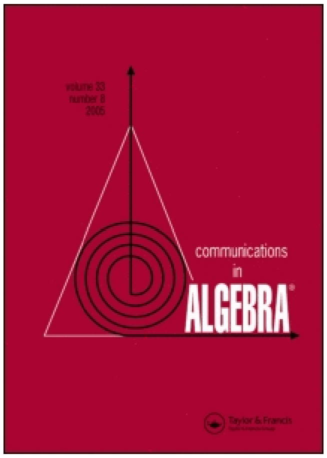

Communications in Algebra
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713597239

Ideals of some matrix rings

F. Kuzucuoglu [a]; V.M. Levchuk [b]
[a] Department of Mathematics, Hacettepe University, Beytepe-Ankara, Turkey
[b] Department of Mathematics, Krasnoyarsk State University, Krasnoyarsk, Russia

Online Publication Date: 01 January 2000
To cite this Article: Kuzucuoglu, F. and Levchuk, V.M. (2000) 'Ideals of some matrix rings ', Communications in Algebra, 28:7, 3503 - 3513
To link to this article: DOI: 10.1080/00927870008827036
URL: http://dx.doi.org/10.1080/00927870008827036

COMMUNICATIONS IN ALGEBRA, 28(7), 3503–3513 (2000)

IDEALS OF SOME MATRIX RINGS[1]

F. KUZUCUOĞLU

Department of Mathematics
Hacettepe University
06532 Beytepe-Ankara
Turkey
e-mail: feridek@eti.cc.hun.edu.tr

V.M. LEVCHUK

Department of Mathematics
Krasnoyarsk State University
av. Svobodny 79,
Krasnoyarsk 660041
Russia
E-mail:levchuk@math.kgu.krasnoyarsk.su

Introduction

Let $NT_n(K)$ be the ring of all (lower niltriangular) $n \times n$ matrices over an associative ring K with zeros on and above the main diagonal. Let $M_n(J)$ be the ring of all $n \times n$ matrices over an ideal J of K. In this paper we investigate ideals and structural connections of the ring $NT_n(K) + M_n(J)$, which is denoted by $R_n(K, J)$.

It was shown in [7] that for $R = NT_n(K)$ and $K = K^2$ the class $\Omega_G(R)$ of all normal subgroups of the adjoint group of the ring R coincides with the class

[1]This research of both authors are supported by TUBITAK.
The second author is partially supported by Russian fund of fundamental researches. He thanks also to Hacettepe University for the hospitality during the preparation of this article.

$\Omega_L(R)$ of all ideals of associated Lie ring. The question about characterization of all associative radical rings R satisfying $\Omega_L(R) = \Omega_G(R)$ (see [5, question 10.19]) is still open. It is clear that if J is a quasi-regular ideal, then $R_n(K, J)$ is a radical (Jacobson) ring. If the quasi-regular ideal J contains an element x satisfying $x^2 \neq 0$, then Example 1.1 below shows that there exists a normal subgroup of the adjoint group which is not an ideal of the groupoid $(R_n(K, J), \star)$ with respect to associated Lie multiplication $\alpha \star \beta = \alpha\beta - \beta\alpha$. On the other hand we show in Section 1 that Example 1.1 gives also a new counterexample to the question in [7, Remark 1] and [4, question 6.19]. The first counterexample to this question was constructed by E.I. Khukhro [cf. comments to the question 6.19 in [4]).

Notice that all ideals of any radical ring R are placed in the intersection $\Omega_G(R) \cap \Omega_L(R)$. R. Dubish and S. Perlis [1, Thm. 9] gave a uniform construction of all ideals of the algebra $NT_n(K)$ over a field K. Similar construction of ideals of the ring $NT_n(K)(= R_n(K, 0))$ over a division ring K has been found in (V. M. Levchuk [7, sect. 2]). It is impossible to give a similar description of the ideals of $NT_n(K)$ for the case $K = Z$ (see [7]).

The aim of Section 2 is to get a description of all ideals of $R_n(K, J)$ in an effective way within the line pointed in [1] and [7]. In this section we assume that K is a commutative ring with identity. Our main Theorem 2.2 describes all ideals of $R_n(K, J)$ when J is a strongly maximal ideal of K (Definition 2.1), in particular, when $K = Z$ or $K = Z_m$, $m > 1$ and J is a maximal ideal of K. This description is similar to the ones in [1], [7] and coincides with them, if K is a field and $J = 0$. At the end of Section 2 we give an example of a maximal ideal which is not strongly maximal.

In Section 3 we describe all maximal abelian ideals of the ring $R_n(K, J)$ when $K = Z_{p^m}$ and J is the strongly maximal ideal (p). Theorem 3.1 shows that if m is even, then $M_n(J^{m/2})$ is a unique maximal abelian'ideal of $R_n(K, J)$. But if m is an odd integer, then the ring $R_n(K, J)$ has precisely $(n-2)p + 1$ of maximal abelian ideals. In proof of Theorem 3.1 we also use the known description of maximal abelian ideals of the ring $NT_n(F)$ over a field F (see [7]).

We denote by e_{ij}, matrices unit and by π_{km}, the canonical projection on $M_n(K)$ see [2]. Thus, $\pi_{km}(e_{ij}) = 1$ if $(k, m) = (i, j)$ and $\pi_{km}(e_{ij}) = 0$ if $(k, m) \neq (i, j)$.

§1. Structural connections of associated Lie ring and adjoint group

Recall that an ideal H of the associative ring R is called quasi-regular, if H is a group with respect to the adjoint multiplication $a \circ b = a + b + ab$, cf [2]. A ring

R is called a radical (Jacobson) ring, if $(R, \circ)$ is a group (adjoint group), that is, for each element $a \in R$ there exists an element $a' \in R$ such that $a \circ a' = a' \circ a = 0$. If a is a nilpotent element, then $a' = -a + a^2 - a^3 + \cdots$. On the other hand, every associative ring R is also associated Lie ring $(R, +, \star)$, where $\star$ is the associated Lie multiplication $a \star b = ab - ba$. It is clear that each ideal of the radical ring is a normal subgroup of the adjoint group and an ideal of the associated Lie ring, simultaneously. The following question is still open:

Characterize radical rings R such that the class $\Omega_L(R)$ of all ideals of associated Lie ring coincides with the class $\Omega_G(R)$ of all normal subgroups of the adjoint group [5, question 10.19].

Other relations between the adjoint group of a radical ring and the associated Lie ring were investigated by A.I. Mal'cev [9] and S.A. Jennings [3].

Taking into account relations

$$(xe_{ii})' = x'e_{ii}, \ (xe_{ij})' = -xe_{ij}, \ \ i \neq j,$$

we can say that the ring $R_n(K, J) = NT_n(K) + M_n(J)$ is radical if and only if J is a quasi-regular ideal of the associative ring K. It was shown in [7] that $\Omega_L(NT_n(K)) = \Omega_G(NT_n(K))$, if the ring K is generated by the set $\{xy \mid x, y \in K\}$, i.e, $K = K^2$. It does not hold when $K = 2Z$, $n \geq 5$. Examples of non-nilpotent radical rings R with the equality $\Omega_L(R) = \Omega_G(R)$ was constructed in [8]. On the other hand, we have

Example 1.1: Let J be a quasi-regular ideal of an associative commutative ring K with identity and $R = R_n(K, J)$. Let e be an identity $n \times n$ matrix. The following map $\pi : \alpha \to e + \alpha$ $(\alpha \in R)$ is a monomorphism between the adjoint group of the ring R and $GL_n(K)$. Assume $H = \pi^{-1}(\pi(R) \cap SL_n(K))$. It is clear that H is a normal subgroup of the adjoint group of R. If $ae_{11} + be_{22} \in H$, then $1 = (1+a)(1+b) = 1 + a \circ b$ and $b = a'$. Thus $a(e_{11} - e_{22}) = ae_{12} \star e_{21} \in H$ $(a \in J)$ holds only when $a' = -a$ and hence $a^2 = 0$. Therefore, if the ideal J contains an element x satisfying $x^2 \neq 0$, then the normal subgroup H of the adjoint group of the ring R is not an ideal of the associated Lie ring and moreover it is not an ideal of the groupoid $(R, \star)$.

It was shown in [7, Lemma 2.3] that for arbitrary subgroup H of the adjoint group of the ring $R = NT_n(K)$ over any associative ring K the following two conditions are equivalent:

(i) H is a normal subgroup of adjoint group $(R, \circ)$;

(ii) H is an ideal of the groupoid $(R, \star)$.

In [7, Remark 1] and in [4, question 6.19] it was asked whether the above conditions (i) and (ii) are equivalent for any subgroup H of the adjoint group of any nilpotent associative ring R.

It was shown that this question has a negative answer in general case. E.I. Khukhro (cf. comments to the question 6.19 in [4]) gave the first counterexample in the case of R to be a free nilpotent algebra of nilpotency class 3 over the field $GF(2)$ generated by two elements, see also [6]. Example 1.1 shows that for rings $R_n(K, J)$ with $J \neq 0$ the answer to the question is usually negative.

§2. The construction of ideals

Throughout this section K will denote a commutative (associative) ring with identity and J will be an ideal of K. First we will distinguish some special ideals of the ring $R_n(K, J)$. Consider following ordered sets of matrices positions

$$\mathcal{L} = \{(i_1, j_1), (i_2, j_2), \cdots, (i_r, j_r)\}, \quad r \geq 1, \tag{1}$$
$$1 \leq j_1 < j_2 < \cdots < j_r \leq n, \quad 1 \leq i_1 < i_2 < \cdots < i_r \leq n;$$

$$\mathcal{L}' = \{(k_1, m_1), (k_2, m_2), \cdots, (k_q, m_q)\}, \quad q \geq 0, \tag{2}$$
$$j_r < m_1 < m_2 < \cdots < m_q \leq n, \quad 1 \leq k_1 < k_2 < \cdots < k_q < i_1.$$

We refer to $\mathcal{L}$ as a "set of corners" of degree n (compare with [1, Sect. 7], [7, Sect. 2]). We also allow the possibility that $\mathcal{L}' = \phi$. Choose a J-submodule T of K. We now consider all J-submodules A of the ring $R_n(K, J)$ with the following conditions (i)-(iv):

(i) There exist sets $\mathcal{L}$ and $\mathcal{L}'$ defined by (1) and (2) such that

$JB \subset A \subset B$, where $B = \sum_{(i,j)\in\mathcal{L}} Te_{ij} + \sum_{(k,m)\in\mathcal{L}'} (JT)e_{km}$;

(ii) If $JT = JTJ$ then $\mathcal{L}' = \phi$ and also $\mathcal{L} = \{(1, n)\}$ when $JT = T$;

(iii) The ideal of K generated by $\pi_{ij}(A)$ is equal to T if $(n, 1) \neq (i, j) \in \mathcal{L}$ and $\pi_{n1}(A) = T$, if $\mathcal{L} = \{(n, 1)\}$;

(iv) $\pi_{km}(A)$ generates the ideal JT for all $(k, m) \in \mathcal{L}'$.

We call every J-submodule A with (i)-(iv) a T-boundary in $R_n(K, J)$. Notice that we can determine a T-boundary $A = A(T; \mathcal{L}, \mathcal{L}')$ with $\mathcal{L} = \{(n, 1)\}$ for any

J-submodule T of K. If $\mathcal{L} \neq \{(n,1)\}$ then T is an ideal of K. It is clear that either $T \subset J$ or $i > j$ for all $(i,j) \in \mathcal{L}$ because $A \subset R_n(K,J)$.

We now describe the ideal of $R_n(K,J)$ which is generated by any T-boundary A. Regard the $n \times n$ matrix as a square array of n^2 points (i,j) (matrix position). Consider the following partial ordering: $(i,j) < (k,m)$ (or $(k,m) > (i,j)$), if $i \leq k$, $m \leq j$ and $(i,j) \neq (k,m)$. For any additive subgroup L of K we denote by $N_{ij}(L)$ (resp. by $Q_{ij}(L)$) the additive group generated by sets Le_{km} for all $(k,m) \geq (i,j)$ (resp. $(k,m) > (i,j)$). It can be easily shown that the ideal of the ring $R_n(K,J)$ generated by any T-boundary $A = A(T;\mathcal{L},\mathcal{L}')$ is equal to

$$I(A) = A + \sum_{(i,j)\in\mathcal{L}} Q_{ij}(T) + N_{i_1 n}(TJ) + N_{1 j_r}(JT) +$$

$$+ M_n(JTJ) + \sum_{(k,m)\in\mathcal{L}'} Q_{km}(JT). \tag{3}$$

We associate "staircases" with sets $\mathcal{L}$ and $\mathcal{L}'$ as in [1], [7]. Then we can compare with the ideal $I(A)$ the following matrix

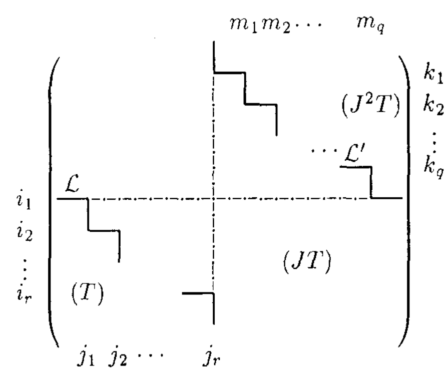

Figure 1.

Now we shall show that in certain cases all ideals of the ring $R_n(K,J)$ are ideals $I(A)$.

Definition 2.1: An ideal J of a ring K is called a strongly maximal ideal if for any J-submodule L of K every ideal of K which is between a J-submodule L and JL, is equal to L or JL.

Let J be a strongly maximal ideal of a ring K.

Theorem 2.2: If H is any ideal of the ring $R_n(K,J)$, $n \geq 2$, then there exists a unique $\pi_{n1}(H)$-boundary A in $R_n(K,J)$ such that $H = I(A)$.

We now require following lemmas.

Lemma 2.3: Let J be an arbitrary ideal of K and let H be an ideal of the ring $R_n(K,J)$ and $H_{uv} = \pi_{uv}(H)$. Then $JH_{uv} \subset H_{iv} \cap H_{uj}$ for all i, j, u, v, and

$$J^2T \subset H_{uv} \subset K\ H_{uv} \subset H_{ij} \subset T \quad \text{for all} \quad (u,v) < (i,j), \tag{4}$$

where $T = H_{n1}$. In particular, all projections H_{uv} are J-submodules of K and

$$M_n(J^2T) \subset H \subset M_n(T). \tag{5}$$

Proof: Notice that the product of matrices $e_{ki}\alpha$ can have nonzero entries only in k-th row which is equal to i-th row of α. Using the inclusion $(Ke_{mi})H \subset H$ it follows $KH_{ij} \subset H_{mj}$ for $m > i$. Similarly, we have $KH_{sm} \subset H_{si}$ for $m > i$. Combining these inclusions we get $KH_{uv} \subset H_{ij} \subset T$ for all $(u,v) < (i,j)$ and hence $H \subset M_n(T)$. The ideal H contains a set $J\ (e_{iu}H + He_{vj})$ and hence $JH_{uv} \subset H_{iv} \cap H_{uj}$. Since the additive group of $M_n(J^2T)$ is generated by its elementary matrices and

$$(J^2H_{uv})e_{ij} = (Je_{iu})H(Je_{vj}) \subset H$$

we derive $M_n(J^2T) \subset H$ and (4) which conclude the proof.

Let $\mathcal{L}(H)$ be the set of all minimal (with respect to the relation $\leq$) matrices positions (i,j) such that H_{ij} generates an ideal of K which contains $T = H_{n1}$. It is clear that the set $\mathcal{L}(H)$ is defined uniquely for H.

Lemma 2.4: Keeping notation of Lemma 2.3 the following hold:

(i) The set $\mathcal{L} = \mathcal{L}(H)$ is a set of corners which is defined by (1);

(ii) $H_{in} \cap H_{1j} \supset JT$ and $H_{km} = T$ for all $(i,j) \in \mathcal{L}$ and $(k,m) > (i,j)$;

(iii) The J-submodule T is an ideal of K when $\mathcal{L} \neq \{(n,1)\}$.

Proof: Since $T = H_{n1} \subset KT$, the set $\mathcal{L}(H)$ is non-empty. Suppose that $(i,j) \in \mathcal{L}(H)$. Taking into account (4) and the inclusion $T \subset K\ H_{ij}$, we obtain

$$KH_{ij} = H_{km} = T \text{ for all } (k,m) > (i,j).$$

Therefore the set $\mathcal{L} = \mathcal{L}(H)$ is a set of corners as in (1). Also, if $\mathcal{L} \neq \{(n,1)\}$, then T is an ideal of K. Using Lemma 2.3 and the equality $H_{n1} = T$ we obtain $H_{in} \cap H_{1j} \supset JH_{ij} = JKH_{ij} = JT$ for all $(i,j) \in \mathcal{L}$ and our lemma is proved.

Proof of Theorem 2.2: Suppose that $H_{uv} = \pi_{uv}(H)$, $T = H_{n1}$ and $\mathcal{L} = \mathcal{L}(H)$. If $\mathcal{L} = \{(1,n)\}$, in particular when $T = JT$, then we have $H = H_{1n}e_{1n} + Q_{1n}(T)$ since

$$H \supset K(e_{ui}H + He_{jv}) \supset (KH_{ij})e_{uv}, \quad i < u, v < j. \tag{6}$$

Therefore, in this case $H_{1n}e_{1n}$ is a unique T-boundary A such that $H = I(A)$. Assume that $\mathcal{L} \neq \{(1,n)\}$. Then we have $T \neq JT$. Choose an arbitrary matrix position (s,t) which is placed in the set

$$\{(i_1 - 1, 1), (i_2 - 1, j_1 + 1), \cdots, (i_r - 1, j_{r-1} + 1), (n, j_r + 1)\}. \tag{7}$$

It is clear that $(s,t) \geq (1, j_r)$ or $(s,t) \geq (i_1, n)$. Hence $T \supset K\ H_{st} \supset H_{st} \supset JT$ by (4) and by Lemma 2.4 (ii). By definition of $\mathcal{L}(H)$ the J-submodule H_{st} generates the ideal of K which does not contain T. Now, assume that J is a strongly maximal ideal of K. Then we conclude $KH_{st} = JT = H_{st}$.

Let (k,m) be an arbitrary matrix position which is placed above staircase $\mathcal{L}(H)$. Then there exists a matrix position $(s,t) \geq (k,m)$ which is in the set (7). Therefore $JT = H_{st} \supset K\ H_{km} \supset H_{km} \supset J(JT)$ by (4). This implies that either the set H_{km} generates the ideal JT or $H_{km} = J^2T$. Notice that if $(k,m) \geq (i_1, n)$ or $(k,m) \geq (1, j_r)$, then $H_{km} \supset JT$ by Lemma 2.4 (ii) and (4).

If $JT \neq J^2T$ we define $\mathcal{L}'(H)$ to be the set of all minimal matrices positions (k,m) such that $1 \leq k < i_1$, $j_r < m \leq n$ and that the J-submodule H_{km} generates the ideal JT. If $JT = J^2T$ then $\mathcal{L}' = \phi$. Thus, for H the set $\mathcal{L}'(H)$ is defined uniquely and $\mathcal{L}' = \mathcal{L}'(H)$ is a set (2).

Suppose that $(i,j) \in \mathcal{L}$. Then $JH_{ij} = JKH_{ij} = JT$ and hence

$$(Je_{1i})H = (JT)e_{1j} \bmod Q_{1j}(JT) + M_n(J^2T).$$

Taking into account (6) we obtain $N_{1j}(JT) \subset H$ and similarly, $N_{in}(JT) \subset H$. Therefore $N_{i_1n}(JT) + N_{1j_r}(JT) + M_n(J^2T) \subset H$. Using (6) it is easy to show that

$$Q_{ij}(T) + Q_{km}(JT) \subset H \quad \text{for all} \quad (i,j) \in \mathcal{L} \text{ and } \quad (k,m) \in \mathcal{L}'.$$

We choose the set A of all matrices $\alpha = \| a_{uv} \| \in H$ with $a_{uv} = 0$ if $(u,v) \notin$

$\mathcal{L} \cup \mathcal{L}'$. It is clear that the set $A = A(T; \mathcal{L}, \mathcal{L}')$ is a J-submodule of $R_n(K,J)$ with conditions (i)-(iv). Therefore, the set A is a T-boundary and $H = I(A)$. Finally, T-boundary A is defined uniquely for H. This completes the proof.

Thus, Theorem 2.2 describes all ideals of the ring $R_n(K,J)$ when J is a strongly maximal ideal of K. The next proposition indicates examples of such ideals.

Proposition 2.5: The zero ideal of any field and every maximal ideal of the rings Z and Z_m, $m > 1$, are strongly maximal ideals.

Proof: Straightforward.

Observe that the case $J = 0$ of Theorem 2.2 gives well known description of ideals of the ring or algebra $NT_n(K)$ over a field K (see [1, Thm. 9] and [7, Sect. 2]).

It is clear that any strongly maximal ideal of a ring K, which is not equal to K, is maximal. We conclude this section with an example.

Example 2.6: Consider the ring $K = Z[x]$ of polynomials in one indeterminate x over Z. The ideal $J = pZ + xK$ for an arbitrary prime p is maximal and

$$J^s = p^s Z + p^{s-1} x Z + \cdots + p x^{s-1} Z + x^s K.$$

Let $T = J^s$ and $s \geq 1$. The quotient-ring T/JT is a ring with zero multiplication and with elementary abelian additive group of order p^{s+1}. Thus between T and JT we can find chains of ideals of K with arbitrary finite length varying s suitably. Hence, the ideal J of K is not strongly maximal.

§3. Abelian ideals of $R_n(K,J)$

In this section we apply Theorem 2.2 in order to describe maximal abelian ideals of the ring $R_n(K,J)$ when $K = Z_p m$ and $J = (p)$. It is convenient now to assume that always $J^0 = K$.

Theorem 3.1: Suppose that $K = Z_p m$ and $J = (p)$. If m is even, then $M_n(J^{m/2})$ is unique maximal abelian ideal of the ring $R_n(K,J)$, $n \geq 2$. If $m = 2s+1$ is an odd integer, then the ring $R_n(K,J)$ has $(n-2)p+1$ maximal abelian ideals which have the form:

$$N_{i\,i-1}(J^s) + M_n(J^{s+1}), \qquad 1 < i \leq n, \text{ if } n = 2 \text{ and } s = 0 \text{ or } n > 2;$$

$$J^s e + N_{21}(J^s) + M_2(J^{s+1}) \text{ if } n = 2 \text{ and } s > 0;$$

$$N_{i+1\,i-1}(J^s) + M_n(J^{s+1}) + J^s(e_{i1} + c e_{ni}), \;\; 1 < i < n, \;\; 1 \leq c < p.$$

Proof: Let H be an arbitrary maximal abelian ideal of $R_n(K,J)$ and $T = \pi_{n1}(H)$. Since

$$(\sum_{k,u} a_{ku}e_{ku})e_{1n}(\sum_{v,t} b_{vt}e_{vt}) = \sum_{k,t}(a_{k1}b_{nt})e_{kt}$$

we obtain $\pi_{n1}(H\star(Je_{1n}H)) = \pi_{n1}(H(Je_{1n})H) = TJT$. However, $H\star(Je_{1n}H) = 0$ since H is an abelian ideal. Hence $T^2J = 0$. Taking into account inclusions (5) we obtain $HM_n(JT) = M_n(JT)H = 0$ and

$$M_n(JT) \subset H \subset M_n(T) \tag{8}$$

since H is a maximal abelian ideal.

Now we find centralizer $C(M_n(T))$ of $M_n(T)$ in the ring $R_n(K,J)$. Let $Ann_K T$ be the annihilator of T in K, and $\alpha = \| a_{uv} \| \in R_n(K,J)$ Then α is in the centralizer $C(Te_{km})$ of Te_{km} in $R_n(K,J)$ if and only if $a_{kk} = a_{mm}$ mod $(Ann_K T)$ and all other elements of k-th column and m-th row of α are contained in $Ann_K T$. Therefore

$$C(M_n(T)) = Je + M_n(Ann_K T) \cap R_n(K,J)$$

where e is the identity matrix. It is clear that $M_n(T)$ is an abelian ideal if and only if $T^2 = 0$. If $Ann_K T = T \subset J$ then the ideal $M_n(T)$ of $R_n(K,J)$ is maximal abelian because any ideal of the ring $R_n(K,J)$ which is between $M_n(T)$ and $Je + M_n(T)$ $(= C(M_n(T)))$ is equal to $M_n(T)$.

Each ideal of our ring K is equal to J^t for some t, $0 \le t \le m$, and $Ann_K J^t = J^{m-t}$. It follows that if m is even then $M_n(J^{m/2})$ is a maximal abelian ideal of $R_n(K,J)$. Suppose that $T = J^s$. Then we have $J^{2s+1} = JT^2 = 0$ and hence $2s+1 \ge m$. If d is the integer part of $(m+1)/2$ then $M_n(J^d) \subset M_n(JT)$ or $H \subset M_n(T) \subset M_n(J^d)$. Since $M_n(J^d)$ is an abelian ideal we have $H \supset M_n(J^d)$ and so $(m-1)/2 \le s \le d$. When m is even we obtain $d = m/2$ and $H = M_n(J^{m/2})$.

Assume that m is an odd integer. Then we have $m = 2s+1$ and $T = J^s$, $J^{s+1} = JT = Ann_K T$. When $s = 0$ the conclusion of our theorem holds by [7, Thm 3]. Let s be a positive integer. By 2.2 and by (8) there exists a set of corners $\mathcal{L}$ as in (1) such that

$$H = A + \sum_{(i,j)\in\mathcal{L}} Q_{ij}(T) + M_n(JT)$$

where $A \subset \sum_{(i,j)\in\mathcal{L}} Te_{ij}$. It is not diffucult to show that

$$C(N_{ij}(T)) = Je + \{N_{j+1\,i-1}(K) + M_n(Ann_K T)\} \cap R_n(K,J).$$

Suppose that $(i,j) \in \mathcal{L}$ and $i \leq j$. If $i < n$ then $N_{i+1i}(T) \subset H$ and so

$$H \subset C(N_{i+1i}(T)) \cap M_n(T) = Te + N_{i+1i}(T) + M_n(JT).$$

Hence $i = j$ and $n = 2$. For $j > 1$ we obtain the same result. Clear that $Te + N_{21}(T) + M_2(JT)$ for $T = J^s$ is a maximal abelian ideal of $R_2(K,J)$. As above all ideals $N_{i+1i}(T) + M_n(JT)$ are maximal abelian in the ring $R_n(K,J)$ for $n > 2$.

We now consider the case when $N_{i+1i}(T) \not\subset H$ for all i. It is clear that $H \subset NT_n(T) + M_n(JT)$ and if $(i_1,j_1),(i_r,j_r) \in \mathcal{L}$ as in (1) then $1 < i_1 \leq j_r < n$. Suppose, if possible, that $xe_{i1} \in H$ for some $i_1 \leq i$ and $x \in (T - (JT))$. Then we have $H \subset C(xe_{i1})$ and so $\pi_{ui}(H) \subset Ann_K x = JT$ for all u. Consequently $i_1 = j_r$ and if $i = i_1$ then $H \cap (Te_{i1}) = JTe_{i1}$ and $H \cap (Te_{ni}) = JTe_{ni}$. Therefore H is placed in the abelian ideal $T(e_{i1} + ce_{ni}) + N_{i+1i-1}(T) + M_n(JT)$ for some $c \in K$ such that $Tc = T$. Clear that we can choose c such that $1 \leq c < p$.

Finally note that the number of all maximal abelian ideals in the ring $R_n(K,J)$ for odd m is equal to $(n-1) + (n-2)(p-1) = (n-2)p + 1$. Theorem 3.1 is proved.

REFERENCES

[1] R. Dubish and S. Perlis, On total nilpotent algebras, Amer. J. Math., 73 (1951) 439-452.

[2] T.W. Hungerford, Algebra (Winston, New York, 1974).

[3] S.A. Jennings, Radical rings with nilpotent associated groups, Trans. Roy Soc. Canada (1) 49 (1955) 31-38.

[4] The Kourovka Notebook (unsolved problems in group theory), 7-th Ed., Institute of Math. USSR. Acad. Sci. Sib. Dept. Novosibirsk, 1980.

[5] The Kourovka Notebook (unsolved problems in group theory), 12-th Ed., Institute of Math. SO RAN, Novosibirsk, 1992.

[6] F. Kuzucuoğlu and V.M. Levchuk, On structural connections of nilpotent associative rings and its associated algebraic systems, in: Symmetry in natural science (Proc. Int. Conf.); Inst. Computation Modelling, Krasnoyarsk, (1998) 74.

[7] V. M. Levchuk, Connections between the unitriangular group and certain rings, Algebra and Logic (5) 15 (1976) 348 - 360.

[8] V.M. Levchuk, Some locally nilpotent rings and their adjoint groups, Math. Zametki (5) 42 (1987) 848 - 853.

[9] A.I. Mal'cev, Generalized nilpotent algebras and their adjoint groups, Matem. Sbornik, (3) 25 (1949) 347-366.

Received: April 1999