Journal of Statistical Planning and Inference 139 (2009) 2552 – 2558

Contents lists available at ScienceDirect

Journal of Statistical Planning and Inference

journal homepage: www.elsevier.com/locate/jspi

Ratio and product estimators in stratified random sampling

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ARTICLE INFO

Article history:
Received 6 March 2008
Accepted 20 November 2008
Available online 30 November 2008

Keywords:
Ratio estimator
Product estimator
Regression estimator
Auxiliary information
Mean square error
Efficiency

ABSTRACT

Khoshnevisan et al. [2007. A general family of estimators for estimating population mean using known value of some population parameter(s). Far East Journal of Theoretical Statistics 22, 181–191] have introduced a family of estimators using auxiliary information in simple random sampling. They have showed that these estimators are more efficient than the classical ratio estimator and that the minimum value of the mean square error (*MSE*) of this family is equal to the value of *MSE* of regression estimator. In this article, we adapt the estimators in this family to the stratified random sampling and motivated by the estimator in Searls [1964. Utilization of known coefficient of kurtosis in the estimation procedure of variance. Journal of the American Statistical Association 59, 1225–1226], we also propose a new family of estimators for the stratified random sampling. The expressions of bias and *MSE* of the adapted and proposed families are derived in a general form. Besides, considering the minimum cases of these *MSE* equations, the efficient conditions between the adapted and proposed families are obtained. Moreover, these theoretical findings are supported by a numerical example with original data.

1. Introduction and notations

Auxiliary variable is commonly used in survey sampling to improve the precision of estimates. Whenever there is auxiliary information available, the researchers want to utilize it in the method of estimation to obtain the most efficient estimator. In some cases, in addition to mean of auxiliary variable, various parameters related to auxiliary variable such as standard deviation, coefficient of variation, skewness, kurtosis, correlation coefficient, etc. may also be known. For these cases, many authors such as Upadhyaya and Singh (1999), Sisodia and Dwivedi (1981), Singh and Tailor (2003) developed various estimators to improve the ratio estimators in the simple random sampling. Kadilar and Cingi (2003) adapted the estimators in Upadhyaya and Singh (1999) to the stratified random sampling. Singh and Vishwakarma (2008)proposed a new family of estimators in stratified random sampling.

In this study, under stratified random sampling without replacement scheme, we suggest two general families of estimators to estimate the population mean of the study variable, $\bar{Y}$, by considering the estimators in Searls (1964) and Khoshnevisan et al. (2007) and the optimum cases of these suggested families of estimators are also examined.

Consider a finite population $U=(u_1,u_2,\ldots,u_N)$ of size N and let y and x, respectively, be the study and auxiliary variables associated with each unit u_j $(j=1,2,\ldots,N)$ of the population. Let the population of size, N, is stratified into L strata with h-th stratum containing N_h units, where $h=1,2,\ldots,L$ such that $\sum_{h=1}^{L}N_h=N$. A simple random sample of size n_h is drawn without replacement from the h-th stratum such that $\sum_{h=1}^{L}n_h=n$. Let (y_{hi},x_{hi}) denote the observed values of y and x on i-th unit of the h-th stratum, where $i=1,2,\ldots,N_h$ and $h=1,2,\ldots,L$. Moreover, let $\bar{y}_h=\sum_{i=1}^{n_h}y_{hi}/n_h$, $\bar{y}_{st}=\sum_{h=1}^{L}W_h\bar{y}_h$, and $\bar{Y}_h=\sum_{i=1}^{N_h}y_{hi}/N_h$,

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doi:10.1016/j.jspi.2008.11.009

$\bar{Y}=\bar{Y}_{st}=\sum_{h=1}^{L}W_h\bar{Y}_h$ be the sample and population means of y, respectively, where $W_h=N_h/N$ is the stratum weight. Similar expressions for x can also be defined.

To obtain the bias and mean square error (MSE), let us define $e_0=(\bar{y}_{st}-\bar{Y})/\bar{Y}$ and $e_1=(\bar{x}_{st}-\bar{X})/\bar{X}$. Using these notations,

$$E(e_0)=E(e_1)=0,$$

$$V_{r,s}=\sum_{h=1}^{L}W_h^{r+s}\frac{E\lfloor(\bar{x}_h-\bar{X}_h)^r(\bar{y}_h-\bar{Y}_h)^s\rfloor}{\bar{X}^r\bar{Y}^s}. \tag{1}$$

From (1), we can write

$$E(e_0^2)=\frac{\sum_{h=1}^{L}W_h^2\gamma_hS_{yh}^2}{\bar{Y}^2}=V_{0,2},$$

$$E(e_1^2)=\frac{\sum_{h=1}^{L}W_h^2\gamma_hS_{xh}^2}{\bar{X}^2}=V_{2,0},$$

$$E(e_0e_1)=\frac{\sum_{h=1}^{L}W_h^2\gamma_hS_{xyh}}{\bar{X}\bar{Y}}=V_{1,1},$$

where

$$S_{yh}^2=\frac{\sum_{i=1}^{N_h}(y_{hi}-\bar{Y}_h)^2}{N_h-1},\quad S_{xh}^2=\frac{\sum_{i=1}^{N_h}(x_{hi}-\bar{X}_h)^2}{N_h-1},\quad S_{xyh}=\frac{\sum_{i=1}^{N_h}(y_{hi}-\bar{Y}_h)(x_{hi}-\bar{X}_h)}{N_h-1},\quad \gamma_h=\frac{1-f_h}{n_h}\quad\text{and}\quad f_h=\frac{n_h}{N_h}.$$

2. Adapted family of estimators

Khoshnevisan et al. (2007) introduced a general family of estimators for population mean using known value of some population parameters in simple random sampling given by

$$t=\bar{y}\left[\frac{a\bar{X}+b}{\alpha(a\bar{x}+b)+(1-\alpha)(a\bar{X}+b)}\right]^g. \tag{2}$$

Motivated by Khoshnevisan et al. (2007), we adapt this family to the stratified random sampling as

$$k=\bar{y}_{st}\left[\frac{a_{st}\bar{X}+b_{st}}{\alpha(a_{st}\bar{x}_{st}+b_{st})+(1-\alpha)(a_{st}\bar{X}+b_{st})}\right]^g, \tag{3}$$

where α is a suitable constant, $a_{st}(\neq 0)$ and b_{st} are either real numbers or the functions of the known parameters of the auxiliary variable, x, such as $\psi_1=\sum_{h=1}^{L}W_hS_{xh}$, $\psi_2=\sum_{h=1}^{L}W_hC_{xh}$, $\psi_3=\sum_{h=1}^{L}W_h\beta_{1h}(x)$, $\psi_4=\sum_{h=1}^{L}W_h\beta_{2h}(x)$, and $\psi_5=\sum_{h=1}^{L}W_h\rho_h$. We would like to remark that for various values of parameters in (3), we get nine ratio estimators and nine product estimators, as shown in Table 1. Note that k_1, in Table 1, is the classical ratio estimator and k_2 is the classical product estimator in the stratified random sampling.

Expressing k in terms of e_i ($i=0,1$), we can write (3) as

$$k=\bar{Y}(1+e_0)[1+\alpha\upsilon e_1]^{-g}, \tag{4}$$

where $\upsilon=a_{st}\bar{X}/(a_{st}\bar{X}+b_{st})$. Suppose $|\alpha\upsilon e_1|<1$ so that $[1+\alpha\upsilon e_1]^{-g}$ is expandable. Expanding the right hand side of (4) to the first order of approximation, we obtain

$$k-\bar{Y}=\bar{Y}\left[-g\alpha\upsilon e_1+\frac{g(g+1)}{2}\alpha^2\upsilon^2e_1^2+e_0-g\alpha\upsilon e_0e_1+\frac{g(g+1)}{2}\alpha^2\upsilon^2e_0e_1^2\right]. \tag{5}$$

Taking expectation of both sides in (5), we get the bias of the estimator k as

$$B(k)=\bar{Y}\left[\frac{g(g+1)}{2}\alpha^2\upsilon^2V_{2,0}-g\alpha\upsilon V_{1,1}\right]. \tag{6}$$

Squaring both sides of (5) and then taking expectation, we get the MSE of the estimator k, to the first order approximation, as

$$MSE(k)=\bar{Y}^2(g^2\alpha^2\upsilon^2V_{2,0}+V_{0,2}-2g\alpha\upsilon V_{1,1}). \tag{7}$$

Table 1
Some members of the family of estimators of k.

$g=1$ ratio estimators	$g=-1$ product estimators	α	a_{st}	b_{st}
$k_1=\bar{y}_{st}\left(\frac{\bar{X}}{\bar{x}_{st}}\right)$	$k_2=\bar{y}_{st}\left(\frac{\bar{x}_{st}}{\bar{X}}\right)$	1	1	0
$k_3=\bar{y}_{st}\left(\frac{\bar{X}+\psi_2}{\bar{x}_{st}+\psi_2}\right)$	$k_4=\bar{y}_{st}\left(\frac{\bar{x}_{st}+\psi_2}{\bar{X}+\psi_2}\right)$	1	1	$\psi_2=\sum_{h=1}^{L}W_hC_{xh}$
$k_5=\bar{y}_{st}\left(\frac{\psi_4\bar{X}+\psi_2}{\psi_4\bar{x}_{st}+\psi_2}\right)$	$k_6=\bar{y}_{st}\left(\frac{\psi_4\bar{x}_{st}+\psi_2}{\psi_4\bar{X}+\psi_2}\right)$	1	$\psi_4=\sum_{h=1}^{L}W_h\beta_{2h}(x)$	$\psi_2=\sum_{h=1}^{L}W_hC_{xh}$
$k_7=\bar{y}_{st}\left(\frac{\psi_2\bar{X}+\psi_4}{\psi_2\bar{x}_{st}+\psi_4}\right)$	$k_8=\bar{y}_{st}\left(\frac{\psi_2\bar{x}_{st}+\psi_4}{\psi_2\bar{X}+\psi_4}\right)$	1	$\psi_2=\sum_{h=1}^{L}W_hC_{xh}$	$\psi_4=\sum_{h=1}^{L}W_h\beta_{2h}(x)$
$k_9=\bar{y}_{st}\left(\frac{\bar{X}+\psi_1}{\bar{x}_{st}+\psi_1}\right)$	$k_{10}=\bar{y}_{st}\left(\frac{\bar{x}_{st}+\psi_1}{\bar{X}+\psi_1}\right)$	1	1	$\psi_1=\sum_{h=1}^{L}W_hS_{xh}$
$k_{11}=\bar{y}_{st}\left(\frac{\psi_3\bar{X}+\psi_1}{\psi_3\bar{x}_{st}+\psi_1}\right)$	$k_{12}=\bar{y}_{st}\left(\frac{\psi_3\bar{x}_{st}+\psi_1}{\psi_3\bar{X}+\psi_1}\right)$	1	ψ_3	$\psi_1=\sum_{h=1}^{L}W_hS_{xh}$
$k_{13}=\bar{y}_{st}\left(\frac{\psi_4\bar{X}+\psi_1}{\psi_4\bar{x}_{st}+\psi_1}\right)$	$k_{14}=\bar{y}_{st}\left(\frac{\psi_4\bar{x}_{st}+\psi_1}{\psi_4\bar{X}+\psi_1}\right)$	1	$\psi_4=\sum_{h=1}^{L}W_h\beta_{2h}(x)$	$\psi_1=\sum_{h=1}^{L}W_hS_{xh}$
$k_{15}=\bar{y}_{st}\left(\frac{\bar{X}+\psi_5}{\bar{x}_{st}+\psi_5}\right)$	$k_{16}=\bar{y}_{st}\left(\frac{\bar{x}_{st}+\psi_5}{\bar{X}+\psi_5}\right)$	1	1	$\psi_5=\sum_{h=1}^{L}W_h\rho_h$
$k_{17}=\bar{y}_{st}\left(\frac{\bar{X}+\psi_4}{\bar{x}_{st}+\psi_4}\right)$	$k_{18}=\bar{y}_{st}\left(\frac{\bar{x}_{st}+\psi_4}{\bar{X}+\psi_4}\right)$	1	1	$\psi_4=\sum_{h=1}^{L}W_h\beta_{2h}(x)$

Minimization of (7) with respect to $t=g\alpha\upsilon$ yields its optimum value as

$$t^*=g\alpha\upsilon=\frac{V_{1,1}}{V_{2,0}}. \tag{8}$$

Substitution of (8) in (7) yields the minimum value of $MSE(k)$ as

$$MSE_{\min}(k)=\bar{Y}^2\left[V_{0,2}-\frac{V_{1,1}^2}{V_{2,0}}\right]=V(\bar{y}_{st})(1-\rho_{st}^2), \tag{9}$$

which is equal to the MSE of combined regression estimator.

For ratio estimators given in Table 1, we can express the MSE equation in (7) as

$$MSE(k_i)=\begin{cases}\bar{Y}^2(V_{0,2}+V_{2,0}-2V_{1,1}), & i=1,\\ \bar{Y}^2[V_{0,2}-2\upsilon_{(i-1)/2}V_{1,1}+\upsilon_{(i-1)/2}^2V_{2,0}], & i=3,5,7,\ldots,17\end{cases} \tag{10}$$

and for product estimators, the MSE equation is

$$MSE(k_j)=\begin{cases}\bar{Y}^2(V_{0,2}+V_{2,0}+2V_{1,1}), & j=2,\\ \bar{Y}^2[V_{0,2}+2\upsilon_{(j/2)-1}V_{1,1}+\upsilon_{(j/2)-1}^2V_{2,0}], & j=4,6,8,\ldots,18,\end{cases} \tag{11}$$

where

$$\upsilon_1=\frac{\bar{X}}{\bar{X}+\psi_2},\quad \upsilon_2=\frac{\psi_4\bar{X}}{\psi_4\bar{X}+\psi_2},\quad \upsilon_3=\frac{\psi_2\bar{X}}{\psi_2\bar{X}+\psi_4},\quad \upsilon_4=\frac{\bar{X}}{\bar{X}+\psi_1},$$

$$\upsilon_5=\frac{\psi_3\bar{X}}{\psi_3\bar{X}+\psi_1},\quad \upsilon_6=\frac{\psi_4\bar{X}}{\psi_4\bar{X}+\psi_1},\quad \upsilon_7=\frac{\bar{X}}{\bar{X}+\psi_5},\quad \upsilon_8=\frac{\bar{X}}{\bar{X}+\psi_4}.$$

3. Suggested family of estimators

Motivated by Searls (1964), we propose a new family of estimators given by

$$\eta=\lambda\bar{y}_{st}\left[\frac{a\bar{X}+b}{\alpha(a\bar{x}_{st}+b)+(1-\alpha)(a\bar{X}+b)}\right]^g, \tag{12}$$

where λ is a constant. We would like to remark that for various values of parameters in (12), we develop nine ratio estimators and nine product estimators, as shown in Table 2. Note that η_1, in Table 2, is the estimator proposed by Kadilar and Cingi (2005).

Table 2
Some members of the family of estimators of η.

$g=1$ ratio estimators	$g=-1$ product estimators	α	a_{st}	b_{st}	λ
$\eta_1=\lambda\bar{y}_{st}\left(\frac{\bar{X}}{\bar{x}_{st}}\right)$	$\eta_2=\lambda\bar{y}_{st}\left(\frac{\bar{x}_{st}}{\bar{X}}\right)$	1	1	0	λ
$\eta_3=\lambda\bar{y}_{st}\left(\frac{\bar{X}+\psi_2}{\bar{x}_{st}+\psi_2}\right)$	$\eta_4=\lambda\bar{y}_{st}\left(\frac{\bar{x}_{st}+\psi_2}{\bar{X}+\psi_2}\right)$	1	1	$\psi_2=\sum_{h=1}^{L}W_hC_{xh}$	λ
$\eta_5=\lambda\bar{y}_{st}\left(\frac{\psi_4\bar{X}+\psi_2}{\psi_4\bar{x}_{st}+\psi_2}\right)$	$\eta_6=\lambda\bar{y}_{st}\left(\frac{\psi_4\bar{x}_{st}+\psi_2}{\psi_4\bar{X}+\psi_2}\right)$	1	$\psi_4=\sum_{h=1}^{L}W_h\beta_{2h}(x)$	$\psi_2=\sum_{h=1}^{L}W_hC_{xh}$	λ
$\eta_7=\lambda\bar{y}_{st}\left(\frac{\psi_2\bar{X}+\psi_4}{\psi_2\bar{x}_{st}+\psi_4}\right)$	$\eta_8=\lambda\bar{y}_{st}\left(\frac{\psi_2\bar{x}_{st}+\psi_4}{\psi_2\bar{X}+\psi_4}\right)$	1	$\psi_2=\sum_{h=1}^{L}W_hC_{xh}$	$\psi_4=\sum_{h=1}^{L}W_h\beta_{2h}(x)$	λ
$\eta_9=\lambda\bar{y}_{st}\left(\frac{\bar{X}+\psi_1}{\bar{x}_{st}+\psi_1}\right)$	$\eta_{10}=\lambda\bar{y}_{st}\left(\frac{\bar{x}_{st}+\psi_1}{\bar{X}+\psi_1}\right)$	1	1	$\psi_1=\sum_{h=1}^{L}W_hS_{xh}$	λ
$\eta_{11}=\lambda\bar{y}_{st}\left(\frac{\psi_3\bar{X}+\psi_1}{\psi_3\bar{x}_{st}+\psi_1}\right)$	$\eta_{12}=\lambda\bar{y}_{st}\left(\frac{\psi_3\bar{x}_{st}+\psi_1}{\psi_3\bar{X}+\psi_1}\right)$	1	$\psi_3=\sum_{h=1}^{L}W_h\beta_{1h}(x)$	$\psi_1=\sum_{h=1}^{L}W_hS_{xh}$	λ
$\eta_{13}=\lambda\bar{y}_{st}\left(\frac{\psi_4\bar{X}+\psi_1}{\psi_4\bar{x}_{st}+\psi_1}\right)$	$\eta_{14}=\lambda\bar{y}_{st}\left(\frac{\psi_4\bar{x}_{st}+\psi_1}{\psi_4\bar{X}+\psi_1}\right)$	1	$\psi_4=\sum_{h=1}^{L}W_h\beta_{2h}(x)$	$\psi_1=\sum_{h=1}^{L}W_hS_{xh}$	λ
$\eta_{15}=\lambda\bar{y}_{st}\left(\frac{\bar{X}+\psi_5}{\bar{x}_{st}+\psi_5}\right)$	$\eta_{16}=\lambda\bar{y}_{st}\left(\frac{\bar{x}_{st}+\psi_5}{\bar{X}+\psi_5}\right)$	1	1	$\psi_5=\sum_{h=1}^{L}W_h\rho_h$	λ
$\eta_{17}=\lambda\bar{y}_{st}\left(\frac{\bar{X}+\psi_4}{\bar{x}_{st}+\psi_4}\right)$	$\eta_{18}=\lambda\bar{y}_{st}\left(\frac{\bar{x}_{st}+\psi_4}{\bar{X}+\psi_4}\right)$	1	1	$\psi_4=\sum_{h=1}^{L}W_h\beta_{2h}(x)$	λ

Expressing the estimator, η, in terms of e_i $(i=0,1)$, we can write (12) as

$$\eta=\lambda\bar{Y}(1+e_0)[1+\alpha\upsilon e_1]^{-g}. \tag{13}$$

Expanding the right hand side of (13) to first order approximation and subtracting $\bar{Y}$ from both sides we get

$$\eta-\bar{Y}=\lambda\bar{Y}\left[1-g\alpha\upsilon e_1+\frac{g(g+1)}{2}\alpha^2\upsilon^2e_1^2+e_0-g\alpha\upsilon e_0e_1\right]-\bar{Y}. \tag{14}$$

After similar procedure, described in Section 2, is applied to (14), the bias and the *MSE* equations of η can easily be obtained as

$$B(\eta)=\lambda\bar{Y}\left[\frac{g(g+1)}{2}\alpha^2\upsilon^2V_{2,0}-g\alpha\upsilon V_{1,1}\right]+\bar{Y}(\lambda-1), \tag{15}$$

$$MSE(\eta)=\bar{Y}^2\{\lambda^2V_{0,2}+(\lambda^2(2g^2+g)-\lambda(g^2+g))\alpha^2\upsilon^2V_{2,0}-2g\alpha\upsilon(2\lambda^2-\lambda)V_{1,1}+(\lambda-1)^2\}. \tag{16}$$

The $MSE(\eta)$ is minimized for the optimal value of λ given by

$$\lambda^*=\frac{A}{2B}, \tag{17}$$

where

$$A=(g^2+g)\alpha^2\upsilon^2V_{2,0}-2g\alpha\upsilon V_{1,1}+2,$$

$$B=V_{0,2}+(2g^2+g)\alpha^2\upsilon^2V_{2,0}-4g\alpha\upsilon V_{1,1}+1.$$

Thus, the minimum *MSE* of the estimator η is obtained as

$$MSE_{\min}(\eta)=\bar{Y}^2\left\{1-\frac{A^2}{4B}\right\}. \tag{18}$$

For ratio estimators given in Table 2, we can express the *MSE* equation in (16) as

$$MSE(\eta_i)=\begin{cases}\bar{Y}^2\{\lambda^{\bullet2}V_{0,2}+(3\lambda^{\bullet2}-2\lambda^{\bullet})V_{2,0}-2(2\lambda^{\bullet2}-\lambda^{\bullet})V_{1,1}+(\lambda^{\bullet}-1)^2\}, & i=1,\\ \bar{Y}^2\{\lambda^{+2}V_{0,2}+(3\lambda^{+2}-2\lambda^{+})\upsilon_{(i-1)/2}^2V_{2,0}-2\upsilon_{(i-1)/2}(2\lambda^{+2}-\lambda^{+})V_{1,1}+(\lambda^{+}-1)^2\}, & i=3,5,\ldots,17\end{cases} \tag{19}$$

and for product estimators, the *MSE* equation is

$$MSE(\eta_j)=\begin{cases}\bar{Y}^2\{\lambda^{\circ2}V_{0,2}+\lambda^{\circ2}V_{2,0}+2(2\lambda^{\circ2}-\lambda^{\circ})V_{1,1}+(\lambda^{\circ}-1)^2\}, & j=2,\\ \bar{Y}^2\{\lambda^{\tau2}V_{0,2}+\lambda^{\tau2}\upsilon_{(j/2)-1}^2V_{2,0}+2\upsilon_{(j/2)-1}(2\lambda^{\tau2}-\lambda^{\tau})V_{1,1}+(\lambda^{\tau}-1)^2\}, & j=4,6,\ldots,18.\end{cases} \tag{20}$$

The $MSE(\eta_i)$ and $MSE(\eta_j)$ are minimized for the optimal values of λ's given by

$$\lambda^{\bullet} = \frac{1+V_{2,0}-V_{1,1}}{1+3V_{2,0}-4V_{1,1}+V_{0,2}} = \frac{A^{\bullet}}{B^{\bullet}}, \quad \lambda^{+} = \frac{\upsilon^2_{(i-1)/2}V_{2,0}-\upsilon_{(i-1)/2}V_{1,1}+1}{V_{0,2}+3\upsilon^2_{(i-1)/2}V_{2,0}-4\upsilon_{(i-1)/2}V_{1,1}+1} = \frac{A^{+}}{B^{+}},$$

$$\lambda^{\circ} = \frac{1+V_{1,1}}{1+V_{0,2}+V_{2,0}+4V_{1,1}} = \frac{A^{\circ}}{B^{\circ}}, \quad \lambda^{\tau} = \frac{\upsilon_{(j/2)-1}V_{1,1}+1}{V_{0,2}+\upsilon^2_{(j/2)-1}V_{2,0}+4\upsilon_{(j/2)-1}V_{1,1}+1} = \frac{A^{\tau}}{B^{\tau}},$$

$$MSE_{\min}(\eta_i) = \begin{cases} \bar{Y}^2\left\{1-\dfrac{A^{\bullet 2}}{B^{\bullet}}\right\}, & i=1, \\ \bar{Y}^2\left\{1-\dfrac{A^{+2}}{B^{+}}\right\}, & i=3,5,\ldots,17, \end{cases} \tag{21}$$

$$MSE_{\min}(\eta_j) = \begin{cases} \bar{Y}^2\left\{1-\dfrac{A^{\circ 2}}{B^{\circ}}\right\}, & j=2, \\ \bar{Y}^2\left\{1-\dfrac{A^{\tau 2}}{B^{\tau}}\right\}, & j=4,6,\ldots,18. \end{cases} \tag{22}$$

4. Efficiency comparisons

In this section, we first compare the adapted family of estimators, given in (3), with the classical ratio estimator.

$$MSE(k_i) < MSE(k_1), \quad i=3,5,7,\ldots,17,$$

$$\upsilon_{(i-1)/2} < 2\frac{V_{1,1}}{V_{2,0}} - 1 \quad \text{for } \upsilon_{(i-1)/2} > 1,$$
$$\upsilon_{(i-1)/2} > 2\frac{V_{1,1}}{V_{2,0}} - 1 \quad \text{for } \upsilon_{(i-1)/2} < 1. \tag{23}$$

When condition (23) is satisfied, we can infer that the adapted family is more efficient than combined ratio estimator.

Second, we compare the suggested family of estimators, given in (12), with the classical ratio estimator.

$$MSE_{\min}(\eta_i) < MSE(k_1), \quad i=3,5,7,\ldots,17,$$

$$\left\{1-\frac{A^{+2}}{B^{+}}\right\} < (V_{0,2}+V_{2,0}-2V_{1,1}). \tag{24}$$

When condition (24) is satisfied, we can infer that the suggested family is more efficient than combined ratio estimator.

Third, we compare the suggested family of estimators, given in (12), with the ratio estimator proposed by Kadilar and Cingi (2005).

$$MSE_{\min}(\eta_i) < MSE_{\min}(\eta_1), \quad i=3,5,7,\ldots,17,$$

$$\frac{A^{\bullet 2}}{B^{\bullet}} - \frac{A^{+2}}{B^{+}} < 0. \tag{25}$$

When condition (25) is satisfied we can infer that the suggested family of estimators is more efficient than the ratio estimator proposed by Kadilar and Cingi (2005).

Finally, we compare the suggested family of estimators, given in (12), with the adapted family of estimators, given in (3).

$$MSE_{\min}(\eta_1) < MSE(k_1),$$

$$\left\{1-\frac{A^{\bullet 2}}{B^{\bullet}}\right\} < (V_{0,2}+V_{2,0}-2V_{1,1}), \tag{26}$$

$$MSE_{\min}(\eta_i) < MSE(k_i), \quad i=3,5,7,\ldots,17,$$

$$\left\{1-\frac{A^{+2}}{B^{+}}\right\} < [V_{0,2}-2\upsilon_{(i-1)/2}V_{1,1}+\upsilon^2_{(i-1)/2}V_{2,0}]. \tag{27}$$

It is clear that for the product estimators similar comparisons can be made and the related conditions can also be obtained. We would also like to note that the efficient condition between the minimum proposed and adapted families of estimators is obtained as

$$MSE_{\min}(\eta) < MSE_{\min}(k),$$

$$1 - \frac{A^2}{4B} < V_{0,2}(1-\rho_{st}^2).$$

5. Numerical example

In this section, we use the data concerning the number of teachers as study variable and the number of students as auxiliary variable in both primary and secondary schools for 923 districts at six regions (as 1: Marmara 2: Agean 3: Mediterranean 4: Central Anatolia 5: Black Sea 6: East and Southeast Anatolia) in Turkey in 2007 (Source: Ministry of Education, Republic of Turkey). The design of the sampling is formed by random selection of the districts from each region and the numbers of these selected districts are computed by Neyman allocation as

$$n_h = n\frac{N_h S_h}{\sum_{h=1}^{L} N_h S_h}. \tag{28}$$

Note that we take the sample of size, n, as 180 and the sample of sizes in each stratum, n_h, which are computed by (28), are shown in Table 3. In Table 3, we observe that the correlations between auxiliary and study variables are positive. Therefore, we use ratio estimators for the estimation of the population mean in this section.

The *MSE* values of the adapted and the proposed estimators have been obtained using (10) and (21), respectively. These values are given in Table 4. When we examine Table 4, we observe that the 13th adapted (k_{13}) and proposed (η_{13}) estimators have the smallest *MSE* values among their own family of estimators. From this result, we can infer that the 13th adapted and proposed estimators are more efficient than both the classical (k_1) and the Kadilar–Cingi estimator (η_1) for this data set. When we further examine Table 4, we see that $MSE(\eta_{13}) < MSE(k_i)$, where $i = 1, 3, 5, \ldots, 17$. From this result, we can conclude that the proposed estimators are more efficient than the adapted estimators for this data set. However, these results are expected results since conditions (23)–(27) are satisfied, as shown in Table 5.

We would also like to remark that the value of the $MSE_{\min}(k)$, which is equal to the value of the *MSE* of the combined regression estimator, is obtained as 194.2832 by (9). It should be noticed that the value of $MSE(\eta_{13})$ is less than this value, as shown in Table 4. Consequently, we can say that the 13th proposed ratio estimator is also more efficient than the combined regression estimator for this data set.

Table 3
Data statistics.

$N_1 = 127$	$N_2 = 117$	$N_3 = 103$
$N_4 = 170$	$N_5 = 205$	$N_6 = 201$
$n_1 = 31$	$n_2 = 21$	$n_3 = 29$
$n_4 = 38$	$n_5 = 22$	$n_6 = 39$
$S_{y1} = 883.835$	$S_{y2} = 644.922$	$S_{y3} = 1033.467$
$S_{y4} = 810.585$	$S_{y5} = 403.654$	$S_{y6} = 711.723$
$\bar{Y}_1 = 703.74$	$\bar{Y}_2 = 413$	$\bar{Y}_3 = 573.17$
$\bar{Y}_4 = 424.66$	$\bar{Y}_5 = 267.03$	$\bar{Y}_6 = 393.84$
$C_{y1} = 1.256$	$C_{y2} = 1.562$	$C_{y3} = 1.803$
$C_{y4} = 1.909$	$C_{y5} = 1.512$	$C_{y6} = 1.807$
$S_{x1} = 30\,486.751$	$S_{x2} = 15\,180.769$	$S_{x3} = 27\,549.697$
$S_{x4} = 18\,218.931$	$S_{x5} = 8497.776$	$S_{x6} = 23\,094.141$
$\bar{X}_1 = 20\,804.59$	$\bar{X}_2 = 9211.79$	$\bar{X}_3 = 14\,309.30$
$\bar{X}_4 = 9478.85$	$\bar{X}_5 = 5569.95$	$\bar{X}_6 = 12\,997.59$
$C_{X1} = 1.465$	$C_{X2} = 1.648$	$C_{X3} = 1.925$
$C_{X4} = 1.922$	$C_{X5} = 1.526$	$C_{X6} = 1.777$
$S_{xy1} = 25\,237\,153.52$	$S_{xy2} = 9\,747\,942.85$	$S_{xy3} = 28\,294\,397.04$
$S_{xy4} = 14\,523\,885.53$	$S_{xy5} = 3\,393\,591.75$	$S_{xy6} = 15\,864\,573.97$
$\rho_1 = 0.936$	$\rho_2 = 0.996$	$\rho_3 = 0.994$
$\rho_4 = 0.983$	$\rho_5 = 0.989$	$\rho_6 = 0.965$
$\beta_2(x_1) = 4.593$	$\beta_2(x_2) = 18.543$	$\beta_2(x_3) = 15.446$
$\beta_2(x_4) = 10.162$	$\beta_2(x_5) = 21.947$	$\beta_2(x_6) = 23.114$
$\beta_2(y_1) = 2.158$	$\beta_2(y_2) = 16.392$	$\beta_2(y_3) = 14.979$
$\beta_2(y_4) = 12.167$	$\beta_2(y_5) = 21.088$	$\beta_2(y_6) = 20.254$
$w_1 = 0.138$	$w_2 = 0.127$	$w_3 = 0.112$
$w_4 = 0.184$	$w_5 = 0.222$	$w_6 = 0.218$
$\beta_1(x_1) = 2.164$	$\beta_1(x_2) = 3.867$	$\beta_1(x_3) = 3.748$
$\beta_1(x_4) = 3.121$	$\beta_1(x_5) = 4.084$	$\beta_1(x_6) = 4.411$

Table 4
MSE values of estimators.

$MSE(k_1) = 216.4183$	$MSE(\eta_1) = 215.3553$
$MSE(k_3) = 216.3485$	$MSE(\eta_3) = 215.2877$
$MSE(k_5) = 216.4141$	$MSE(\eta_5) = 215.3512$
$MSE(k_7) = 216.0243$	$MSE(\eta_7) = 214.9739$
$MSE(k_9) = 905.6034$	$MSE(\eta_9) = 904.7041$
$MSE(k_{11}) = 318.3222$	$MSE(\eta_{11}) = 318.3032$
$MSE(k_{13}) = 194.2852$[a]	$MSE(\eta_{13}) = 194.0831$[b]
$MSE(k_{15}) = 216.3783$	$MSE(\eta_{15}) = 215.3166$
$MSE(k_{17}) = 215.7490$	$MSE(\eta_{17}) = 214.7074$

[a]Represents the most efficient estimator among k_i estimators.
[b]Represents the most efficient estimator among η_i estimators.

Table 5
Efficiency conditions.

Estimators	Cond. (23) $MSE(k_i) < MSE(k_1)$	Cond. (24) $MSE_{\min}(\eta_i) < MSE(k_1)$	Cond. (25) $MSE_{\min}(\eta_i) < MSE_{\min}(\eta_1)$	Cond. (26) $MSE_{\min}(\eta_1) < MSE(k_1)$	Cond. (27) $MSE_{\min}(\eta_i) < MSE(k_i)$
k_1				$0.001131 < 0.001136$	
k_3	0.999850976[a]				
k_5	0.999990959[a]				
k_7	0.999155641[a]				
k_9	0.370169324				
k_{11}	0.681985585				
k_{13}	0.90644646[a]				
k_{15}	0.999914605[a]				
k_{17}	0.998561085[a]				
η_1					
η_3		0.001130267[a]	−3.54731E−07[a]		$0.0011303 < 0.001136$[a]
η_5		0.001130601[a]	−2.15366E−08[a]		$0.0011306 < 0.001136$[a]
η_7		0.00112862[a]	−2.00246E−06[a]		$0.0011286 < 0.001134$[a]
η_9		0.004749725	0.003619103		$0.0047497 < 0.004754$[a]
η_{11}		0.001671102	0.00054048		$0.0016711 < 0.0016712$[a]
η_{12}		0.001018943[a]	−0.000111679[a]		$0.0010189 < 0.00102$[a]
η_{13}		0.001130419[a]	−2.03338E−07[a]		$0.0011304 < 0.001136$[a]
η_{15}		0.001127221[a]	−3.40168E−06[a]		$0.0011272 < 0.001133$[a]
	$0.811111 < \upsilon_{(i-1)/2} < 1$	< 0.001136	< 0		

[a]Represents that conditions are satisfied.

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