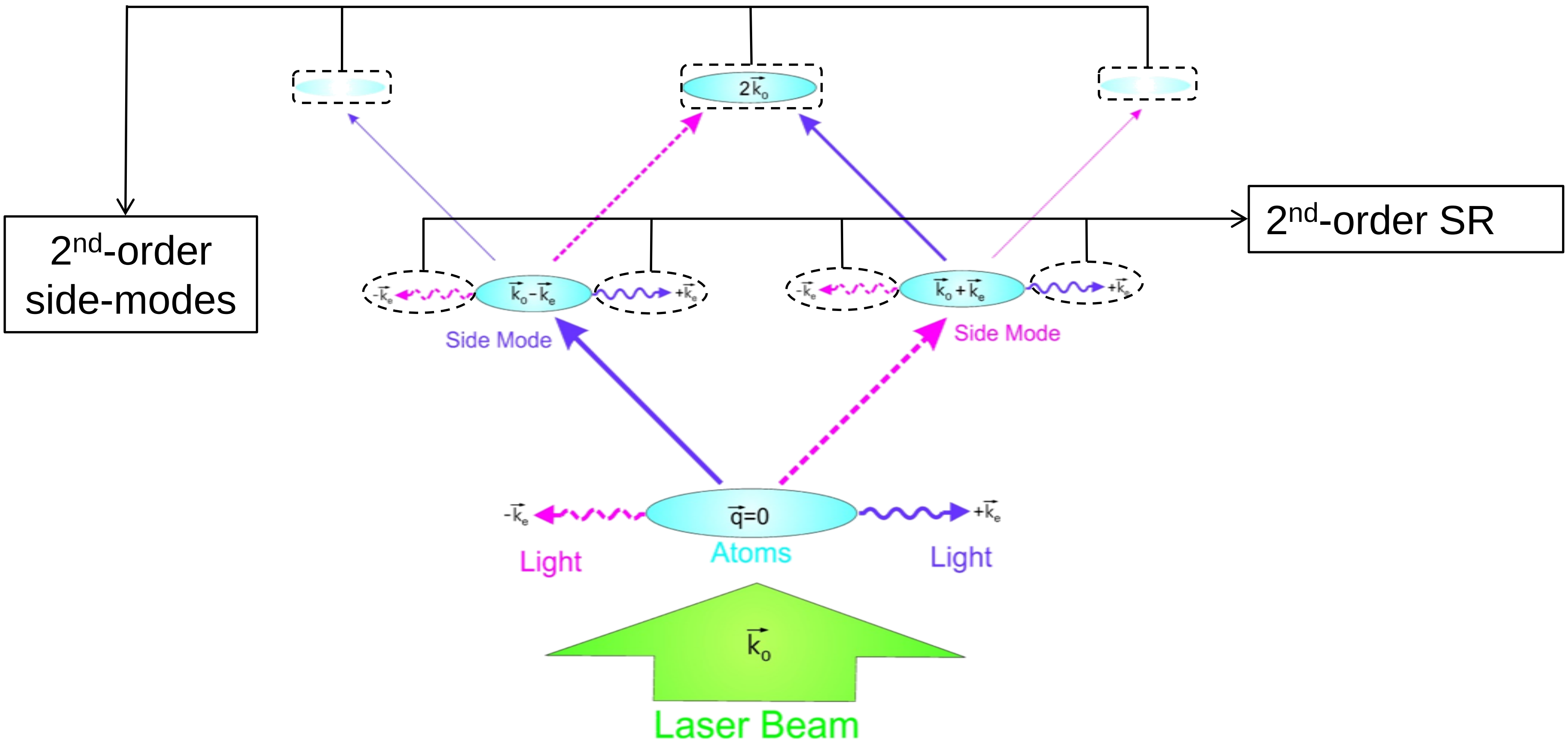
•End-fire mode ($+\vec{k}_e$) ⟺ Atomic side-mode ($\vec{k}_0-\vec{k}_e$)

•End-fire mode ($-\vec{k}_e$) ⟺ Atomic side-mode ($\vec{k}_0+\vec{k}_e$)



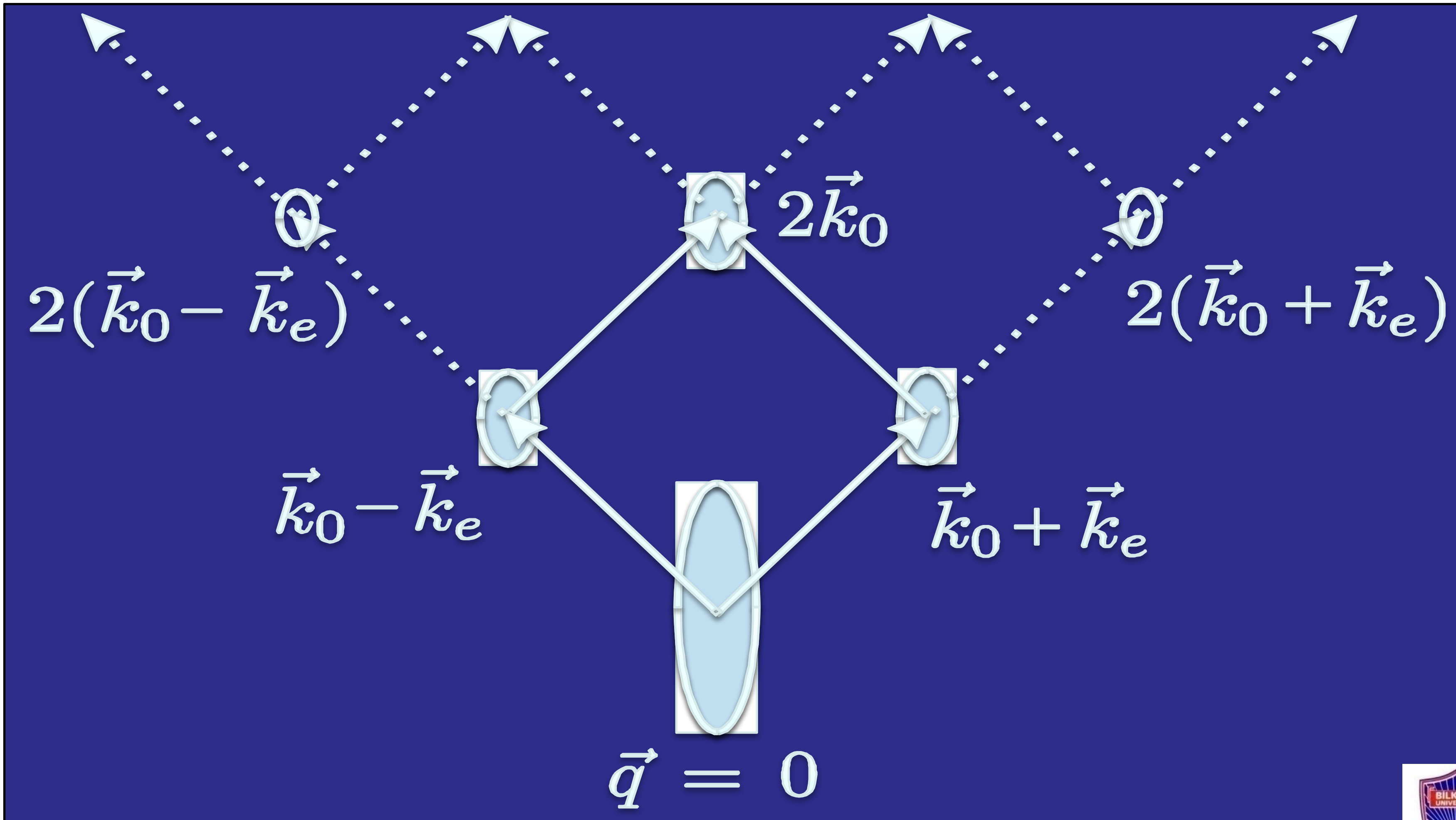
- End-fire mode ($+\vec{k}_e$) ⟺ Atomic side-mode ($\vec{k}_0 - \vec{k}_e$)
- End-fire mode ($-\vec{k}_e$) ⟺ Atomic side-mode ($\vec{k}_0 + \vec{k}_e$)

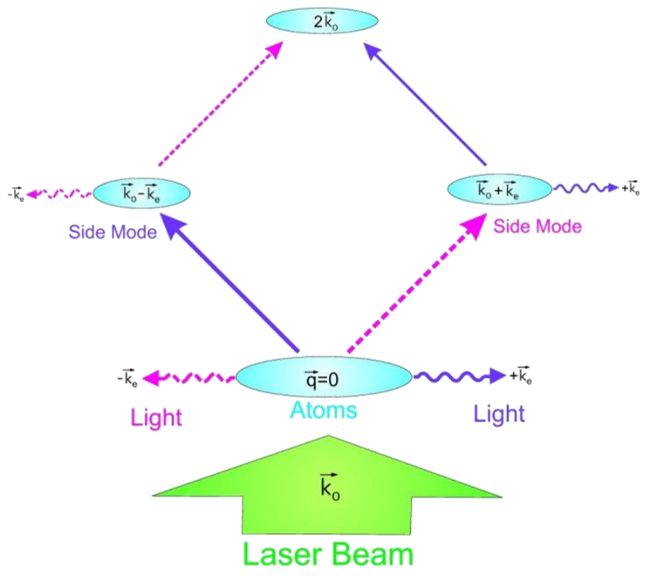
BEC Superradiance (sequential SR)

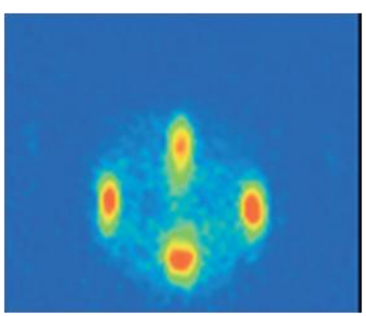


1st-order side-modes highly occupied → 1st-order side-modes superradiates → forms 2nd-order side-modes

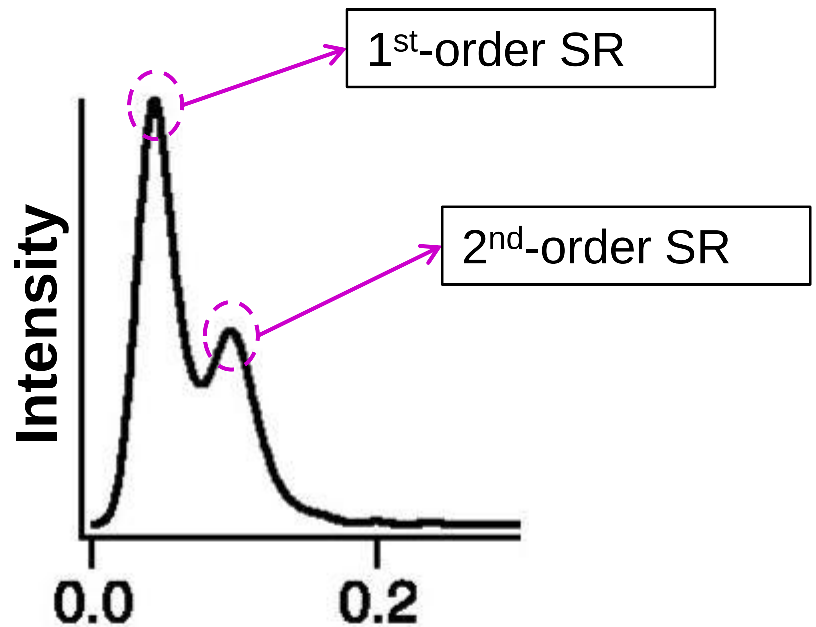
Lattice of side-modes **p**-space







$\tau_p = $ 75μs



- normal SR: Single peak
- sequential SR: Two peaks

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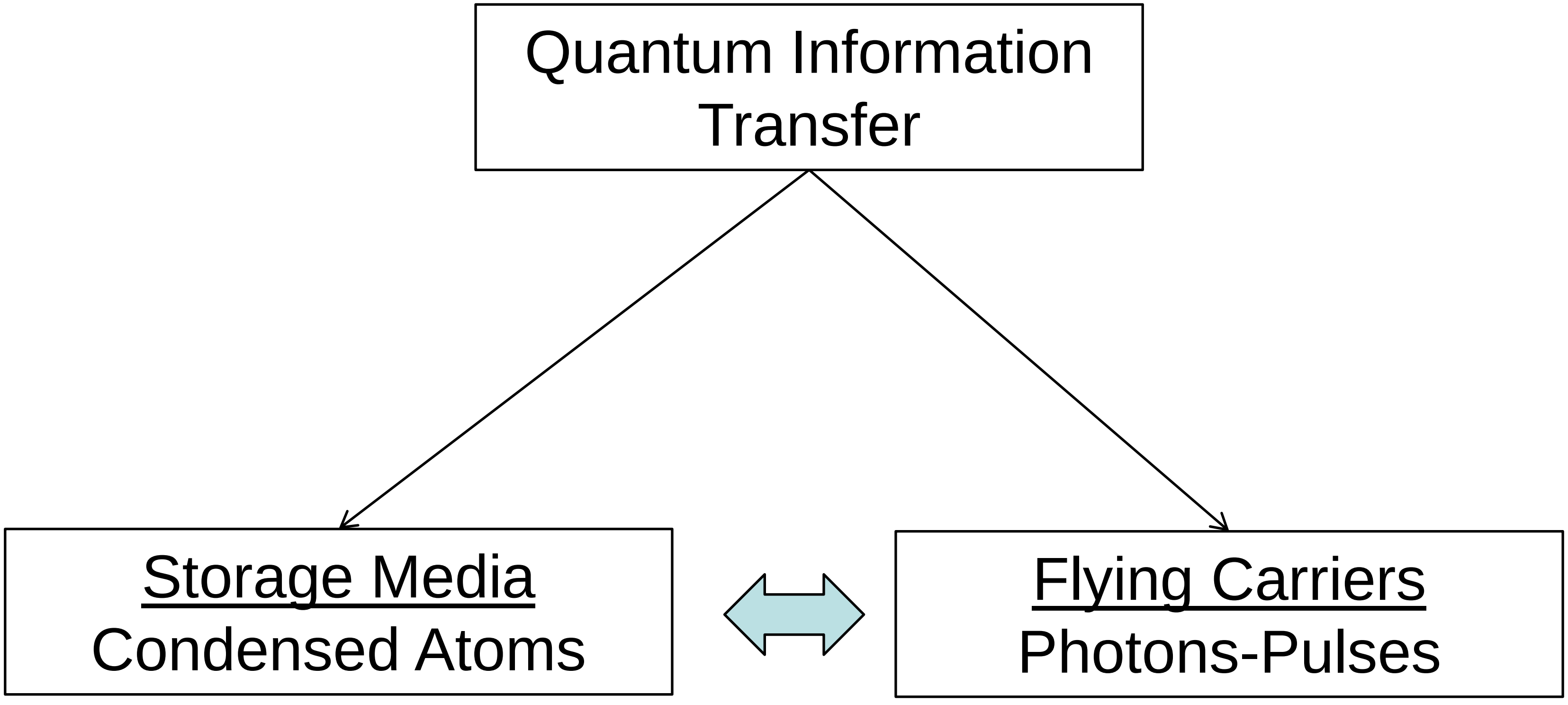
Motivation-Purpose 1



Motivation-Purpose

BEC

Normal Scattering
(linear regime)

Superradiant Scattering
(nonlinear regime)

Entanglement of
(single atom)-(single photon)

Entanglement of
(atomic wave)-(end-fire pulse)

many atoms | many photons

Discrete-variable entanglement
i.e. (atom spin)-(photon polarization)

Continuous-variable entanglement
i.e. Electric-fields of two pulses

[M.G. Moore and P. Meystre, PRL **85**, 5026 (2000).]

[M.E. Taşgın, M.Ö. Oktel, L. You, and Ö.E. Müstecaplıoğlu, PRA **79**, 0536 (2009).]

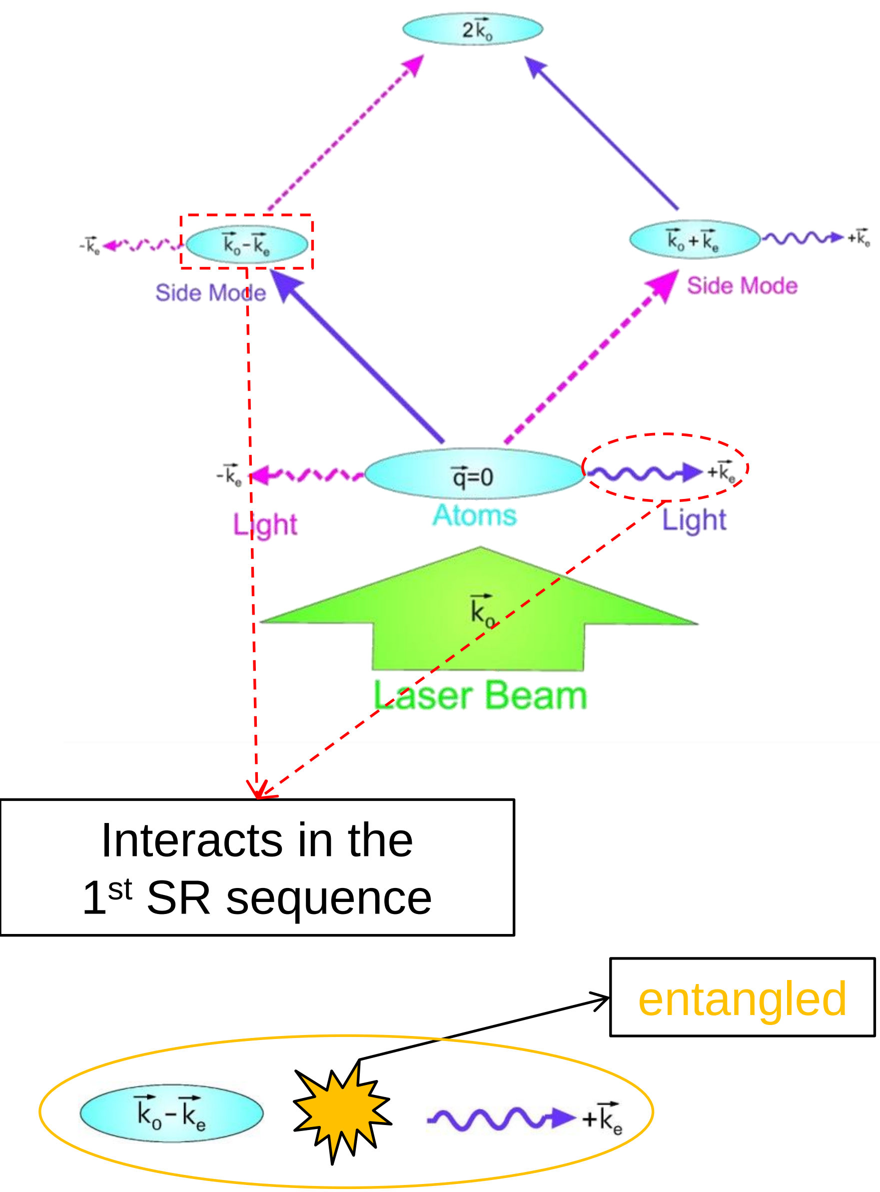
Motivation-Purpose

Interested in the

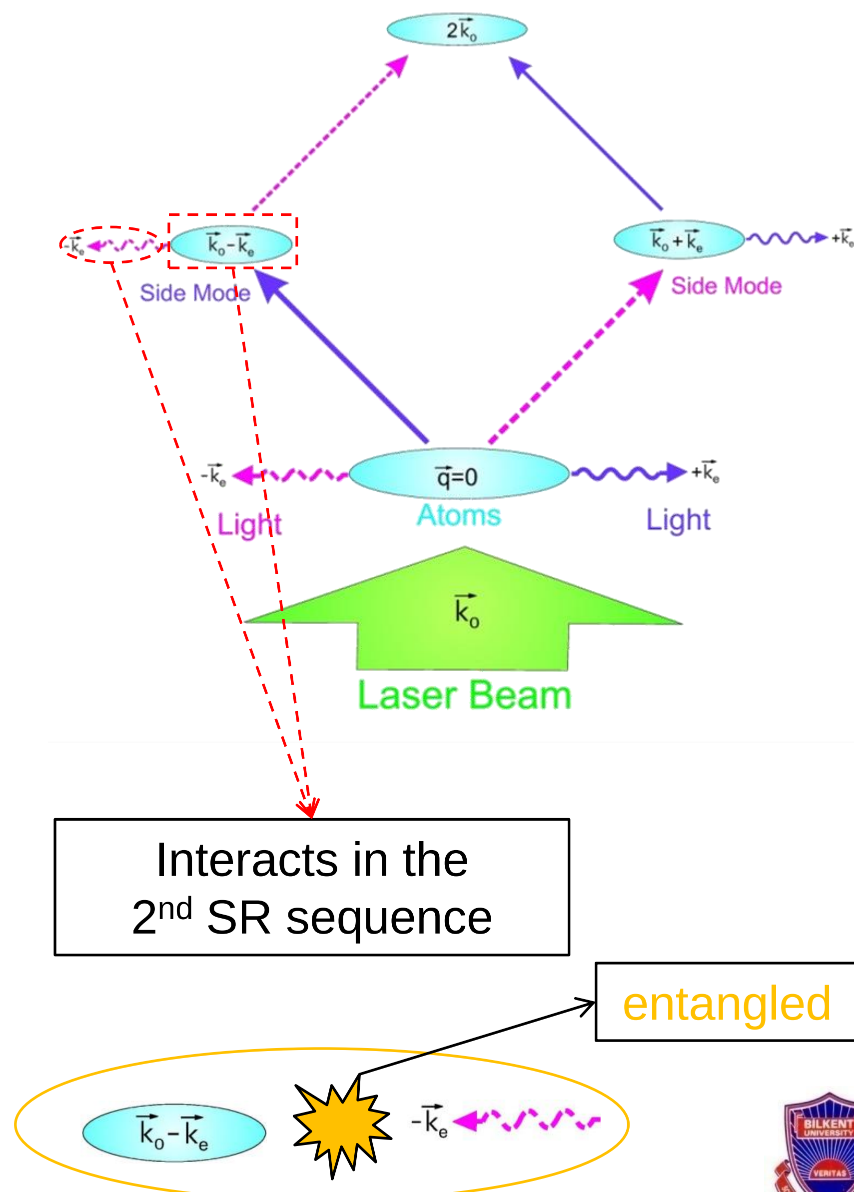
Continuous-Variable ($\vec{E}$-fields) Entanglement

of

cross-propagating end-fire pulses.

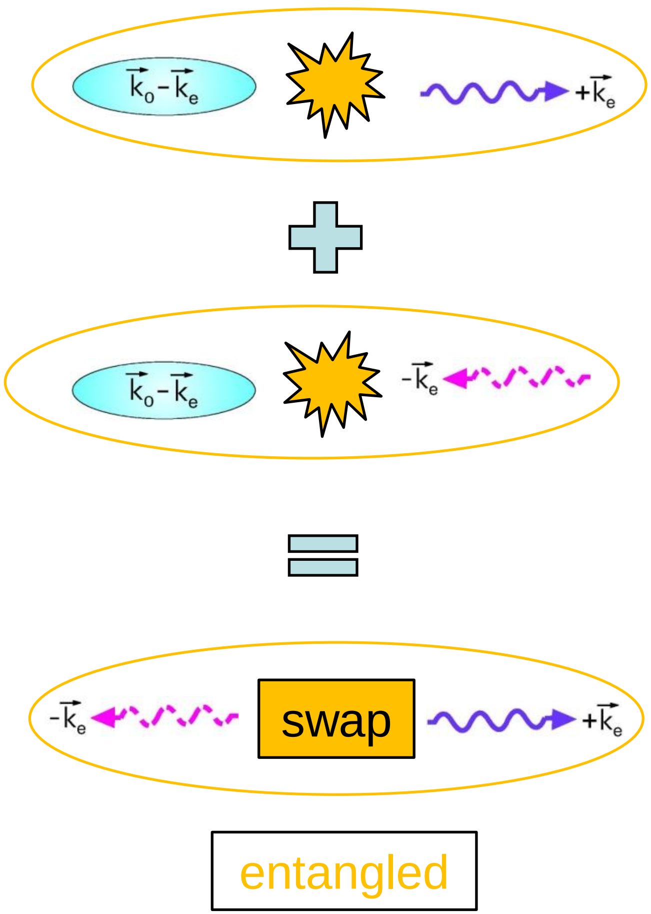


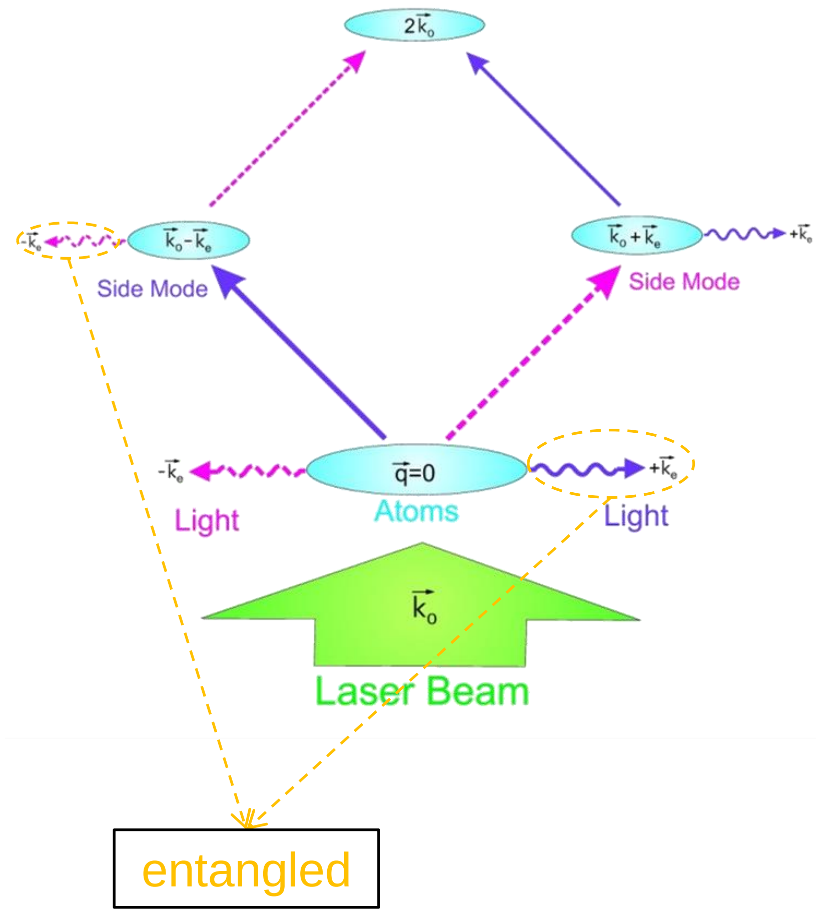
$2\vec{k}_o$
$\vec{k}_o-\vec{k}_e$
$-\vec{k}_e$
Side Mode
$\vec{k}_o+\vec{k}_e$
$+\vec{k}_e$
Side Mode
$\vec{q}=0$
Atoms
$-\vec{k}_e$
Light
$+\vec{k}_e$
Light
$\vec{k}_o$
Laser Beam
Interacts in the
1st SR sequence
entangled
$\vec{k}_o-\vec{k}_e$
$+\vec{k}_e$



$2\vec{k}_o$
$\vec{k}_o-\vec{k}_e$
$-\vec{k}_e$
Side Mode
$\vec{k}_o+\vec{k}_e$
$+\vec{k}_e$
Side Mode
$\vec{q}=0$
Atoms
$-\vec{k}_e$
Light
$+\vec{k}_e$
Light
$\vec{k}_o$
Laser Beam
Interacts in the
2nd SR sequence
entangled
$\vec{k}_o-\vec{k}_e$
$-\vec{k}_e$

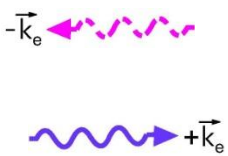
Motivation (entanglement-swap)

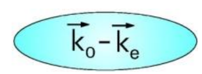




Motivation (entanglement-swap)

Entanglement swap: Entangle systems that never before interacted.

Both 

interact with 

at different times.

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Hamiltonian

Full second-quantized Hamiltonian of Laser-BEC:

$$\widehat{H} = \int d^3\boldsymbol{k}\,\hbar\omega(\boldsymbol{k})\hat{a}(\boldsymbol{k})^\dagger\hat{a}(\boldsymbol{k}) + \sum_{\boldsymbol{q}} \hbar\omega_{\boldsymbol{q}}\hat{c}_{\boldsymbol{q}}{}^\dagger\hat{c}_{\boldsymbol{q}}$$

$$-\frac{g(\boldsymbol{k}_0)}{\Delta}\sum_{\boldsymbol{q},\boldsymbol{q}'}\int d^3\boldsymbol{k}\rho_{\boldsymbol{q},\boldsymbol{q}'}(\boldsymbol{k})\hbar g^*(\boldsymbol{k})\hat{c}_{\boldsymbol{q}}{}^\dagger\hat{a}_{\boldsymbol{k}}{}^\dagger\hat{a}_{\boldsymbol{k}_0}\hat{c}_{\boldsymbol{q}'}$$

$\hat{a}^\dagger_{\mathbf{k}}$: creates photon of momentum $\vec{\mathbf{k}}$, energy $\hbar\omega_{\mathbf{k}} = ck.$

$\hat{c}^\dagger_{\mathbf{k}}$: creates atom(boson) in side-mode $\vec{\mathbf{q}}$, energy $\hbar\omega_{\mathbf{q}} = \dfrac{\hbar^2 q^2}{2M}.$

$g(\mathbf{k}) = \left(ckd^2 / 2\hbar\varepsilon_0\right)^{1/2}$: dipole coupling

Δ : laser detuning

$\rho_{\mathbf{q},\mathbf{q}'}(\mathbf{k},\mathbf{k}') = \int d\mathbf{r}\left|\phi_0(\mathbf{r})\right|^2 e^{i[(\mathbf{k}+\mathbf{q})-(\mathbf{k}'+\mathbf{q}')]\bullet\mathbf{r}}$: structure factor of BEC.

Hamiltonian 2

1) Move rotating frame.

2) Assume

- laser pulse
- end-fire pulse
- scattered atoms (side-modes)

$\rightarrow$ single mode.

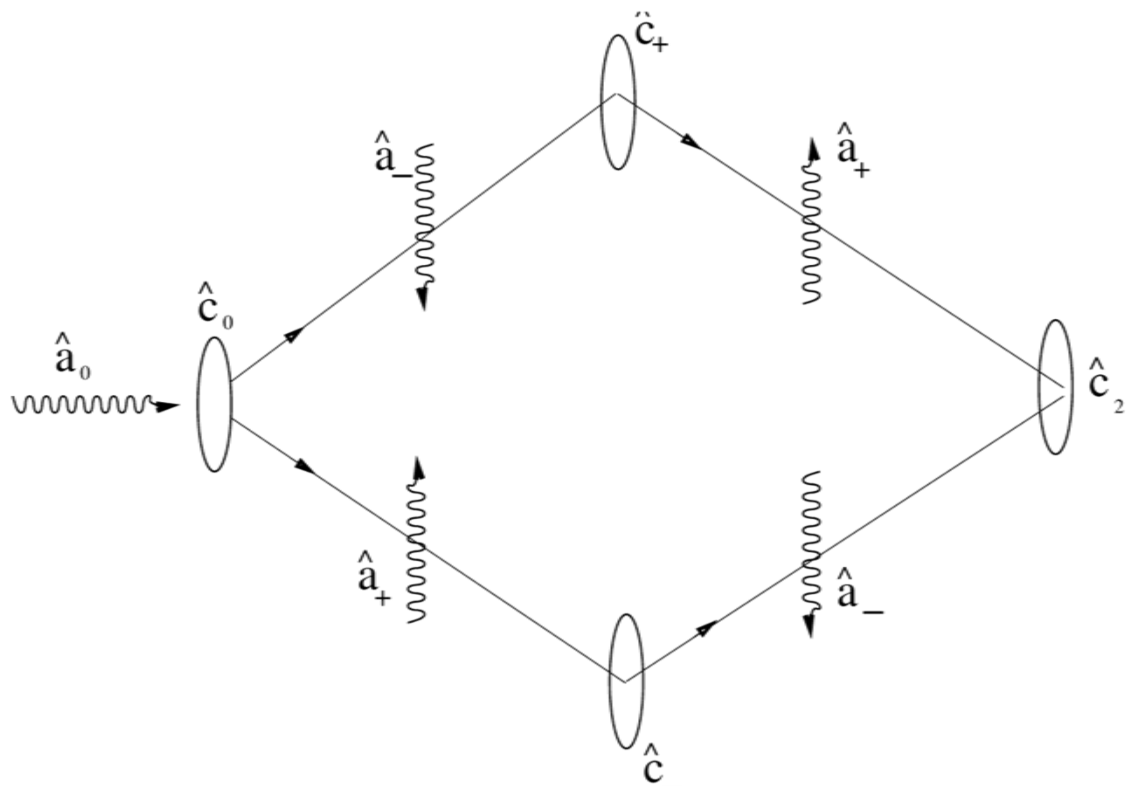
effective Hamiltonian:

$$\hat{H} = -\hbar\frac{g^2}{\Delta}\left(\hat{c}_+^\dagger\hat{a}_-^\dagger\hat{a}_0\hat{c}_0 + \hat{c}_-^\dagger\hat{a}_+^\dagger\hat{a}_0\hat{c}_0 + \hat{c}_2^\dagger\hat{a}_-^\dagger\hat{a}_0\hat{c}_- + \hat{c}_2^\dagger\hat{a}_+^\dagger\hat{a}_0\hat{c}_+\right) + H.c.$$

$$\hat{a}_\pm \equiv \hat{a}_{\pm\boldsymbol{k}_e}\ ,\quad \hat{a}_0 \equiv \hat{a}_{\pm\boldsymbol{k}_0}\ ,\quad \hat{c}_\pm \equiv \hat{c}_{(\boldsymbol{k}_0\pm\boldsymbol{k}_e)}\ ,\quad \hat{c}_2 \equiv \hat{c}_{2\boldsymbol{k}_0}$$

Schematic acts of operators:

$$\widehat{H} = -\hbar \frac{g^2}{\Delta} \left(\hat{c}_+^\dagger \hat{a}_-^\dagger \hat{a}_0 \hat{c}_0 + \hat{c}_-^\dagger \hat{a}_+^\dagger \hat{a}_0 \hat{c}_0 + \hat{c}_2^\dagger \hat{a}_-^\dagger \hat{a}_0 \hat{c}_- + \hat{c}_2^\dagger \hat{a}_+^\dagger \hat{a}_0 \hat{c}_+ \right) \; + H.c.$$



$$\hat{a}_\pm \equiv \hat{a}_{\pm k_e} \;\; , \;\; \hat{a}_0 \equiv \hat{a}_{\pm k_0} \;\; , \;\; \hat{c}_\pm \equiv \hat{c}_{(k_0 \pm k_e)} \;\; , \;\; \hat{c}_2 \equiv \hat{c}_{2k_0}$$

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Entanglement parameter

Separability and Entanglement

If density-matrix is inseparable

⇨ it cannot written as $$\rho = \sum_r p_r \rho_r^1 \otimes \rho_r^2$$

⇨ subsystems *1,2* are entangled.

Aim : Define a parameter to test entanglement.

Entanglement parameter 2

<u>Separability and Entanglement</u>

[L.M. Duan *et al.*, PRL **84**, 2722 (2000).] showed:

$$\langle \Delta\hat{u}^2 \rangle + \langle \Delta\hat{v}^2 \rangle \geq \left(c^2 + \frac{1}{c^2} \right)$$ ⇒ density-matrix separable ⇒ subsystems not entangled

$$\underbrace{\left| c^2 - \frac{1}{c^2} \right|}_{\text{uncertainty limit}} \leq \langle \Delta\hat{u}^2 \rangle + \langle \Delta\hat{v}^2 \rangle \leq \underbrace{\left(c^2 + \frac{1}{c^2} \right)}_{\text{separability limit}}$$ ⇒ density-matrix inseparable ⇒ subsystems <u>entangled</u>

$$\hat{u} = |c|\hat{x}_1 + \hat{x}_2 / c$$
$$\hat{v} = |c|\hat{p}_1 - \hat{p}_2 / c$$

are EPR operators with

$$\hat{x}_{1,2} = \left(\hat{a}_{\pm} + \hat{a}_{\pm}^{\dagger}\right) / \sqrt{2}$$
$$\hat{p}_{1,2} = \left(\hat{a}_{\pm} - \hat{a}_{\pm}^{\dagger}\right) / i\sqrt{2}$$

Separability and Entanglement

[L.M. Duan *et al.*, PRL **84**, 2722 (2000).] showed:

$$\left\langle \Delta\hat{u}^2 \right\rangle + \left\langle \Delta\hat{v}^2 \right\rangle \geq \left(c^2 + \frac{1}{c^2} \right) \Rightarrow \text{density-matrix separable} \Rightarrow \text{subsystems not entangled}$$

$$\underbrace{\left| c^2 - \frac{1}{c^2} \right|}_{\text{uncertainty limit}} \leq \left\langle \Delta\hat{u}^2 \right\rangle + \left\langle \Delta\hat{v}^2 \right\rangle \leq \underbrace{\left(c^2 + \frac{1}{c^2} \right)}_{\text{separability limit}} \Rightarrow \text{density-matrix inseparable} \Rightarrow \text{subsystems \underline{entangled}}$$

$$\lambda(t) = \left\langle \Delta\hat{u}^2 \right\rangle + \left\langle \Delta\hat{v}^2 \right\rangle - \left(c^2 + \frac{1}{c^2} \right)$$

$\lambda(t) < 0 \Rightarrow$ entangled

Entanglement parameter

$$\lambda(t) = \left\langle \Delta\hat{u}^2 \right\rangle + \left\langle \Delta\hat{v}^2 \right\rangle - \left(c^2 + \frac{1}{c^2} \right)$$

$\lambda(t) < 0$ ⇨ entangled

$\hat{a}_+ \leftrightarrow \hat{a}_-$ symmetry ⇨ $c^2 = 1$

lowest possible λ is: $\lambda_{\mathrm{low}} = -2$ → (uncertainty limit)

$c^2 = 1$ ⇨

$$x \equiv \vec{\mathbf{E}} - \text{field}$$

$$p \equiv \vec{\mathbf{H}} - \text{field}$$

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Swap Mechanism (analytical treatment) 1

$$\hat{H} = -\hbar\frac{g^2}{\Delta}\left(\hat{c}_+^\dagger\hat{a}_-^\dagger\hat{a}_0\hat{c}_0 + \hat{c}_-^\dagger\hat{a}_+^\dagger\hat{a}_0\hat{c}_0 + \hat{c}_2^\dagger\hat{a}_-^\dagger\hat{a}_0\hat{c}_- + \hat{c}_2^\dagger\hat{a}_+^\dagger\hat{a}_0\hat{c}_+\right) + H.c.$$

but not exactly solvable.

Seems innocent,

Even numerical simulation is hard.
(Keep lots of analytical expressions by hand.)

- First, investigate H approximately. (general behavior)

- Illustrate swap mechanism, analytically.

$$\hat{H} = -\hbar\frac{g^2}{\Delta}\left(\hat{c}_+^\dagger\hat{a}_-^\dagger\hat{a}_0\hat{c}_0 + \hat{c}_-^\dagger\hat{a}_+^\dagger\hat{a}_0\hat{c}_0 + \hat{c}_2^\dagger\hat{a}_-^\dagger\hat{a}_0\hat{c}_- + \hat{c}_2^\dagger\hat{a}_+^\dagger\hat{a}_0\hat{c}_+\right) + H.c.$$

Approximation

Initial Times

$$\hat{a}_0 \to \sqrt{M}e^{i\theta_0} \ , \ \hat{c}_0 \to \sqrt{N}e^{i\phi_1}$$

$$\hat{H}_1 = -\hbar\chi_1\left[e^{i\theta_1}\left(\hat{a}_+^\dagger\hat{c}_-^\dagger + \hat{a}_-^\dagger\hat{c}_+^\dagger\right) + \text{H.c.}\right]$$

couples

$$|\hat{c}_-\rangle \longleftrightarrow |\hat{a}_+\rangle$$

Later Times

$$\hat{a}_0 \to \sqrt{M}e^{i\theta_0} \ , \ \hat{a}_2 \to \sqrt{N_2}e^{i\phi_2}$$

$$\hat{H}_2 = -\hbar\chi_2\left[e^{i\theta_2}\left(\hat{a}_-^\dagger\hat{c}_- + \hat{a}_+^\dagger\hat{c}_+\right) + \text{H.c.}\right]$$

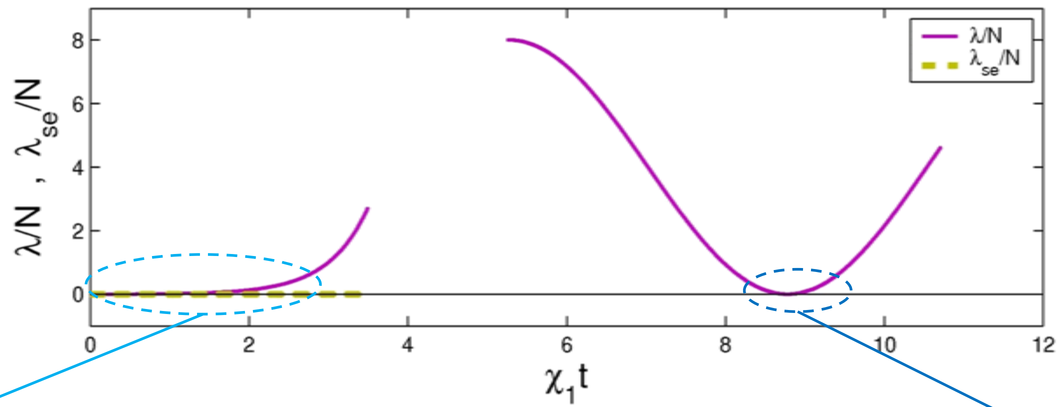
couples

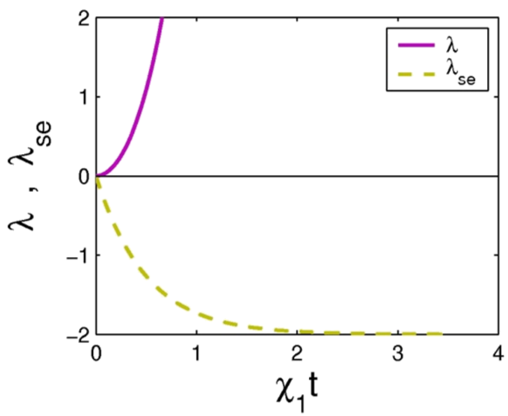
$$|\hat{a}_-\rangle \longleftrightarrow |\hat{c}_-\rangle$$

couples

$$|\hat{a}_+\rangle \longleftrightarrow |\hat{a}_-\rangle$$

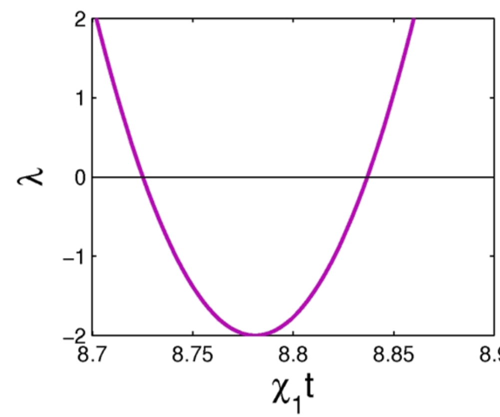
Swap Mechanism (analytical treatment)





λ_{se} (side-mode)-(end-fire)

λ (end-fire)-(end-fire)



Initial
atom-photon
entanglement

$\lambda_{se} < 0$

is
swapped
to

Later
photon-photon
entanglement

$\lambda < 0$

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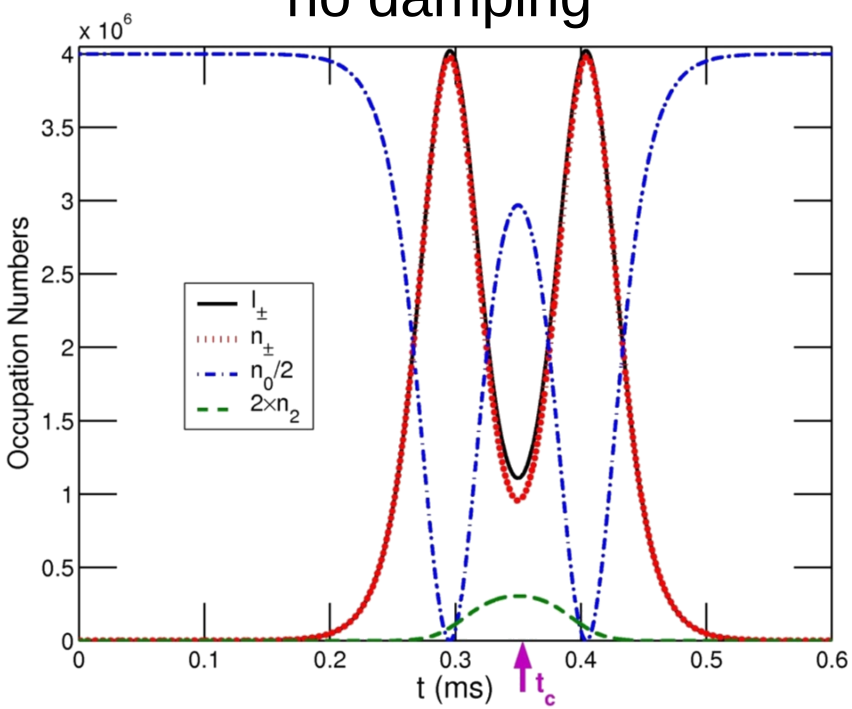
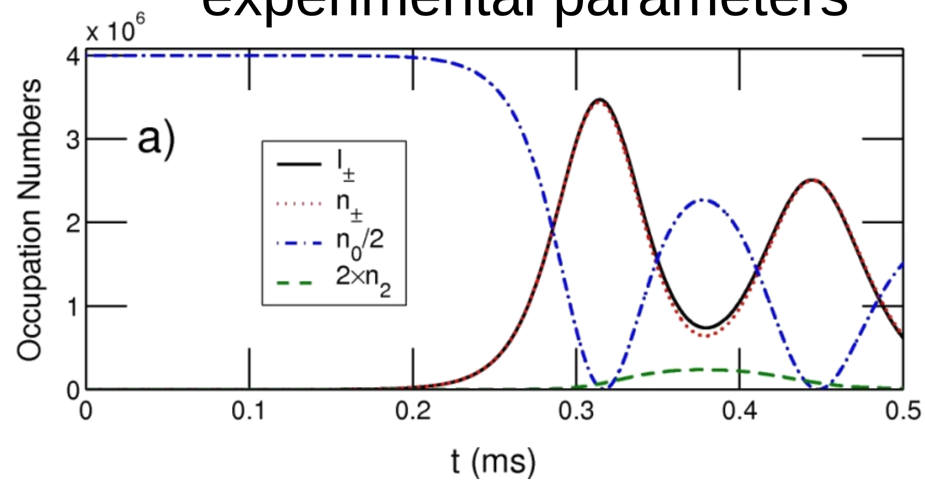
End-fire Intensity and Side-mode Occupations

no damping



experimental parameters



MIT 1999 experiment

$$N = 8\times10^6$$

$$M = 2\times10^8$$

decoherence: $\gamma_\perp = 1.3\times10^4\,\mathrm{Hz}$

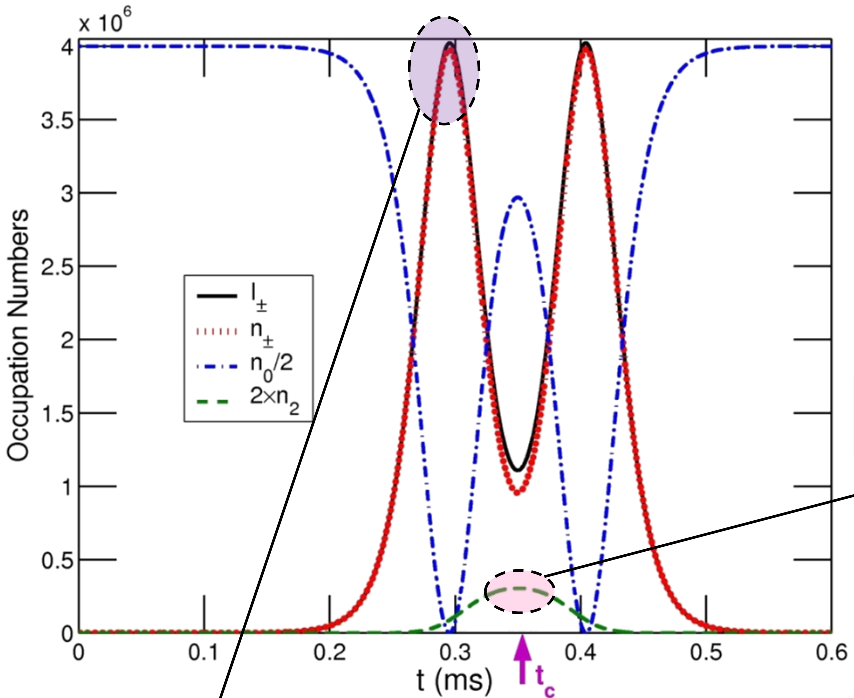
$I_\pm$: Intensity of end-fire modes

$n_0, n_\pm, n_2$: Occupation of side-modes

Simulations (intensity-occupations) 2

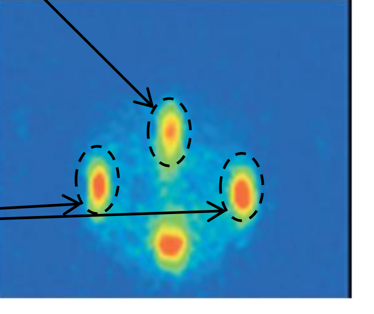
$I_{\pm}$: Intensity of end-fire modes

$n_0, n_{\pm}, n_2$: Occupation of side-modes



2nd-order SR

2nd-order side-modes occupied



1st-order SR

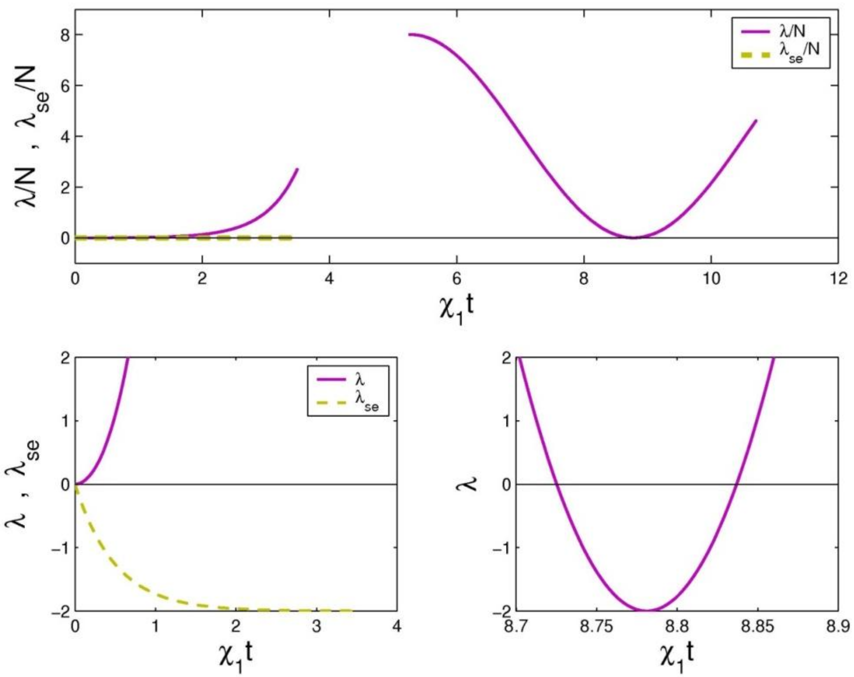
1st-order side-modes occupied

$\tau_p = 75\mu s$

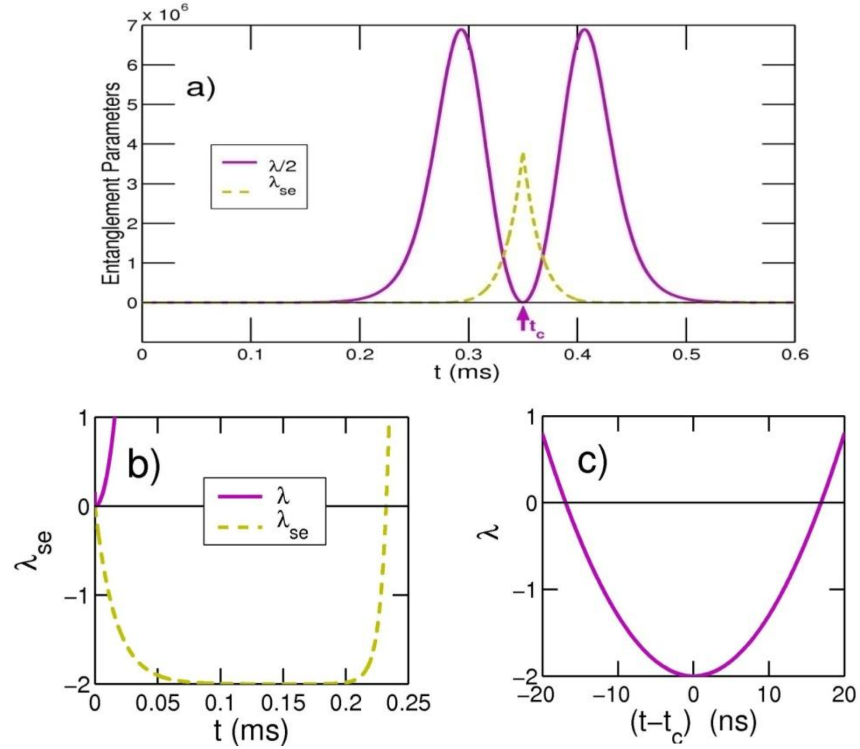
- Similar behavior when decoherence introduced.

Simulations (quantum-correlations) 3

<u>Analytical</u>

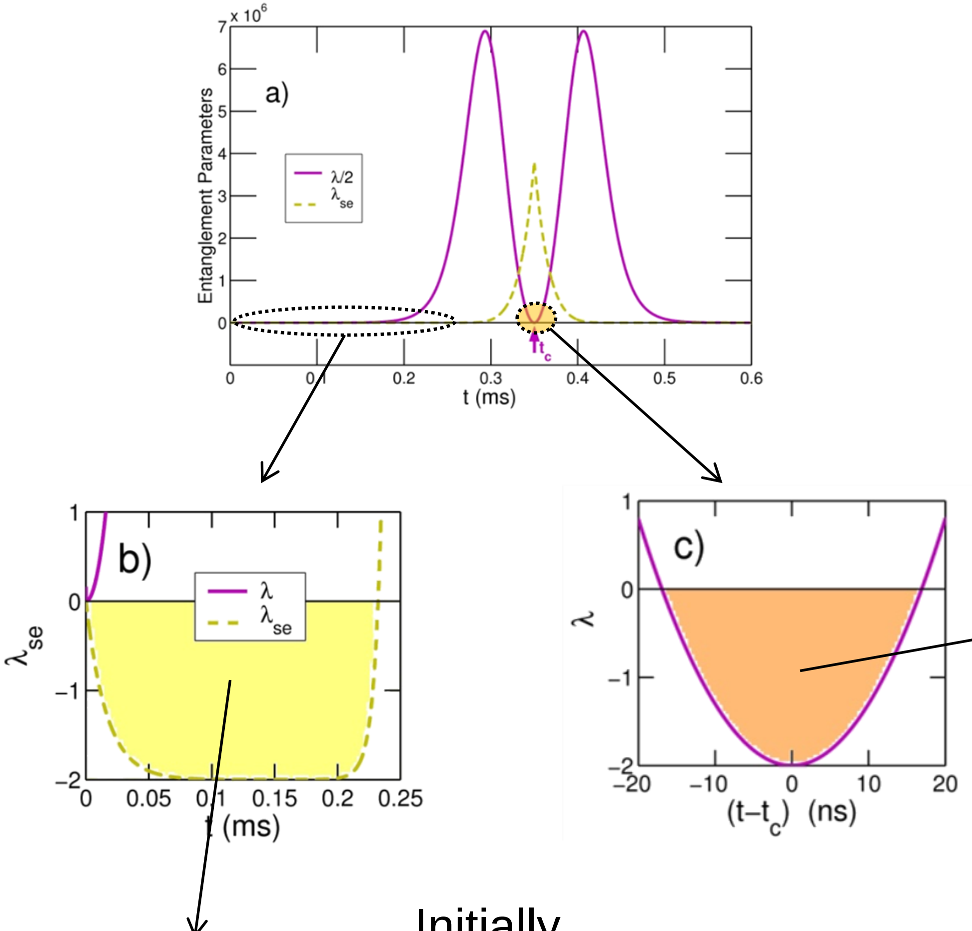


<u>Numerical</u>



- (Numerical simulations) parallel (analytical predictions).
- Simulations only fill in the blanks.

Evolution of Quantum Correlation

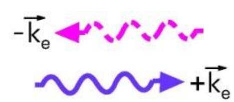


λ_{se} - - - - (side-mode)-(end-fire) entang.

λ —— (end-fire)-(end-fire) entang.

Later on

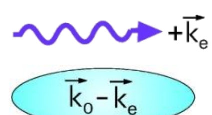
$$\lambda(t) < 0 \quad \text{for} \quad \Delta t = 30\text{ns}$$



(entangled)

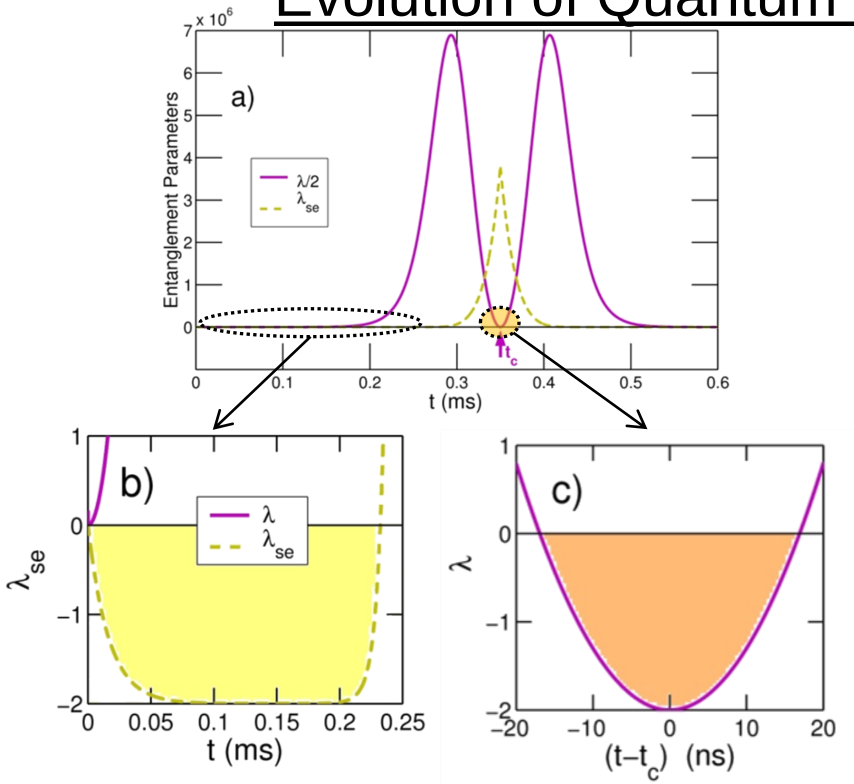
Initially

$$\lambda_{se}(t) < 0 \quad \text{for} \quad \Delta t = 23\text{ms}$$



(entangled)

Evolution of Quantum Correlation



λ_{se} - - - - - - (side-mode)-(end-fire) entang.

λ ——— (end-fire)-(end-fire) entang.

- λ takes on the lowest possible value.

Interesting scale difference

$\lambda_{max} \sim 10^{+7}$

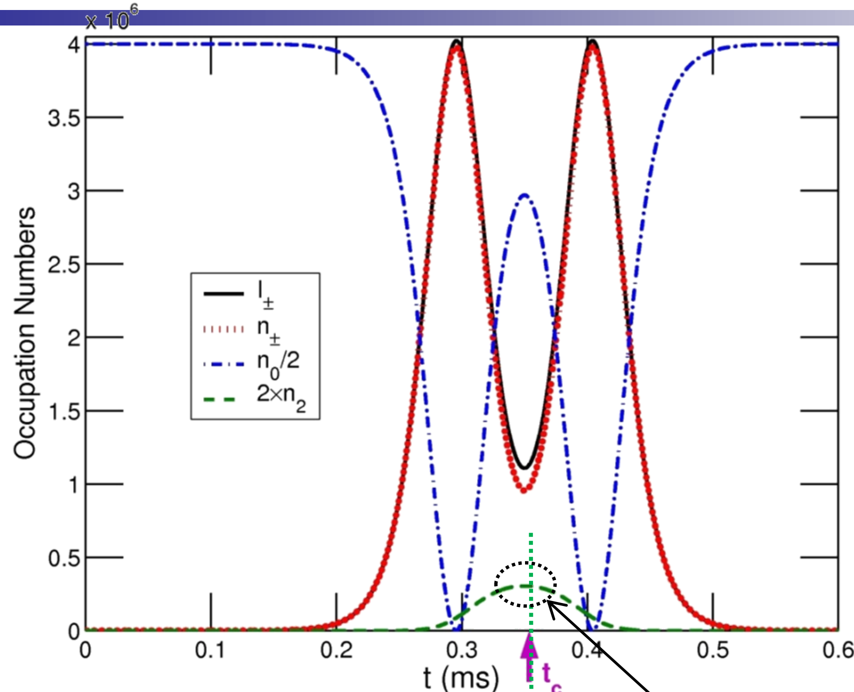
drops to

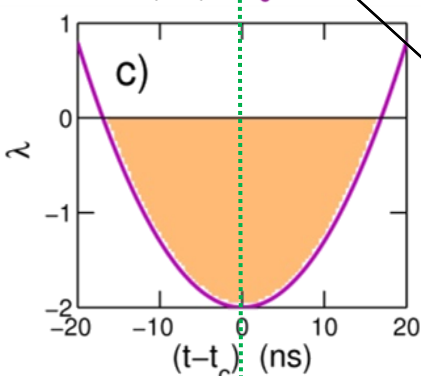
$\lambda_{min} = -2$

$\lambda_{low} = -2$

Reaches the uncertainty limit.

Simulations (quantum-correlations)





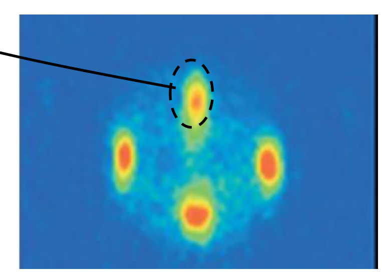
λ_{se} (side-mode)-(end-fire) entang.

λ (end-fire)-(end-fire) entang.

(end-fire)-(end-fire) entanglement $\left(\lambda(t)<0\right)$ takes place

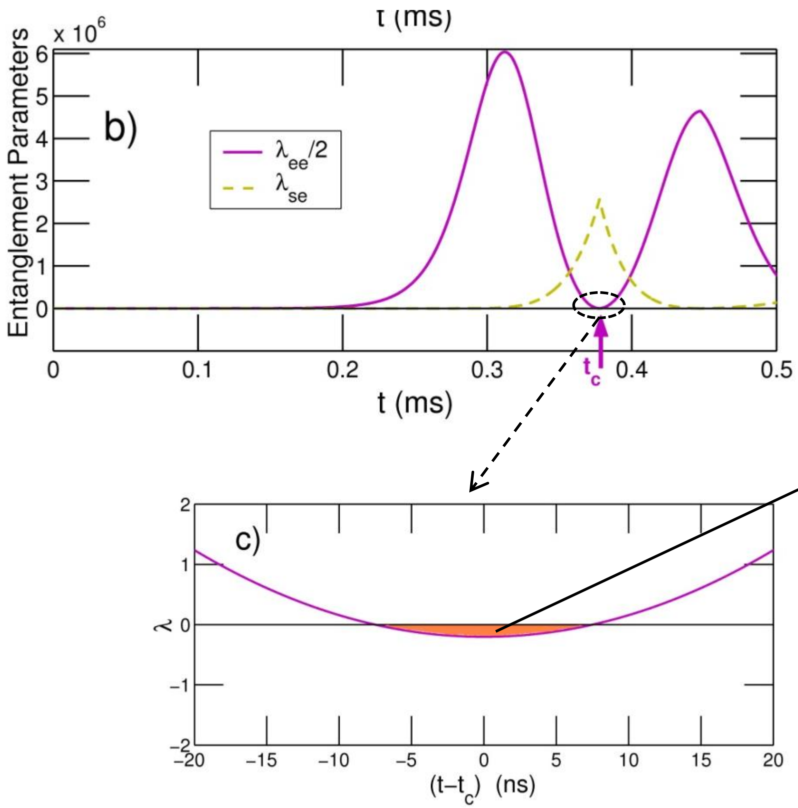
after

$\left|c_2\right\rangle$ side-mode occupied
or
2nd-order SR occurs.



$\max\left(n_2\right)$ coincides with λ_{min}

Correlations with decoherence



λ_{se} (side-mode)-(end-fire) entang.

λ (end-fire)-(end-fire) entang.

$\lambda(t) < 0$ for $\Delta t = 10\text{ns}$

$\lambda_{min} = -0.2 \neq \lambda_{low} = -2$

- Smaller entanglement time.
- Less negative λ.

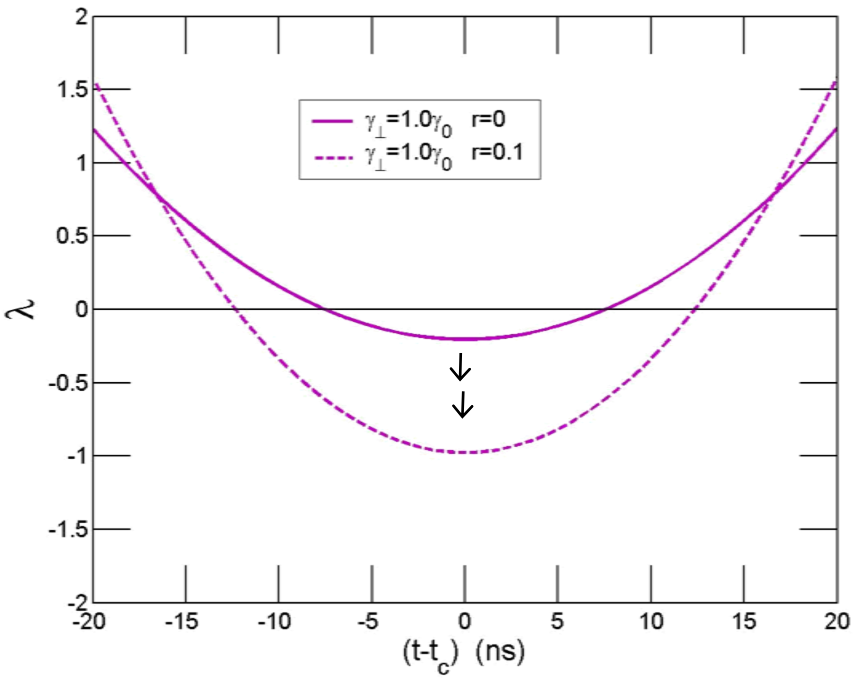
➢ Decoherence destroys entanglement.

$$\underbrace{|\xi\rangle}_{\text{squeezed-vacuum}} = e^{\xi^* \hat{a}_1\hat{a}_2 - \xi \hat{a}_1^\dagger \hat{a}_2^\dagger} \underbrace{|\text{vacuum}\rangle}_{\text{Fock-vacuum}}, \qquad \xi = re^{i\theta}$$

r : squeezing strength

•Initialize in two-mode (two end-fire modes) squeezed vacuum.

<u>(squeezed-vacuum) vs. (decoherence)</u>

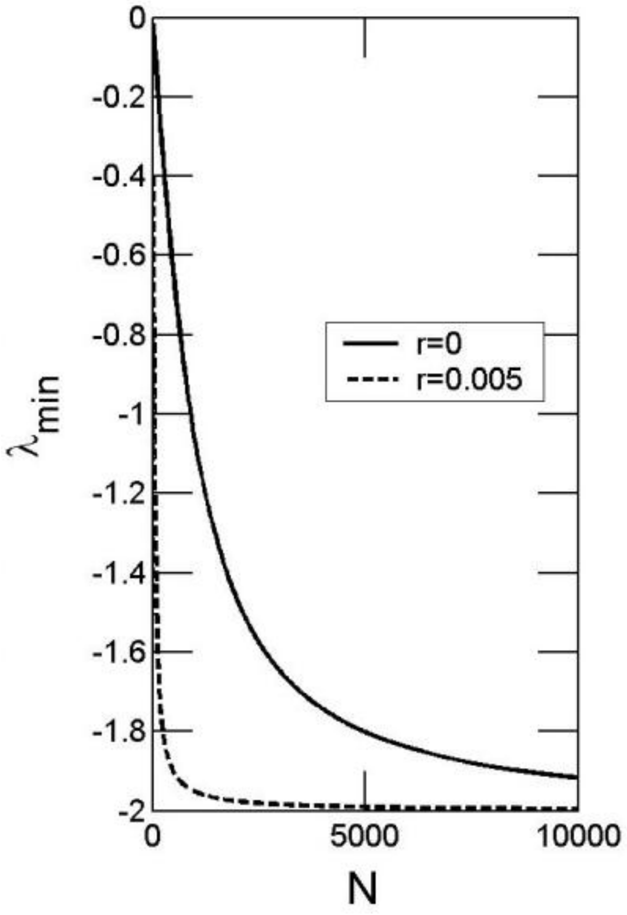


$$\lambda_{\min} = -0.2 \xrightarrow{\text{shifts to}} \lambda_{\min} = -1$$

Entanglement enhanced

against

decoherence



$$\left|\xi\right\rangle = e^{\xi^{*}\hat{a}_1\hat{a}_2 - \xi\hat{a}_1^{\dagger}\hat{a}_2^{\dagger}}\left|\text{vacuum}\right\rangle,$$

$$\xi = re^{i\theta}$$

r : squeezing strength

Increase number of atoms in BEC enhances entanglement.

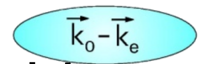
Squeezing further enhances entanglement.

Outline

- Superradiance and BEC Superradiance
- Motivation: Entanglement of scattered pulses.
- Our Model Hamiltonian
- Entanglement parameter
- Swap Mechanism
- Simulations
- Conclusions

Conclusions

- We investigated the quantum-correlations in a Superradiant(SR) BEC.



- Initially; scattered BEC wave (side-mode) entangles with the SR end-fire pulse.

- Later-times; two end-fire pulses become entangled due to entanglement-swap.

- Decorence destroys the entanglement.

- Squeezed vacuum injection for the two end-fire modes, and increasing number of condensate atoms enhances the entanglement.

Thanks

Thank you
for your
attention!