Magnetotransport in modulation-doped In$_{0.53}$Ga$_{0.47}$As/In$_{0.52}$Al$_{0.48}$As heterojunctions: in-plane effective mass, quantum and transport mobilities of 2D electrons

E. TIRAS, M. CANKURTARAN†, H. ÇELIK
Hacettepe University, Faculty of Engineering, Department of Physics, Beytepe, 06532 Ankara, Turkey

A. BOLAND THOMS, N. BALKAN
Semiconductor Optoelectronics Group, Department of Electronic Systems Engineering, University of Essex, Colchester, U.K.

(Received 29 September 2000)

Shubnikov–de Haas (SdH) and Hall effect measurements, performed in the temperature range between 3.3 and 20 K and at magnetic fields up to 2.3 T, have been used to investigate the electronic transport properties of lattice-matched In$_{0.53}$Ga$_{0.47}$As/In$_{0.52}$Al$_{0.48}$As heterojunctions. The spacer layer thickness ($t_S$) in modulation-doped samples was in the range between 0 and 400 Å. SdH oscillations indicate that two subbands are already occupied for all samples except for that with $t_S = 400$ Å. The carrier density in each subband, Fermi energy and subband separation have been determined from the periods of the SdH oscillations. The in-plane effective mass ($m^*$) and the quantum lifetime ($\tau_q$) of 2D electrons in each subband have been obtained from the temperature and magnetic field dependences of the amplitude of SdH oscillations, respectively. The 2D carrier density ($N_1$) in the first subband decreases rapidly with increasing spacer thickness, while that ($N_2$) in the second subband, which is much smaller than $N_1$, decreases slightly with increasing spacer thickness from 0 to 200 Å. The in-plane effective mass of 2D electrons is similar to that of electrons in bulk In$_{0.51}$Ga$_{0.47}$As and show no dependence on spacer thickness. The quantum mobility of 2D electrons is essentially independent of the thickness of the spacer layer in the range between 0 and 200 Å. It is, however, markedly higher for the samples with a 400 Å thick spacer layer. The quantum mobility of 2D electrons is substantially smaller than the transport mobility which is obtained from the Hall effect measurements at low magnetic fields. The transport mobility of 2D electrons in the first subband is substantially higher than that of electrons in the second subband for all samples with double subband occupancy. The results obtained for transport-to-quantum lifetime ratios suggest that the scattering of electrons in the first subband is, on average, forward displaced in momentum space, while the electrons in the second subband undergo mainly large-angle scattering.

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†E-mail: cankur@hacettepe.edu.tr
Key words: two-dimensional electron gas, In$_{0.53}$Ga$_{0.47}$As/In$_{0.52}$Al$_{0.48}$As heterojunctions, Shubnikov–de Haas effect, effective mass, quantum lifetime, transport lifetime, alloy scattering, remote ionized impurity scattering, background impurity scattering, intersubband scattering.

1. Introduction

In a two-dimensional (2D) structure the confinement in the growth direction leads to quantization of the electron energy into discrete subbands where the electron motion is restricted to the plane of the 2D electron gas. If the electron density is low, only the first subband is occupied; when the electron density increases, the Fermi level moves above the second subband energy and electrons start populating this subband. Shubnikov–de Haas (SdH) effect and Hall effect measurements are very useful tools in characterizing the electronic transport properties of a multiple carrier system such as a two subband populated 2D electron gas. The SdH effect measurements provide the carrier density, effective mass and quantum lifetime (single-particle scattering time) of 2D electrons, while the low-field Hall effect measurements provide the sheet carrier density and transport lifetime (momentum relaxation time). The transport lifetime ($\tau_t$) is weighted by the scattering angle and hence contains no contribution from forward scattering and only a little contribution from small-angle scattering [1]. The quantum lifetime ($\tau_q$) is given by the total scattering rate, in which every scattering event is equally weighted, and is dominated by small-angle scattering [1]. Useful information about the nature of the scattering of 2D electrons in the system can be achieved by comparing the values of $\tau_t$ and $\tau_q$. For instance, remote ionized impurity scattering in nonalloy systems, such as high-mobility GaAs/Al$_x$Ga$_{1-x}$As heterostructures, usually results in a scattering time ratio $\tau_t/\tau_q \gg 1$, indicative of the long-range nature of the scattering potential [1–7]. On the other hand, short-range scattering from an alloy disorder potential results in $\tau_t/\tau_q \sim 1$ [4, 7]. When alloy scattering and ionized impurity scattering are simultaneously present, as in the case of modulation-doped In$_{0.53}$Ga$_{0.47}$As/In$_{0.52}$Al$_{0.48}$As heterojunctions, a more complicated behaviour for $\tau_t/\tau_q$ would be expected depending on the sample parameters such as carrier density, spacer layer thickness and impurity concentration. Recent theoretical calculations [7] have shown that, for $\delta$-doped In$_{0.53}$Ga$_{0.47}$As/InP quantum wells of single subband occupancy, the presence of alloy scattering in addition to ionized impurity scattering leads to a significant reduction in the ratio $\tau_t/\tau_q$, when compared to that of material systems where alloy scattering does not exist.

Several reports have been published concerning the measurement of transport and/or quantum lifetimes of electrons in In$_{0.53}$Ga$_{0.47}$As/InP quantum wells [8–12] and in other In$_x$Ga$_{1-x}$As HEMT structures (δ-doped pseudomorphic GaAs/Ga$_{0.8}$In$_{0.2}$As/Ga$_{0.75}$In$_{0.25}$As heterostructures [13] and In$_{0.18}$Ga$_{0.82}$As/Al$_{0.20}$Ga$_{0.80}$As quantum wells [14]) where alloy scattering is expected to be significant. Recently, by using SdH effect measurements in combination with a persistent photoconductivity effect, Lo et al. [15] observed the second subband population in δ-doped In$_{0.53}$Ga$_{0.47}$As/In$_{0.52}$Al$_{0.48}$As heterojunctions with a 60 Å thick spacer layer. They noted that the quantum lifetime of electrons in the second subband is longer than that of electrons in the first subband, however, they did not provide actual values [15].

To our knowledge there is no systematic study to date concerning the effects of the spacer layer thickness on the carrier density, effective mass and quantum and transport lifetimes of 2D electrons in In$_{0.53}$Ga$_{0.47}$As/In$_{0.52}$Al$_{0.48}$As heterojunctions. In this paper we present the results of SdH effect and Hall effect measurements in lattice-matched, modulation-doped In$_{0.53}$Ga$_{0.47}$As/In$_{0.52}$Al$_{0.48}$As heterojunctions with spacer layer thickness in the range between $l_S = 0$ and 400 Å. In all but the samples with $l_S = 400$ Å, the first two subbands are already occupied at liquid He temperatures. The carrier density, in-plane effective mass, quantum and transport lifetimes of 2D electrons in each subband of the samples determined experimentally are presented. The results shed further light on the understanding of the relative importance of various
scattering mechanisms that limit the transport and quantum mobilities of electrons in the modulation-doped In$_{0.53}$Ga$_{0.47}$As/In$_{0.52}$Al$_{0.48}$As heterojunctions.

2. Experimental procedures

The modulation-doped In$_{0.53}$Ga$_{0.47}$As/In$_{0.52}$Al$_{0.48}$As heterojunctions were grown by the MBE technique. The samples were fabricated in Hall-bar geometry and ohmic contacts were formed by diffusing Au/Ge/In alloy. Layer structure and doping parameters of all the samples (Fig. 1) were nominally identical with the exception of the undoped In$_{0.52}$Al$_{0.48}$As spacer layer thickness ($t_S$) which was varied from 0 (no spacer layer) to 400 Å. Table 1 lists both the growth characteristics and transport parameters of all the samples studied.

Magnetoresistance and Hall resistance measurements were carried out simultaneously as a function of temperature in the range between $T = 3.3$ and 20 K in a three-stage closed-cycle refrigeration system (HS-4 Heliplex, APD Cryogenics) using a conventional dc technique in combination with a constant current source (Keithley 220) and a nanovoltmeter (Keithley 182). The current flow was in the plane of the 2D electron gas and the current through the sample was kept low enough (ranging form 1 to 10 µA depending on the resistance of the sample) to ensure ohmic conditions. Steady magnetic fields ($B$) up to 2.3 T were applied perpendicular to the plane of the samples. The data were taken at equal intervals of $1/B$. All the measurements were performed in the dark. In order to check the 2D nature of the electron gas giving rise to SdH oscillations, magnetoresistance measurements were also performed as a function of the angle ($\theta$) between the normal to the plane of the 2D electron gas and the applied magnetic field. It was found that both the peak positions and the oscillation periods shift with a factor of $\cos \theta$ and the oscillations disappear at $\theta = 90^\circ$. This observation is a characteristic of the 2D electron gas (see for instance [16]).

3. Results and discussion

Figures 2–5 show typical examples for the magnetoresistance ($R_{xx}$) and Hall resistance ($R_{xy}$) measurements in the modulation-doped In$_{0.53}$Ga$_{0.47}$As/In$_{0.52}$Al$_{0.48}$As heterojunctions. It is evident from Figs 2–4 that the oscillatory component of the magnetoresistance for the samples with $t_S = 0$, 100 and 200 Å consists of two superimposed sets of oscillations with different periods indicating the occupation of two subbands [17–20]. The amplitude of the long-period oscillations which are due to the electrons in the second subband is much larger than the amplitude of the short-period oscillations due to the first-subband electrons. For the sample with $t_S = 400$ Å, however, the SdH oscillations contain only one period and are superimposed on a rising positive background magnetoresistance (Fig. 5A). Hall effect measurements on this sample show that the $R_{xy}(B)$ data deviate from the classical straight line (Fig. 5A). These observations, as quoted previously by other researchers [13, 21–23], evince for the presence of a parallel conduction channel in this

<table>
<thead>
<tr>
<th>Layer</th>
<th>Type</th>
<th>Thickness (Å)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GaAs</td>
<td>undoped</td>
<td>50</td>
</tr>
<tr>
<td>In$<em>{0.52}$Al$</em>{0.48}$As</td>
<td>$n^+ \sim 10^{24}$ m$^{-3}$ (Si)</td>
<td>250</td>
</tr>
<tr>
<td>In$<em>{0.52}$Al$</em>{0.48}$As</td>
<td>undoped</td>
<td>($t_S = 0$, 100, 200, and 400 Å)</td>
</tr>
<tr>
<td>In$<em>{0.53}$Ga$</em>{0.47}$As</td>
<td>undoped</td>
<td>6000</td>
</tr>
<tr>
<td>In$<em>{0.53}$Ga$</em>{0.47}$As</td>
<td>$p \sim 10^{23}$ m$^3$ (Be)</td>
<td>2000</td>
</tr>
<tr>
<td>InP (Fe) substrate</td>
<td>Semi-insulating</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 1. The structure of the modulation-doped In$_{0.53}$Ga$_{0.47}$As/In$_{0.52}$Al$_{0.48}$As heterojunction samples used in the studies.
Table 1: Sample parameters and electronic transport properties of the modulation-doped In$_{0.53}$Ga$_{0.47}$As/In$_{0.52}$Al$_{0.48}$As heterojunctions determined at 3.3 K.

<table>
<thead>
<tr>
<th>Sample</th>
<th>MV572A</th>
<th>MV576A</th>
<th>MV577</th>
<th>MV578A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spacer thickness, $t_S$ (Å)</td>
<td>0</td>
<td>100</td>
<td>200</td>
<td>400</td>
</tr>
<tr>
<td>Subband index</td>
<td>2D carrier</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>density, $N_c(10^{16}$ m$^{-2}$)</td>
<td>1.67</td>
<td>0.33</td>
<td>1.09</td>
<td>0.21</td>
</tr>
<tr>
<td>Total 2D carrier density, $N_1 + N_2(10^{16}$ m$^{-2}$)</td>
<td>±</td>
<td>±</td>
<td>±</td>
<td>±</td>
</tr>
<tr>
<td>Sheet carrier density, $N_H(10^{16}$ m$^{-2}$)</td>
<td>1.95</td>
<td>1.10</td>
<td>0.91</td>
<td>1.13</td>
</tr>
<tr>
<td>Effective mass, $m^*(m_0)$</td>
<td>—</td>
<td>0.039</td>
<td>0.041</td>
<td>0.037</td>
</tr>
<tr>
<td>Fermi energy, $E_F - E_i$ (meV)</td>
<td>±</td>
<td>±</td>
<td>±</td>
<td>±</td>
</tr>
<tr>
<td>$E_2 - E_1$ (meV)</td>
<td>82</td>
<td>51</td>
<td>45</td>
<td>—</td>
</tr>
<tr>
<td>$\mu_H$ (m$^2$ V$^{-1}$ s$^{-1}$)</td>
<td>4.8</td>
<td>8.1</td>
<td>6.7</td>
<td>4.1</td>
</tr>
<tr>
<td>Transport mobility, $\mu_T$ (m$^2$ V$^{-1}$ s$^{-1}$)</td>
<td>4.91</td>
<td>3.75</td>
<td>7.54</td>
<td>3.30</td>
</tr>
<tr>
<td>Transport lifetime, $\tau_T(10^{-12}$ s)</td>
<td>1.08</td>
<td>0.83</td>
<td>1.75</td>
<td>0.70</td>
</tr>
<tr>
<td>Quantum mobility, $\mu_{qi}$ (m$^2$ V$^{-1}$ s$^{-1}$)</td>
<td>—</td>
<td>0.90</td>
<td>0.68</td>
<td>1.18</td>
</tr>
<tr>
<td>Quantum lifetime, $\tau_{qi}(10^{-12}$ s)</td>
<td>—</td>
<td>0.20</td>
<td>0.15</td>
<td>0.25</td>
</tr>
<tr>
<td>Lifetime ratio, $\tau_i/\tau_q$</td>
<td>—</td>
<td>4.2</td>
<td>9.7</td>
<td>2.8</td>
</tr>
</tbody>
</table>
sample. Furthermore, the 2D carrier density obtained from the oscillation period of the $R_{xx}(B)$ data is almost half of the sheet carrier density ($N_H$) determined from the low-field Hall effect measurements (Table 1). This also indicates the presence of a parallel conduction channel with a similar 3D carrier density. Well-defined plateaus are seen clearly in the Hall resistance measurements at higher magnetic fields for the samples with $t_S = 100$ and 400 Å (Figs 3A and 5A), while no such plateaus are visible in the $R_{xx}(B)$ data for the samples with $t_S = 0$ and 200 Å (Figs 2A and 4A).

In order to exclude the effects of the background magnetoresistance and to extract the SdH oscillations, the negative second derivative of the raw experimental data with respect to the magnetic field ($-\partial^2 R_{xx}/\partial B^2$) has been calculated. The technique [24, 25] suppresses the amplitude of the long-period oscillations and amplifies the short-period oscillations as shown in Fig. 2B: the quantum oscillations in $-\partial^2 R_{xx}/\partial B^2$ arising from the electrons in each subband are resolved clearly. To separate the oscillatory components with different periods, digital filtering [26] has been first applied to the raw $R_{xx}(B)$ data and then the second derivative has been calculated. A typical plot of the two components extracted by digital filtering along with the raw $R_{xx}(B)$ data are shown in Fig. 3 for the sample with $t_S = 100$ Å.

The periods of SdH oscillations provide direct information about the carrier density in each of the populated subbands. However, the amplitude of the short-period component of the oscillations originating from the first subband of the sample with $t_S = 0$ Å is much smaller compared to that from the second subband (Fig. 2B) and diminishes with increasing temperature. We were, therefore, able to determine the in-plane effective mass and quantum lifetime of electrons only in the second subband of this sample. Because of the lack of prominent oscillation peaks from the second subband of the sample with $t_S = 200$ Å (Fig. 4A), the in-plane effective mass and quantum lifetime of electrons can only be obtained for the first subband.

### 3.1. The carrier density of 2D electrons in each subband and Fermi energy

Contributions from all the occupied subbands of a 2D electron gas appear in magnetoresistance measurements as SdH oscillations with different periodicity. Fast Fourier transform (FFT) of the SdH oscillations data for the samples with $t_S = 0$, 100 and 200 Å yields two peaks (see the insert in Figs 2B, 3A and 4A) conforming that two subbands are populated in these samples. The periods of the SdH oscillations ($\Delta_i(1/B)$) have been obtained from the FFT analysis of the experimental data and also from the plots of the reciprocal magnetic field ($1/B_n$), at which the $n$th peak occurs, against the peak number $n$. The carrier density ($N_i$) in each subband has been determined from the measured period of corresponding SdH oscillations using [15, 25]

$$\Delta_i \left( \frac{1}{B} \right) = \frac{e}{\pi h N_i} = \frac{eh}{m^*(E_F - E_i)},$$

(1)

where $i (=1, 2)$ is the subband index, and $E_F - E_i$ is the energy difference between the Fermi level and the bottom of the $i$th subband. The results thus obtained for $N_1$ and $N_2$ are given in Table 1. The carrier density in each subband is found to be essentially independent of temperature in the range from $T = 3.3$ to 20 K. The Fermi energies with respect to the subband energy ($E_F - E_i$) for all the samples have been obtained from the oscillation period using eqn (1) together with the in-plane effective mass $m^*$ of 2D electrons as obtained from the temperature dependence of SdH oscillations (see below). The energy separation of the two populated subbands ($E_2 - E_1$) has been evaluated from the electron densities $N_1$ and $N_2$ using eqn (1). The results for $E_F - E_1$ and $E_2 - E_1$ are also included in Table 1. The experimental results obtained for the spacer thickness dependence of $E_2 - E_1$ are in qualitative agreement with those estimated using the Airy functions [27] and the formalism described in [28, 29].

The 2D carrier density in each subband ($N_i$) and the total 2D carrier density ($N_1 + N_2$) in the modulation-doped In$_{0.53}$Ga$_{0.47}$As/In$_{0.52}$Al$_{0.48}$As heterojunctions, determined from the SdH oscillations, and the sheet carrier density ($N_H$), obtained from the low-field Hall effect measurements, are plotted in Fig. 6 as a function of spacer layer thickness. It can be seen that, for the samples with $t_S = 0$, 100 and 200 Å, the total carrier
Fig. 2. A, Magnetoresistance ($R_{xx}$) and Hall resistance ($R_{xy}$) as a function of magnetic field for sample MV572A measured at 3.32 K and B, the negative second derivative with respect to magnetic field of the $R_{xx}(B)$ data presented in A. The insert in B shows the fast Fourier spectrum of the SdH oscillations. The full curves through the experimental data points are intended as a guide to the eye.
Fig. 3. A, Magnetoresistance ($R_{xx}$) and Hall resistance ($R_{xy}$) as a function of magnetic field for sample MV576A measured at 3.25 K. The straight line superimposed on the $R_{xy}(B)$ curve displays the classical Hall resistance. The insert shows the fast Fourier spectrum of the SdH oscillations in magnetoresistance. B, The negative second derivative with respect to magnetic field of the $R_{xx}(B)$ data filtered through a high-pass filter to extract the oscillations arising from the first subband. C, The negative second derivative with respect to magnetic field of the $R_{xx}(B)$ data filtered through a low-pass filter to obtain the oscillations from the second subband. The full curves through the experimental data points are intended as a guide to the eye.
density of 2D electrons determined independently from the SdH oscillations and Hall effect measurements are in good agreement. This indicates that any parallel conduction due to carriers outside the 2D channel is negligible for these samples at low temperatures. The carrier density \(N_1\) in the first subband decreases rapidly with increasing the spacer thickness from 0 to 400 Å, while that \(N_2\) in the second subband decreases only slightly with increasing the spacer thickness from 0 to 200 Å. A decrease in 2D carrier density with increasing spacer thickness is expected from theoretical considerations involving the self-consistent solutions of Schrödinger and Poisson equations \[30, 31\]. In principle, as the undoped spacer layer becomes wider, the electron transfer from the Si-doped layer to the 2D channel becomes less efficient, and hence the 2D carrier density is reduced progressively. The carrier density in the second subband is a small fraction of the total carrier density in the 2D channel: about 17%, 16% and 12% of 2D electrons populate the second subband of the samples with \(t_S = 0, 100\) and 200 Å, respectively. The present results for the fractional populations of the two subbands are in accord with those found previously for modulation-doped \(\text{In}_{0.53}\text{Ga}_{0.47}\text{As/In}_{0.52}\text{Al}_{0.48}\text{As}\) heterojunctions with \(t_S = 80\) Å \[17, 18\] and \(t_S = 100\) Å \[19\], \(\delta\)-doped \(\text{In}_{0.53}\text{Ga}_{0.47}\text{As/In}_{0.52}\text{Al}_{0.48}\text{As}\) heterojunctions with \(t_S = 60\) Å \[15, 20, 32\], and with the results of the calculations of subband populations in \(\text{In}_{0.53}\text{Ga}_{0.47}\text{As}\) MODFETs \[33\].

3.2. The in-plane effective mass of 2D electrons

The in-plane effective mass \(m^*\) of 2D electrons can be obtained from the temperature dependence of the amplitude of the SdH oscillations at constant magnetic field using \[34, 35\]

\[
\frac{A(T, B_n)}{A(T_0, B_n)} = \frac{T \cdot \sinh\left(\frac{2\pi^2k_Bm^*T}{\hbar^2B_n}\right)}{T_0 \cdot \sinh\left(\frac{2\pi^2k_Bm^*T_0}{\hbar^2B_n}\right)}.
\]
Fig. 4. A, Magnetoresistance ($R_{xx}$) and Hall resistance ($R_{xy}$) as a function of magnetic field for sample MV577 measured at 3.33 K. The straight line superimposed on the $R_{xy}(B)$ curve displays the classical Hall resistance. The insert shows the fast Fourier spectrum of the SdH oscillations in magnetoresistance. B, The negative second derivative with respect to magnetic field of the $R_{xx}(B)$ curve, which corresponds to the oscillations from the first subband. The full curves through the experimental data points are intended as a guide to the eye.
Fig. 5. A, Magnetoresistance ($R_{xx}$) and Hall resistance ($R_{xy}$) as function of magnetic field for sample MV578A measured at 3.33 K. The straight line displays the classical Hall resistance. The insert shows the fast Fourier spectrum of the SdH oscillations in magnetoresistance. B, The negative second derivative with respect to magnetic field of the $R_{xx}(B)$ data, which shows the population of only one subband. The full curves through the experimental data points are intended as a guide to the eye.
Fig. 6. Spacer thickness dependence of the carrier density in modulation-doped In$_{0.53}$Ga$_{0.47}$As/In$_{0.52}$Al$_{0.48}$As heterojunctions. The data points represented by the open circles, full circles and open diamonds correspond to the carrier density in the first subband ($N_1$), second subband ($N_2$) and the total carrier density ($N_1 + N_2$) determined from the periods of the SdH oscillations, respectively. The full squares correspond to the sheet carrier density ($N_H$) determined from low-field Hall effect measurements. The full curves through the experimental data points are intended as a guide to the eye.

where $A(T, B_n)$ and $A(T_0, B_n)$ are the amplitudes of the oscillation peaks observed at a magnetic field $B_n$ and at temperatures $T$ and $T_0$, respectively. In the derivation of eqn (2) the quantum lifetime of electrons is assumed to be independent of temperature and the effects of higher harmonics are neglected [25, 34].

A typical example for the variation of SdH oscillations with temperature is shown in Fig. 7, where only the data sets measured at three different temperatures are presented for clarity. The relative amplitude $A(T, B_n)/A(T_0, B_n)$ decreases with increasing temperature (Fig. 8) in accordance with the usual thermal damping factor [34]. The in-plane effective mass of 2D electrons has been determined by fitting the experimental data for the temperature dependence of $A(T, B_n)/A(T_0, B_n)$ to eqn (2). A similar analysis made for all the oscillation peaks observed in the magnetic-field range from 1.1 to 2.3 T have established that the in-plane effective mass of 2D electrons in each sample (Table 1) is essentially independent of magnetic field.

It should be noted that the determination of $m^*$ from the temperature dependence of the amplitude of SdH oscillations becomes complicated in the samples where two subbands are populated. We were, however, able to determine the in-plane effective mass of the electrons in the first and second subbands of the sample with $t_S = 100$ Å as shown in Table 1.

The effective mass of 2D electrons in the modulation-doped In$_{0.53}$Ga$_{0.47}$As/In$_{0.52}$Al$_{0.48}$As heterojunctions with different spacer thickness is constant within about 8% (Table 1). Our results for $m^*$ of 2D electrons in both the first and second subbands are in good agreement with the bulk effective mass in In$_{0.53}$Ga$_{0.47}$As where $m^*$ was reported [36, 37] to be in the range between 0.039$m_0$ and 0.041$m_0$. This indicates that both the nonparabolicity of the conduction band of In$_{0.53}$Ga$_{0.47}$As and the wavefunction penetration into the In$_{0.52}$Al$_{0.48}$As barrier/spacer layer have no significant effects on $m^*$ of 2D electrons in our samples. However,
Fig. 7. Experimental results showing the effects of temperature on the SdH oscillations in magnetoresistance observed in sample MV576A. The measurements were made at 3.25 K (full triangles), 7.00 K (open circles), and 12.01 K (full circles). The full curves through the experimental data points are intended as a guide to the eye.

the values obtained for \( m^* \) of electrons in the second subband of the samples with \( t_S = 0 \) and 100 Å are somewhat smaller than that \((=0.045 \pm 0.003)m_0\) found [15, 32] for electrons in the second subband of \( \delta \)-doped \( \text{In}_{0.53}\text{Ga}_{0.47}\text{As/In}_{0.52}\text{Al}_{0.48}\text{As} \) heterojunctions with \( t_S = 60 \) Å. The discrepancy could be due to variations in the nominal compositional parameters of the samples used in each study.

3.3. The quantum and transport lifetimes of 2D electrons

The SdH oscillations and Hall effect measurements allow the determination of both the quantum and transport lifetimes of electrons in each subband of modulation-doped \( \text{In}_{0.53}\text{Ga}_{0.47}\text{As/In}_{0.52}\text{Al}_{0.48}\text{As} \) heterojunctions. Hence the relative importance of various scattering mechanisms including ionized impurity scattering, alloy scattering and intersubband scattering can be investigated. The quantum lifetime \( (\tau_q) \) can be determined from the magnetic-field dependence of the amplitude of the SdH oscillations (i.e. Dingle plots) at a constant temperature provided that the electron effective mass is known [34, 35]. Figure 9 shows examples for the Dingle plots for all the samples investigated. There is a good agreement between the experimental data and the straight line described by [25]

\[
\ln \left[ \frac{A(T, B_n) \cdot B_n^{-1/2} \cdot \sinh \chi}{\chi} \right] = C - \frac{\pi m^*}{e \tau_q} \frac{1}{B_n},
\]

where \( \chi = 2\pi^2 k_B T/(\hbar \omega_c) \), \( \omega_c(= eB_n/m^*) \) is the cyclotron frequency and \( C \) is a constant. The quantum lifetime has been obtained from the slope of the Dingle plot using eqn (3) together with the measured values of \( m^* \) (Table 1). It is evident from Fig. 9 that the deviation of experimental data from linearity is insignificant. This indicates that \( \tau_q \) is independent of magnetic field as assumed in eqns (2) and (3). The good linearity
Fig. 8. Temperature dependence of the normalized amplitude of the oscillation extremum at $B_n$ measured for samples: A, MV576A and B, MV578A. The data points represented by the full circles and open circles correspond to the oscillations arising from the first and second subbands, respectively. The full and dashed curves are the best fits of eqn (2) to the experimental data for the first and second subbands, respectively.
Fig. 9. Determination of the quantum lifetime. The data points represented by the full squares, full circles, open circles, full inverted triangles and full diamonds correspond to the samples MV572A (second subband), MV576A (first subband), MV576A (second subband), MV577 (first subband) and MV578A (first subband), respectively. The straight lines are the least-squares fits of eqn (3) to each set of the experimental data.

of the Dingle plots corresponding to the SdH oscillations arising from the first subband of the samples with \( t_S = 100 \) and 200 Å (see Fig. 9) also implies that intermodulation [24, 38–40] of the first-subband oscillations due to the population of the second subband is unimportant in these samples. Indeed, for each sample studied, the quantum lifetime is found to remain constant within 2% in the whole temperature and magnetic-field ranges of the measurements. For the samples with \( t_S = 0, 100 \) and 200 Å, the amplitude of the long-period oscillations originating from the electrons in the second subband is much larger than the amplitude of the short-period oscillations from the first subband (see Figs 2A, 3A and 4A). This implies that the quantum lifetime of electrons in the second subband is longer than that of electrons in the first subband.

It is possible to extract quantitative information about the quantum lifetime of electrons in both populated subbands in only one of the samples studied as shown in Table 1. This gives a value of \( \tau_{q2}/\tau_{q1} \approx 1.7 \) for the sample with \( t_S = 100 \) Å. It is worth noting that in modulation-doped GaAs/Al\(_x\)Ga\(_{1-x}\)As heterojunctions a much higher value \( (\tau_{q2}/\tau_{q1} \approx 3.0) \) was reported [38, 40–44] and explained (see for instance [41, 44]) by assuming that the scattering is due to the linearly screened electrostatic potential of the ionized donors in the Al\(_x\)Ga\(_{1-x}\)As barrier. The relatively smaller value obtained here for the ratio \( \tau_{q2}/\tau_{q1} \) of modulation-doped In\(_{0.53}\)Ga\(_{0.47}\)As/In\(_{0.52}\)Al\(_{0.48}\)As heterojunctions could be attributed to the presence of alloy scattering in addition to remote ionized impurity scattering. The markedly higher value of \( \tau_q \) for the sample with \( t_S = 400 \) Å (Table 1) is in accord with the theoretical predictions [45] that the broadening of the Landau levels decreases (i.e. \( \tau_q \) increases) due to enhanced screening effects in the samples with wide spacer layer. However, it should be noted that intersubband scattering effects are absent in this sample. This may also lead to the observed enhancement in the value of \( \tau_q \) when compared to the other samples studied.

Since the effective mass and carrier density of 2D electrons in the two subbands are available (Table 1),
the transport mobility \( \mu_i \) of electrons in each subband can be estimated from the magnetoresistance measurements in the classical regime, as explained elsewhere (see for instance [46–50]). Figure 10A shows the magnetoresistance \( R_{xx}(B) \) at low magnetic fields well below the onset of quantum oscillations. It can be seen that \( R_{xx}(B) \) is approximately constant for the samples with \( t_S = 0 \) and 100 Å. For the samples with \( t_S = 200 \) and 400 Å, however, a positive magnetoresistance is observed, which indicates the presence of a parallel conduction channel in these samples. The resistivity \( \rho_{xx} \) versus \( B \) curves for the samples with \( t_S = 0 \) and 100 Å are plotted in Fig. 10B on an expanded scale. It is now evident that all the samples studied exhibit positive magnetoresistance at low magnetic fields, the magnitude of which varies from one sample to another. Since \( (N_1 + N_2) \approx N_H \) and the variation of magnetoresistance with magnetic field is very small for the samples with \( t_S = 0, 100 \) and 200 Å, we believe that the parallel conduction in these samples is due to 2D electrons in the two populated subbands. For the sample with a 400 Å thick spacer layer, however, only one subband is populated, the 2D carrier density is almost half of the sheet carrier density \( (N_1 \approx N_H/2) \) and the magnetoresistance shows much stronger variation with magnetic field when compared to the other samples. Hence, the parallel conduction in this sample is assumed to be due to 2D electrons in the first subband and bulk carriers outside the 2D channel.

The two-band model predicts a variation in the resistivity \( \rho_{xx} \) with magnetic field \( B \) given by [46–48]

\[
\rho_{xx} = \frac{(D_1 + D_2)/[(D_1 + D_2)^2 + (A_1 + A_2)^2]}
\]

where

\[
D_i = \frac{N_i e^2}{m^*_i \left( \frac{\tau_i}{1 + \omega_i^2 \tau_i^2} \right)}
\]

is the diagonal term of the conductivity matrix for the \( i \)th conducting channel, \( A_i = \omega_i^2 \tau_i D_i \) is the off-diagonal term and \( \tau_i \) is the transport lifetime of electrons in the \( i \)th conducting channel. In the case of
Fig. 10.—continued.
the samples with $t_S = 0, 100$ and $200$ Å, the subscript $i (= 1, 2)$ refers to the first and second subbands, respectively. For the sample with $t_S = 400$ Å, the single subband populated by 2D electrons and the parallel conducting channel containing the 3D carriers are labelled by $i = 1$ and $i = 2$, respectively. The experimental $\rho_{xx}(B)$ data can be accurately fitted to eqn (4) using a standard software package with the measured values of carrier density and effective mass (see Table 1) as input parameters and by taking the transport lifetimes $\tau_{t1}$ and $\tau_{t2}$ as adjustable parameters. There is an excellent fit between eqn (4) and the experimental data as shown in Fig. 10B. The transport lifetimes $\tau_{t1}$ and $\tau_{t2}$ (and hence the transport mobilities $\mu_{t1}(= e\tau_{t1}/m_i^* \mu)$ and $\mu_{t2}(= e\tau_{t2}/m_i^*)$) of 2D electrons in the first and second subbands of the samples with $t_S = 0, 100$ and $200$ Å are obtained from the curve fitting procedure with an accuracy of better than 0.1% and presented in Table 1 along with the Hall mobility ($\mu_H$). The transport mobility of electrons in the first subband is substantially higher than that of electrons in the second subband (Table 1). Intersubband scattering is expected to reduce substantially the mobility of the first subband from that in the single subband case [30, 51–53] and consequently the average mobility ($\mu_H$) becomes smaller when the second subband becomes populated by electrons.

The case for the sample with $t_S = 400$ Å is, however, completely different from that of the samples in which two subbands are populated. The best fit of eqn (4) to the experimental $\rho_{xx}(B)$ data for the sample with $t_S = 400$ Å, obtained by using $N_1 = 5.1 \times 10^{15}$ m$^{-2}$, $N_2 = (N_H - N_1) = 6.2 \times 10^{15}$ m$^{-2}$, $m_1^* = 0.042m_0$, and $m_2^* = 0.089m_0$ (the effective mass of electrons in bulk In$_{0.52}$Al$_{0.48}$As [36]) as input parameters, yields $\mu_{t1} = 9.16$ m$^2$ V$^{-1}$ s$^{-1}$ for 2D electrons and $\mu_{t2} = 0.052$ m$^2$ V$^{-1}$ s$^{-1}$ for 3D charge carriers in the parallel conducting channel. The transport mobility $\mu_{t1}$ is almost twice that of the Hall mobility $\mu_H$. On the other hand, the value ($=0.052$ m$^2$ V$^{-1}$ s$^{-1}$) obtained for $\mu_{t2}$ is comparable to that of the mobility measured [54, 55] for 3D electrons in Si-doped In$_{0.52}$Al$_{0.48}$As epitectal layers with a doping level similar to our samples. Therefore we conclude that the parallel conduction giving rise to the strong positive magnetoresistance for the sample with $t_S = 400$ Å is due to undepleted donors in the Si-doped In$_{0.52}$Al$_{0.48}$As barrier [22, 46–48], the concentration of which increases as the spacer thickness is increased [22]. It is also clear from Fig. 10B that the parallel conduction due to the two populated subbands of a 2D electron gas, which have different but comparable transport mobilities, produce much smaller positive magnetoresistance when compared with the case of the usual parallel conduction due to 2D electrons at the heterointerface and 3D electrons in undepleted donors in the barrier. The results also indicate that the spacer layer thickness in modulation-doped heterojunctions is an important parameter that plays a role in the determination of the onset of parallel conduction due to charge carriers outside the 2D channel [22].

The observation that the transport mobility of electrons in the first subband is higher than that of electrons in the second subband of modulation-doped In$_{0.55}$Ga$_{0.47}$As/In$_{0.52}$Al$_{0.48}$As heterojunctions with $t_S = 0, 100$ and $200$ Å is in line with the results obtained previously for modulation-doped GaAs/Al$_x$Ga$_{1-x}$As heterojunctions [43, 49] and GaAs/Al$_x$Ga$_{1-x}$As as multiple quantum wells [56]. The argument for the observed higher electron mobility in the first subband was based on the larger Fermi wavevector and reduced Coulomb scattering in the first subband [56]. However, contradictory results were also reported [42] for modulation-doped GaAs/Al$_x$Ga$_{1-x}$As heterojunctions. These were explained by considering remote ionized impurity scattering. Since, on average, the electrons in the first subband are closer to the interface and hence closer to the ionized donors in the barrier, they would be scattered more strongly than those in the second subband. The discrepancy between the results of different studies was often attributed to the differences in spacer thickness and background impurity concentrations of the samples used in each work and to the way in which the second subband was populated [43]. Fletcher et al. [5] reported that the second-subband transport mobility can be greater or smaller than that of the first subband of modulation-doped GaAs/Al$_x$Ga$_{1-x}$As heterojunctions, depending on the illumination conditions. Zaremba [50] showed that the relative magnitudes of the transport mobilities in the two subbands of modulation-doped GaAs/Al$_x$Ga$_{1-x}$As heterojunctions obtained by Van Houten et al. [49] could be reversed when intersubband scattering is included in the calculations. We
have carried out a theoretical fit to our \( \mu_{xx}(B) \) results for the modulation-doped In\(_{0.53}\)Ga\(_{0.47}\)As/In\(_{0.52}\)Al\(_{0.48}\)As heterojunctions with two subband occupancy using the model proposed in Ref. [50]. The results indicate that there is not any significant difference in the \( \mu_{t1} \) and \( \mu_{t2} \) values from those obtained using eqn (4).

The quantum mobility \( \mu_{q1} = e\tau_{q1}/m^* \) and the transport mobility \( \mu_{t1} \) of 2D electrons in each subband and the Hall mobility \( \mu_H \) are plotted versus spacer layer thickness in Fig. 11. It is clear from the figure that the values obtained for \( \mu_{q1} \) and \( \mu_{q2} \) are substantially smaller than those of \( \mu_{t1} \) and \( \mu_H \). It is also worth noting that the quantum mobility does not vary much with increasing the spacer thickness from 0 to 200 Å, and it becomes markedly higher for the sample with a 400 Å thick spacer layer. The initial enhancement of \( \mu_{t1} \) and \( \mu_H \) can be attributed to the reduction in remote ionized impurity scattering with increasing spacer thickness, since alloy scattering is expected to be insensitive to variations in the spacer layer thickness [30, 31]. A comparison between the spacer-thickness dependences of the quantum mobility and transport mobility suggests that the dominant low-temperature scattering mechanisms (remote ionized impurity scattering and alloy scattering) have markedly different effects on \( \mu_q \) and \( \mu_t \) of 2D electrons in modulation-doped In\(_{0.53}\)Ga\(_{0.47}\)As/In\(_{0.52}\)Al\(_{0.48}\)As heterojunctions.

Theoretical calculations and interpretation of the difference between the quantum lifetime and transport lifetime of electrons occupying only one subband have been reported often in the literature [1–7, 27, 28]. However, such calculations are not yet available for the case of two subband occupancy. Therefore we could only compare the present results with the predictions of the existing theoretical models based on single subband occupancy. The transport-to-quantum lifetime ratios of electrons in the first subband \( (\tau_{t1}/\tau_{q1}) \) of the samples with \( t_S = 100, 200 \) and \( 400 \) Å are 9.7, 9.1 and 5.2, respectively; while those for the second subband \( (\tau_{t2}/\tau_{q2}) \) of the samples with \( t_S = 0 \) and \( 100 \) Å are 4.2 and 2.8, respectively (Table 1). Theoretical calcula-
tions relating the transport lifetime to the quantum lifetime predict a $\tau_1/\tau_q$ ratio much greater than unity for small-angle scattering and equal to or smaller than unity for large-angle scattering in the extreme quantum limit for single subband occupancy [1–4, 6]. This implies that in our samples the scattering of electrons in the first subband with remote ionized impurities and alloy disorder is on average forward displaced in momentum space. The relatively smaller values found for the ratio $\tau_{12}/\tau_{q2}$ of the second subband suggest mainly large-angle scattering [25].

Recently, Kearney et al. [7] calculated the ratio $\tau_1/\tau_q$ for In$_{0.53}$Ga$_{0.47}$As/InP quantum wells of single subband occupancy where they considered alloy scattering, scattering from uniform background ionized impurities, and remote ionized impurities with a sheet charge distribution. By taking the combined effects of alloy scattering and remote ionized impurity scattering into account, they estimated $\tau_1/\tau_q$ to be in the range from about 3.0 to 4.5 for quantum wells with a sheet carrier density of $1 \times 10^{16}$ m$^{-2}$. Kearney et al. [7] also estimated a value of about 1.5 for $\tau_1/\tau_q$ due to alloy scattering and background impurity scattering. Both sets of values are comparatively smaller than those (Table 1) obtained for the modulation-doped In$_{0.53}$Ga$_{0.47}$As/In$_{0.52}$Al$_{0.48}$As heterojunctions with a similar sheet carrier density. It appears therefore that in our samples background impurity scattering is not significant but alloy scattering and remote ionized impurity scattering are the dominant mechanisms which determine the transport and quantum mobilities.

It is also instructive to compare the present results for the transport and quantum lifetimes of electrons in modulation-doped In$_{0.53}$Ga$_{0.47}$As/In$_{0.52}$Al$_{0.48}$As heterojunctions with those reported previously for In$_{0.53}$Ga$_{0.47}$As/InP quantum wells [8–12] and other In$_x$Ga$_{1-x}$As-based HEMT structures [13, 14] in which only one subband is populated and alloy disorder scattering is also expected to be significant. Based on the value of $\tau_1/\tau_q$ it was suggested [11, 12] that, for modulation-doped In$_{0.53}$Ga$_{0.47}$As/InP narrow quantum wells, interface roughness scattering is a more effective scattering mechanism than ionized impurity scattering. Wiesner et al. [10], however, pointed out that ionized impurity scattering is the dominant scattering mechanism in modulation-doped, narrow quantum wells. By using SdH effect measurements in combination with a persistent photoconductivity effect, Van der Burgt et al. [13] studied the carrier density dependence of both the transport and quantum lifetimes of 2D electrons in the first subband of δ-doped pseudomorphic GaAs/Ga$_{0.8}$In$_{0.2}$As/Ga$_{0.75}$In$_{0.25}$As heterostructures with a 50 Å thick spacer layer. They found that $\tau_1$ is larger than $\tau_q$ by a factor of about 4.5 to 8.5 depending on the carrier density. They concluded that the transport lifetime in these structures is determined mainly by large-angle scattering such as cluster scattering [57] due to the nonuniform distribution of In in the Ga$_{0.8}$In$_{0.2}$As quantum well, while the quantum lifetime is dominated by the small-angle scattering mainly such as remote ionized impurity scattering. Recently, Diaz-Paniagua et al. [14] reported a value of about 4 for the ratio $\tau_1/\tau_q$ of In$_{0.18}$Ga$_{0.82}$As/Al$_{0.20}$Ga$_{0.80}$As quantum wells with a 20 Å thick spacer layer and noted that large-angle scattering mechanisms such as In-cluster scattering should be taken into account. Despite the effects of intersubband scattering which reduce primarily the transport lifetime of electrons in the first subband [30, 51–53], the values obtained for $\tau_{11}/\tau_{q1}$ of our samples with $t_S = 100$ and 200 Å (see Table 1) are relatively larger than those found for pseudomorphic GaAs/Ga$_{0.8}$In$_{0.2}$As/Ga$_{0.75}$In$_{0.25}$As heterostructures [13] and In$_{0.18}$Ga$_{0.82}$As/Al$_{0.20}$Ga$_{0.80}$As quantum wells [14] where only the first subband was populated. Therefore, we believe that In-cluster scattering does not have a significant effect on the transport mobility of 2D electrons in modulation-doped In$_{0.53}$Ga$_{0.47}$As/In$_{0.52}$Al$_{0.48}$As heterojunctions.

4. Conclusions

The carrier density, effective mass, quantum mobility and transport mobility of 2D electrons in modulation-doped In$_{0.53}$Ga$_{0.47}$As/In$_{0.52}$Al$_{0.48}$As heterojunctions have been determined as a function of the spacer layer thickness ($t_S$) in the range between 0 and 400 Å by SdH effect and Hall effect measurements. The 2D carrier density in the first subband decreases rapidly with increasing the spacer layer thickness from 0 to 400 Å, while that in the second subband being much smaller than that in the first subband decreases slightly with increasing
spacer thickness. The values found for the effective mass of 2D electrons in all the samples are similar to that of 3D electrons in bulk In$_{0.53}$Ga$_{0.47}$As, indicating that the effects of both the nonparabolicity of the conduction band of In$_{0.53}$Ga$_{0.47}$As and the wavefunction penetration into the In$_{0.52}$Al$_{0.48}$As barrier/spacer layer are not significant. The quantum mobility is approximately constant for the samples with spacer layer thickness in the range from 0 to 200 Å and becomes markedly higher for the sample with a 400 Å thick spacer layer. The quantum mobility of electrons in the first subband is substantially smaller than that of electrons in the second subband, while the quantum mobility of the first subband is smaller than that of the second subband. The results obtained for the transport-to-quantum lifetime ratios of the respective subbands indicate that the scattering of electrons in the first subband with remote ionized impurities and alloy disorder is on average forward displaced in momentum space, while the electrons in the second subband undergo mainly large-angle scattering.

Acknowledgements—We are grateful to TÜBITAK (TBAG-1676) and Hacettepe University Research Fund (project No: 99.02.602.004) for financial support.

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