Fourth Edition

CHAPTER

5

MECHANICS OF MATERIALS

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Analysis and Design of Beams for Bending

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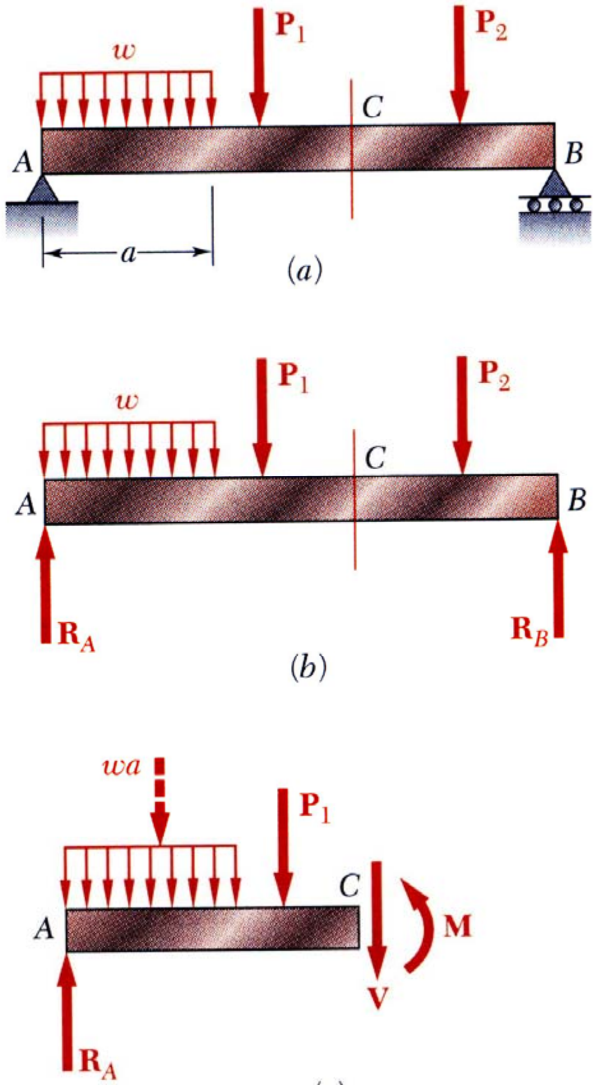
Introduction



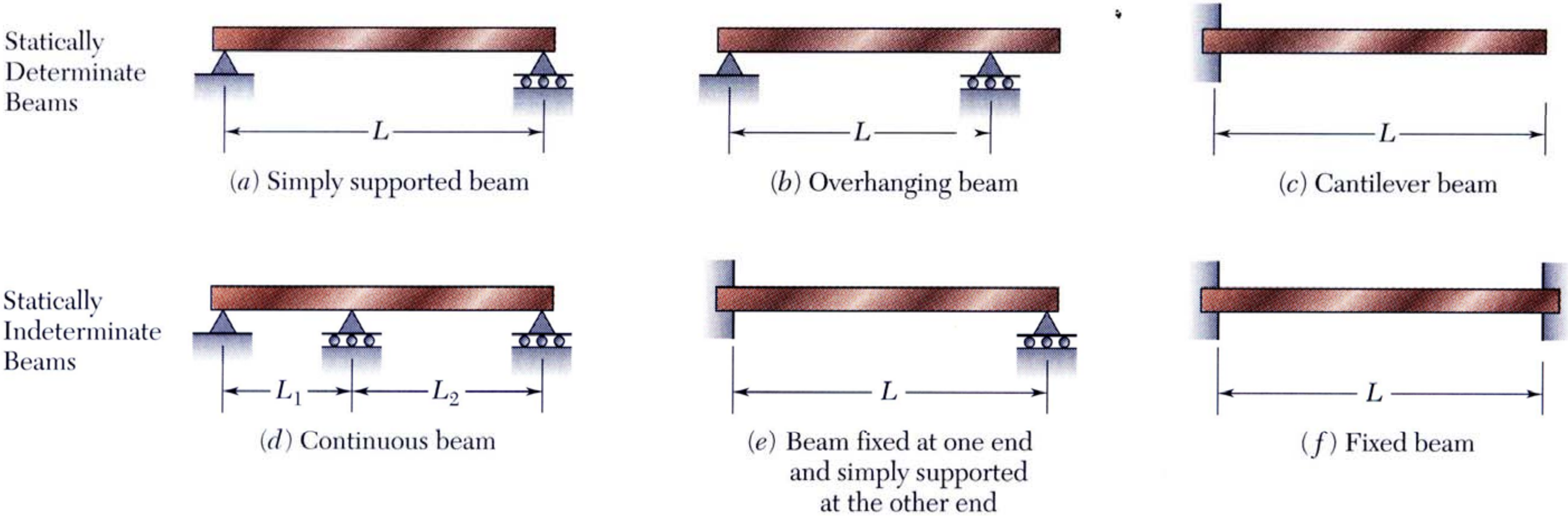
- Objective - Analysis and design of beams
- *Beams* - structural members supporting loads at various points along the member
- Transverse loadings of beams are classified as *concentrated* loads or *distributed* loads
- Applied loads result in internal forces consisting of a shear force (from the shear stress distribution) and a bending couple (from the normal stress distribution)
- Normal stress is often the critical design criteria

$$\sigma_x = -\frac{My}{I} \qquad \sigma_m = \frac{|M|c}{I} = \frac{|M|}{S}$$

Requires determination of the location and magnitude of largest bending moment

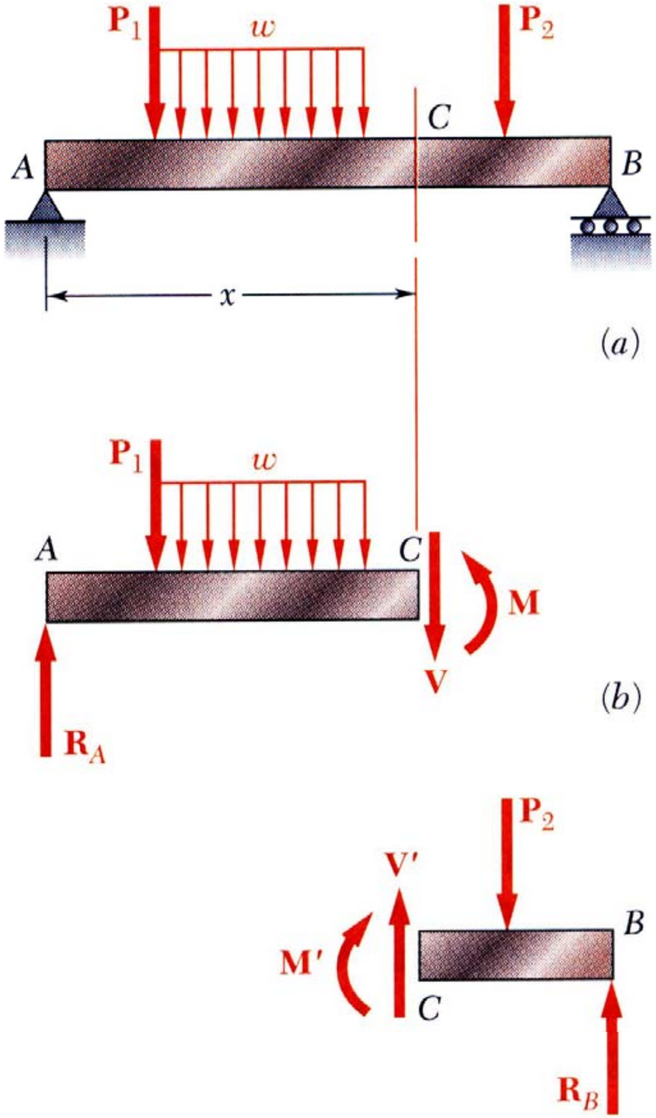
Introduction

Classification of Beam Supports

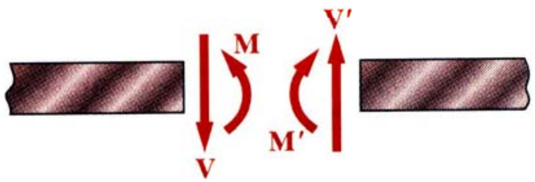


(*a*) Simply supported beam
(*b*) Overhanging beam
(*c*) Cantilever beam
(*d*) Continuous beam
(*e*) Beam fixed at one end and simply supported at the other end
(*f*) Fixed beam

Shear and Bending Moment Diagrams

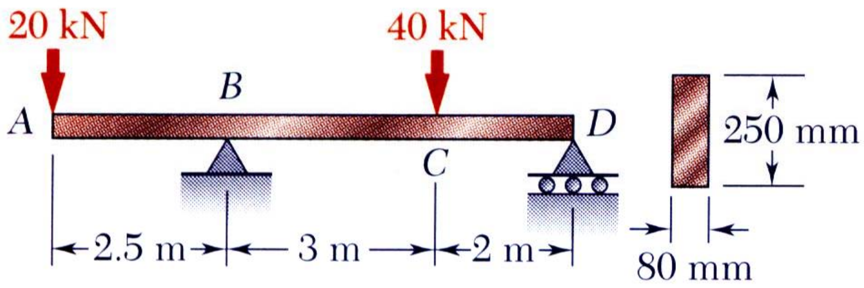


- Determination of maximum normal and shearing stresses requires identification of maximum internal shear force and bending couple.
- Shear force and bending couple at a point are determined by passing a section through the beam and applying an equilibrium analysis on the beam portions on either side of the section.
- Sign conventions for shear forces *V* and *V'* and bending couples *M* and *M'*



(*a*) Internal forces
(positive shear and positive bending moment)

Sample Problem 5.1

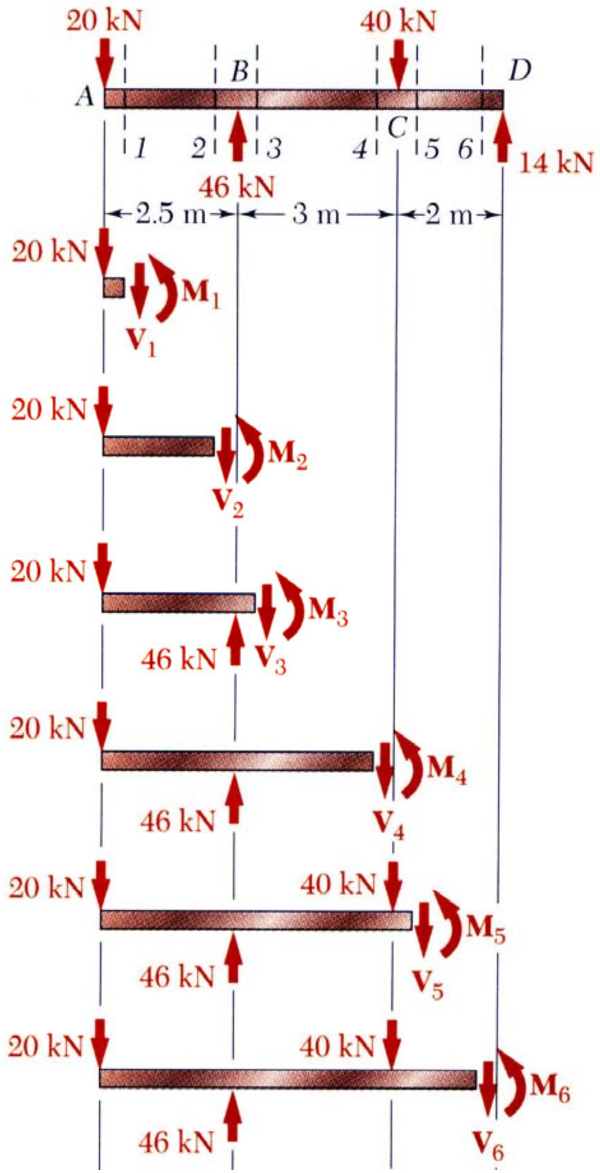


For the timber beam and loading shown, draw the shear and bend-moment diagrams and determine the maximum normal stress due to bending.

SOLUTION:

- Treating the entire beam as a rigid body, determine the reaction forces
- Section the beam at points near supports and load application points. Apply equilibrium analyses on resulting free-bodies to determine internal shear forces and bending couples
- Identify the maximum shear and bending-moment from plots of their distributions.
- Apply the elastic flexure formulas to determine the corresponding maximum normal stress.

Sample Problem 5.1



SOLUTION:

- Treating the entire beam as a rigid body, determine the reaction forces
  from $\sum F_y = 0 = \sum M_B : \quad R_B = 46\,\mathrm{kN} \quad R_D = 14\,\mathrm{kN}$

- Section the beam and apply equilibrium analyses on resulting free-bodies

$$\sum F_y = 0 \quad -20\text{ kN} - V_1 = 0 \qquad V_1 = -20\,\mathrm{kN}$$

$$\sum M_1 = 0 \quad (20\,\mathrm{kN})(0\,\mathrm{m}) + M_1 = 0 \quad M_1 = 0$$

$$\sum F_y = 0 \quad -20\text{ kN} - V_2 = 0 \qquad V_2 = -20\,\mathrm{kN}$$

$$\sum M_2 = 0 \quad (20\,\mathrm{kN})(2.5\,\mathrm{m}) + M_2 = 0 \quad M_2 = -50\,\mathrm{kN}\cdot\mathrm{m}$$

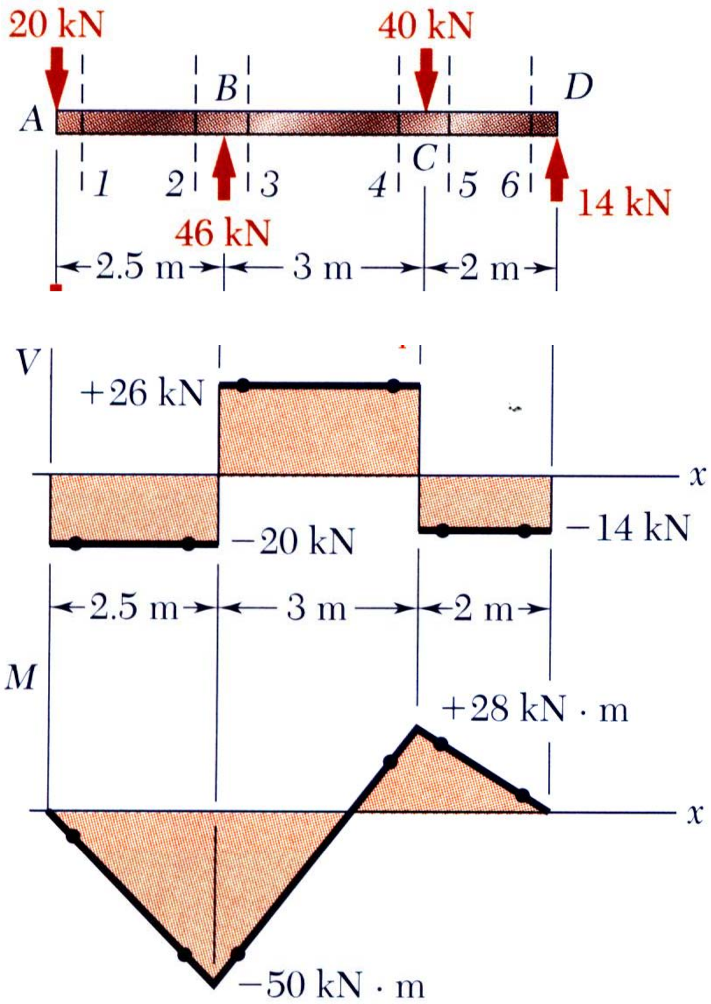
$$V_3 = +26\,\mathrm{kN} \quad M_3 = -50\,\mathrm{kN}\cdot\mathrm{m}$$

$$V_4 = +26\,\mathrm{kN} \quad M_4 = +28\,\mathrm{kN}\cdot\mathrm{m}$$

$$V_5 = -14\,\mathrm{kN} \quad M_5 = +28\,\mathrm{kN}\cdot\mathrm{m}$$

$$V_6 = -14\,\mathrm{kN} \quad M_6 = 0$$

Sample Problem 5.1



- Identify the maximum shear and bending-moment from plots of their distributions.

$$V_m = 26\,\text{kN} \quad M_m = |M_B| = 50\,\text{kN}\cdot\text{m}$$

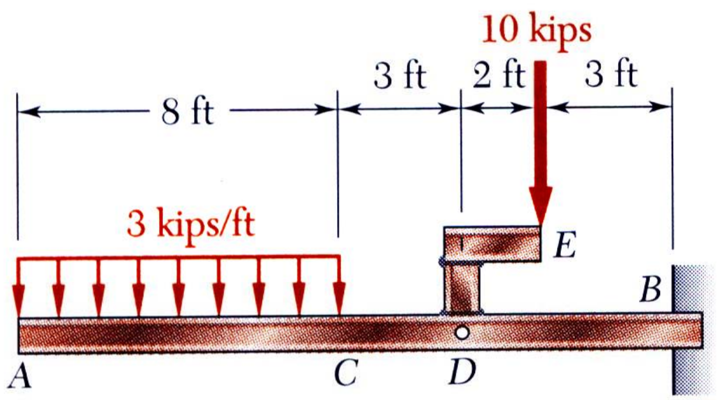
- Apply the elastic flexure formulas to determine the corresponding maximum normal stress.

$$S = \tfrac{1}{6} b h^2 = \tfrac{1}{6}(0.080\,\text{m})(0.250\,\text{m})^2$$
$$= 833.33\times10^{-6}\,\text{m}^3$$

$$\sigma_m = \frac{|M_B|}{S} = \frac{50\times10^3\,\text{N}\cdot\text{m}}{833.33\times10^{-6}\,\text{m}^3}$$

$$\boxed{\sigma_m = 60.0\times10^6\,\text{Pa}}$$

Sample Problem 5.2



The structure shown is constructed of a W10x112 rolled-steel beam. (*a*) Draw the shear and bending-moment diagrams for the beam and the given loading. (*b*) determine normal stress in sections just to the right and left of point *D*.

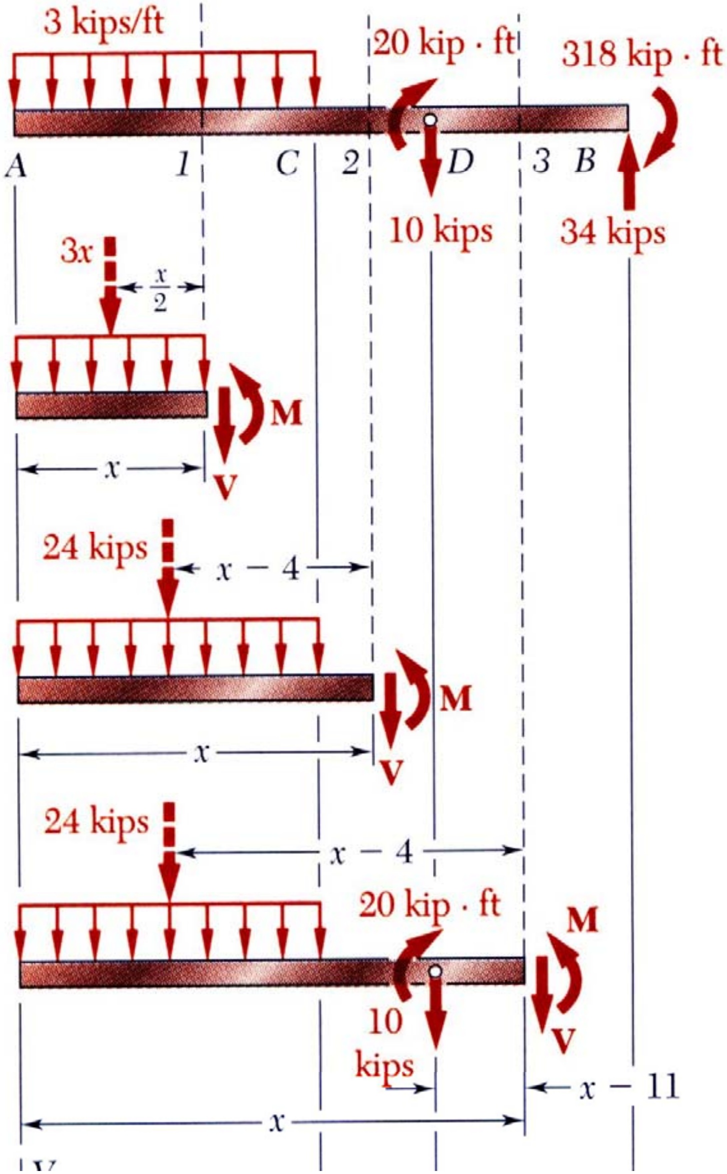
SOLUTION:

- Replace the 10 kip load with an equivalent force-couple system at *D*. Find the reactions at *B* by considering the beam as a rigid body.

- Section the beam at points near the support and load application points. Apply equilibrium analyses on resulting free-bodies to determine internal shear forces and bending couples.

- Apply the elastic flexure formulas to determine the maximum normal stress to the left and right of point *D*.

Sample Problem 5.2



SOLUTION:

- Replace the 10 kip load with equivalent force-couple system at *D*. Find reactions at *B*.
- Section the beam and apply equilibrium analyses on resulting free-bodies.

From A to C :

$$\sum F_y = 0 \quad -3x - V = 0 \qquad V = -3x \text{ kips}$$

$$\sum M_1 = 0 \quad (3x)\left(\tfrac{1}{2}x\right) + M = 0 \qquad M = -1.5x^2 \text{ kip} \cdot \text{ft}$$

From C to D :

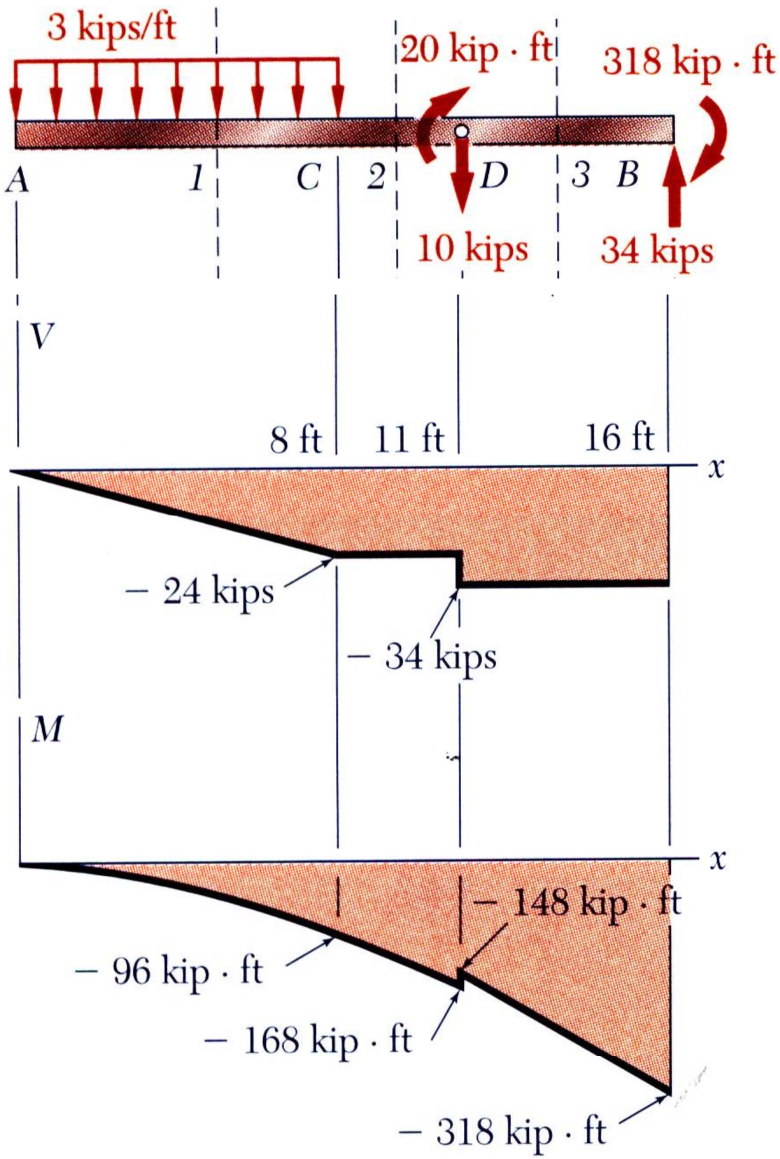
$$\sum F_y = 0 \quad -24 - V = 0 \qquad V = -24 \text{ kips}$$

$$\sum M_2 = 0 \quad 24(x-4) + M = 0 \qquad M = (96 - 24x) \text{ kip} \cdot \text{ft}$$

From D to B :

$$V = -34 \text{ kips} \qquad M = (226 - 34x) \text{ kip} \cdot \text{ft}$$

Sample Problem 5.2



- Apply the elastic flexure formulas to determine the maximum normal stress to the left and right of point *D*.

From Appendix C for a W10x112 rolled steel shape, S = 126 in^3 about the X-X axis.

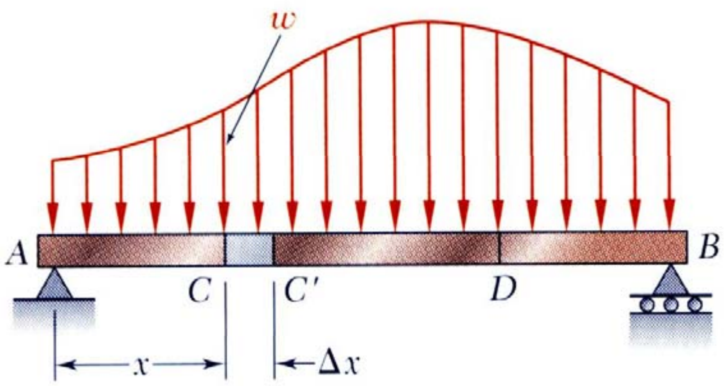
To the left of D:

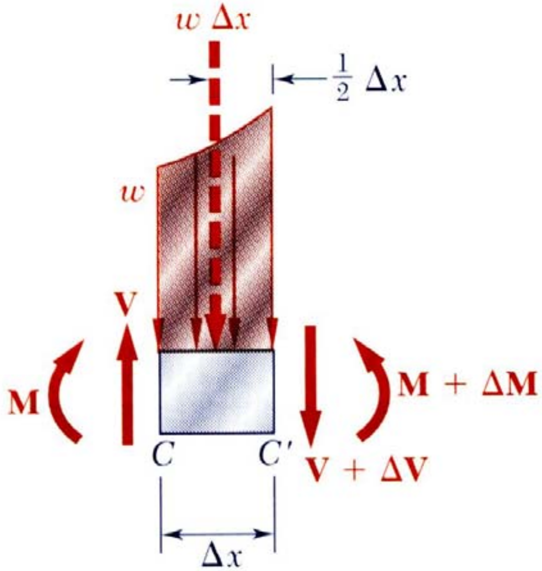
$$\sigma_m = \frac{|M|}{S} = \frac{2016\,\text{kip}\cdot\text{in}}{126\,\text{in}^3} \qquad \boxed{\sigma_m = 16.0\,\text{ksi}}$$

To the right of D:

$$\sigma_m = \frac{|M|}{S} = \frac{1776\,\text{kip}\cdot\text{in}}{126\,\text{in}^3} \qquad \boxed{\sigma_m = 14.1\,\text{ksi}}$$

Relations Among Load, Shear, and Bending Moment





- Relationship between load and shear:

$$\sum F_y = 0: \quad V - (V + \Delta V) - w\,\Delta x = 0$$

$$\Delta V = -w\,\Delta x$$

$$\frac{dV}{dx} = -w$$

$$V_D - V_C = -\int_{x_C}^{x_D} w\,dx$$

- Relationship between shear and bending moment:

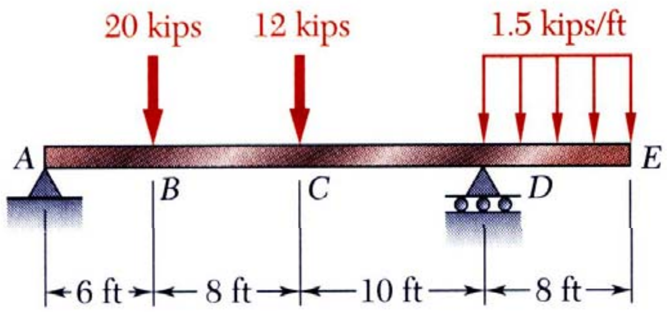
$$\sum M_{C'} = 0: \quad (M + \Delta M) - M - V\,\Delta x + w\Delta x\frac{\Delta x}{2} = 0$$

$$\Delta M = V\,\Delta x - \tfrac{1}{2} w (\Delta x)^2$$

$$\frac{dM}{dx} = V$$

$$M_D - M_C = \int_{x_C}^{x_D} V\,dx$$

Sample Problem 5.3

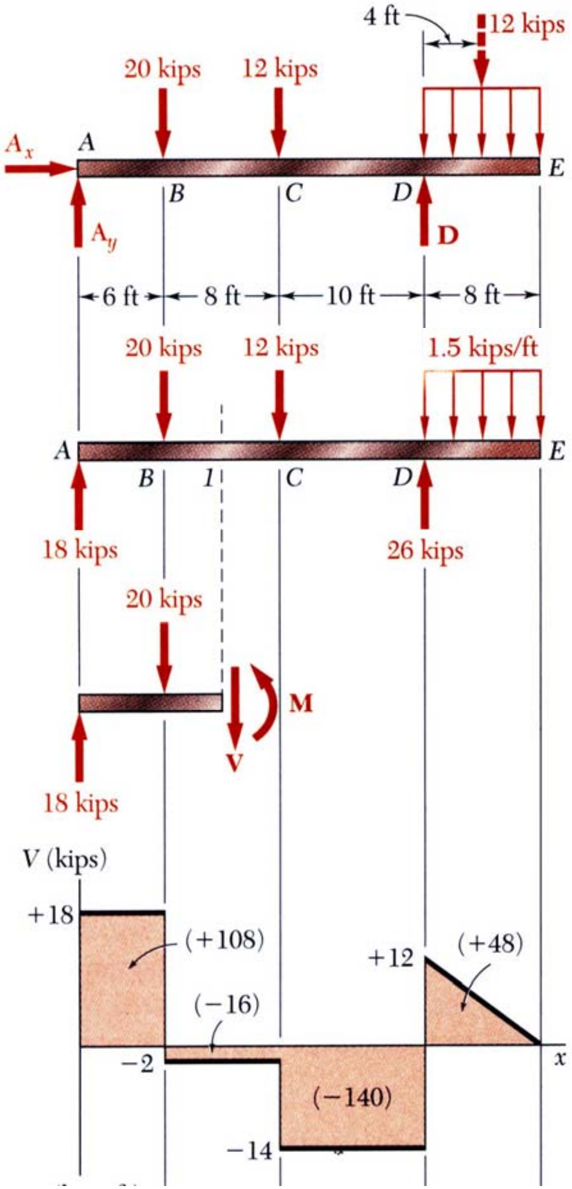


Draw the shear and bending moment diagrams for the beam and loading shown.

SOLUTION:

- Taking the entire beam as a free body, determine the reactions at *A* and *D*.
- Apply the relationship between shear and load to develop the shear diagram.
- Apply the relationship between bending moment and shear to develop the bending moment diagram.

Sample Problem 5.3



SOLUTION:

- Taking the entire beam as a free body, determine the reactions at *A* and *D*.

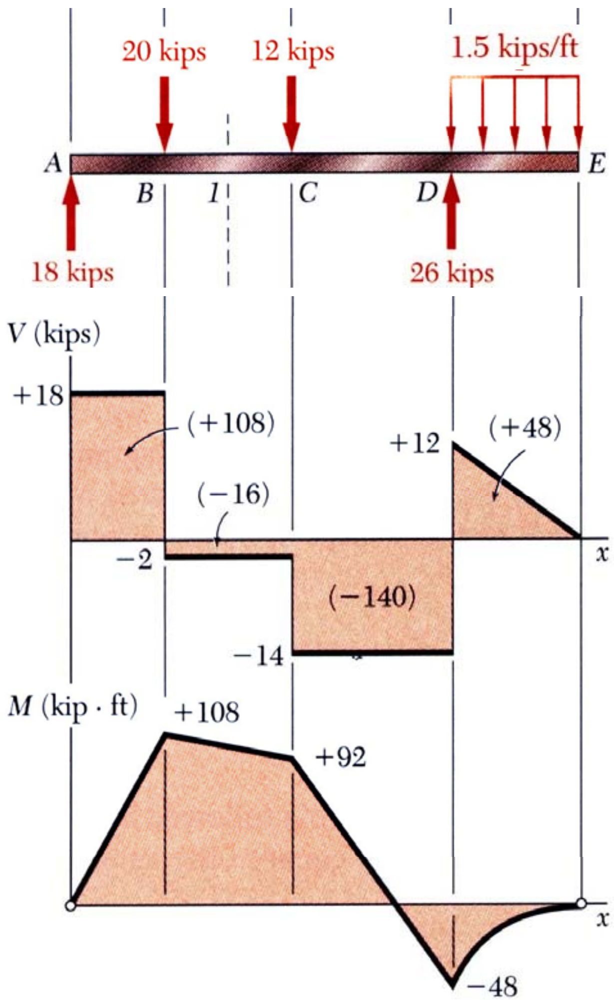
$$\sum M_A = 0$$
$$0 = D(24\,\text{ft}) - (20\,\text{kips})(6\,\text{ft}) - (12\,\text{kips})(14\,\text{ft}) - (12\,\text{kips})(28\,\text{ft})$$
$$D = 26\,\text{kips}$$
$$\sum F_y = 0$$
$$0 = A_y - 20\,\text{kips} - 12\,\text{kips} + 26\,\text{kips} - 12\,\text{kips}$$
$$A_y = 18\,\text{kips}$$

- Apply the relationship between shear and load to develop the shear diagram.

$$\frac{dV}{dx} = -w \qquad dV = -w\,dx$$

- zero slope between concentrated loads
- linear variation over uniform load segment

Sample Problem 5.3

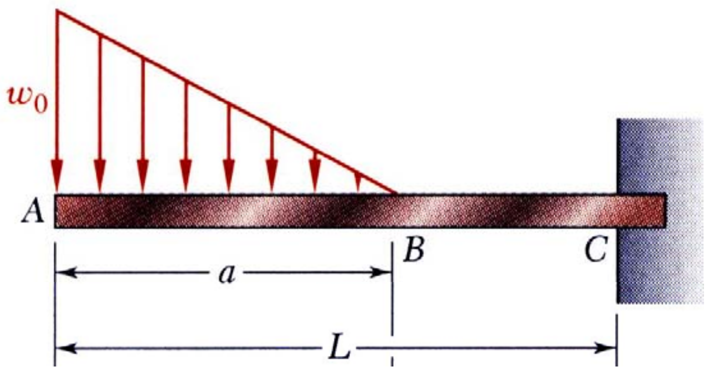


- Apply the relationship between bending moment and shear to develop the bending moment diagram.

$$\frac{dM}{dx} = V \qquad dM = V\,dx$$

- bending moment at *A* and *E* is zero
- bending moment variation between *A*, *B*, *C* and *D* is linear
- bending moment variation between D and *E* is quadratic
- net change in bending moment is equal to areas under shear distribution segments
- total of all bending moment changes across the beam should be zero

Sample Problem 5.5

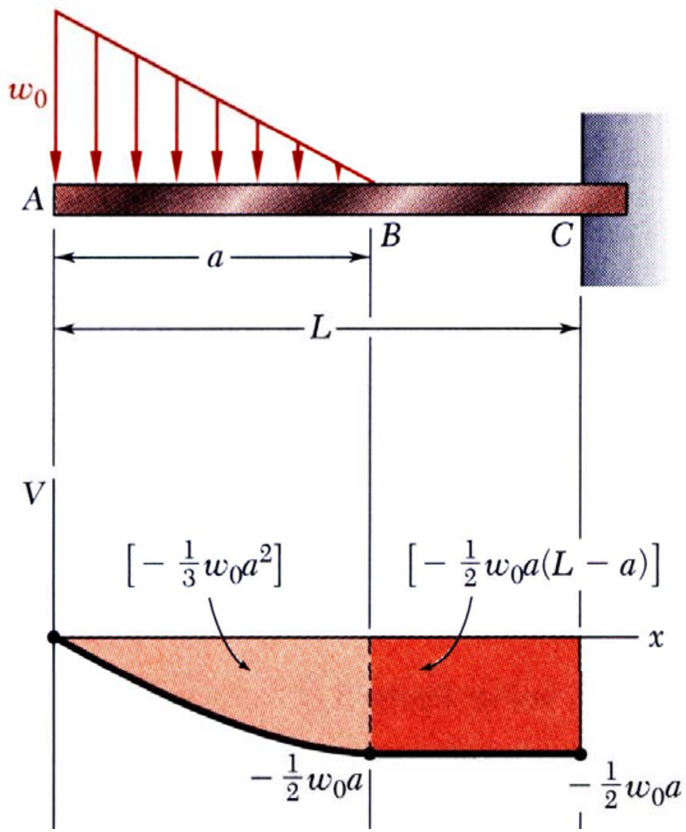


Draw the shear and bending moment diagrams for the beam and loading shown.

SOLUTION:

- Taking the entire beam as a free body, determine the reactions at *C*.
- Apply the relationship between shear and load to develop the shear diagram.
- Apply the relationship between bending moment and shear to develop the bending moment diagram.

Sample Problem 5.5



SOLUTION:

- Taking the entire beam as a free body, determine the reactions at *C*.

$$\sum F_y = 0 = -\tfrac{1}{2}w_0a + R_C \qquad R_C = \tfrac{1}{2}w_0a$$

$$\sum M_C = 0 = \tfrac{1}{2}w_0a\left(L-\frac{a}{3}\right) + M_C \qquad M_C = -\tfrac{1}{2}w_0a\left(L-\frac{a}{3}\right)$$

Results from integration of the load and shear distributions should be equivalent.

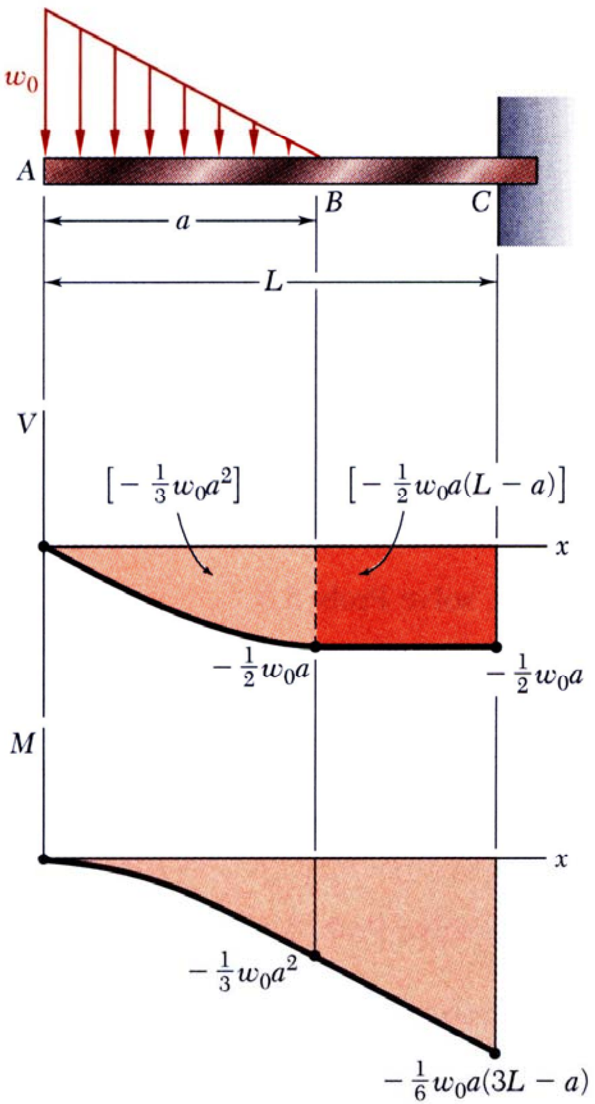
- Apply the relationship between shear and load to develop the shear diagram.

$$V_B - V_A = -\int_0^a w_0\left(1-\frac{x}{a}\right)dx = -\left[w_0\left(x-\frac{x^2}{2a}\right)\right]_0^a$$

$$V_B = -\tfrac{1}{2}w_0a = -(\mathit{area\ under\ load\ curve})$$

- No change in shear between *B* and *C*.
- Compatible with free body analysis

Sample Problem 5.5



- Apply the relationship between bending moment and shear to develop the bending moment diagram.

$$M_B - M_A = \int_0^a \left(-w_0\left(x - \frac{x^2}{2a}\right)\right)dx = \left[-w_0\left(\frac{x^2}{2} - \frac{x^3}{6a}\right)\right]_0^a$$

$$M_B = -\tfrac{1}{3} w_0 a^2$$

$$M_B - M_C = \int_a^L \left(-\tfrac{1}{2} w_0 a\right)dx = -\tfrac{1}{2} w_0 a(L-a)$$

$$M_C = -\tfrac{1}{6} w_0 a(3L-a) = \frac{a\,w_0}{2}\left(L - \frac{a}{3}\right)$$

Results at *C* are compatible with free-body analysis