Problem Set 1

Part A

The following problems are from Ward Cheney and David Kincaid's text *Linear Algebra: Theory and Applications*.

1. Solve the following systems of linear equations:

$$\begin{bmatrix} 3 & 6 & 6 \\ 2 & 4 & 5 \\ 2 & 5 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 21 \\ 16 \\ 17 \end{bmatrix} \qquad \begin{bmatrix} 1 & 3 & 7 \\ 2 & 1 & 0 \\ 11 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 42 \\ 6 \\ 76 \end{bmatrix}$$

2. Consider the following system of linear equations:

$$\begin{bmatrix} \alpha & 3 \\ \gamma & -5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ -4 \end{bmatrix}$$

For what values of α and γ does the system have a (nontrivial) solution?

3. Consider the following system of linear equations:

$$\begin{bmatrix} 0.780 & 0.563 \\ 0.913 & 0.659 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0.217 \\ 0.254 \end{bmatrix}$$

Which solution is better? $(x, y) = (0.341, -0.087)$ or $(x, y) = (0.999, -1.001)$. Explain.

4. Consider the equation $3x(2y+5) + \ln(x^2) - 2y(6+3x) = 13 + 2\ln(x)$. Explain why this is really a *linear* equation.

5. Describe the general solution, if any, of the following system of linear equations:

$$\begin{bmatrix} -12 & 0 & -1 & 2 \\ 16 & 3 & 1 & 0 \\ 20 & 3 & 2 & 4 \\ 12 & 3 & 1 & 3 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

6. Three planes in $\mathbb{R}^3$ are given as $3x + y = 6$, $5x + y + z = 6$ and $3x + z = 1$. Do these planes have a point in common? If so, find this point. If not, explain why not.

Part B

The following problems are from Chiang and Wainwright's text (4th edition).

1. From Exercise 4.2 (page 58): 1 and 2
2. From Exercise 4.3 (page 66): 2
3. From Exercise 4.4 (page 70): 4 and 6
4. From Exercise 4.5 (page 72): 1 and 2
5. From Exercise 4.6 (page 78): 4
6. From Exercise 5.2 (page 93): 4 and 6
7. From Exercise 5.3 (page 98): 4
8. From Exercise 5.4 (page 102): 4, 6, and 7

Part C

Let a linear economic model be given as

$$Ax + By = z$$

where $x \in \mathbb{R}^m$ is the vector of exogenous variables, $y \in \mathbb{R}^n$ is the vector of endogenous variables and $z \in \mathbb{R}^n$ is the vector of some constants.

1. Suppose that x, z, A and B are known. Which matrix should be nonsingular for the system to have a solution.
2. Write down the solution for y.
3. Now suppose that y, z, A and B are known but x is unknown. Does this new system have a solution if $m > n$? Why or why not?