Optimization of GPS Networks with Respect to Accuracy and Reliability Criteria

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Abstract

Optimal design of geodetic GPS networks with respect to accuracy and reliability criteria is an essential part of most geodesy related projects. Whether the datum and point locations of a network was known, the process of determining the optimal baseline configuration and their optimal weights with respect to the selected design criteria can be achieved by optimizing observational plan using Second Order Design (SOD). The scalar design criteria can only satisfy to the limited demands for a network. Thus, criterion matrices, which can be defined as the computed variance-covariance matrix in designing stage that meet many of the accuracy demands, is mostly used.

In this study, observational plan optimization of GPS networks with respect to the accuracy and reliability criteria was aimed to carry out. Taylor-Karman structured criterion matrix was constituted. Direct approximation of the inverse criterion matrix (U,m approximation) method was applied to the criterion matrix to compute the optimal weights of observations. In the first step of the solution, optimal baseline configuration was found out by removing the baselines of which weights were negative or near zero from the observational plan. In the second step of this solution, optimal weights of the remaining baselines in the observational plan were computed. In the last step, redundancy numbers, internal and external reliability values were calculated, so as to put forth the sensitivity of the network configuration to model errors for consideration, in other words the reliability of the network was determined and new baselines were planned to the baselines of which reliability values were not found out sufficient. As a result of the study, in designing stage to ensure the precision and reliability demands from GPS networks, applicable solutions have been suggested.

Introduction

Geodetic networks are to satisfy certain accuracy and reliability criteria as to serve the aim of network establishment. In designing stage of a GPS network postulated accuracy and reliability criteria can be formulated and applied by certain optimization procedures. The main optimization purpose of geodetic networks is to establish them with high reliability, good accuracy and low cost. To fulfill the demands of homogeneity and isotropy conditions in all directions for a network with respect to accuracy, second order design (SOD), which is defined as the problem of finding the observational weight matrix from the configuration of the network, is applicable. The SOD Problem has become rather popular in geodesy after its initial treatment by Grafarend in 1974. Since then many new algorithms have been developed with simulated example and real application, where the criterion matrices with Taylor-Karman-structured are used in SOD for terrestrial geodetic networks in terms of achieving maximum accuracy and minimum cost. Generally, three basic kinds of approximation methods are used in the SOD procedure. These are the direct approximation of the criterion matrix (direct-HR), iterative approximation of the criterion matrix (iterative-HR) and direct approximation of the inverse criterion matrix (U,m). Analytical methods used to realize optimization that serves to approach the criterion matrix are: sequential least squares, linear and non-linear programming.

In order to increase the capability of detecting model errors and outliers in a geodetic network, it has to be optimized. Baarda in 1968 distinguishes "*internal reliability*" and "*external reliability*". While internal reliability of a control network measures the marginal undetectable error in the measurements, external reliability measures the effect of an undetectable gross error on the network coordinates and on quantities computed from them.

Method

♦ **Accuracy Optimization of Geodetic GPS Networks**

$$C_{xx}=\begin{bmatrix} d^2 & 0 & 0 & d^2-2c^2S_{ij} & 0 & 0 \\ 0 & d^2 & 0 & 0 & d^2-2c^2S_{ij} & 0 \\ 0 & 0 & 4d^2 & 0 & 0 & 4(d^2-2c^2S_{ij}) \\ d^2-2c^2S_{ij} & 0 & 0 & d^2 & 0 & 0 \\ 0 & d^2-2c^2S_{ij} & 0 & 0 & d^2 & 0 \\ 0 & 0 & 4(d^2-2c^2S_{ij}) & 0 & 0 & 4d^2 \end{bmatrix}$$

The **Taylor-Karman (TK) structured criterion matrix** where ***d***, is the semi axes of point error ellipsoids, ***c*** is an arbitrary constant that provides all the values to be positive and $\boldsymbol{S_{ij}}$ is the vector length between two arbitrary points.

♦ **U,m approximation method of Second Order Design**

$$A^T PA \doteq \overline{Q}_{xx}^{+}$$

The basic equation for the approximation method where ***A*** is the design matrix, ***P*** is the unknown weight matrix of observations, $\boldsymbol{Q_{xx}}$ is the datum dependent criterion matrix.

$$(A^T \odot A^T)p \doteq q$$

Matrix equation above is converted to a **set of linear equations** where ⊙ is the notation of the Khatri-Rao product, ***q*** is the vectorised criterion matrix, ***p*** is a vector composed of the diagonal elements of ***P***.

$$(A^T \odot A^T)p = q + d$$

By adding inconsistency parameters (***d***) to the linear equation system given above,

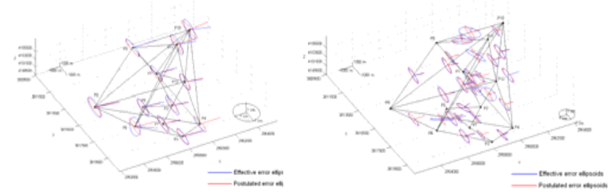
and applying **least squares method providing the condition**, $\Omega = d^T d = e^T\{[(A^T PA) - \overline{Q}_{xx}^{+}]^*[(A^T PA) - \overline{Q}_{xx}^{+}]\}e \Rightarrow \min$

unknown **weights of baseline vector components** $p = [(A^T \odot A^T)^T (A^T \odot A^T)]^{+} (A^T \odot A^T)^T q$,are produced.

Results

♦ **Accuracy Optimization of GPS Networks**

Effective and postulated, point and relative error ellipsoids of final (fourth) observational plan derived from U,m approximation method



Approximation quality values of U,m approximation method for each iteration

	First iteration	Second iteration	Third iteration	Fourth iteration
The number of baselines input for the optimization	66*3 = 198	36*3 = 108	29*3=87	27*3=81
Global test value ($d^T d$)	2.523	12.856	18.602	18.779
Maximum lamda value of matrix B (λ_{max})	3.253	5.508	5.437	5.670
The number of baselines decided to eliminate from observational plan	30*3 = 90	7*3=21	2*3=6	0*0=0

Method

♦ **Reliability Optimization of Geodetic GPS Networks**

The reliability of a network is considered high when the network can identify even small gross errors. Gross errors in the measurements affect the adjustment parameters; therefore, the reliability of the network is useful as a design criterion.

Certain scalar reliability objective functions which are to ensure the limit of the critical values

Reliability objective functions		Critical Values
Individual redundancy	$Z = r_j = (Q_{vv})_j P_j$	$Z = r_j > 0.4$
Internal reliability	$Z = \lvert\Delta_{0j}\rvert = m_0\sqrt{\frac{w_0}{P_j r_j}}$	$Z = \Delta_{0j} < 6\ m_j$
External reliability	$Z = \delta_{0j}^2 = \frac{1-r_j}{r_j} w_0$	$Z = \delta_{0j} < 6$

In the equations above, $\boldsymbol{Q_{vv}}$ is the cofactor matrix of the residuals, ***P*** is the weight matrix of the observations, $\boldsymbol{m_0}$ is the standard deviation of unit weight and $\boldsymbol{w_0}$ is the lower bound for non-centrality parameter in dependency of the significance level ($\boldsymbol{\alpha_0}$) and the required minimum power of the test ($\boldsymbol{1-\beta_0}$).

A geodetic GPS network can be optimized depending on the reliability objective functions from table above as the following steps:

-The global relative redundancy number (r_0) of the network is and individual redundancy numbers of the baselines are (r_j) calculated from the first observation plan.

-If some of the individual redundancy numbers are lower than the global relative redundancy it is decided that the concerned baselines cannot be checked adequately by the other baselines. For that reason, new baselines are planned perpendicular to the relevant baselines.

-Internal reliability criteria (Δ_{0j}) and external reliability criteria ($\boldsymbol{\delta_{0j}}$) are calculated for the network.

-The baselines of which internal reliability values are above the critical values ($\boldsymbol{6m_j}$) are decided that they cannot be checked adequately by the other baselines. For that reason, new baselines are planned perpendicular to the relevant baselines.

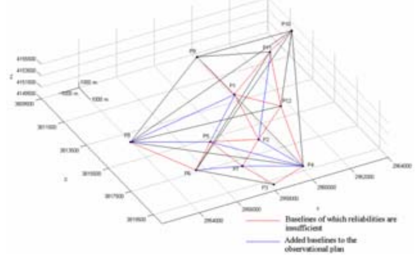
-When the cost of the network is taken into consideration, the baselines of which redundancy numbers are $r_j >> r_0$ would be removed from the network.

- The improved session plan is reviewed lastly if it ensures the selected objective function. Then the design is applied to the field.

Results

♦ **Reliability Optimization of GPS Networks**

The baselines of which reliabilities are insufficient and the added baselines to the observational plan



Internal and external reliability values of the designed GPS network

Baselines	Internal reliabilities and critical values						External reliabilities (Critical value = 6)		
	$\lvert\Delta_{0\Delta X}\rvert$	$6m_{\Delta X}$	$\lvert\Delta_{0\Delta Y}\rvert$	$6m_{\Delta Y}$	$\lvert\Delta_{0\Delta Z}\rvert$	$6m_{\Delta Z}$	$\delta_{0\Delta X_0}$	$\delta_{0\Delta Y_0}$	$\delta_{0\Delta Z_0}$
P1-P2	5.51	8.18	**7.81**	**4.80**	11.45	14.40	0.57	**8.91**	2.60
P1-P9	6.78	6.81	**9.64**	**5.73**	13.07	13.68	4.43	**9.26**	4.11
P1-P11	5.19	6.82	5.22	8.68	**13.09**	**10.71**	2.21	0.10	**6.14**
P1-P12	**7.20**	**5.36**	6.22	7.19	10.82	14.13	**7.00**	3.31	2.26
P2-P5	**6.39**	**4.93**	6.59	7.23	10.40	12.77	**6.66**	3.73	2.81
P2-P7	5.02	5.54	5.37	6.57	**12.10**	**8.63**	3.67	2.84	**7.40**
P2-P12	6.48	6.65	5.43	7.27	**12.80**	**9.81**	4.86	2.01	**6.73**
P3-P4	**8.37**	**5.16**	7.75	**6.86**	**16.19**	**9.14**	**8.87**	**6.47**	**9.84**
P3-P6	6.34	8.07	6.63	7.48	11.93	15.90	2.49	3.51	2.06
P3-P7	**8.31**	**5.19**	**6.97**	**4.78**	**13.56**	**12.34**	**8.73**	**7.78**	**6.24**
P4-P8	6.72	7.86	6.70	8.63	13.42	17.77	3.21	2.39	2.13
P4-P9	6.30	7.85	6.01	8.09	13.05	18.17	2.69	1.96	1.61
P4-P10	6.14	7.75	6.26	7.95	12.08	16.84	2.57	2.50	1.59
P4-P12	5.52	7.29	**6.38**	**5.93**	12.19	13.77	2.16	5.07	3.49
P5-P6	7.50	7.70	6.70	6.99	**17.45**	**8.99**	4.27	4.13	**10.93**
P5-P7	5.06	5.94	**6.99**	**4.55**	12.05	12.09	4.62	**8.31**	4.44
P6-P8	**7.51**	**6.54**	**9.17**	7.46	16.00	16.65	5.60	**6.19**	4.15
P6-P10	5.98	7.52	6.49	9.33	12.58	17.49	2.60	1.21	1.62
P6-P11	5.95	7.71	6.43	9.33	11.79	17.41	2.34	1.03	0.72
P8-P9	6.13	8.10	6.88	8.99	15.17	15.37	2.15	2.25	4.78
P8-P10	5.78	7.91	7.26	9.03	12.47	16.46	1.79	2.70	2.16
P8-P12	4.17	5.58	6.16	8.83	11.94	16.86	1.62	1.22	1.44
P9-P10	5.09	7.09	6.50	8.59	11.10	14.68	1.59	2.15	2.14
P9-P11	6.18	6.29	6.70	8.35	10.48	14.06	4.33	2.68	2.00
P10-P11	7.72	**4.92**	**8.34**	**5.81**	**14.45**	**8.45**	**8.54**	7.62	**9.45**
P10-P12	5.29	7.94	4.93	7.47	10.58	14.23	0.10	0.10	1.98
P11-P12	5.52	7.62	**9.58**	**6.35**	10.52	13.65	1.70	**8.12**	2.32

Conclusion

In this study, SOD of a GPS network was carried out with respect to the accuracy criteria. A criterion matrix was constituted which provides equal sized (homogeneity) and sphere viewed (isotropy) error ellipsoids of points and the radius of the relative error ellipsoids were computed from a function of the distances between points, named chaotic (complete isotropy) Taylor-Karman structured criterion matrix. Then, U,m approximation method was applied to the criterion matrix to compute the optimal weights of observations. Optimal baseline configuration was found out by removing the baselines of which weights were negative or near zero from the observational plan. In the second step of this solution, optimal weights of the remaining baselines in the observational plan were computed. In the last step, redundancy numbers of the baseline components, limit values of the undetected blunders (internal reliability) and the effect of the undetected blunders on the coordinates (external reliability) were calculated, so as to put forth the sensitivity of the network configuration to model errors for consideration, in other words the reliability of the network was determined and new baselines were planned to the baselines of which reliability values were not found out sufficient. So that capability of the network geometry in detecting model errors and outliers was increased. Finally, U,m approximation method was applied to the criterion matrix again with the design matrix of the new observational plan. The last outcomes were the optimal baselines and their optimal weights. The last observational plan was ensured not only the optimal weights of the baselines provided by U,m approximation method of SOD but also the reliability conditions for the network.