Generating small-world networks

- Assign properties to nodes (e.g. spatial location, group membership)

- Add or rewire links according to some rule
  - optimize for a particular property (simulated annealing)
  - add links with probability depending on property of existing nodes, edges (preferential attachment, link copying)
  - simulate nodes as agents ‘deciding’ whether to rewire or add links

Origins of small worlds: efficient network example trade-off between wiring and connectivity

Small worlds: How and Why, Nisha Mathias and Venkatesh Gopal

$$E = \lambda L + (1-\lambda)W$$

$$L = \frac{1}{n(n-1)} \sum_{i \neq j} d_{ij}$$

$$W = \sum_{e_{ij}} \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$$

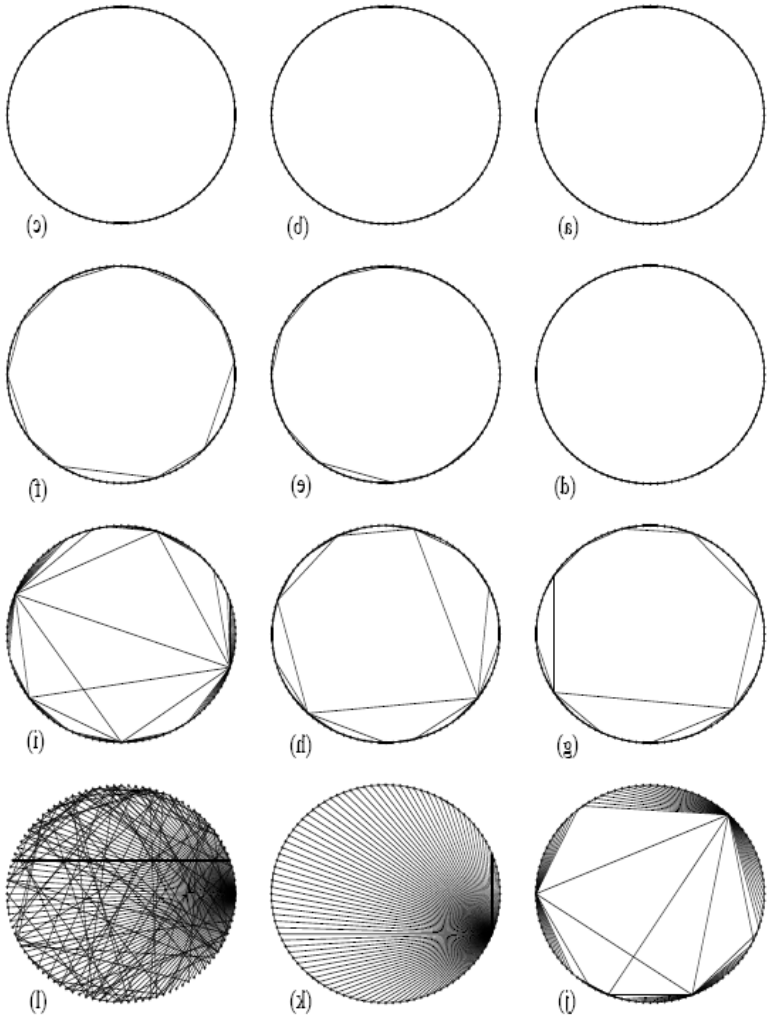
- E is the ‘energy’ cost we are trying to minimize
- L is the average shortest path in ‘hops’
- W is the total length of wire used

Quiz Q:

- Incorporates a person's preference for short distances or a small number of hops
- Relative to setting up the optimization for a road network, the optimization for an airline transportation network, from the passengers' point of view, should:

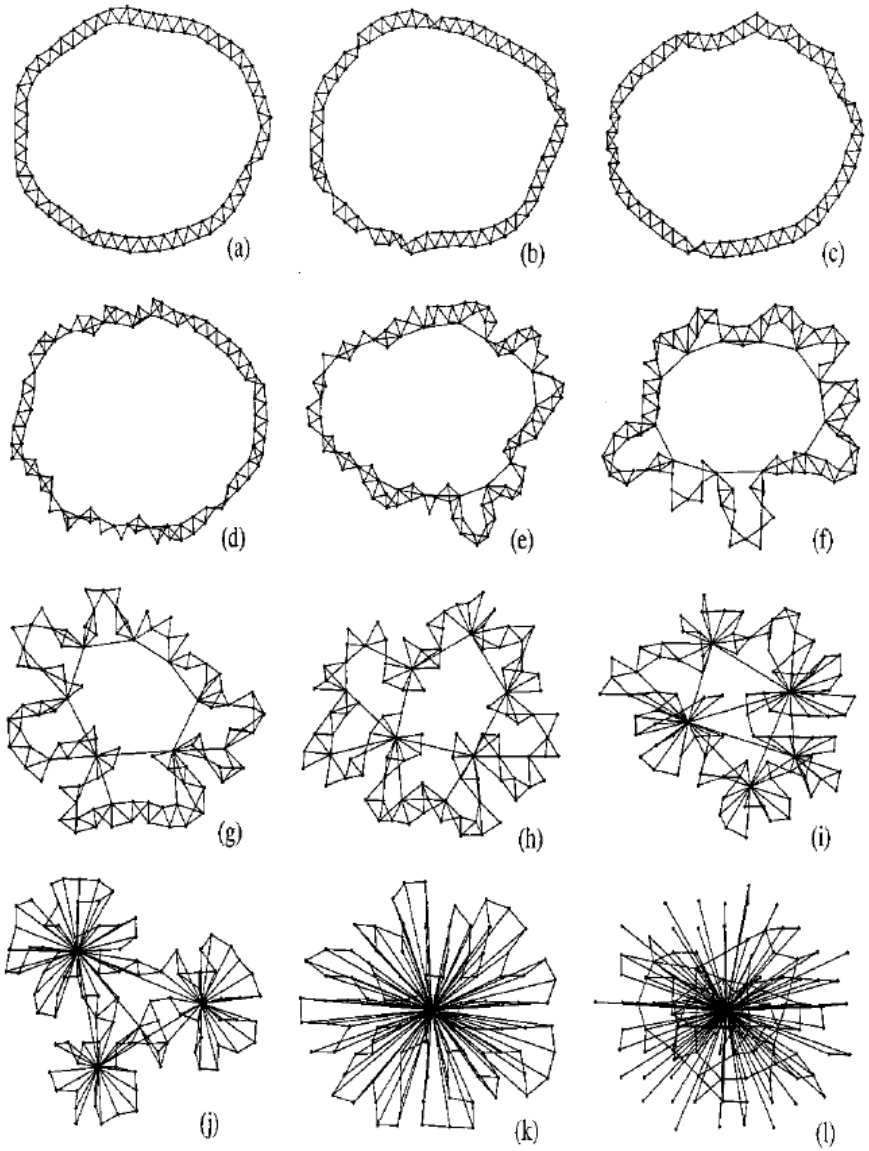
optimized networks

- rewire using simulated annealing
- sequence is shown in order of increasing λ



Source: Small worlds: How and Why, Nisha Mathias and Venkatesh Gopal
http://link.aps.org/doi/10.1103/PhysRevE.63.021117 DOI: 10.1103/PhysRevE.63.021117

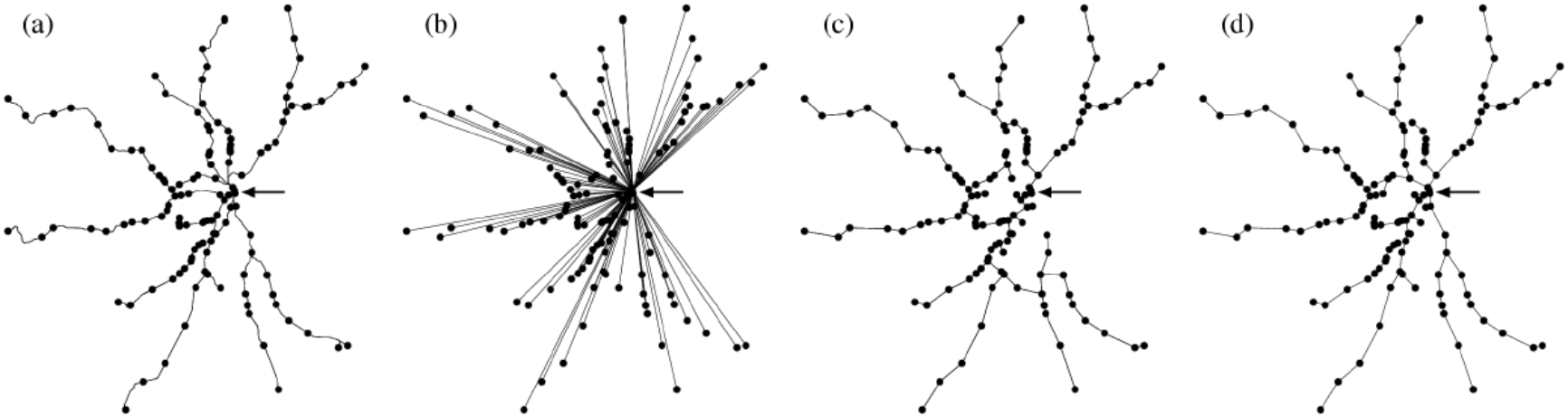
another view of optimized networks



- same networks, but the vertices are allowed to move using a spring layout algorithm
- wiring cost associated with the physical distance between nodes

Source: Small worlds: How and Why, Nisha Mathias and Venkatesh Gopal
http://link.aps.org/doi/10.1103/PhysRevE.63.021117 DOI: 10.1103/PhysRevE.63.021117

optimizing from scratch



(a) Commuter rail network in the Boston area. The arrow marks the assumed root of the network.

(b) Star graph.

(c) Minimum spanning tree.

(d) The model applied to the same set of stations.

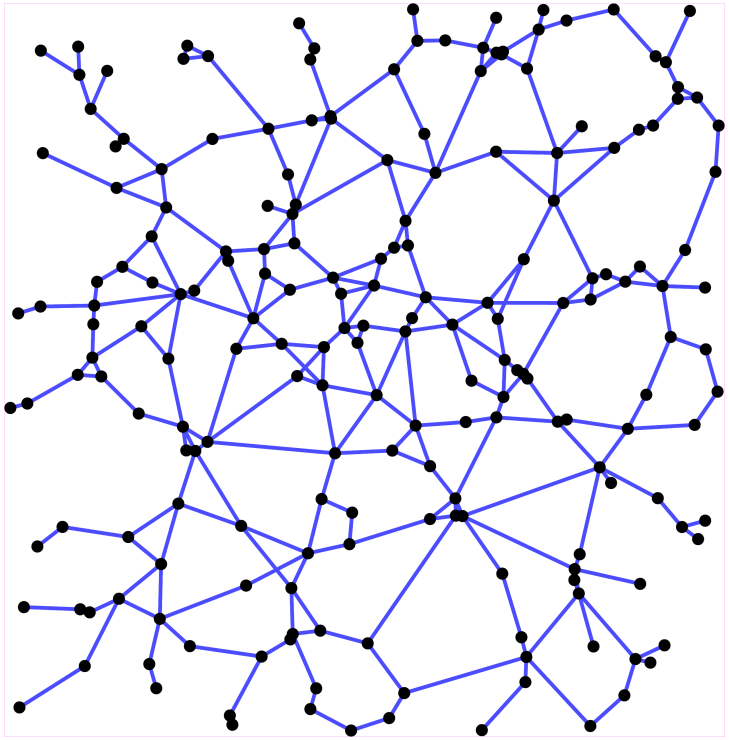
add edge with smallest weight $w'_{ij} = d_{ij} + \beta l_{j0}$

hops to root node

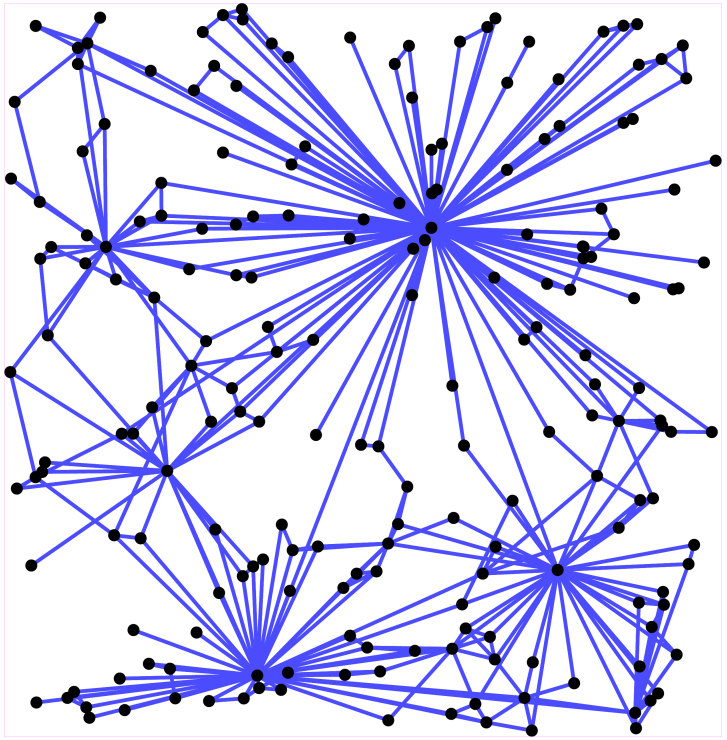
Euclidean distance between i and j

Source: The Spatial Structure of Networks, M. T. Gastner and M. E.J. Newman
http://www.springerlink.com/content/p26t67882668514q DOI: 10.1140/epjb/e2006-00046-8

reminiscent of



Roads



Air routes

Source: The Spatial Structure of Networks, M. T. Gastner and M. E.J. Newman
http://www.springerlink.com/content/p26t67882668514q DOI: 10.1140/epjb/e2006-00046-8

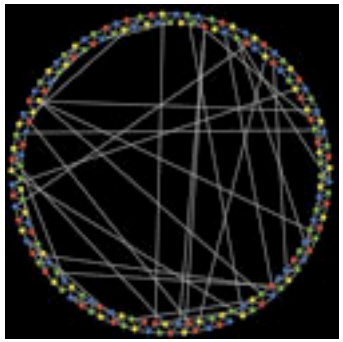
Quiz Q:

- A network that contains many hubs with far reaching edges is indicative of(check all that apply)
  - high cost of distance traveled
  - low cost of distance traveled
  - high cost of making many hops
  - low cost of making many hops

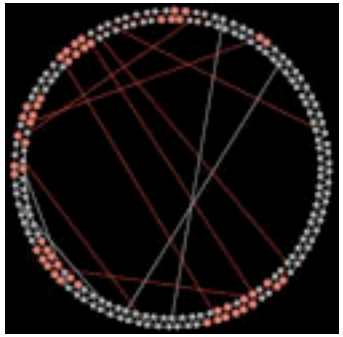
recap

- The world is small if you look at it as a network
- Yet it has lots of interesting local structure
- Watts & Strogatz came up with a simple model to incorporate the two
- Other models incorporate geography and hierarchical social structure
- Small worlds may evolve from different constraints (navigation, constraint optimization, group affiliation)

Next week: learning and diffusing on small world (and other topologies)



Graph coloring



Diffusion