Vortices in trapped boson-fermion mixtures


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We consider a trapped system of atomic boson-fermion mixture with a quantized vortex. We investigate the density profiles of bosonic and fermionic components as functions of the boson-boson and boson-fermion short-range interaction strengths within the mean-field approach. Stability of a vortex and conditions for the phase segregation are studied. We compare and contrast our results with the related system of droplets of $^3$He-$^4$He mixtures.

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1. INTRODUCTION

After the successful achievement of Bose-Einstein condensation in dilute alkali gases\(^1\) under magneto-optical trap potentials, a vast theoretical and experimental activity on cold degenerate quantum gases has followed.\(^2\) More recently, fermionic gases are cooled to quantum degeneracy temperatures facilitated by mixing with cold bosonic gases by a process known as sympathetic cooling. Experimental progress in this direction has culminated in achieving the realization of quantum degenerate Bose-Fermi mixtures by several groups.\(^3\)-\(^8\) Currently there are a number of experiments on boson-fermion mixtures in harmonic traps. In the Paris experiment\(^4\) $^6$Li-$^7$Li mixture with a repulsive boson-fermion scattering length, and in the Florence experiment\(^8\) $^{40}$K-$^{87}$Rb mixture with an attractive boson-fermion scattering length are realized. Furthermore, using Feshbach resonances many groups have tuned the scattering length both for bosons and fermions. Theoretical studies on trapped boson-fermion mixtures employed the mean-field theory at zero temperature to determine the density profiles of respective components.\(^9\)-\(^12\) Related properties such as the stability against phase separation and collapse were also investigated.\(^13,14\) The temperature effects and their role in phase separation were addressed by Akdeniz et al.\(^15\) The critical temperature of the Bose-Einstein condensation in a trapped mixture were
considered by several groups.\textsuperscript{16,17}

Motivated by these recent experiments on boson-fermion mixtures of dilute alkali gases, in this paper, we study the ground state properties of such system in the presence of a single vortex. The quantized vortices are important in establishing the superfluid nature of Bose condensates.\textsuperscript{2} Recently there has been numerous experimental works devoted to the creation and investigation of properties of quantized vortices in trapped condensates.\textsuperscript{18} We are also motivated by the analogies and differences of trapped quantum gases and Helium droplets as prototypes of finite quantum fluids as recently surveyed by Dalfovo and Stringari.\textsuperscript{19} To this end, we make contact with recent theoretical calculations of a vortex state in $^3$He-$^4$He droplets.\textsuperscript{20-22}

We employ the mean-field theory at zero temperature to consider a mixture of Bose condensed atoms and spin-polarized gas of fermions in a harmonic trap. Introducing a single quantized vortex through the Feynman-Onsager ansatz we study the ensuing density profiles of respective species. The density profiles are obtained by solving the mean-field equations for the trapped boson-fermion mixture using a variational ansatz.

\section{Model and Theory}

We consider $N_B$ bosons of mass $m_B$ in the condensed state and $N_F$ fermions of mass $m_F$ in respective trap potentials $V_B = \frac{1}{2}m_B\omega_B^2r^2$ and $V_F = \frac{1}{2}m_F\omega_F^2r^2$ in the form of isotropic harmonic oscillators. $\omega_B$ and $\omega_F$ are the trap frequencies for bosonic and fermionic species, respectively. The ground-state energy functional of a mixture of bosons and fermions in the mean-field approximation is given by

\[ E[n_B(r), n_F(r)] = \int \text{d}r \left( E_B + E_F + E_{BF} \right). \]

The energy density of bosons is

\[ E_B = \frac{\hbar^2}{2m_B} |\nabla \Psi(r)|^2 + V_B(r)n_B(r) + \frac{g}{2}n_B(r)^2, \tag{1} \]

where $\Psi(r)$ is the condensate wavefunction, $n_B(r) = |\Psi(r)|^2$ is the condensate density distribution, and $g$ is the boson-boson interaction strength. Since the fermions are assumed to be noninteracting, we have

\[ E_F = T_F[n_F(r)] + V_F(r)n_F(r), \tag{2} \]

where the kinetic energy functional for fermions with single spin species in the Thomas-Fermi approximation\textsuperscript{9,10,14} is $T_F = (6\pi^2n_F)^{5/3}/20\pi^2m_F$ and
the fermion density distribution is \( n_F(r) = \frac{(2m_B)^{3/2}}{6\pi^2} (\epsilon_F - V_F(r) - \hbar n_B(r))^{3/2} \) with \( \epsilon_F \) the Fermi energy. The boson-fermion interaction energy density is \( E_{BF} = \hbar n_F(r) n_B(r), \) where \( \hbar \) is the boson-fermion interaction strength. The total energy-density for the mixture now becomes

\[
E[n_B, n_F] = \int d^3 r \left[ \frac{\hbar^2}{2m_B} |\nabla \Psi(r)|^2 + V_B(r)|\Psi(r)|^2 + \frac{g}{2} |\Psi(r)|^4 \right] + T_F(n_F) + V_F(r)n_F(r) + \hbar n_F(r)|\Psi(r)|^2.
\] (3)

We have assumed that the fermionic component of the mixture is spin-polarized whereby the \( s \)-wave scattering between the fermions is inhibited by the Pauli principle. \( g \) and \( \hbar \) are the boson-boson and boson-fermion interaction strengths, respectively, related to the \( s \)-wave scattering lengths \( a_{BB} \) and \( a_{BF} \) as measured in experiments, viz. \( g = 4\pi\hbar^2 a_{BB}/m_B \) and \( h = 4\pi\hbar^2 a_{BF}/\mu_{BF} \), where \( \mu_{BF} \) is the reduced mass. We introduce a quantized vortex through the Feynman-Onsager ansatz, \( \Psi(r) = \psi(r)e^{i\phi} \), which amounts to adding a centrifugal energy term \( \frac{\hbar^2}{2m_B r^2} |\psi|^2 \) to the total energy functional. Our goal is to minimize the total energy functional subject to the normalization conditions \( \int dr n_B(r) = N_B \) and \( \int dr n_F(r) = N_F \).

To study the density profiles of boson and fermion components of the mixture, we now introduce the variational wavefunction for the condensate with a vortex, \( \Psi(r, \phi) = A r e^{i\phi} e^{-\alpha r^2} \) where \( A \) is the normalization constant and \( \alpha \) is the variational parameter. The normalization integral for \( N_B \) bosons yields \( A = N_B^{1/2} (128\alpha^5/9\pi^3) \).

3. RESULTS AND DISCUSSION

We have minimized the total energy of the mixture with respect to the variational parameter \( \alpha \) and the fermion density \( n_F(r) \) using the number of particles \( N_B \) and \( N_F \) as constraints. We have assumed the same mass \( m_B = m_F \) for both species and the same trap frequency \( \omega_B = \omega_F \) for simplicity. The boson-boson and boson-fermion interaction strengths are treated as tunable parameters.

In Fig.1 we show the density profile of fermion species \( n_F(r) \) as a function of the radial coordinate for various values of the repulsive boson-fermion interaction strength. For fixed boson-boson repulsive interaction \( (g = 0.005\hbar\omega_{HO}^3 \) in the examples shown) we observe a depletion in the central region of the fermion density as boson-fermion interaction strength increases. The dip in the central region of \( n_F(r) \) coincides with the maximum of the density profile of the condensate with a vortex. Further increase in \( h \)
Fig. 1. Density profile of fermions $n_F(r)$ (dashed lines) for $g = 0.005\hbar\omega a_{HO}^3$ and $N_B = N_F = 10^4$ particles. $h^* = \frac{\hbar}{(\hbar\omega a_{HO}^3)}$. Solid lines indicate the boson density $n_B(r)$.

causes the break up of fermion density into two parts, one filling the vortex core, the other part pushed to the outer region [Fig. 1(c)]. Eventually, when $h$ becomes very large, the fermions disappear from the vortex core region and occupy only the outer region surrounding the condensate [Fig. 1(d)]. This last situation is the phase separated case of two species, similar to the theoretically calculated case of trapped boson-fermion mixtures without a vortex.\(^6\) The small overlap of boson $n_B(r)$ and fermion $n_F(r)$ densities is an artifact of Gaussian variational wavefunction which would give in to a complete phase segregation in more elaborate calculations.

We point out the similarity between our results shown in Fig. 1 and those of Mayol et al.\(^20\) who considered quantized vortices in \(^3\)He-\(^4\)He droplets. They have found that even a small number of \(^3\)He atoms fills the vortex
Vortices in Trapped Boson-Fermion Mixtures

Fig. 2. Density profile of fermions \( n_F(r) \) (dashed lines) for \( g = 0.005 \hbar \omega a_{HO}^3 \) and \( N_B = N_F = 10^4 \) particles. \( h^* = \hbar / (\hbar \omega a_{HO}^3) \). Solid lines indicate the boson density \( n_B(r) \).

core provided by the quantized vortex in a \(^4\)He condensate. Whereas in the case of \(^3\)He-\(^4\)He mixtures the strong interaction potential between He atoms is fixed, the interactions between the alkali atoms can be tuned by Feshbach resonances to study a wider range of density profiles and possible phase separations.

We next consider attractive interactions between bosons and fermions. As shown in Fig. 2 an attractive boson-fermion interaction strength causes the central region of the fermion density \( n_F(r) \) to increase. At a critical value of \( h \) the system becomes unstable and the fermionic component collapses much like the situation in vortex-free boson-fermion mixtures studied previously.\(^7,8\)

Our variational calculations employing a Gaussian ansatz may be im-
proved by choosing better variational wavefunctions or numerically solving the coupled Euler-Lagrange equations for the mixture. We surmise, however, the results reported here should be qualitatively correct.

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REFERENCES