Different Tropospheric Mapping Functions and Cut off Angles Investigated by Processing VLBI CONT05 Sessions

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VLBI GEOMETRIC MODEL

\[ c(\tau_{\text{obs}} - (\tau_{\text{clock}} + \tau_{\text{trop}} + \tau_{\text{ionos}} + \tau_{\text{rel}})) = c(t_2 - t_1) = \hat{k} [r_2(t_1) - r_1(t_1)] + \hat{k} \beta [t_2 - t_1] \]

\( \hat{k} \): direction of the wave front
\( \beta \): mean velocity vector of #2

* Vectors are defined in Solar-System-Barycentric
**VLBI TROPOSPHERE MODEL**

*Total Delay = Ionospheric Delay + Neutral Delay*

*Zenith Neutral Delay = Zenith Hydrostatic Delay + Zenith Wet Delay*

\[
\Delta L(e) = ZHD.mf_h(e) + ZWD.mf_w(e)
\]

\[
\Delta L(e) : \text{Zenith neutral path delay}
\]

\[
mf_{h,w} : \text{Hydrostatic, wet tropospheric mapping functions}
\]

\[
ZHD = f(\varphi, H, P) : \text{Zenith Hydrostatic delay}
\]

\[
ZWD = \text{Estimated in VLBI analysis by piecewise linear function : Zenith wet delay}
\]

\[
\tau_{troposphere} = -\frac{1}{c} (\Delta L_2(e) - \Delta L_1(e)) \quad \text{(for the station 1 and 2)}
\]
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TROPOSPHERIC MAPPING FUNCTIONS

\[ m_{h,w}(e) = \frac{1 + \frac{a_i}{b_i}}{1 + \frac{c_i}{1 + \ldots}} \]

\[ \sin(e) + \frac{a_i}{\sin(e) + \frac{b_i}{\sin(e) + \frac{c_i}{\sin(e) + \ldots}}} \]

\( e \) : Elevation cut off angle
\( a_i, b_i, c_i, \ldots = f(\varphi, H, \text{doy}, t, \alpha, \ldots) \)
\( \varphi \) : station latitude
\( H \) : station orthometric height
\( \text{doy} \) : day of year
\( P \) : surface total pressure
\( t \) : surface temperature
\( \alpha \) : temperature lapse rate

Niell Mapping Function
Isobaric Mapping Function
Vienna Mapping Function

Some other mapping functions:
Chao, Ifadis, Davis, MTT, B&E, F&K, UNBabc, UNBab
\[ \delta \rho_{trp}(z) = mf(z) \cdot \delta \rho_{trp}(0) \sim 1 / \cos(z) \cdot \delta \rho_{trp}(0) \]

\[ \delta \rho_{h}(z) = \cos(z) \cdot \delta \rho_{h}(0) ; \delta \rho_{h}(0) = \delta h \]

\[ \delta \rho_{clk}(z) = \delta \rho_{clk}(0) = c \cdot \delta t_R \]
CONT05 was a two-week campaign of continuous VLBI sessions, scheduled for observing during September 2005.

The plan for the CONT05 campaign is to acquire state of the art VLBI data over a two-week period to demonstrate the highest accuracy of which VLBI is capable.
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Regression function of the baseline length repeatabilities and fitting a curve

\[
\sigma = \sqrt{\frac{\sum_{i=1}^{n} (L_i - L_0)^2}{n-2}}
\]

Formal errors as baseline length repeatabilities

Regression function for baseline length repeatabilities

\[
y = a^2 + b^2 ppb^2 L^2
\]

LSM Application

\[
y = \begin{bmatrix} \text{rms}_1^2 \\ \text{rms}_2^2 \\ \vdots \\ \text{rms}_n^2 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & ppb^2 L_1^2 \\ 1 & ppb^2 L_2^2 \\ \vdots & \vdots \\ 1 & ppb^2 L_n^2 \end{bmatrix}, \quad x = \begin{bmatrix} a^2 \\ b^2 \end{bmatrix}, \quad W = \begin{bmatrix} 1/s_1^2 & 0 \\ 0 & \vdotsc \end{bmatrix}
\]

\[
x = (A^TWA)^{-1} A^T W y
\]
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### Values of the estimated parameters by LSM

<table>
<thead>
<tr>
<th>Mapping Functions</th>
<th>Parameters of the function for different cut off angles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5° (6156)</td>
</tr>
<tr>
<td></td>
<td>a (cm)</td>
</tr>
<tr>
<td>VMI</td>
<td>0.505</td>
</tr>
<tr>
<td>GMF</td>
<td>0.524</td>
</tr>
<tr>
<td>NMF</td>
<td>0.528</td>
</tr>
<tr>
<td></td>
<td>10° (5502)</td>
</tr>
<tr>
<td></td>
<td>a (cm)</td>
</tr>
<tr>
<td>VMI</td>
<td>0.501</td>
</tr>
<tr>
<td>GMF</td>
<td>0.500</td>
</tr>
<tr>
<td>IMF</td>
<td>0.500</td>
</tr>
</tbody>
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CONT05 baseline length repeatabilities
(cut off : 5°)

Number of observables (CONT05)
6156
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CONT05 baseline length repeatabilities (cut off : 7°)

Number of observables (CONT 05)

5907
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CONT05 baseline length repeatabilities (cut off : 10°)

Number of observables (CONT 05) 5502
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CONT05 baseline length repeatabilities (cut off : 20°)

Number of observables (CONT 05)

3906
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CONT05 baseline length repeatabilities (VMF1)

Number of observables (CONT 05)

- 6156
- 6028
- 5907
- 5818
- 5646
- 5502
- 5207
- 4730
- 3906
- 2491
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CONT05 baseline length repeatabilities (VMF1)

Number of observables (CONT 05)

Baseline length uncertainty (m km)

Baseline length (10^6 m)
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Values of the estimated parameters by LSM

Parameter “a” fixed to 0.5 cm

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<td></td>
<td>5°(6156)</td>
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<tr>
<td></td>
<td>a(cm)</td>
</tr>
<tr>
<td>VM1</td>
<td>0.5</td>
</tr>
<tr>
<td>GMF</td>
<td>0.5</td>
</tr>
<tr>
<td>NMF</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>10°(5502)</td>
</tr>
<tr>
<td></td>
<td>a(cm)</td>
</tr>
<tr>
<td>VM1</td>
<td>0.5</td>
</tr>
<tr>
<td>GMF</td>
<td>0.5</td>
</tr>
<tr>
<td>IMF</td>
<td>0.5</td>
</tr>
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Comparison of the parameter “b”

![Graph comparing different tropospheric mapping functions and cut-off angles](image-url)
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**CONT05 baseline length repeatabilities (VMF1)**

Parameter “a“ fixed to 0.5 cm

Number of observables (CONT 05)

- 6156
- 6028
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**Simulation Formula**

\[ \Delta \tau : \text{Observed group delay is simulated} \]

\[ \Delta \tau = \Delta \tau_{\text{computed}} + (WZD_2 \text{ mfw}_2(e) + \text{cl}_2) - (WZD_1 \text{ mfw}_1(e) + \text{cl}_1) + \text{wn}_{\text{bsl}(1-2)} \]

\[ \sum_{j=1}^{m} (\text{rep}_{\text{real}(j)} - \text{rep}_{\text{simulated}(j)})^2 \Rightarrow \text{min} \]
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Comparison of baseline length repeatabilities derived from simulated and real CONT05 NGS files

\[ \sum_{j=1}^{n} d_j d_j = 2.72 \text{ cm} \]

\[ \text{Baseline Length (10^6 m)} \]

\[ \text{Baseline length uncertainty (\(\text{mm}, \text{cm}\))} \]
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### Simulation Results

<table>
<thead>
<tr>
<th>Parameters</th>
<th>1st Simulation (A05)</th>
<th>2nd Simulation (B05)</th>
<th>3rd Simulation (D05)</th>
</tr>
</thead>
<tbody>
<tr>
<td>white noise</td>
<td>8psec (2.4 mm)</td>
<td>12psec (3.6mm)</td>
<td>8psec (2.4mm)</td>
</tr>
<tr>
<td>predicted clock</td>
<td>1e-15@15min</td>
<td>1e-15@15min</td>
<td>1e-15@15min</td>
</tr>
<tr>
<td>predicted wet zenith delay</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HARTRAO</td>
<td>0.1</td>
<td>HARTRAO</td>
<td>0.1</td>
</tr>
<tr>
<td>KOOKEE</td>
<td>0.8</td>
<td>KOOKEE</td>
<td>0.8</td>
</tr>
<tr>
<td>TSUKUB32</td>
<td>0.6</td>
<td>TSUKUB32</td>
<td>0.6</td>
</tr>
<tr>
<td>The rest of all stations</td>
<td>0.5</td>
<td>The rest of all stations</td>
<td>0.5</td>
</tr>
<tr>
<td>All stations</td>
<td></td>
<td></td>
<td>0.5</td>
</tr>
</tbody>
</table>
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Conclusions

• Similar baseline uncertainty values for cut off angles 5 to 10 degrees but not for 12 to 30 degrees.

• Inspite of the small differences, VM1 gives always the best results.

• If the same amount of observables for simulations with the real ones, cut off angle 7 gives approximately the best outcomes.

• It has been succeeded to create overlapped simulation outcomes with the real ones for cut off angle 7 degree.
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**Outlook**

- No need to observe quasars below the cut off angle 7 unless the wet zenith delay parameters will be measured accurately and the related models will be improved.

- Future simulations should use the turbulence model for the wet zenith delays.

- Down-weighting of low elevation observations should be tested.
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Thank You ...
VLBI GEOMETRIC MODEL

Station \#2 (t_2)

Earth center

r_2(t_2)

\beta \tau

r_2(t_1)

#2 (t_1)

baseline vector

R_2(t_2)

R_2(t_1)

R_c(t_1)

R_1(t_1)

Source

\[ c \tau = c (t_2 - t_1) = |R_C (t_1) + r_2 (t_1) + \beta \tau| - |R_C (t_1) + r (t_1)| \]