A.1. $\int 3x\sqrt{1-2x^2}\,dx$. Letting $u=1-2x^2$, we have $du=-4x\,dx$, and so

$$\int 3x\sqrt{1-2x^2}\,dx = \int 3\sqrt{u}\left(-\frac{du}{4}\right) = -\frac{3}{4}\int u^{1/2}\,du$$

$$= -\frac{3}{4}\,\frac{2}{3}\,u^{3/2}+C$$

$$= -\frac{1}{2}(1-2x^2)^{3/2}+C.$$

A.2. $$\int \frac{(1+x)^2}{\sqrt{x}}\,dx = \int \frac{1+2x+x^2}{x^{1/2}}\,dx$$

$$= \int \left(x^{-1/2}+2x^{1/2}+x^{3/2}\right)dx$$

$$= 2x^{1/2}+\frac{4}{3}x^{3/2}+\frac{2}{5}x^{5/2}+C.$$

A.3. $$\int \frac{x+2}{x+1}\,dx = \int \left(1+\frac{1}{x+1}\right)dx = x+\ln|x+1|+C.$$

A.4. $\int \frac{e^{2/x}}{x^2}\,dx$. Let $u=2/x$. Then $du=-(2/x^2)\,dx$ and

$$\int \frac{e^{2/x}}{x^2}\,dx = \int e^u\left(-\frac{du}{2}\right) = -\frac{1}{2}\int e^u\,du$$

$$= -\frac{1}{2}e^u+C$$

$$= -\frac{1}{2}e^{2/x}+C.$$

A.5. $\int \frac{dx}{e^x+1} = \int \frac{dx}{e^x(1+e^{-x})} = \int \frac{e^{-x}dx}{1+e^{-x}}$. If we set $u = 1+e^{-x}$, then

$$\int \frac{dx}{e^x+1} = \int \frac{-du}{u} = -\ln|u| + C = -\ln|1+e^{-x}| + C$$

$$= \ln\left(\frac{e^x}{e^x+1}\right) + C.$$

A.6. $\int \tan x \, dx = \int \frac{\sin x}{\cos x} dx = \int \frac{-du}{u}$, where $u = \cos x \Rightarrow$

$$\int \tan x \, dx = -\ln|\cos x| + C = \ln|\sec x| + C.$$

A.7. $\int \frac{dx}{1+\cos x} = \int \frac{(1-\cos x)dx}{1-\cos^2 x} = \int \frac{1-\cos x}{\sin^2 x} dx$

$$= \int \csc^2 x \, dx - \int \frac{\cos x}{\sin^2 x} dx$$

We know that $\int \csc^2 dx = -\cot x + C$. For the second integral we make the substitution $u = \sin x$, and get

$$\int \frac{\cos x}{\sin^2 x} dx = \int u^{-2} du = -u^{-1} + C = -\csc x + C.$$

Thus,

$$\int \frac{dx}{1+\cos x} = -\cot x + \csc x + C.$$

A.8. $\int \csc u \, du = \int \frac{1}{\sin u} du = \int \frac{2\,du}{\sin\frac{u}{2}\cos\frac{u}{2}}$

$$= \int \frac{2\sin\frac{u}{2}\,du}{\sin^2\frac{u}{2}\cos\frac{u}{2}}$$

$$= \int \frac{2\csc^2\frac{u}{2}}{\cot\frac{u}{2}}\,du$$

$\left(\text{Let } t = \cot\frac{u}{2}\right) \longrightarrow \quad = \int \frac{2(-2dt)}{t}$

$$= -4\ln|t| + C$$

$$= -4\ln\left|\cot\frac{u}{2}\right| + C$$

ALTERNATIVE WAY:

$$\int \csc u\,du = \int \frac{\csc u(\cot u - \csc u)}{\cot u - \csc u}\,du = \int \frac{\csc u\cot u - \csc^2 u}{\cot u - \csc u}\,du$$

$= \int \frac{dt}{t}$, where $t = \cot u - \csc u$

$= \ln|t| + C = \ln|\cot u - \csc u| + C.$

A.9. $\int \frac{dx}{x\sqrt{x^4-1}}$. Let $u=x^2$. Then $du=2x\,dx$

and so

$$\int \frac{dx}{x\sqrt{x^4-1}} = \int \frac{x\,dx}{x^2\sqrt{x^4-1}} = \int \frac{\frac{1}{2}\,du}{u\sqrt{u^2-1}} = \frac{1}{2}\operatorname{arcsec}(u)+C$$

$$= \underline{\frac{1}{2}\operatorname{arcsec}(x^2)+C}.$$

A.10. $$\int \frac{dx}{e^x+e^{-x}} = \int \frac{e^x\,dx}{e^{2x}+1}$$

$$= \int \frac{du}{u^2+1}, \quad \text{where } u=e^x$$

$$= \arctan u + C = \underline{\arctan(e^x)+C}.$$

A.11. $\int \frac{dx}{\sqrt{20+8x-x^2}} = I$. $20+8x-x^2 = -(x^2-8x-20)$

$\uparrow$ say!

$$= -\left[(x^2-8x+16)-36\right]$$

$$= -\left[(x-4)^2-36\right]$$

$$= 36-(x-4)^2$$

$$\xRightarrow{u=x-4} I = \int \frac{du}{\sqrt{36-u^2}} = \arcsin\left(\frac{u}{6}\right)+C = \arcsin\left(\frac{x-4}{6}\right)+C.$$

A.12. $\int \frac{dx}{2x^2+2x+5} = I.$

$$2x^2+2x+5 = 2\left(x^2+x+\frac{5}{2}\right) = 2\left[\left(x+\frac{1}{2}\right)^2+\frac{9}{4}\right] \xRightarrow{u=x+\frac{1}{2}}$$

$$I = \int \frac{du}{2\left(u^2+\frac{9}{4}\right)} = \frac{1}{2}\int \frac{du}{u^2+9/4} = \frac{1}{2}\cdot\frac{2}{3}\tan^{-1}\left(\frac{2u}{3}\right)+C$$

$$= \frac{1}{3}\tan^{-1}\left(\frac{2x+1}{3}\right)+C$$

A.13. $\int \frac{x+1}{x^2-4x+8}\,dx = I.$

$$x^2-4x+8 = (x-2)^2+4 \xRightarrow{u=x-2} I = \int \frac{u+3}{u^2+4}\,du$$

$$= \int \frac{u\,du}{u^2+4} + 3\int \frac{du}{u^2+4}$$

Now, we know that $\int \frac{du}{u^2+4} = \frac{1}{2}\tan^{-1}\left(\frac{u}{2}\right)+C$. Also, letting $t=u^2+4$, we have $dt = 2u\,du$ and hence, $\int \frac{u\,du}{u^2+4} = \frac{1}{2}\int \frac{dt}{t} = \frac{1}{2}\ln|t|+C = \frac{1}{2}\ln|u^2+4|+C$. Consequently, we obtain

$$I = \frac{1}{2}\ln|u^2+4| + \frac{3}{2}\tan^{-1}\left(\frac{u}{2}\right)+C = \underline{\frac{1}{2}\ln|x^2-4x+8| + \frac{3}{2}\tan^{-1}\left(\frac{x-2}{2}\right)+C}$$

A.14 AND A.15.

Similar to A.13.

A.16. $\int \frac{e^x - 1}{e^x + 1}\,dx = \int \frac{e^x}{e^x+1}\,dx - \int \frac{1}{e^x+1}\,dx$

$= \ln|e^x+1| - \ln\left|\frac{e^x}{e^x+1}\right| + C$ (see A.5)

$= \ln\frac{(e^x+1)^2}{e^x} + C = \underline{2\ln(e^x+1) - x + C}$

A.17. $\int \frac{dx}{x + x^{1/3}} = \int \frac{dx}{x^{1/3}(x^{2/3}+1)} = \int \frac{x^{-1/3}\,dx}{x^{2/3}+1}$

$= \int \frac{\frac{3}{2}\,du}{u}$ where $u = x^{2/3}+1$

$= \frac{3}{2}\ln|u| + C$

$= \underline{\frac{3}{2}\ln(x^{2/3}+1) + C}$.

A.18. $\int \tan^2 x\,dx = \int [(1+\tan^2 x) - 1]\,dx = \underline{\tan x - x + C}$.

A.19. $\displaystyle\int_0^{\pi/4} \tan^3 x\,dx = \int_0^{\pi/4} \frac{\tan^2 x}{\sec x} \tan x \sec x\,dx$

$$= \int_0^{\pi/4} \frac{\sec^2 x - 1}{\sec x} \tan x \sec x\,dx$$

$$= \int_1^{\sqrt{2}} \frac{u^2-1}{u}\,du, \text{ where } u = \sec x$$

$$= \int_1^{\sqrt{2}} \left(u - \frac{1}{u}\right) du = \left(\frac{u^2}{2} - \ln|u|\right)\Big|_1^{\sqrt{2}}$$

$$= 1 - \ln\sqrt{2} - \frac{1}{2}$$

$$= \underline{\frac{1}{2}(1 - \ln 2)}$$

A.20. $\displaystyle\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}} = \int_1^4 \frac{du}{\sqrt{u}}, \text{ where } u = \ln x$

$$= \int_1^4 u^{-1/2}\,du = 2u^{1/2}\Big|_1^4 = \underline{2}.$$