Journal of Computational and Applied Mathematics 229 (2009) 152–157

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

Modified estimators for the change point in hazard function

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ARTICLE INFO

Article history:
Received 29 August 2007
Received in revised form 25 October 2007

Keywords:
Change point
Constant hazard
Failure rate
Survival analysis

ABSTRACT

We propose the consistent estimators for the change point in hazard function by improving the estimators in [A.P. Basu, J.K. Ghosh, S.N. Joshi, On estimating change point in a failure rate, in: S.S. Gupta, J.O. Berger (Eds.), Statistical Decision Theory and Related Topics IV, vol. 2, Springer-Verlag, 1988, pp. 239–252] and [H.T. Nguyen, G.S. Rogers, E.A. Walker, Estimation in change point hazard rate model, Biometrika 71 (1984) 299–304]. By a simulation study, we show that the proposed estimators are more efficient than the original estimators in many cases.

1. Introduction

Researchers in the medical area are concerned with a test of a constant failure rate against the alternative for a failure rate involving a single change point in a hazard model. Let T denote independent identically distributed random variable of survival times. The hazard model of T is given by

$$h(t) = \begin{cases} \alpha & 0 \le t \le \tau \\ \beta & t > \tau \end{cases} \tag{1}$$

where $\alpha > 0, \beta > 0, \tau > 0, \alpha$ and β are hazard rates and τ is the change point. Here, the hazard function, $h(t)$, is assumed to be a constant α until time τ and a constant β after time τ. The change point is a parameter of interest in medical and biological researches. In these fields, some of the recent studies in literature can be given as Gupta et al. [1], Tabnak et al. [2], Faucett et al. [3], Gijbels and Gurler [4] etc.

There are some important studies examining the hazard model in literature such as Nguyen et al. [5], Basu et al. [6], Ghosh and Joshi [7], Ghosh et al. [8,9] etc. These studies are described in the next section and the modified estimators are presented in the third section. A simulation study is performed and the results of this simulation are discussed in the fourth section.

2. Traditional estimators

It is well known that the probability density function and survival function of a random variable T are, respectively, given by

$$f(t) = \begin{cases} \alpha\, \mathrm{e}^{-\alpha t} & 0 \le t \le \tau \\ \beta\, \mathrm{e}^{-(\alpha-\beta)\tau-\beta t} & t > \tau \end{cases} \tag{2}$$

and

$$S(t) = \begin{cases} \mathrm{e}^{-\alpha t} & 0 \le t \le \tau \\ \mathrm{e}^{-(\alpha-\beta)\tau-\beta t} & t > \tau. \end{cases} \tag{3}$$

Note that $f(t)$ and $S(t)$ have a jump point at τ [10–12].

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doi:10.1016/j.cam.2008.10.019

2.1. NRW estimator

Nguyen et al. [5] developed a consistent estimator for the change point in (1) as follows:

$$X_n(t) = \frac{\sqrt{\upsilon(t)}}{n}\left\{[n - R(t)]\log\frac{n}{n - R(t)} - R(t)\right\} + \frac{R(t)\,E(t)}{n} - \frac{\bar{T}}{n}\log\left(\frac{n}{n - R(t)}\right) \tag{4}$$

where $\bar{T} = \frac{T_1 + T_2 + \cdots + T_n}{n}$ is the mean of the sample; $R(t) = \sum_{i=1}^{n} I_{[T_i \le t]}$ is the number of left-hand portion of the sample; $E(t) = \frac{\sum_{i=R(t)+1}^{n} T_i}{n - R(t)}$ and $\upsilon(t) = \frac{\sum_{i=R(t)+1}^{n} T_i^2}{n - R(t)} - \{E(t)\}^2$ are the mean and variance of the right-hand portion of the sample, respectively.

Here I is an indicator function and the survival times, $T_1, \ldots, T_n$, are ordered as $T_1 \le \cdots \le T_n$. A value of t, such that $X_n(t)$ is close to 0, is a candidate for the estimate of τ.

2.2. BGJ estimators

Basu et al. [6] proposed two estimators for the change point as

$$\hat{\tau}_{BGJ1} = \operatorname{Inf}\{t > 0 : y_n(t + h_n) - y_n(t) \le h_n\hat{\beta} + \varepsilon_n\} \tag{5}$$

$$\hat{\tau}_{BGJ2} = \operatorname{Inf}\{t > 0 : -y_n(t) - \log(1 - p_0) \le \hat{\beta}(\hat{\xi}_{p_0} - t) + \varepsilon_n\}$$

where $\hat{\beta}$ and $\hat{\xi}_{p_0}$ are the estimates of β and ξ_{p_0}, respectively; $p_0 < 1$; $y_n(t) = -\log[S_n(t)]$; $\varepsilon_n = \frac{c}{\sqrt{n}}(\log n)$; $h_n = \frac{1}{\sqrt[4]{n}}$. Here ξ_{p_0} is the p_0-th population quartile and c is a constant. Note that $\alpha > \beta$ in (1) for this method.

As Basu et al. [6] note that $y_n(t + h_n) - y_n(t) \le h_n$ is an estimate of the hazard rate $\tau(t)$ at t, the Eq. (5) also implies a test, for each fixed t, as

$$H_{0t} : h(t) = \beta \text{ vs } H_{1t} : h(t) > \beta$$

using the acceptance region $\{y_n(t + h_n) - y_n(t) \le h_n\hat{\beta} + \varepsilon_n\}$. According to (5), τ is estimated as the smallest t for which H_{0t} is accepted. In order to accept the null hypothesis, the following condition

$$\hat{h}_n(t) \le \hat{\beta} + \varepsilon_n \tag{6}$$

should be satisfied. Here $\hat{h}_n(t) = \frac{y_n(t + h_n) - y_n(t)}{h_n}$ [8].

Basu et al. [6] show that the estimates, $\hat{\tau}_{BGJ1}$ and $\hat{\tau}_{BGJ2}$, are consistent for τ. In addition, they find $\hat{\tau}_{BGJ1}$ more efficient than $\hat{\tau}_{BGJ2}$ by simulation, so we consider only $\hat{\tau}_{BGJ1}$ for the simulation study in this article.

Ghosh and Joshi [7] also investigate the asymptotic distribution of $\hat{\tau}_{BGJ1}$ and $\hat{\tau}_{BGJ2}$.

2.3. Bayesian estimator

Defining the prior function as

$$\pi(\alpha, \beta, t) = \frac{1}{\alpha\beta}. \tag{7}$$

Ghosh et al. [8] consider the following likelihood function:

$$L(\alpha, \beta, t \setminus \boldsymbol{D}) = \alpha^{R(t)} \mathrm{e}^{-\alpha Q(t)} \beta^{[n - R(t)]} \mathrm{e}^{-\beta[T_{\text{tot}} - Q(t)]}, \tag{8}$$

where $\mathbf{D}$ denotes the data $\{T_1, \ldots, T_n\}$ as $T_0 = 0$ and $T_{n+1} = \infty$; $A(t) = \sum_{i=1}^{n} T_i I_{[T_i \le t]}$ is the sum of survival times for the left-hand portion of the sample; $Q(t) = A(t) + \{n - R(t)\}t$ and $T_{\text{tot}} = \sum_{i=1}^{n} T_i$.

Multiplying (7) with (8), Ghosh et al. [8] obtain the posterior function as

$$\pi(\alpha, \beta, t \setminus \boldsymbol{D}) \propto \alpha^{R(t) - 1} \mathrm{e}^{-\alpha Q(t)} \beta^{[n - R(t) - 1]} \mathrm{e}^{-\beta[T_{\text{tot}} - Q(t)]} \tag{9}$$

and using (9), the marginal posterior function for the change point is given by

$$\pi(t \setminus \boldsymbol{D}) \propto \frac{(i - 1)!}{[Q(t)]^i} \mathrm{e}^{-T_{\text{tot}}\beta_0} \sum_{j=0}^{i-1}\left[\left\{\sum_{k=0}^{n-i+j-1} \frac{(T_{\text{tot}}\beta_0)^k}{k!}\right\} \frac{(n - i + j - 1)!}{j!} \frac{[Q(t)]^j}{T_{\text{tot}}^{n-i+j}}\right]. \tag{10}$$

A value of t, which maximizes (10), is a candidate for the estimate of τ.

3. Suggested estimators

In this section, considering the studies of Basu et al. [6] and Nguyen et al. [5], we focus on improving the estimation of the change point in the hazard function.

Motivated in [6], we propose to use $y_n^*(t) = -\log[F_n(t)]$ instead of $y_n(t)$ in the Eq. (5) in *BGJ*1 estimator, where $F_n(t)$ is the distribution function of T. Note that

$$0 < F(t) < 1. \tag{11}$$

Thus, this modification changes $\hat{h}_n(t)$ in the condition (6) as

$$\hat{h}_n^*(t) = \frac{y_n(t) - y_n(t+h_n)}{h_n}$$

so the acceptance region is

$$y_n(t) - y_n(t+h_n) \le h_n\hat{\beta} + \varepsilon_n \tag{12}$$

and we assume that $0.2 < h_n < 0.5$.

Under these conditions, the smallest t for which H_{0t} is accepted, $\hat{\tau}_1$, is the estimation of τ.

Theorem 1. *Let (1) and (11) hold. Then the proposed estimator, $\hat{\tau}_1$, is a consistent estimator for τ.*

Proof. Let

$$\sqrt{n}\,[y_n(t) - y(t)] = Op(1) \tag{13}$$

and

$$\sqrt{n}\left(\hat{\beta} - \beta\right) = O_p(1) \tag{14}$$

(see [6] for details).

For sufficiently small $\varepsilon > 0$ and for sufficiently large n, we can write

$$\begin{aligned} y_n(\tau+\varepsilon) - y_n(\tau+\varepsilon+h_n) &= y(\tau+\varepsilon) - y(\tau+\varepsilon+h_n) + Op\left(n^{-\frac{1}{2}}\right), \quad \text{using (13)} \\ &= h_n\hat{\beta} + O_p\left(n^{-\frac{1}{2}}\right), \quad \text{using (12) and (14).} \end{aligned} \tag{15}$$

In addition, it is clear that

$$\{y_n(\tau+\varepsilon) - y_n(\tau+\varepsilon+h_n) \le h_n\hat{\beta} + \varepsilon_n\} \Rightarrow \left(\hat{\tau}_1 \le \tau+\varepsilon\right). \tag{16}$$

Thus, using (15) and (16), we have

$$P\left(\hat{\tau}_1 \le \tau+\varepsilon\right) \to 1. \tag{17}$$

For sufficiently small $\varepsilon > 0$ we have $\tau - \varepsilon > 0$, hence using (13), we can write

$$\begin{aligned} y_n(t) - y_n(t+h_n) &= \log F(t+h_n) - \log F(t) + Op\left(n^{-\frac{1}{2}}\right), \quad \text{uniformly in } 0 \le t \le \tau-\varepsilon \\ &\ge h_n\alpha + O_p\left(n^{-\frac{1}{2}}\right) \quad (\text{Note that } \alpha > \beta) \\ &> h_n\beta + h_n\delta_\varepsilon + O_p\left(n^{-\frac{1}{2}}\right), \quad \text{using (12) and (14) where } \delta\varepsilon < \alpha - \beta. \end{aligned}$$

Thus,

$$P\left(\hat{\tau}_1 \ge \tau-\varepsilon\right) \to 1. \tag{18}$$

The relations between (17) and (18) prove the consistency of $\hat{\tau}_1$. □

Motivated in [5], we propose second estimator as

$$X_n^*(t) = \frac{X_n(t)}{\sum_{t=1}^{n} E\ (t) - \sum_{t=1}^{n} \sqrt{\upsilon\ (t)} - n\bar{T}}, \tag{19}$$

where $X_n(t)$ is computed by using (4). A value of t, $\hat{\tau}_2$, such that $X_n^*(t)$ is close to 0, is a candidate for the estimate of τ. Note that $X_n^*(t) > 0$, otherwise it is not taken in the analysis.

Theorem 2. . *Second proposed estimator, $\hat{\tau}_2$, is a consistent estimator for τ.*

The proof of this theorem is the same as the proof in [5] when $X_n(t)$ is replaced with $X_n^*(t)$.

4. Simulation

In this section, we try to find out which estimator has the smallest mean square error (MSE) under different conditions. To fulfill this aim, we coded a program for the simulation study in Visual Basic 6.0 using the following steps:

Table 1
Estimations of the change point and their MSE values.

α	β	τ	n	NRW	BGJ	Bayesian	Proposed1	Proposed2
1.5	1	2.5	25	0.0276[a]	2.5189	1.5215	1.9487	1.5059
				(6.1136)	(1.038)	(1.094)	(**0.4189**)	(1.1441)
			50	0.0135	2.8385	1.8447	2.1501	1.9689
				(6.1829)	(0.9066)	(0.5072)	(**0.1856**)	(0.5139)
			100	0.0065	2.9481	2.1788	2.2906	2.4522
				(6.2178)	(0.7323)	(0.1560)	(**0.0724**)	(0.3448)
2.8	0.5	3	25	0.0155	1.0787	0.8185	1.3368	0.8046
				(8.9118)	(4.2402)	(4.8011)	(**2.9462**)	(4.8622)
			50	0.0072	1.4541	1.0735	1.5820	1.0402
				(8.9568)	(2.9407)	(3.7612)	(**2.1794**)	(3.8948)
			100	0.0035	1.7272	1.3145	1.8089	1.2582
				(8.9793)	(2.2023)	(2.8909)	(**1.5667**)	(3.0978)
2.5	1.5	1.5	25	0.0166	1.5369	0.9129	1.1692	0.9040
				(2.2009)	(0.4339)	(0.3938)	(**0.1508**)	(0.4121)
			50	0.0081	1.7359	1.1072	1.2900	1.1859
				(2.2259)	(0.3938)	(0.1823)	(**0.0668**)	(0.1876)
			100	0.0039	1.8014	1.3078	1.3744	1.4881
				(2.2384)	(0.3236)	(0.0563)	(**0.0261**)	(0.1399)
3.8	1.5	2	25	0.0109	0.9561	0.6031	0.9741	0.5929
				(3.9567)	(1.2637)	(1.9742)	(**1.1384**)	(2.0031)
			50	0.0053	1.1661	0.7910	1.1478	0.7665
				(3.9788)	(0.8699)	(1.4887)	(**0.8032**)	(1.5509)
			100	0.0025	1.3598	0.9686	1.3107	0.9271
				(3.9898)	(0.5991)	(1.0910)	(**0.5413**)	(1.1858)
2	1	0.5	25	0.0207	0.6166	0.7142	0.4877	1.7841
				(0.2302)	(0.0303)	(0.0706)	(**0.0044**)	(1.9809)
			50	0.0101	0.5559	0.7673	0.4789	2.4926
				(0.2401)	(0.0065)	(0.0848)	(**0.0012**)	(4.3684)
			100	0.0048	0.5274	0.7968	0.4864	3.1601
				(0.2452)	(0.0015)	(0.0935)	(**0.0004**)	(7.4877)
2	0.5	1	25	0.0207	1.7257	0.9501	0.9094	1.7330
				(0.9595)	(1.4249)	(0.0413)	(**0.0354**)	(1.4904)
			50	0.0107	1.3679	1.0603	0.9347	3.0165
				(0.9792)	(0.3371)	(0.0367)	(**0.0078**)	(5.6085)
			100	0.0055	1.1690	1.1215	0.9668	4.3613
				(0.9895)	(0.0607)	(0.0351)	(**0.0020**)	(12.8699)
3.5	1.5	1	25	0.0118	1.1740	0.6494	0.8015	0.6489
				(0.9766)	(0.3977)	(0.1469)	(**0.0561**)	(0.1565)
			50	0.0058	1.2914	0.7780	0.8729	0.8771
				(0.9885)	(0.3598)	(0.0622)	(**0.0251**)	(0.0859)
			100	0.0028	1.2848	0.8985	0.9273	1.1638
				(0.9945)	(0.2614)	(0.0170)	(**0.0089**)	(0.1615)
3	1	0.5	25	0.0138	0.7171	0.5578	0.4573	1.2982
				(0.2366)	(0.1047)	(0.0215)	(**0.0049**)	(0.9552)
			50	0.0067	0.6026	0.6142	0.4726	2.0052
				(0.2434)	(0.0233)	(0.0248)	(**0.0014**)	(2.6572)
			100	0.0032	0.5472	0.6513	0.4856	2.6806
				(0.2468)	(0.0045)	(0.0283)	(**0.0004**)	(5.1479)
4.5	2.5	3	25	0.0092	–	0.5093	0.8393	0.5007
				(8.9449)	(–)	(6.2199)	(**4.7476**)	(6.2632)
			50	0.0045	–	0.6679	0.9962	0.6472
				(8.9731)	(–)	(5.4577)	(**4.0940**)	(5.5564)
			100	0.0022	–	0.8179	1.1493	0.7829
				(8.9871)	(–)	(4.7808)	(**3.5045**)	(4.9403)
4.8	1	1.5	25	0.0086	0.8042	0.4775	0.7682	0.4694
				(2.2243)	(0.6213)	(1.0599)	(**0.5871**)	(1.0767)
			50	0.0042	0.9659	0.62622	0.9012	0.6068
				(2.2374)	(0.4599)	(0.7805)	(**0.4020**)	(0.8162)
			100	0.0020	1.1256	0.7660	1.0227	0.7340
				(2.2440)	(0.3907)	(0.5554)	(**0.2627**)	(0.6086)
4	2	1	25	0.0103	0.9812	0.5718	0.7600	0.5643
				(0.9795)	(0.2139)	(0.2033)	(**0.0780**)	(0.2117)
			50	0.0051	1.1409	0.7057	0.8462	0.7386
				(0.9899)	(0.2172)	(0.0992)	(**0.0349**)	(0.1026)
			100	0.0024	1.2327	0.8373	0.9054	0.9259
				(0.9952)	(0.2059)	(0.0359)	(**0.0142**)	(0.0642)

(*continued on next page*)

Table 1 (*continued*)

α	β	τ	n	NRW	BGJ	Bayesian	Proposed1	Proposed2
5.8	0.5	4	25	0.0071	–	0.3952	0.6512	0.3884
				(15.9430)	(–)	(13.0048)	(**11.2620**)	(13.0533)
			50	0.0035	–	0.5183	0.7729	0.5022
				(15.9721)	(–)	(12.1342)	(**10.4616**)	(12.2474)
			100	0.0017	–	0.6346	0.8917	0.6074
				(15.9867)	(–)	(11.3376)	(**9.7094**)	(11.5245)
5	2.5	2.5	25	0.0083	–	0.4584	0.7554	0.4506
				(6.2087)	(–)	(4.1815)	(**3.1077**)	(4.2134)
			50	0.0040	–	0.6012	0.8965	0.5825
				(6.2298)	(–)	(3.6212)	(**2.6348**)	(3.6937)
			100	0.0019	–	0.7361	1.0343	0.7046
				(6.2403)	(–)	(3.1269)	(**2.2124**)	(3.2435)
6.5	1	2.5	25	0.0064	–	0.3526	0.5811	0.3466
				(6.2182)	(–)	(4.6192)	(**3.7202**)	(4.6450)
			50	0.0031	–	0.4624	0.6897	0.4481
				(6.2345)	(–)	(4.1609)	(**3.3151**)	(4.2204)
			100	0.0015	–	0.5663	0.7956	0.5420
				(6.2426)	(–)	(3.7487)	(**2.9428**)	(3.8456)
6.5	2	0.5	25	0.0064	0.6888	0.3478	0.41114	0.3547
				(0.2437)	(0.2102)	(0.0298)	(**0.0118**)	(0.0345)
			50	0.0036	0.7396	0.4103	0.4449	0.5102
				(0.2466)	(0.1979)	(0.0116)	(**0.0049**)	(0.0414)
			100	0.0017	0.6657	0.4667	0.4691	0.7362
				(0.2483)	(0.0838)	(0.0029)	(**0.0017**)	(0.1375)
5.5	2	1.5	25	0.0075	0.5460	0.4167	0.6797	0.4096
				(2.2275)	(1.0372)	(1.1845)	(**0.7189**)	(1.1999)
			50	0.0037	0.7349	0.5465	0.8045	0.5296
				(2.2390)	(0.7054)	(0.9221)	(**0.5269**)	(0.9558)
			100	0.0018	0.8688	0.6692	0.9192	0.6406
				(2.2447)	(0.5085)	(0.7032)	(**0.3751**)	(0.7552)
6	2.5	1	25	0.0069	0.6344	0.3820	0.5987	0.3755
				(0.9863)	(0.2047)	(0.3912)	(**0.1877**)	(0.3993)
			50	0.0034	0.7621	0.4996	0.6962	0.4854
				(0.9933)	(0.1412)	(0.2606)	(**0.1129**)	(0.2765)
			100	0.0016	0.8899	0.6092	0.7787	0.5874
				(0.9968)	(0.1225)	(0.1619)	(**0.0631**)	(0.1845)
7	2	1	25	0.0059	0.5488	0.3274	0.5257	0.3218
				(0.9882)	(0.2577)	(0.4591)	(**0.2487**)	(0.4667)
			50	0.0029	0.6580	0.4294	0.6165	0.4161
				(0.9942)	(0.1829)	(0.3336)	(**0.1671**)	(0.3496)
			100	0.0014	0.7643	0.5251	0.6992	0.5033
				(0.9972)	(0.1444)	(0.2333)	(**0.1063**)	(0.2570)
7.8	2	1.5	25	0.0053	–	0.2938	0.4842	0.2888
				(2.2341)	(–)	(1.4603)	(**1.0581**)	(1.4724)
			50	0.0026	–	0.3854	0.5747	0.3734
				(2.2422)	(–)	(1.2488)	(**0.8823**)	(1.2762)
			100	0.0012	–	0.4719	0.6630	0.4517
				(2.2463)	(–)	(1.0635)	(**0.7269**)	(1.1072)
7.5	3.5	2	25	0.0055	–	0.3056	0.5036	0.3004
				(3.9779)	(–)	(2.8769)	(**2.2677**)	(2.8946)
			50	0.0027	–	0.4008	0.5977	0.3883
				(3.9892)	(–)	(2.5645)	(**1.9948**)	(2.6049)
			100	0.0013	–	0.4908	0.6896	0.4697
				(3.9948)	(–)	(2.2848)	(**1.7458**)	(2.3506)

The number in parenthesis is the MSE value of the estimation. The bold number represents the smallest MSE value.

[a] Estimation value for the change point.

Step 1. We generate 1000 samples of size n using (2) with various values for the parameters.

Step 2. We use the data from 1000 samples in Step 1 to obtain the value of $\hat{\tau}$. Thus, we find 1000 values of $\hat{\tau}$ from 1000 samples for each n and for each parameter in (1). Let $\hat{\tau}$ represent both the estimators mentioned in Section 2 and the proposed estimators presented in Section 3.

Step 3. For each n and different values of parameters, the MSE of $\hat{\tau}$ is computed by $\mathrm{MSE}\left(\hat{\tau}\right) = \frac{1}{1000}\sum_{i=1}^{1000}\left(\hat{\tau} - \tau\right)^2$.

In this simulation study, we take sample sizes $n = 25, 50, 100$ and various values for parameters in (1), as shown in Table 1. The computed MSE values of the traditional estimators and the proposed estimators are also given in Table 1. From Table 1, we observe that first proposed estimator always has the smallest MSE, although the second proposed estimator has a smaller MSE than the NRW estimator generally, except for a few cases such as $\alpha = 2$, $\beta = 1$, $\tau = 0.5$; $\alpha = 2$, $\beta = 0.5$,

$\tau = 1$; $\alpha = 3$, $\beta = 1$, $\tau = 0.5$. Therefore, we can infer that the modified NRW estimator (second proposed estimator) is more efficient than the original NRW estimator in general and that the modified BGJ estimator (first proposed estimator) is the most efficient estimator in all conditions, whereas original BGJ estimator cannot find a suitable t that satisfies the condition (6) in some cases where we put hyphen in Table 1. In addition, when we further observe Table 1, we see that the efficiency of first proposed estimator gets higher when sample size gets larger.

5. Conclusion

This article emphasizes the problem of the change point estimation in hazard functions and paves the way for future work in this important area. Simulation results show that first proposed estimator can be used to obtain the most accurate estimate of the change point and also show that a modification of the NRW estimator improves the efficiency of this estimator.

Acknowledgment

The authors are grateful to Prof. Dr. Lawrence Michael Brown in Department of Mathematics at Hacettepe University for improving the grammar and mathematical issues of the entire paper.

References

[1] R.C. Gupta, N. Kannan, A. Raychaudhuri, Analysis of lognormal survival data, Mathematical Biosciences 139 (1997) 103–115.

[2] F. Tabnak, H.G. Muller, J.L. Wang, J.M. Chiou, R.K.P. Sun, A change-point model for reporting delays under change of AIDS case definition, European Journal of Epidemiology 16 (2000) 1135–1141.

[3] C.L. Faucett, N. Schenker, J.M.G. Taylor, Survival analysis using auxiliary variables via multiple imputation, with application to AIDS clinical trial data, Biometrics 58 (2002) 37–47.

[4] I. Gijbels, U. Gurler, Estimation of a change point in a hazard function based on censored data, Lifetime Data Analysis 9 (2003) 395–411.

[5] H.T. Nguyen, G.S. Rogers, E.A. Walker, Estimation in change point hazard rate model, Biometrika 71 (1984) 299–304.

[6] A.P. Basu, J.K. Ghosh, S.N. Joshi, On estimating change point in a failure rate, in: S.S. Gupta, J.O. Berger (Eds.), Statistical Decision Theory and Related Topics IV, vol. 2, Springer-Verlag, 1988, pp. 239–252.

[7] J.K. Ghosh, S.N. Joshi, On the asymptotic distribution of an estimate of the change point in a failure rate, Communications in Statistics: Theory and Methods 21 (1992) 3571–3588.

[8] J.K. Ghosh, S.N. Joshi, C. Mukhopadhyay, A Bayesian approach to the estimation of change point in a hazard rate, in: A.P. Basu (Ed.), Advances in Reliability, Elsevier Science Publications, 1993, pp. 141–170.

[9] J.K. Ghosh, S.N. Joshi, C. Mukhopadhyay, Asymptotics of a Bayesian approach to estimating change-point in a hazard rate, Communications in Statistics: Theory and Methods 25 (1996) 3147–3166.

[10] D.E. Matthews, V.T. Farewell, On a singularity in the likelihood for a change-point hazard rate model, Biometrika 72 (1985) 703–704.

[11] J.A. Achcar, S. Loibel, Constant hazard function models with a change point: A Bayesian analysis using markov chain Monte Carlo methods, Biometrical Journal 40 (1998) 543–555.

[12] S. Ghosal, J.K. Ghosh, T. Samanta, Approximation of the posterior distribution in a change-point problem, Annals of the Institute of Statistical Mathematics 51 (1999) 479–497.