Corrigendum to: A Generalization of Semiregular and Almost Principally Injective Rings

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Abstract. Some errors were detected in Example 3.5 and Example 3.8 of [1]. We replace Example 3.5 with a new example and correct the proof of Example 3.8.

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In [1, Example 3.5], we claimed that the ring R is not right almost S_l-semiregular. There is an error in the proof of this example. We delete this example and its proof, and replace it with the example below:

Example 1 There exists a right almost semiregular ring that is not right almost S_l-semiregular (S_r-semiregular).

Proof. Let $R = \mathbb{Z}_{(p)}$ be the localization of the ring of integers $\mathbb{Z}$ at a prime p. Since R is a local ring, it is semiregular whence it is right almost semiregular. We claim that the ring R is not right almost S_l-semiregular. Take a non-zero element a in the Jacobson radical $J(R)$. Since a is non-zero, we have $l_R r_R(a) = R$. Because R is indecomposable as a left R-module, the only decomposition is $l_R r_R(a) = R = R \oplus 0$. Because a is non-unit in R, we have $Ra \neq R$. On the other hand, if R was right almost S_l-semiregular, then we would have $Ra \subseteq S_l$ by the definition of the almost S_l-semiregularity. But this is a contradiction since $S_l = 0$. □

We delete the last two sentences of Example 3.8, and give the proof below in order to show that the ring R in Example 3.8 is not right almost semiregular.

Example 2 There exists a right almost δ_l (or δ_r)-semiregular ring that is not right almost semiregular.

Proof. Let F be a field and $I = \begin{bmatrix} F & F \\ 0 & F \end{bmatrix}$. Consider the ring

$$R = \{(x_1, x_2, \ldots, x_n, x, x, \ldots) \mid n \in \mathbb{N},\, x_i \in M_2(F),\, x \in I\}.$$

We claim that R does not satisfy $(C2)$ condition as a right R-module. Take the element $\alpha = (x, x, \ldots)$ of R and the idempotent $g = (e, e, \ldots)$, where $x = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ and $e = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$. Then $\alpha R \cong gR$. One can observe that the idempotents in αR is of the form $f = (f_1, f_2, \ldots, f_n, 0, 0, \ldots)$, where $f_i = 0$ or $f_i = \begin{bmatrix} 1 & d \\ 0 & 0 \end{bmatrix}$, $d \in F$ for $i = 1, 2, \ldots, n$. Hence, $fR \neq \alpha R$ for each idempotent $f \in \alpha R$. Thus, R_R does not satisfy $(C2)$. By [1, Theorem 3.14], R is not right almost semiregular. $\square$

References

[1] Özcan A.Ç., Aydoğdu P., (2010), A generalization of semiregular and almost-principally injective rings, *Algebra Colloq.* 17(Spec 1):905-916.