Estimation of geodetic and geodynamical parameters with VieVS

Hana Spicakova, Johannes Böhm, Sigrid Böhm, Tobias Nilsson, Andrea Pany, Lucia Plank, Kamil Teke, Harald Schuh

Institute of Geodesy and Geophysics, Vienna University of Technology

Contact author: Hana Spicakova, e-mail: `hana@mars.hg.tuwien.ac.at`

Abstract

Since 2008 the VLBI group at the Institute of Geodesy and Geophysics at TU Vienna has focused on the development of a new VLBI data analysis software called VieVS (Vienna VLBI Software). One part of the program, currently under development, is a unit for parameter estimation in so-called global solutions, where the connection of the single sessions is done by stacking at the normal equation level. We can determine time independent geodynamical parameters such as Love and Shida numbers of the solid Earth tides. Apart from the estimation of the constant nominal values of Love and Shida numbers for the second degree of the tidal potential, it is possible to determine frequency dependent values in the diurnal band together with the resonance frequency of Free Core Nutation in VieVS. In this paper we show first results obtained from the 24-hour IVS R1 and R4 sessions.

1. Introduction

The Vienna VLBI Software VieVS (Boehm et al., 2009 [1]) is a data analysis software, which is being developed by the VLBI group at the Vienna University of Technology. The software is written in Matlab with the plan to make it compatible with equivalent non-commercial software, e.g., Octave. One of our main goals is to develop an easy-to-handle software, which will be attractive to our students and thus allow us to easily involve more students in VLBI research. In the modelling part of the software (vie_mod) the theoretical VLBI observable (group delay) is computed following the most recent IERS Conventions (McCarthy and Petit, 2004 [7]) and IVS standards like the treatment of thermal deformation of VLBI radio telescopes (Nothnagel, 2009 [9]). The time varying parameters (such as Earth orientation or troposphere parameters) are estimated using piecewise linear offsets at integer fractions of integer hours with an least-squares algorithm to easily allow comparison and combination with estimates from other space geodetic techniques. The VieVS software is extended by an extra program unit for parameter estimation in so-called global solutions, where the connection of the single sessions is done by stacking of the normal equations. In this paper we focus on the determination of geodynamical parameters of the solid Earth tides: frequency dependent Love and Shida numbers in the diurnal tidal band together with the Free Core Nutation period.

2. Global solution in VieVS

Because of the limited computer memory capacity it is essential to keep the equation system small. In the VLBI technique there are parameters in the observation equations which cannot be fixed to a priori values, even if we are not interested in them, e.g., clock parameters. Therefore, a reduction algorithm is used which is based on a division of the normal equation system into two parts. The first part contains parameters which we want to estimate and the second part the parameters which will be reduced:

$$\begin{pmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{pmatrix} \cdot \begin{pmatrix} dx_1 \\ dx_2 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \quad , \tag{1}$$

where $N = A^T PA$ and $b = A^T Pl$. The reduction of dx_2 is done by executing the matrix operation in Equation (1) and introducing the second equation into the first one:

$$\left(N_{11} - N_{12}N_{22}^{-1}N_{21}\right) \cdot dx_1 = b_1 - N_{12}N_{22}^{-1}b_2 \quad \Longleftrightarrow \quad N_{reduc} \cdot dx_1 = b_{reduc} \quad . \tag{2}$$

Stacking is used for combining normal equation systems if a parameter is contained in at least two normal equation systems and only one value in the resulting combined system should be estimated. For a combined solution of the identical parameters (dx), the normal matrices (N_{reduc}) and the right hand side vectors (b_{reduc}) from n single sessions have to be summed up:

$$N_{REDUC} = N_{reduc\_1} + N_{reduc\_2} + ... + N_{reduc\_n} \quad , \tag{3}$$

$$b_{REDUC} = b_{reduc\_1} + b_{reduc\_2} + ... + b_{reduc\_n} \quad . \tag{4}$$

The final solution is obtained by an inversion of the normal matrix:

$$dx_1 = N_{REDUC}^{-1} \cdot b_{reduc} \quad . \tag{5}$$

3. Love numbers and Free Core Nutation period

Solid Earth deformation arises from the variations in the Earth's gravitational field caused by the Moon and the Sun. Love and Shida numbers ($h,\ l$) are dimensionless parameters, which reflect the amount by which the surface of the Earth responds to the tide-generating potential (V^{tid}). Considering the Earth being spherical, non-rotating, elastic and isotropic, what is the most basic model, the tidal displacement (u) at a given latitude and longitude ($\varphi,\ \lambda$) in the topocentric system is described by the following equations, where g stands for gravity acceleration and n is the degree of the tide-generating potential.

$$u_R = \sum_{n=2}^{\infty} h_n \cdot \frac{1}{g} \cdot V_n^{tid} \quad , \tag{6}$$

$$u_E = \sum_{n=2}^{\infty} l_n \cdot \frac{1}{g \cdot \cos\varphi} \cdot \frac{\partial V_n^{tid}}{\partial \lambda} \quad , \tag{7}$$

$$u_N = \sum_{n=2}^{\infty} l_n \cdot \frac{1}{g} \cdot \frac{\partial V_n^{tid}}{\partial \varphi} \quad . \tag{8}$$

When a more precise model of the Earth is considered, i.e. ellipticity, rotation and the elastic mantle – fluid core boundary is taken into account, the relationship between the tide-generating potential and the displacement becomes more complicated. The displacement vector is composed of deformational parts (δu) of specific harmonic degree and order, and specific frequency inside the band (Wahr, 1981 [14]).

$$\delta u_{R(f)}^{(21)} = -\frac{3}{2}\sqrt{\frac{5}{24\pi}} H_f \delta h_{21(f)} \sin(2\varphi) \sin(\theta_f + \lambda) \qquad , \qquad (9)$$

where H_f is the Cartwright-Tayler amplitude of the tidal term, $\delta h_{21(f)}$ stands for the difference of the frequency dependent Love number from the constant second degree Love number h_2, and θ_f is a tidal harmonic argument. The strong frequency dependence in the diurnal tidal response of the solid Earth is caused by inhomogeneities in the Earth's interior. This variation of Love numbers with frequency in the diurnal band arises from the resonance behaviour of the Earth, which is caused by the presence of the fluid core. The rotational axis is slightly inclined w.r.t. the axis of rotation of the mantle. In this situation forces arise at the elliptical core-mantle boundary, which try to realign the two axes. In the terrestrial reference frame, this phenomenon is seen as a diurnal motion, in the celestial frame it appears as a retrograde motion of the celestial pole with a period of approximately 430 days and is designated as Free Core Nutation (FCN). Because this effect is a free motion with time-varying excitation and damping, resulting in a variable amplitude and phase, an FCN model is not included in the recent precession-nutation model adopted by the International Astronomical Union (IAU 2006/2000A). It means that after taking into account the precession-nutation model a quasi-periodic unmodeled motion of the celestial pole in the celestial frame at the 0.1 – 0.3 mas level still remains in the measured data (e.g., Lambert, 2007 [6]).

3.1. Estimation of Love numbers and FCN in VieVS

Basically, three approaches can be used to determine the period of the Free Core Nutation motion. The first option is an indirect estimation from analysis of the celestial pole offsets obtained from VLBI measurements (e.g., Herring et al., 1986 [4], Vondrak and Ron, 2006 [13]). The FCN effects are also seen in gravity data, where the first determination of the resonant period was done by Neuberg et al. (1987) [8] and was followed by, e.g., Sato et al. (1994, 2004) [11], [12] or Ducarme et al. (2007) [2]. In the VieVS software we implemented the possibility to determine the FCN period directly from analysis of the observed solid Earth tidal displacement of the VLBI antennas, as it was introduced by Haas and Schuh (1996) [3]. For the estimation of the FCN period a resonance formula for the Love numbers in the diurnal band is used, which was published by Wahr (1981) [14]:

$$h_{21}(\omega_T) = h_{21}(\omega_{O1}) + h_{RS} \cdot \frac{\omega_T - \omega_{O1}}{\omega_{FCN} - \omega_T} \qquad . \qquad (10)$$

$h_{21}(\omega_{O1})$ is the Love number of the O1 tidal wave, whose frequency ω_{O1} is used as reference which is far enough from the resonance frequency of the FCN ω_{FCN}. This formula is applied in the least-squares adjustment as a condition equation constraining the estimates of diurnal Love numbers at the tidal waves T, by means of fitting them to the resonance curve, for which also the resonance strength factor h_{RS} was estimated. Because the equation is not linear w.r.t. the ω_{FCN}, iterations have to be carried out.

4. Data and Results

For this paper we used all IVS R1 and R4 sessions, i.e. 24-hour observing sessions which have been performed two times per week since January 2002 using global VLBI networks. Before the analysis we removed outliers from the observations and excluded sessions with an a posteriori variance of unit weight higher than 1.5. Clock parameters, zenith wet delays, troposphere gradients and Earth orientation parameters were reduced from the normal equation system and estimated implicitly. The station coordinates and velocities of the 18 VLBI radio telescopes together with the radio source coordinates were fixed to their a priori catalogue values, ITRF2005 and ICRF2, respectively. We solved for Love numbers of 10 diurnal tidal waves using partial derivatives from Equation (9) and the resonance parameters (i.e. FCN period with the resonance strength factor h_{RS} were estimated after 5 iterations from the condition equation (10).

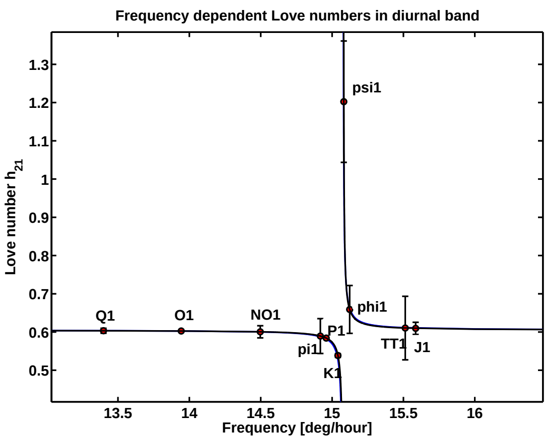

Figure 1. Estimates of the diurnal Love numbers h_{21} and the resonance curve.

Our estimates of the Love numbers are in a very good agreement with the recently adopted IERS values. Larger differences can be seen mainly for the tidal waves in the vicinity of the resonance period (phi1, psi1) and for the tides with lower amplitudes (e.g., NO1, pi1, J1). For the resonance period we obtained 413 ± 20 sidereal days in the celestial frame. The large standard deviation of the FCN period might be due to using only 8 years of data and the high correlation with the resonance strength factor, as was pointed out in Haas and Schuh (1996) [3]. An apparent time-variation of the FCN period was discussed in Roosbeek et al. (1999) [10] or Hinderer et al. (2000) [5].

Table 1. Estimates of frequency dependent Love numbers for 10 diurnal tidal waves, FCN period and resonance strength factor compared with the IERS values [7] and with the results from Haas and Schuh (1996) [3].

		Love number h_{21}		
Frequency [°/h]	Tide	IERS values [7]	this paper	Haas and Schuh (1996) [3]
13.3987	Q1	0.6033	0.6033 ± 0.0065	0.560 ± 0.012
13.9430	O1	0.6026	0.6026 ± 0.0017	0.606 ± 0.002
14.4967	NO1	0.6004	0.6005 ± 0.0158	0.435 ± 0.030
14.9179	pi1	0.5882	0.5895 ± 0.0456	0.623 ± 0.090
14.9589	P1	0.5823	0.5842 ± 0.0027	0.574 ± 0.005
15.0411	K1	0.5261	0.5385 ± 0.0049	0.496 ± 0.002
15.0821	psi1	1.0439	1.2023 ± 0.1587	-0.136 ± 0.228
15.1232	phi1	0.6623	0.6590 ± 0.0626	0.702 ± 0.121
15.5126	TT1	0.6113	0.6104 ± 0.0830	0.934 ± 0.158
15.5854	J1	0.6105	0.6096 ± 0.0159	0.538 ± 0.031
FCN period		432 sid. days	413 ± 20 sid. days	426.3 ± 20.3 sid.days
h_{RS}		–	-0.00217 ± 0.00015	-0.00162 ± 0.00016

5. Summary

The VLBI group at Vienna University of Technology is developing a new data analysis software, called VieVS (Vienna VLBI Software). The first version of the software has been released and can be downloaded by registered users. In this paper we focused on a new module for global adjustment of more sessions, which is implemented into the software. Besides from estimation of a new terrestrial reference frame, celestial reference frame and Earth orientation parameters, the program unit allows to determine geophysical parameters such as frequency dependent Love and Shida numbers and the Free Core Nutation period from solid Earth tidal deformations.

References

[1] Boehm, J., H. Spicakova, L. Plank, K. Teke, A. Pany, J. Wresnik, S. Englich, H. Schuh, T. Hobiger, R. Ichikawa, Y. Koyama, T. Gotoh, T. Otsubo, T. Kubooka, Plans for the Vienna VLBI Software VieVS, Proceedings of the 19th European VLBI for Geodesy and Astrometry Working Meeting, edited by G. Bourda, P. Charlot, A. Collioud, pp. 161–164, 2009.

[2] Ducarme, B., H.-P. Sun, J.-Q. Xu, Determination of the free core nutation period from tidal gravity observations of the GGP superconducting gravimeter network, J Geod. 81, pp. 179–187, 2007.

[3] Haas, R., H. Schuh, Determination of frequency dependent Love and Shida numbers from VLBI data, Geophys. Res. Lett. Vol. 23 No. 12/1996, pp. 1509–1512, 1996.

[4] Herring, T. A., C. R. Gwinn, I. I. Shapiro, Geodesy by radio interferometry: studies of the forced nutations of the Earth, J Geophys Res 91(B5), pp. 4745–4754, 1986.

[5] Hinderer, J., J. P. Boy, P. Gegout, P. Defraigne, F. Roosbeek, V. Dehant, Are the free core nutation parameters variable in time?, Physics of the Earth and Planetary Interiors 117, pp. 37–49, 2000.

[6] Lambert, S., Empirical modeling of the retrograde Free Core Nutation, available at ftp://hpiers.obspm.fr/eop-pc/models/fcn/notice.pdf

[7] McCarthy, D. D., and G. Petit, IERS Conventions 2003, IERS Technical Note 32, Verlag des Bundesamtes fuer Kartographie und Geodaesie. 2004. + electronic updates.

[8] Neuberg, J., J. Hinderer, W. Zuern, Stacking gravity tide observations in central Europe for the retrieval of the complex eigenfrequency of the Nearly Diurnal Free Wobble, Geophys. J. R. Astr. Soc. 91, pp. 853–868, 1987.

[9] Nothnagel, A., Conventions on thermal expansion modelling of radio telescopes for geodetic and astrometric VLBI, Journal of Geodesy, 83, pp. 787–792, 2009.

[10] Roosbeek, F., P. Defraigne, M. Feissel, V. Dehant, The free core nutation period stays between 431 and 434 sidereal days, Geophys. Res. Letters., 26, 1, pp. 131–134, 1999.

[11] Sato, T., Y. Tamura, T. Higashi, S. Takemoto, I. Nakagawa, N. Morimoto, Y. Fukuda, J. Segawa, N. Seama, Resonance parameters of Nearly Diurnal Free Core Nutation measured from three superconducting gravimeters in Japan, J. Geomagn. Geoelectr. 46, pp. 571–586, 1994.

[12] Sato, T., Y. Tamura, K. Matsumoto, Y. Imanishi, H. McQueen, Parameters of the fluid core resonance inferred from superconducting gravimeter data, J. Geodyn. 38, pp. 375–389, 2004.

[13] Vondrak, J., C. Ron, Resonant period of free core nutation – its observed changes and excitations. Acta Geodyn. Geomater. 3 (143), pp. 53–60, 2006.

[14] Wahr, J. M., Body tides on an elliptical rotating, elastic and oceanless Earth, Geophys. J. R. Astron. Soc., 64, pp. 677–703, 1981.