Vie_LSM and Vie_LSM_scan (version 2.1)

Kamil Teke and Johannes Böhm
“vie_lsm” is a module of “VieVS”, which estimates geodetic parameters with least squares adjustment from VLBI observations.

All the parameters can be estimated as continuous piece-wise linear offsets (CPWLO) in sub-daily and daily temporal resolution.
Estimated parameters are:

- Clocks (offset (cm), rate (cm/day), quadratic term (cm/day$^2$), CPWLO (cm)),
- Zenith wet delays (cm) as CPWLO,
- Troposphere gradients (cm) as CPWLO,
- EOP (mas and ms) as CPWLO,
- Antenna coordinates in TRF (cm) as one offset per session or as CPWLO,
- Source coordinates in CRF (declinations in mas and right ascensions in ms) as CPWLO.
continuous piece-wise linear offsets (CPWLO)

\[ x_i = x_1 + \frac{t - t_1}{t_2 - t_1} (x_2 - x_1) \]
Partial derivatives of the delay model w.r.t. a sub-daily parameter

\[
\frac{\partial \tau(t)}{\partial x_1} = \frac{\partial \tau(t)}{\partial x_i} \cdot \frac{\partial x_i}{\partial x_1}
\]
\[
\frac{\partial x_i}{\partial x_1} = 1 - \frac{t - t_j}{t_{j+1} - t_j}
\]
\[
\frac{\partial \tau(t)}{\partial x_2} = \frac{\partial \tau(t)}{\partial x_i} \cdot \frac{\partial x_i}{\partial x_2}
\]
\[
\frac{\partial x_i}{\partial x_2} = \frac{t - t_j}{t_{j+1} - t_j}
\]

\[t_j < t < t_{j+1}\]
\[
A = \begin{bmatrix} A(1).sm & \cdots & A(15).sm \end{bmatrix}
\]
design matrix of real observation equations

\[
H = \begin{bmatrix}
H(1).sm & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & H(15).sm
\end{bmatrix}
\]
design matrix of pseudo-observation equations (constraints)

\[
N = \begin{bmatrix} A^T PA + H^T P_H H & C^T \\
C & 0 \end{bmatrix}
\]
\[
b = \begin{bmatrix} A^T P_{oc} + H^T P_{och} \\
b_c \end{bmatrix}
\]

bc is a zero vector (due to NNT and NNR conditions)

\[
x = N^{-1} b \\
m_0 = (v^T P v + v^T_{H H} P_{H H} v_H) / (n_{obs} + n_{constr} - n_{unk})
\]

\[
K_x = m_0 N^{-1}
\]
variance-covariance matrix of the estimates
The first solution is meant to remove large clock offsets (and rates and quadratic terms) for numerical reasons. (Clock offsets can be several kilometers.)
Parametrization for least squares
VLBI clock error

\[
\Delta \tau_{\text{clk}}^{\text{poly}}(t) = \beta_0 + \beta_1(t-t_0) + \beta_2(t-t_0)^2
\]

quadratic polynomial for each clock

\[
\Delta \tau_{\text{clk}}^{\text{CPWLO}}(t) = x_1 + \frac{t-t_1}{t_2-t_1}(x_2 - x_1)
\]

CPWLO for each clock e.g. at each UTC integer hour \((t_1\text{ and }t_2)\)

\[
\Delta \tau_{\text{clk}}(t) = \Delta \tau_{\text{clk}}^{\text{poly}}(t) + \Delta \tau_{\text{clk}}^{\text{CPWLO}}(t)
\]

total clock error at epoch \(t\)
This combination adds relative constraints on the clock offsets. Actually, observation equations are added to the design matrix which tell that the difference between two adjacent piecewise linear clock offsets is zero ± a certain standard deviation \( \sigma \). (These constraints are mainly important to bridge gaps without observations to avoid singularity of the normal equation system.)
Troposphere delay

\[
\Delta \tau_{\text{trop}} = 10^{-6} \int_{0}^{H_{\text{trop}}} \left[ N_h(s) + N_w(s) \right] ds
\]

\[
\Delta \tau_{\text{trop}}(\alpha, \varepsilon) = ZHD m_h(\varepsilon) + ZWD m_w(\varepsilon) + m_w(\varepsilon) \cot(\varepsilon) \left[ G_n \cos(\alpha) + G_e \sin(\alpha) \right]
\]

- Estimated
- Reduced from observations a priori to the adjustment
Parametrization for ZWD and gradients
If you want to estimate one constant value per session, the recommendation is to set the parameterization as shown above. Very strong relative constraints of $1 \times 10^{-4}$ m(a)s/day take care that the estimates are the same over the session.

Example: The session is from 18 UT to 18 UT. Then, three piecewise linear offsets are set up for each EOP. (They are set up a midnight before the session, at midnight during the session, and at midnight after the session.) The strong constraints take care that all three estimates per session are the same.
Parametrization for Station Coordinates

- Estimate station coordinates as one offset per session by introducing NNT/NNR condition equations
  - No Net Translation (NNT)
  - No Net Rotation (NNR)
  - No Net Scale (NNS)

Non-TRF stations are estimated in any case! Select "Sessionwise parameterization" to force fixing of also non-TRF stations.
Parametrization for Source Coordinates

- Estimate source coordinates as pwl offsets
- Estimation interval [min]: 1440
- Relative constraints [mas]: checked
- Constraints after 1440 minutes: 1.0e-4

Only non-CRF sources are estimated if this checkbox is ticked (select "Sessionwise parameterization" if you want otherwise...).
Session-wise parameterization
Session-wise parameterization

### vie_lsm [ single session first solution ]

**Parameterization for removing large clock errors**

- **Apply first basic solution (only with clock function)**
  - one offset per clock
  - one offset & one rate per clock
  - one offset, one rate, & one quadratic term per clock
- **Use clock breaks (From OPT file)**

**Main solution**

- **Apply main solution**
  - simple outlier test \[ \text{coefficient} \times \text{mo} \]
  - coefficient: 5
  - basic outlier test \[ \text{coefficient} \times \text{mo} \times \text{sqrt(qw)} \]

**Clock/s that have breaks in the session**

- ZELENCHK

**Reference clock for the first solution**

- WETTZELL
  - TSUKUB32
  - WETTZELL
  - SVETLOE
  - ZELENCHK
  - ONSALA60
  - NYALES20
  - HARTRAO
  - KOKEE
  - WESTFORD
  - MEDICINA
  - TIGOCONC
Session-wise parameterization

Table: Clock Parameterization

<table>
<thead>
<tr>
<th>Clock Constraints</th>
<th>Clock Interval</th>
<th>Reference Clock</th>
</tr>
</thead>
<tbody>
<tr>
<td>TSUKUB32</td>
<td>1.3000</td>
<td>60</td>
</tr>
<tr>
<td>WETTZEELL</td>
<td>1.3000</td>
<td>60</td>
</tr>
<tr>
<td>SVETLOE</td>
<td>1.3000</td>
<td>60</td>
</tr>
<tr>
<td>ZELENCHK</td>
<td>1.3000</td>
<td>60</td>
</tr>
<tr>
<td>ONSALA60</td>
<td>1.3000</td>
<td>60</td>
</tr>
<tr>
<td>NYALES20</td>
<td>1.3000</td>
<td>60</td>
</tr>
<tr>
<td>HARTRAO</td>
<td>1.3000</td>
<td>60</td>
</tr>
<tr>
<td>KOKEE</td>
<td>1.3000</td>
<td>60</td>
</tr>
<tr>
<td>WESTFORD</td>
<td>1.3000</td>
<td>60</td>
</tr>
<tr>
<td>MEDICINA</td>
<td>1.3000</td>
<td>60</td>
</tr>
<tr>
<td>TIGOCONC</td>
<td>1.3000</td>
<td>60</td>
</tr>
</tbody>
</table>

- Default reference clock has not any clock break.
- Reference clock is the first clock in the NGS file
- OR if any OPT file of the session exists fixed clock is from OPT file
- Unit of clock estimation intervals is minutes.
- Unit of clock constraints is centimeters
  E.g. 1.3 cm after 1 hour is relatively loose.
Session-wise parameterization

### vie_lsm [ single session troposphere ]

- **Apply relative constraints between tropospheric offset estimates**
  - Introduce relative constraints between pwL ZENITH WET DELAY offsets
  - Introduce relative constraints between pwL tropo. NORTH GRADIENT offsets
  - Introduce relative constraints between pwL tropo. EAST GRADIENT offsets
  - Introduce absolute constraints between pwL tropo. NORTH GRADIENT offsets
  - Introduce absolute constraints between pwL tropo. EAST GRADIENT offsets

<table>
<thead>
<tr>
<th>Station</th>
<th>ZWD coef.</th>
<th>NGR rel. coef.</th>
<th>EGR rel. coef.</th>
<th>NGR abs. coef.</th>
<th>EGR abs. coef.</th>
<th>ZWD int.</th>
<th>NGR int.</th>
<th>EGR int.</th>
<th>est. ZWD</th>
<th>est. NGR</th>
<th>est. EGR</th>
</tr>
</thead>
<tbody>
<tr>
<td>TSUKUB32</td>
<td>1.5000</td>
<td>0.0500</td>
<td>0.0500</td>
<td>0.1000</td>
<td>0.1000</td>
<td>60</td>
<td>360</td>
<td>360</td>
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<td></td>
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</tr>
<tr>
<td>WETTZELL</td>
<td>1.5000</td>
<td>0.0500</td>
<td>0.0500</td>
<td>0.1000</td>
<td>0.1000</td>
<td>60</td>
<td>360</td>
<td>360</td>
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<tr>
<td>SVETLOE</td>
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<td>0.0500</td>
<td>0.1000</td>
<td>0.1000</td>
<td>60</td>
<td>360</td>
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<tr>
<td>ZELENCHK</td>
<td>1.5000</td>
<td>0.0500</td>
<td>0.0500</td>
<td>0.1000</td>
<td>0.1000</td>
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<td>0.0500</td>
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<td>60</td>
<td>360</td>
<td>360</td>
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<tr>
<td>HARTRAO</td>
<td>1.5000</td>
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<td>0.0500</td>
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<td>0.1000</td>
<td>60</td>
<td>360</td>
<td>360</td>
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<tr>
<td>KOKE</td>
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<td>0.0500</td>
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<td>0.1000</td>
<td>0.1000</td>
<td>60</td>
<td>360</td>
<td>360</td>
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<tr>
<td>WESTFORD</td>
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<td>0.0500</td>
<td>0.0500</td>
<td>0.1000</td>
<td>0.1000</td>
<td>60</td>
<td>360</td>
<td>360</td>
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</tr>
<tr>
<td>MEDICINA</td>
<td>1.5000</td>
<td>0.0500</td>
<td>0.0500</td>
<td>0.1000</td>
<td>0.1000</td>
<td>60</td>
<td>360</td>
<td>360</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Unit of estimation intervals is minute.
- Unit of ZWD relative constraints is cm e.g. 1.5 cm after 1 hour is relatively loose.
- Unit of NGR & EGR relative constraints is cm e.g. 0.05 cm after 6 hours is relatively loose.
- Unit of NGR & EGR absolute constraints is cm e.g. 0.1 cm absolutely loose.
Session-wise parameterization

Case 1: NNT/NNR
(one coordinate offset per session)
Session-wise parameterization

Case 2: Fix some station coordinates (one coordinate offset per session)
Session-wise parameterization

Case 3: Fix some station coordinates (CPWLO coordinates)
Session-wise parameterization

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Include Model</th>
<th>Estimation Interval</th>
<th>Use Constraints</th>
<th>Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>Xpol (inter. pole coord. in TRF)</td>
<td>✔️</td>
<td>1440</td>
<td>✔️</td>
<td>1.0000e-04</td>
</tr>
<tr>
<td>Ypol (inter. pole coord. in TRF)</td>
<td>✔️</td>
<td>1440</td>
<td>✔️</td>
<td>1.0000e-04</td>
</tr>
<tr>
<td>dUT1 (rotation angle)</td>
<td>✔️</td>
<td>1440</td>
<td>✔️</td>
<td>1.0000e-04</td>
</tr>
<tr>
<td>nutdx (CIP coord. in celest. long.)</td>
<td>✔️</td>
<td>1440</td>
<td>✔️</td>
<td>1.0000e-04</td>
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<tr>
<td>nutdy (CIP coord. in obliquity)</td>
<td>✔️</td>
<td>1440</td>
<td>✔️</td>
<td>1.0000e-04</td>
</tr>
</tbody>
</table>

- Unit of estimation intervals is minute
- Constraints are mas for EOP
- 2 mas after 1 hour is relatively loose constraints for EOP
### vie_lsm [ single session source coordinates ]

- **Estimate coordinates of sources as pwlo offsets**: All the unselected sources will be fixed to CRF.
- **Introduce relative constraints between pwlo source coordinates**

<table>
<thead>
<tr>
<th>Source Name</th>
<th>Total Observations</th>
<th>Est. Coor.</th>
<th>Constraints</th>
<th>Coord. Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>34</td>
<td>1044+719</td>
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<tr>
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<td>1308+326</td>
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<tr>
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<td>2201+315</td>
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<tr>
<td>52</td>
<td>2121+053</td>
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</tbody>
</table>
Session-wise parameterization
vie_lsm scan-wise update
Scan-wise update of normal equation system

1 A-matrix per scan

Scan 1

\[ A_{s1} \]

\[ \vdots \]

\[ \vdots \]

\[ \vdots \]

\[ \vdots \]

Scan j

\[ A_{sj} \]

\[ \text{DESIGN MATRIX OF REAL OBSERVATIONS (A)} \]

\[ N_{s1} = A_{s1}^T \cdot P_{s1} \cdot A_{s1} \]

\[ N_A = N_{s1} + N_{s2} + \ldots + N_{sj} \]

\[ b_{s1} = A_{s1}^T \cdot P_{s1} \cdot oc_{s1} \]

\[ b_A = b_{s1} + b_{s2} + \ldots + b_{sj} \]

\( j : \text{number of scans in the session} \)

Claudia Tierno Ros, Third VieVS user workshop, 11-13 September, 2012
Scan-wise update of normal equation system

Same procedure with H matrix: 15 H-matrixes

\[ N = A^T \cdot P \cdot A \quad \Rightarrow \quad N = N_A + N_H \]

\[ b = A^T \cdot P \cdot oc \quad \Rightarrow \quad b = b_A + b_H \]

\[ x = N^{-1} \cdot b \]

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vie_mod successfully finished!

| Welcome to VIE_LSM!!!!!! |

---

number of scans : 1151
number of antennas : 11
number of sources : 76
number of obs. : 9917

2. CREATING DEFAULT OPTIONS
3. FORMING THE WEIGHT MATRIX OF THE OBSERVATIONS "Pobserv"

apriori std. dev. of unit weight. : 1.1217

obs. of the antenna TSUKUB32 : 1957
obs. of the antenna WETZELL : 2326
obs. of the antenna SVEILOE : 2457
obs. of the antenna ZELENCHK : 1420
obs. of the antenna ONSALA60 : 2465
obs. of the antenna NYALES20 : 2354
obs. of the antenna HARTRAQ : 1073
obs. of the antenna KOKEE : 1456
obs. of the antenna WESTFORD : 1769
obs. of the antenna MEDICINA : 2269
obs. of the antenna TIGOCONC : 288

4. FORMING THE REDUCED OBSERVATION VECTOR "oc_observ"
clock WETZELL is selected as the ref. clock for the first solution

chi-squared of first solution: 23.3393

5. FORMING THE DESIGN MATRICES "A(i).sm" ...
6. FORMING THE CONSTRAINT MATRIX and WEIGHT MATRIX OF CONSTRAINTS
7. ESTIMATING THE PARAMETERS WITH LEAST SQUARES

!!! some or all station coordinates are not estimated (selected as fixed)!!!
clock WETZELL is selected as the ref. clock for the main solution

chi-squared of main solution vTPv/degOfFreedom: 1.2136
detecting outliers
num. of outliers : 2
writing outliers to file ../DATA/OUTLIER/2008/08AUG12XA_N004.OUT
total 2 outlier observations are found but NOT eliminated

----------

total number of estimated parameters: 735
total clock offsets: 238
total rate and quad. terms of clock funct.: 20
total zenith wet delay offsets: 263
total tropo. north gradients: 53
total tropo. east gradients: 53
total pole coor. (x-pol) offsets: 2
total pole coor. (y-pol) offsets: 2
total dUT1 offsets: 2
total celestial pole (nutation dx) offsets: 2
total celestial pole (nutation dy) offsets: 2
total right ascension offsets of sources : 4
total declination offsets of sources : 4
antenna coor. dx offsets: 30
antenna coor. dy offsets: 30
antenna coor. dz offsets: 30

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estimated parameters are saved as ../VieVS/DATA/LEVEL3/forworkshop/x_08AUG12XA_N004.mat
estimation options are saved as ../VieVS/DATA/LEVEL3/forworkshop/opt_08AUG12XA_N004.mat
normal equation matrix is saved as ../VieVS/DATA/LEVEL3/forworkshop/ateq_08AUG12XA_N004.mat
right hand side vector is saved as ../VieVS/DATA/LEVEL3/forworkshop/atr1_08AUG12XA_N004.mat
residuals are saved as ../VieVS/DATA/LEVEL3/forworkshop/res_08AUG12XA_N004.mat

Data for GLOBAL SOLUTION is saved as ../VieVS/DATA/LEVEL2/forworkshop/08AUG12XA_N004_an_glob.mat
Data for GLOBAL SOLUTION is saved as ../VieVS/DATA/LEVEL2/forworkshop/08AUG12XA_N004_par_glob.mat
Data for GLOBAL SOLUTION is saved as ../VieVS/DATA/LEVEL2/forworkshop/08AUG12XA_N004_Nb_glob.mat

Reduced N and b for SINEX output are saved in ../VieVS/DATA/LEVEL3/forworkshop/SINEX/
Conclusions

vie_lsm corrects clock breaks and detects outlier observations.

vie_lsm provides SINEX input and datum free normal equations for global solutions (vie_glob).

CPWLO estimates of VieVS are in a good agreement with those derived from other space geodetic techniques.

Scan-wise update of normal equation system ensures a successful process of the future sessions with lots of observations.
Thank you so much for your attention!