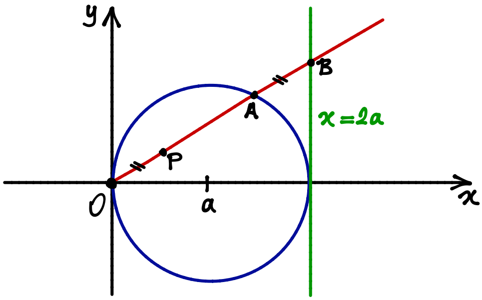
PROBLEM SET I

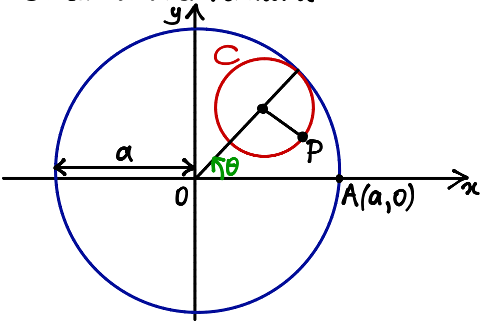
1. Sketch the graph of the curve defined by the equations
$$x = a\cos^3\theta \quad y = a\sin^3\theta,$$
where a is a positive real number which is not zero.

2. Find parametric equations for the set of all points P as shown in the figure $|OP| = |AB|$. This curve is called the cissoid of Diocles after the Greek scholar Diocles, who introduced the cissoid as a graphical method for constructing the edge of a cube whose volume is twice that of a given cube.



Sketch the graph of the curve.

3. A hypocycloid is a curve traced out by a fixed point P on a circle C of radius b as C rolls on the inside of a circle with center O and radius a.



Show that if the initial position of P is $(a,0)$ and the parameter θ is chosen as in the figure, then parametric equations of the hypocycloid are
$$x = (a-b)\cos\theta + b\cos\left(\frac{a-b}{b}\theta\right)$$
$$y = (a-b)\sin\theta - b\sin\left(\frac{a-b}{b}\theta\right).$$

4. A string is wound around a circle and then unwound while being held taut. The curve traced by the point P at the end of the string is called the involute of the circle. If the circle has radius r and center O and the initial position of P is $(r, 0)$, and if the parameter θ is chosen as in the figure, show that parametric e uations o the involute are

$$x = r(\cos\theta + \theta\sin\theta) \qquad y = r(\sin\theta - \theta\cos\theta).$$