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RESEARCH GROUP  
ADVANCED GEODESY  
Institute of Geodesy and Geophysics

**Sixth IVS General Meeting, February 7-13, 2010, Hobart, TAS, Australia**

Session 3:

# **Estimation of Geodetic and Geodynamical Parameters with VieVS**

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Tobias Nilsson, Andrea Pany, Lucia Plank,  
Kamil Teke, Harald Schuh

- VieVS: new geodetic VLBI data analysis software
  - written in Matlab

more details about the theoretical computed delay in:

**Session 3:**

**Comparison Campaign of VLBI Data  
Analysis Software – First Results**

**Lucia Plank**

- **single session solution** – classical least squares adjustment
  - time varying parameters are modelled by piecewise linear offsets at integer fractions of integer hours

The screenshot shows a software window titled "vie\_lsm\_multi\_gui\_tropo" with a sub-header "vie\_lsm [ troposphere ]". It contains several checkboxes for applying relative constraints between tropospheric offset estimates, all of which are checked. A table below lists various coefficients and intervals, with checkmarks indicating which are estimated. At the bottom, there are "Back" and "Next" buttons, with a mouse cursor pointing to "Next".

apply relative constraints between tropospheric offset estimates

- introduce relative constraints between pwl zenith wet delay offsets
- introduce relative constraints between pwl tropo. north gradient offsets
- introduce relative constraints between pwl tropo. east gradient offsets

- all units are minutes for estimation intervals  
- units are picosec<sup>2</sup>/sec for the coefficients of the ZWD constraints  
- units are millimeters/day for the coefficients of the NGR & EGR constraints

| ZWD coef. | NGR coef. | EGR coef. | ZWD int. | NGR int. | EGR int. | est. ZWD                            | est. NGR                            | est. EGR                            |
|-----------|-----------|-----------|----------|----------|----------|-------------------------------------|-------------------------------------|-------------------------------------|
| 0.7000    | 2         | 2         | 60       | 360      | 360      | <input checked="" type="checkbox"/> | <input checked="" type="checkbox"/> | <input checked="" type="checkbox"/> |

Back Next

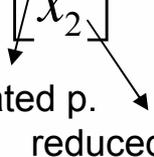
example of troposphere parameterisation

development of program unit for parameter estimation with so-called **global solution**

present state, common parameters: antenna coordinates and velocities, EOPs and Love & Shida numbers (reduction of clock parameters, zwd, troposphere gradients)

- Step 1: **reduction** of parameters

$$\begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$



$$N_{reduc} = N_{11} - N_{12} \cdot N_{22}^{-1} \cdot N_{21}$$

$$b_{reduc} = b_1 - N_{12} \cdot N_{22}^{-1} \cdot b_2$$

- Step 2: **stacking** of the reduced normal equation systems

$$N = N_{reduc\_1} + N_{reduc\_2} + \dots + N_{reduc\_nse}$$

$$b = b_{reduc\_1} + b_{reduc\_2} + \dots + b_{reduc\_nse}$$

$$x = N^{-1} \cdot b$$

# Love & Shida numbers

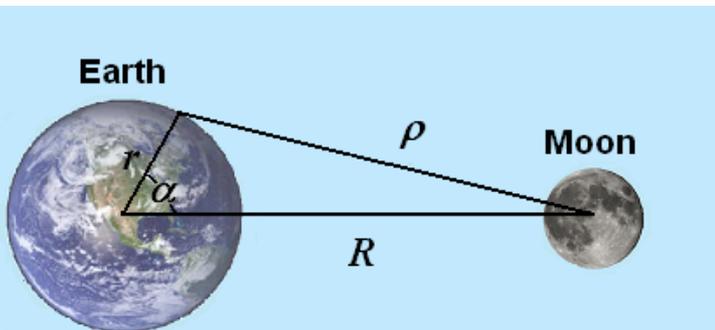
- **Solid Earth tidal deformation** arises from the variations in the Earth's gravitational field caused by the **Moon/Sun** relative to its strength at the geocentre

$$V_{Grav} = \frac{GM_M}{R} \sum_{n=0}^{\infty} \left(\frac{r}{R}\right)^n P_n(\cos \alpha) = V_0 + V_1 + V_2 + \dots$$

tidal potential  $V^{tid}$

- **Love and Shida** numbers  $h$ ,  $l$  are dimensionless **parameters**, which characterize how strong is the **effect** of the potential component **on the displacement**

tidal displacement in REN system



**basic Earth model:**

spherical, non-rotating,  
elastic, isotropic

**$h$  and  $l$  depend only on the degree** of the tidal potential

$$u_R = \sum_{n=2}^{\infty} h_n \cdot \frac{1}{g} \cdot V_n^{tid}$$

$$u_N = \sum_{n=2}^{\infty} l_n \cdot \frac{1}{g} \cdot \frac{\partial V_n^{tid}}{\partial \varphi}$$

$$u_E = \sum_{n=2}^{\infty} l_n \cdot \frac{1}{g \cdot \cos \varphi} \cdot \frac{\partial V_n^{tid}}{\partial \lambda}$$

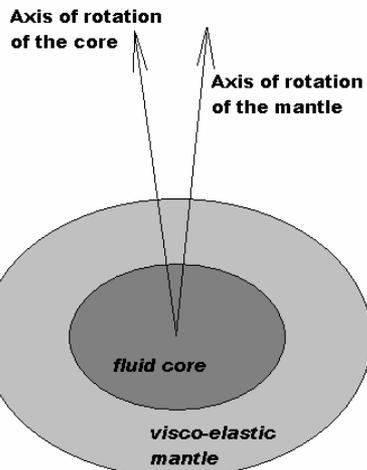
## More precise Earth model with fluid core and elastic mantle

- the tidal response of the Earth becomes frequency dependent in the diurnal band

$$\delta u_{R(f)}^{(21)} = -\frac{3}{2} \sqrt{\frac{5}{24\pi}} H_f \delta h_{21(f)} \sin(2\varphi) \sin(\theta_f + \lambda)$$

the corrections to radial displacement coming from the harmonic terms of the second degree tidal potential in the diurnal band

|                    |  |
|--------------------|--|
| $H_f$              | Cartwright-Tayler amplitude of the tidal term          |
| $\delta h_{21(f)}$ | difference of $h_{21}(f)$ from the nominal value $h_2$ |
| $\theta_f$         | tidal harmonic argument                                |

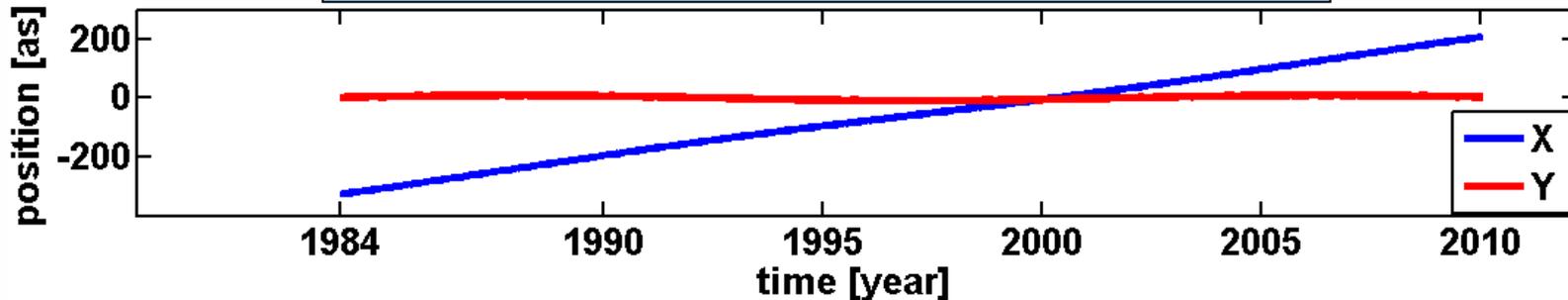


- Resonance** of the tidal force with the forces at the elliptical core-mantle boundary at the Free Core Nutation period
- FCN**: the fluid core rotates around an axis which is slightly inclined w.r.t. the axis of rotation of the mantle → **small periodic motion of the Earth's axis of rotation** (Wahr, 1981)

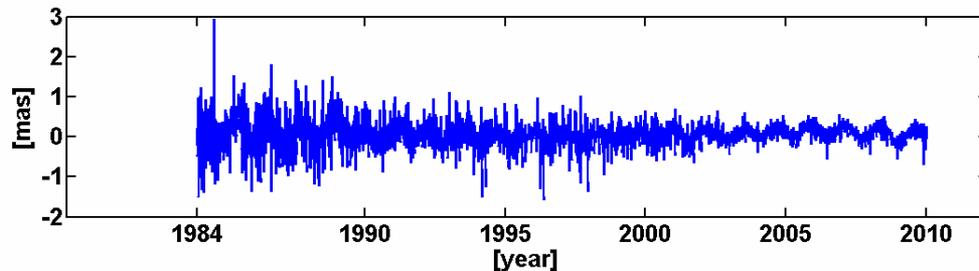
# Free Core Nutation (FCN)

Motion of the Earth's rotational axis in the celestial system

Position of the **Celestial Intermediate Pole** in the **Geocentric Celestial Reference System**



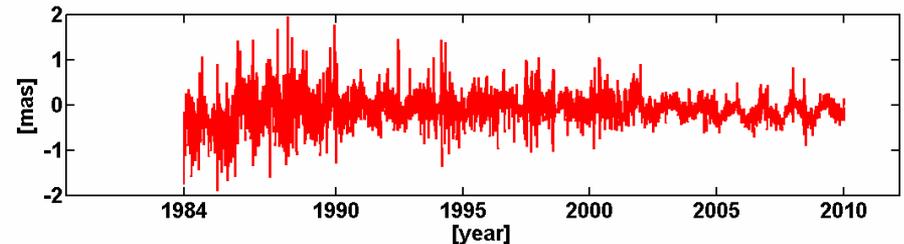
dX



**Celestial pole offsets w.r.t. the IAU 2000A precession-nutation model**

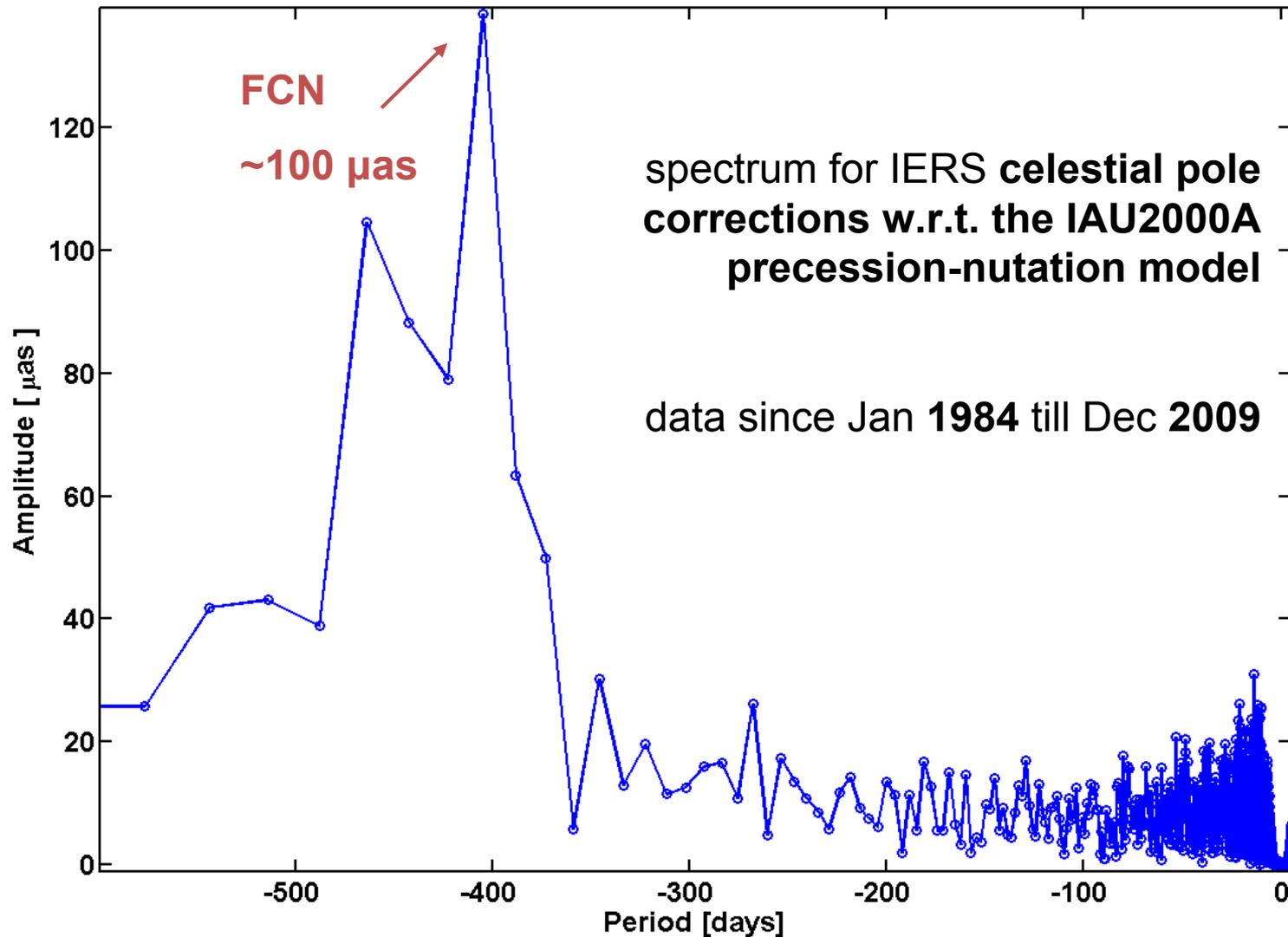
**FCN** is a rotational free mode of the Earth, which **cannot be predicted** rigorously. It is not considered as a part of the a-priori precession-nutation model

dY



# Free Core Nutation (FCN)

P/N C0405 IAU2000A



- Three options for determining the FCN-period

1. analysis of **celestial pole offsets**

- e.g. Herring et al., 1986; Vondrak and Ron, 2006

2. analysis of tidal **gravity data**

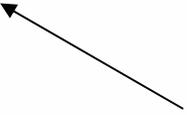
- e.g. Defraigne, 1994; Florsch and Hinderer, 2000; Ducarme et al., 2007

3. analysis of observed solid Earth **tidal displacements**, e.g., of the VLBI antennas

- Haas and Schuh, 1996 – 10 years data

(Determination of frequency dependent Love and Shida numbers from VLBI data. Geophysical Research Letters. Vol. 23 No. 12/1996. p.1509-1512)

motivation for our work



## Present state in VieVS

- degree 2 Love & Shida numbers: **h2, I2**
- **frequency dependent** Love & Shida numbers in the diurnal band
  - 31 diurnal tidal waves (also in dehanttideinel.f (=subroutine provided by V.Dehant))
  - estimate Love & Shida numbers separately or
  - apply condition equation with **FCN-period** and **resonance strength** factor

The screenshot shows a window titled 'guiglob\_hl' with two main sections: 'Love numbers' and 'Shida numbers'. Each section has a 'constant' and a 'diurnal band' sub-section. The 'diurnal band' sections are checked, indicating that diurnal Love and Shida numbers are to be estimated. Below these are 31 checkboxes for specific tidal waves, with many already checked. At the bottom, there are checkboxes for 'apply condition equation', 'estimate resonance factor hRS', and 'estimate FCN', all of which are checked. An 'OK' button is visible in the center-right of the window.

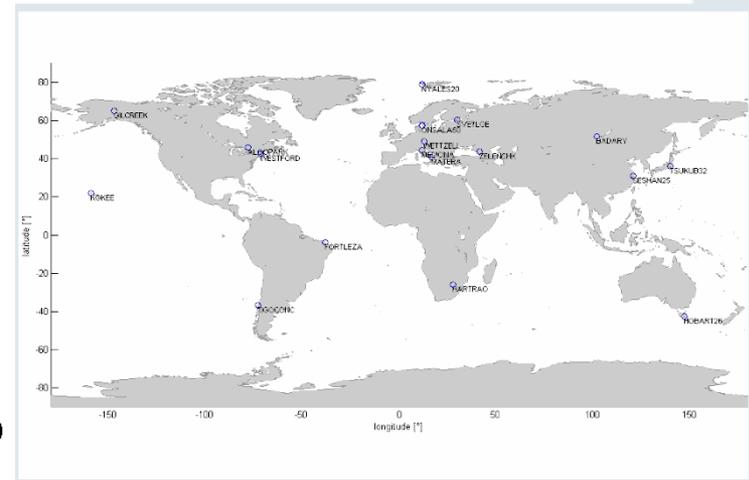
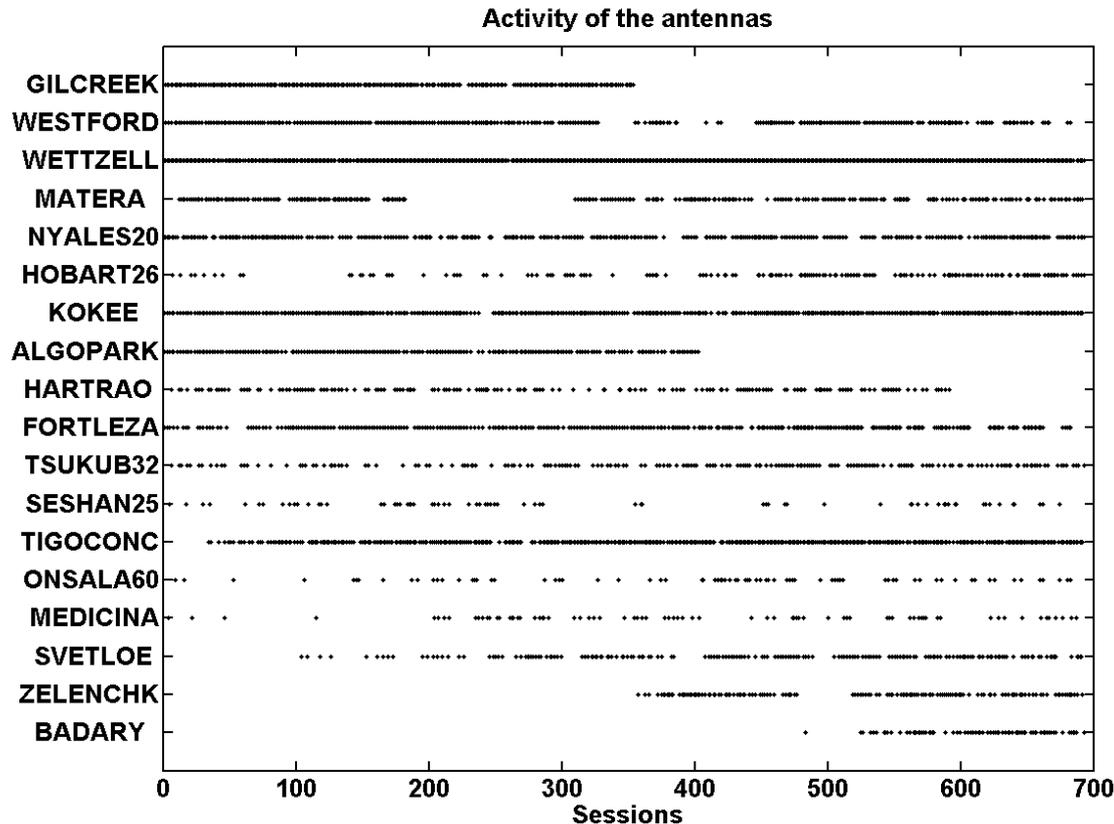
$$h_{21}(\omega_T) = h_{21}(\omega_{O1}) + h_{RS} \frac{\omega_T - \omega_{O1}}{\omega_{FCN} - \omega_T}$$

- R1 and R4 IVS sessions
  - 24-hour sessions
  - since Jan 2002 – Nov 2009 (7 years)

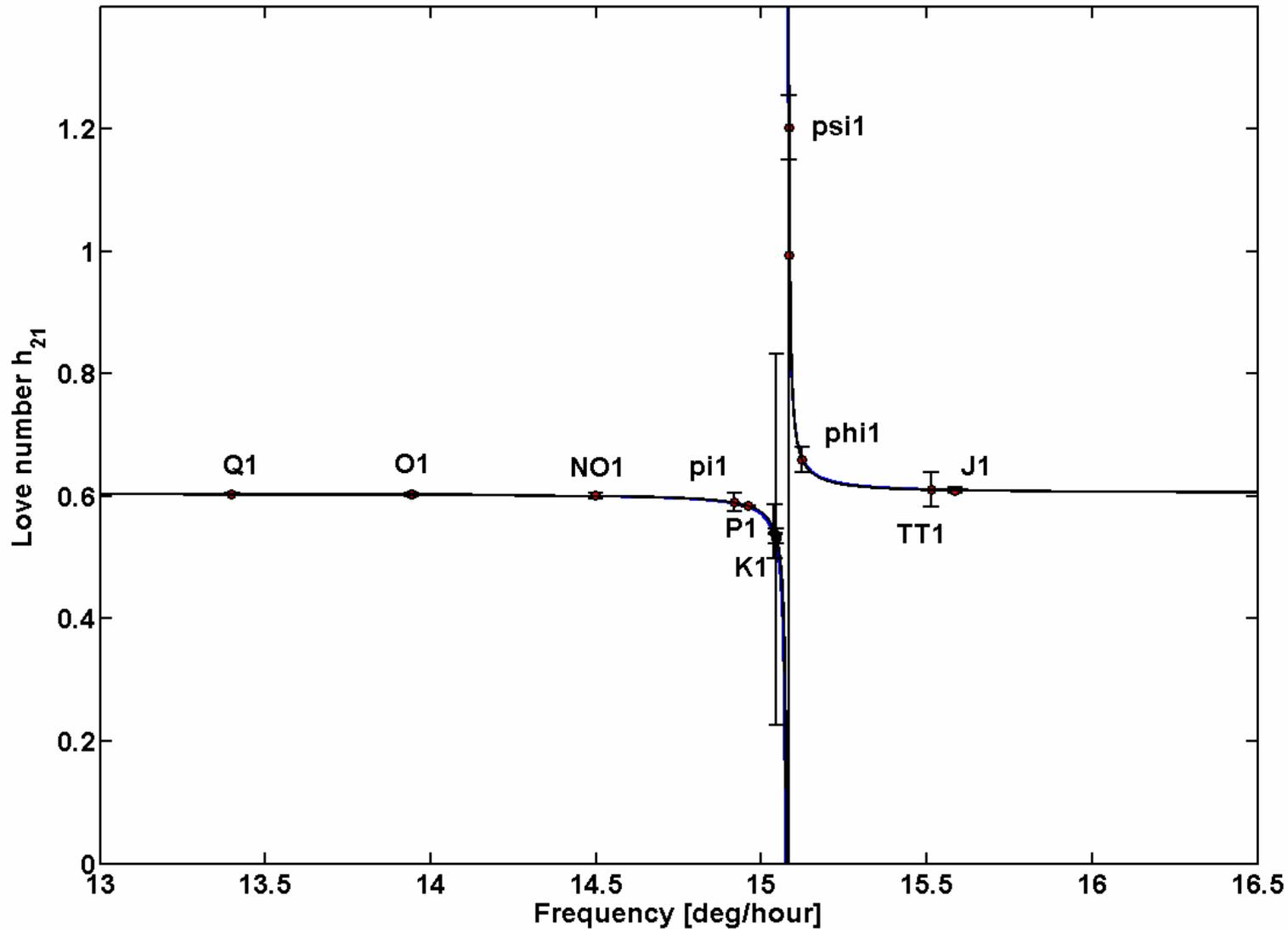
- outliers in observations removed
- only sessions with a posteriori variance of unit weight  $< 1.5$

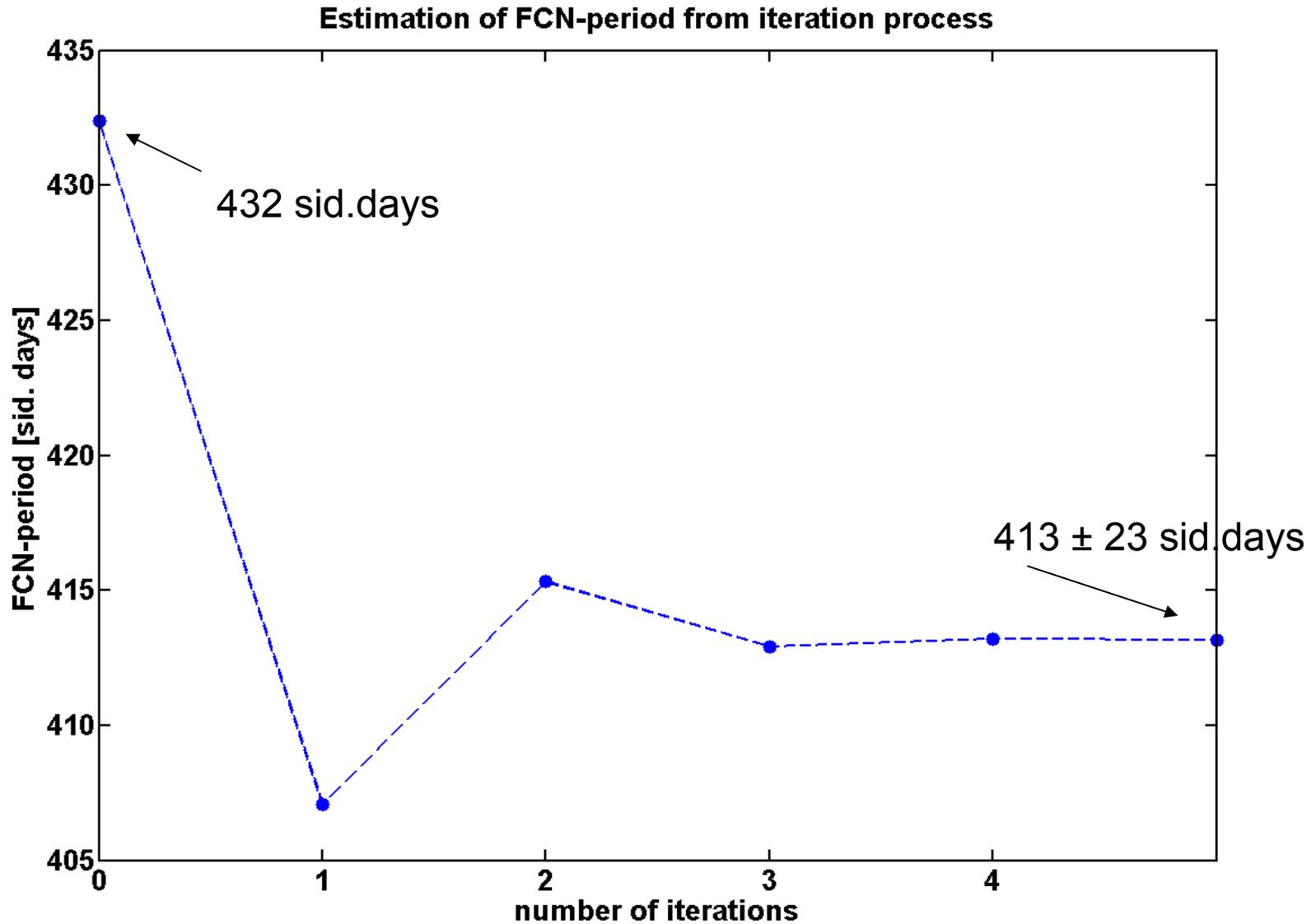
$$\sigma_0^2 = \frac{\mathbf{v}^T \mathbf{P} \mathbf{v}}{n_{obs} - u_{par}}$$

- 692 sessions at total

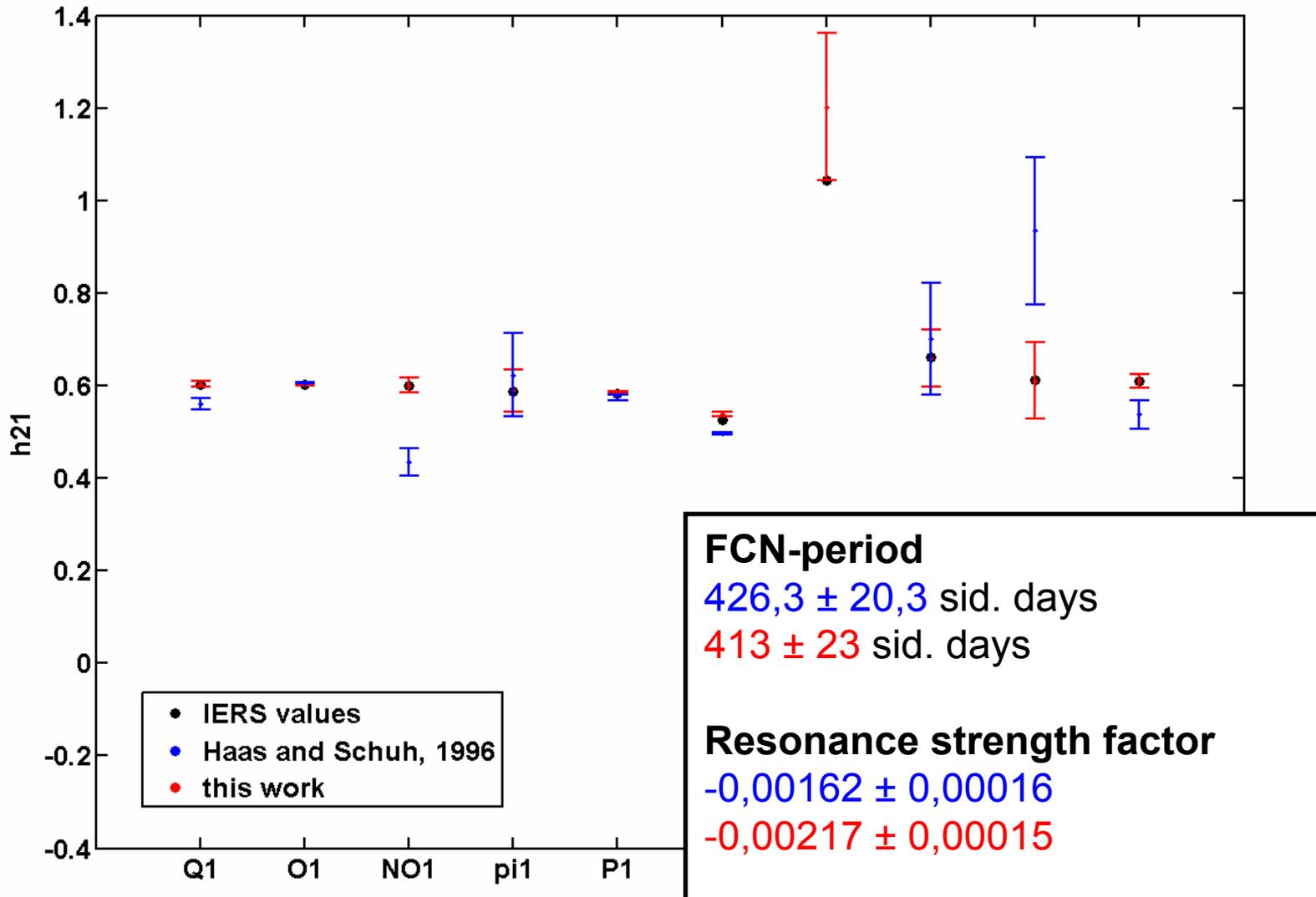


Frequency dependent Love numbers in diurnal band





## Estimates of frequency dependent Love numbers $h_{21}$



- VLBI group at TU Vienna is developing a new data analysis software, called VieVS (Vienna VLBI Software)  
<http://mars.hg.tuwien.ac.at/~vievs/>
- a new modul for global adjustment of more sessions is being implemented into the software
- aside from estimation of a new TRF/CRF or EOPs the global modul allows to determine geophysical parameters such as frequency dependent Love and Shida numbers and the Free Core Nutation period from solid Earth tidal deformations

**Thank you for your attention!**

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Parameters with VieVS

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