Piecewise Linear Offsets for VLBI Parameter Estimation

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Abstract. The Institute of Geodesy and Geophysics of the Vienna University of Technology is developing new software, Vienna VLBI Software, VieVS, for the analysis of Very Long Baseline Interferometry (VLBI) observations. In the parameter estimation high degrees of freedom, proper selection of estimation intervals and appropriate constraints for each parameter should be ensured. In VieVS, all parameters to be estimated are basically modelled with piecewise linear offset functions where offsets are at integer hours, integer fractions or at integer multiples of integer hours. The results of the least squares module of VieVS (*vie_lsm*) are the piecewise linear offsets of clocks, troposphere zenith wet delays, troposphere gradients, Earth orientation parameters, antenna coordinates, and others with their respective covariance matrices.

Keywords. VieVS, least squares, piecewise linear offset function, parameter estimation

1 Introduction

In the scope of creating a new VLBI analysis software called Vienna VLBI Software (VieVS) (Boehm et al., 2009), the least squares (LS) parameter estimation module has been mostly finished. The development of VieVS was not started from scratch but it is based on algorithms used in the software OCCAM (Titov et al., 2004). VieVS is designed to meet the most probable analysis related requirements of VLBI in the future, in particular the huge amount of observations compared to present sessions. Increasing the number of observations with the new VLBI system, VLBI2010, will provide enough degrees of freedom to determine the sub-daily variations of the VLBI estimates more accurately. In the least squares parameter estimation part of VieVS (*vie_lsm*) most of the estimated parameters are modelled by piecewise linear (pwl) offset functions (Boehm et al., 2009), and the required partial derivatives of the design matrix are produced by the preceding module, *vie_mod*. As exchange format between the modules Matlab 'mat' files are used.

2 Least squares parameter estimation in VieVS

VLBI parameters can be estimated according to the Least Squares (LS) adjustment based on the Gauss-Markov model. The complete functional and stochastic model can be formed as

$$\begin{bmatrix} v \\ v_c \end{bmatrix} = \begin{bmatrix} A \\ H \end{bmatrix} dx - \begin{bmatrix} l \\ h \end{bmatrix}, \quad \begin{bmatrix} P & 0 \\ 0 & P_c \end{bmatrix}, \tag{1}$$

where l is the vector of the reduced observations (observed minus computed), h is the vector of the constraint equations as pseudo observations, P is the weight matrix of the observations, P_c denotes to the weight matrix of the constraints. In Eq. (1), A is the upper block matrix of the functional model formed by presently 15 horizontally concatenated sub-matrices,

$$A = \begin{bmatrix} A_1 & A_2 & \cdots & A_{15} \end{bmatrix}, \tag{2}$$

consisting of the partial derivatives of pwl offset functions. Up to now the applied models are: pwl clock offsets (A_1), rate and quadratic terms of clock functions (A_2), pwl offsets of zenith wet delays (A_3), pwl offsets of tropospheric gradients (A_4, and A_5), pwl offsets of Earth orientation parameters (from A_6 to A_{10}), one Love number estimate per session (A_{11}), one Shida number estimate per session (A_{12}), and pwl offsets of antenna coordinates (from A_{13} to A_{15}). All the pwl offsets are estimated at UTC integer hours, integer fractions or at integer multiples of integer hours (Boehm et al., 2009). The length of the estimation intervals of station specific and global pwl offset parameters can be chosen between five minutes and one day. Independent from the number of clock breaks and the number of clocks where clock breaks occur, the treatment of clock breaks is handled properly by fitting clock functions for each interval defined by the clock breaks. The estimation is performed in units of centimeters and milliarcseconds. In order to avoid singularity problems of the normal equation matrix caused by an inadequate number of observations within an estimation interval, loose constraints are introduced (Kutterer, 2003).

The design matrix A_i with the partials of pwl offset functions (an example for zenith wet delays is described in Boehm et al. (2009)) for a scan including three observations, three antennas, and one estimation interval (result in two unknowns for each station) is

$$\begin{bmatrix} \frac{dL^1}{dx_{1,1}} & \frac{dL^1}{dx_{1,2}} & \frac{-dL^1}{dx_{2,1}} & \frac{-dL^1}{dx_{2,2}} & 0 & 0 \\ 0 & 0 & \frac{dL^2}{dx_{2,1}} & \frac{dL^2}{dx_{2,2}} & \frac{-dL^2}{dx_{3,1}} & \frac{-dL^2}{dx_{3,2}} \\ \frac{dL^3}{dx_{1,1}} & \frac{dL^3}{dx_{1,2}} & 0 & 0 & \frac{-dL^3}{dx_{3,1}} & \frac{-dL^3}{dx_{3,2}} \\ & & & \cdots & & \end{bmatrix}, \quad (3)$$

where $\frac{dL^k}{dx_{i,j}}$ are the partial derivatives of the delay L with respect to the offset estimates x, when k is the number of the observation, i the number of the station, and j the number of offset estimate per station. Loose constraints are introduced in order to avoid singularity (rank deficiency) of the design matrix. The constraints are applied as pseudo-observation equations

$$x_{i+1} - x_i = 0 \pm m_{\Delta x}, \quad (4)$$

where x_i is the *ith* pwl offset estimate. $m_{\Delta x}$ denotes the standard deviation of the constraint which can be a function of the estimation interval and a variance rate set before processing *vie_lsm*. The coefficients of the standard deviations can be selected in order to derive the parameters as tightly, loosely, or 'quasi-tightly' constrained. Singularity of the design matrix, which is treated properly in *vie_lsm*, occurs mostly when too short estimation intervals are selected before processing. The constraints Eq. (4) on the offset estimates formed in the lower block Eq. (1) of the design matrix, H, are made up as follows

$$H = \begin{bmatrix} H_1 & 0 & \cdots & 0 \\ 0 & H_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & H_{15} \end{bmatrix}, \quad (5)$$

where H_i is the sub-constrain matrix of H for the respective model, consisting of the partials of Eq. (4), simply formed by

$$H_i = \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & -1 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & -1 \end{bmatrix}. \quad (6)$$

According to analysis requirements, any model can be excluded from the *vie_lsm* processing station-wise or entirely. Besides, constraints on the parameters can be omitted for each model separately or for all models.

The datum definition of the VLBI network can be provided as full-trace-minimum (No Net Translation (NNT) and No Net Rotation (NNR)) over all stations or partial trace minima can be applied for certain stations. The NNT/NNR conditions for antenna coordinates are applied according to Helmert's method as originally presented by F.R. Helmert in 1872 (Pringle and Rayner, 1971), (Rao and Mitra, 1971), (Pelzer, 1974), and (Gotthardt and Schmitt, 1978). The condition equations on the coordinate estimates (C) are added to the normal equation matrix as rows and columns

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ & \cdots & \\ 0 & -z_i^{'} & y_i^{'} \\ z_i^{'} & 0 & -x_i^{'} \\ -y_i^{'} & x_i^{'} & 0 \\ & \cdots & \\ x_i^{'} & y_i^{'} & z_i^{'} \end{bmatrix}, \quad (7)$$

where $x_i^{'}, y_i^{'}, z_i^{'}$ are the a priori coordinates (translated to the geometric center of the stations and scaled) of the stations for which the NNT/NNR conditions are introduced. The overall LS solution is done after forming the matrices

$$dx_{total} = N_{total}^{-1}\, b_{total}, \qquad (8)$$

where the vector of unknowns (dx_{total}) is

$$dx_{total} = \begin{bmatrix} dx \\ x_c \end{bmatrix}. \qquad (9)$$

The normal equation matrix (N_{total}) after introducing the constraints as pseudo observations and condition equations is

$$N_{total} = \begin{bmatrix} A^T P A + H^T P_c\, H & C^T \\ C & 0 \end{bmatrix} \qquad (10)$$

and the normal equation constants (right hand side) vector (b_total) is

$$b_{total} = \begin{bmatrix} A^T Pl + H^T P_c\, h \\ b\_c \end{bmatrix}, \qquad (11)$$

where b_c is the constant vector if any constants of condition equations exist. The a posteriori variance factor for the constrained normal equation system is

$$s0_c = (v^T Pv + v_c^T P_c v_c) \;/\; dof, \qquad (12)$$

where dof is the degrees of freedom of the adjustment. It can easily be computed with

$$dof = n_{obs} + n_{constr} + n_{unk}, \qquad (13)$$

where n_{obs} is the number of observations, n_{constr} is the number of constraints, n_{unk} is the number of unknowns in the adjustment. The variance-covariance matrix of the unknowns of the constrained normal equation system can be computed with

$$K_{xx} = s0_c\, N_{total}^{-1}. \qquad (14)$$

Each estimated value (dx), the standard deviation of the estimate (m_{dx}), the estimation epoch in Modified Julian Date and the respective column number of the normal equation matrix, the options of the least squares parameter estimation, the normal equation matrix (N_{total}) and the vector of the right-hand side (b_{total}) for global solutions are stored in the output of the *vie_lsm* module in a Matlab structure array.

2.1 Future prospects

A proper outlier detection test, elevation dependent weighting of observations, and no net translation conditions for the clocks will be implemented. Pwl offsets of quasar coordinates will be estimated as well, and for specific analyses, certain stations can be excluded from the parameter estimation process, so that the respective station-wise parameters (e.g. observations, respective weights of the observations, partials and a priori values of the estimates related to these stations) will not be taken into account in the parameter estimation. Spectral analysis sub-routines of Matlab will be used for statistical analysis of the time series of the estimates produced by global solutions.

2.2 Estimation strategy

The interval for piecewise linear modelling of the parameters is usually set to values between one day and ten minutes. Due to the limited number of observations in a session, the estimation intervals should be selected accordingly. Too short estimation intervals for every parameter will cause too many unknowns which will decrease the degrees of freedom of the adjustment and may lead to several singularity problems. In order to avoid numerical problems (as e.g. rank deficiencies) and to stabilize the parameter estimation process, constraints (or pseudo-observations) have to be included in intervals with only a small number of observations. The weights of the pseudo-observations have to be chosen according to those of the real observations.

The first 24h session of CONT05 processed by the VieVS module *vie_lsm* showed us that Matlab has enough capacity to estimate VLBI parameters accurately with its built in functions. The elapsed time to process the first 24h CONT05 session with 6195 observations and 879 scans, ranges from 20 to 60 seconds mainly depending on the parameterization of the estimates. This time span includes loading the exchange files provided by the previous module, forming the estimation related matrices, inversion of the normal equation matrix and saving the outputs. In Figures 1, 2, and 3 some of the sub-daily estimates of VieVS are plotted for the first 24h session of CONT05.

3 Conclusions and prospects

From the investigations carried out within this study the following can be drawn:

- Piecewise linear offsets can be applied for almost all parameters of geodetic VLBI, e.g. zenith wet delays, troposphere gradients, clocks, Earth orientation parameters, antenna and quasar coordinates.
- The offsets should be determined at integer days, integer hours, or integer fractions of integer hours, respectively.
- A proper outlier detection test will be implemented.
- For specific analyses, it will be possible to exclude stations from the parameter estimation stage, with other words, the respective station-wise parameters (e.g. observations, weights of the observations, partials and apriori values of the estimates related to these stations) will not be taken into account in the *vie_lsm* module. This will provide us the flexibility of selecting particular baselines answering the needs of the analysis.
- More investigations on the magnitude of the amplitudes and respective sub-daily periods of the estimated parameters will be carried out.

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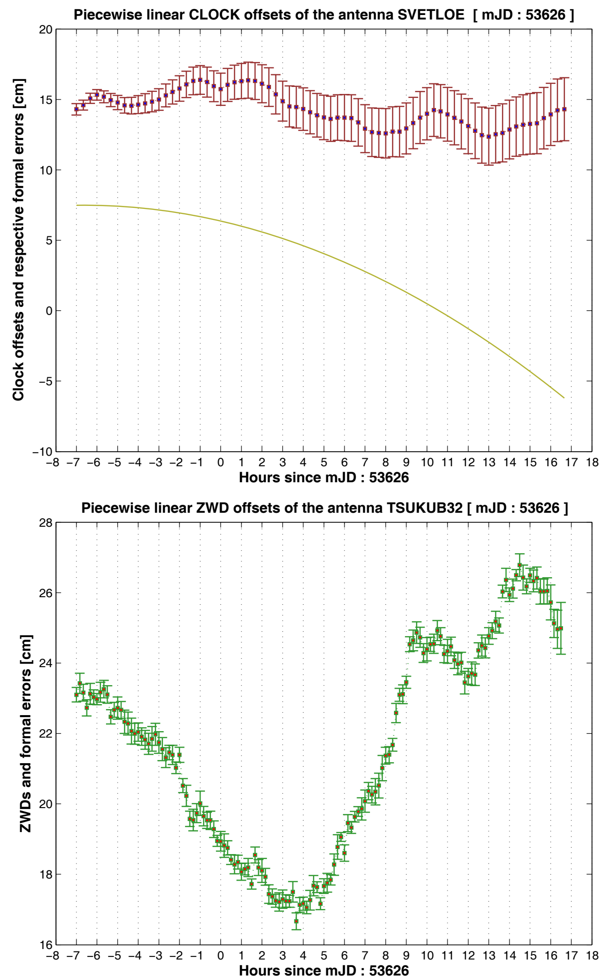

Figure 1. Parameters of clock function and pwl offset estimates of zenith wet delay

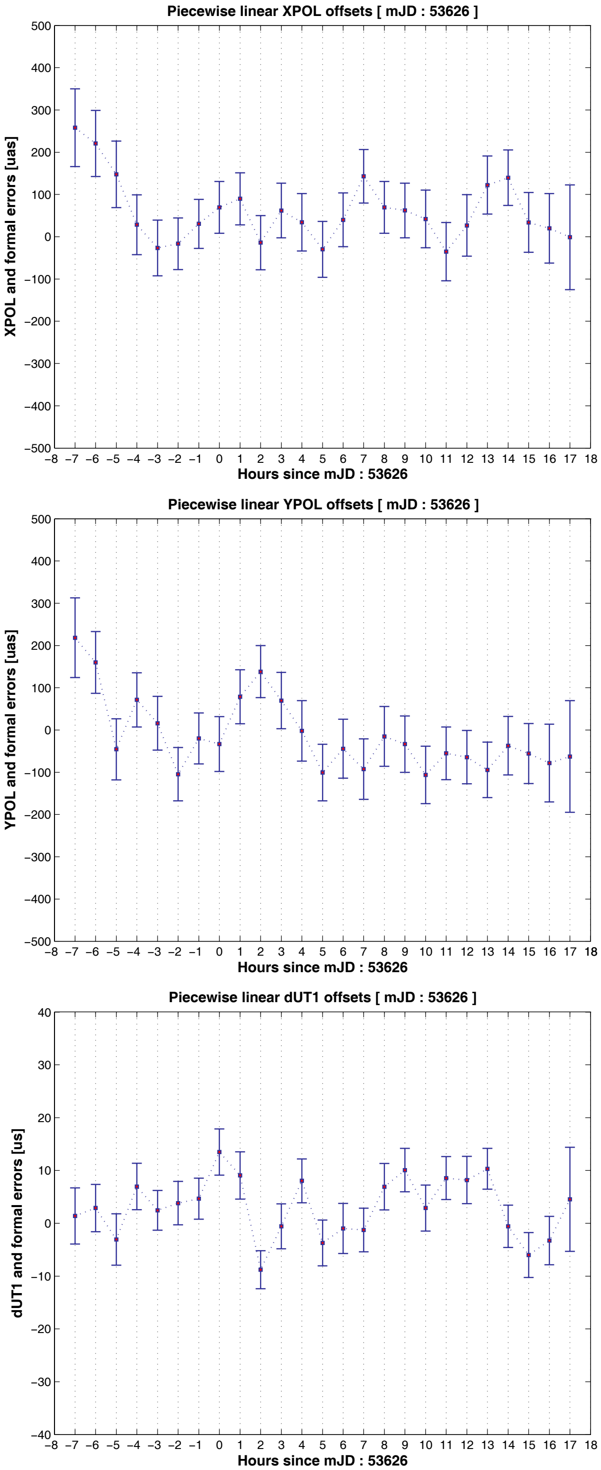

Figure 2. Sub-daily pwl offset estimates of Earth rotation parameters

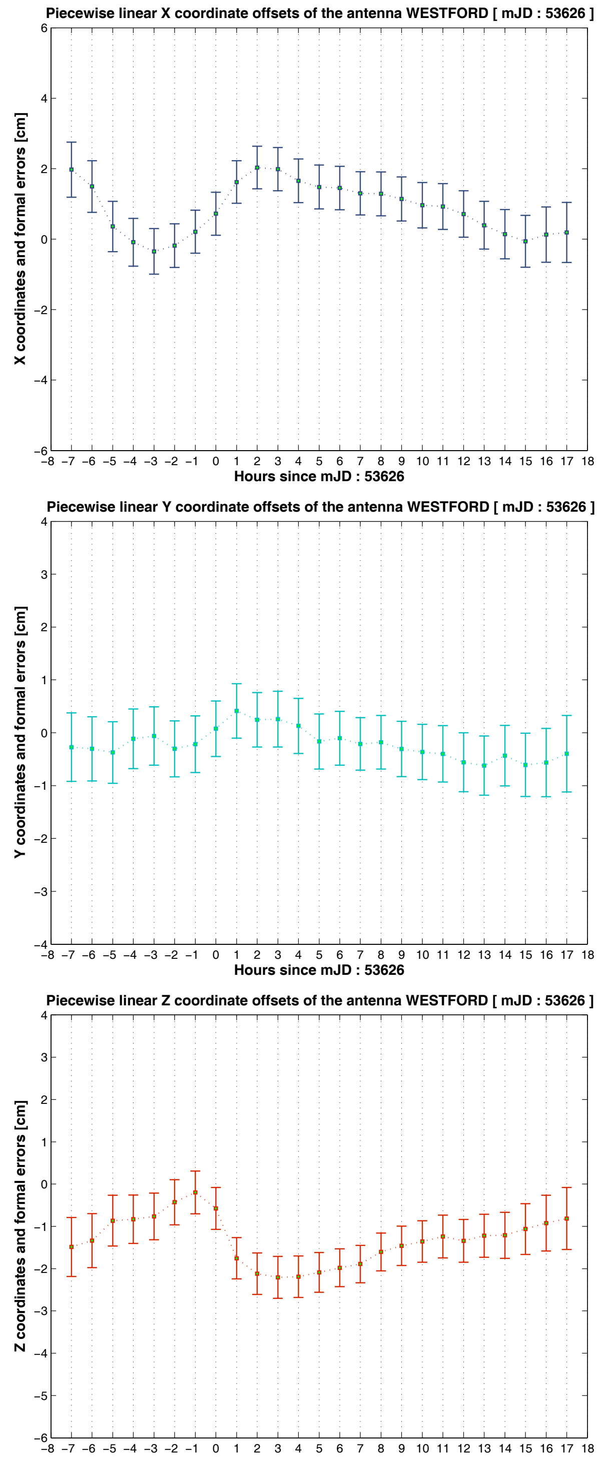

Figure 3. Sub-daily pwl offset estimates of antenna coordinates