

Can a signal propagate
superluminal ($v > c$)
in dispersive medium?

M. Emre Taşgın

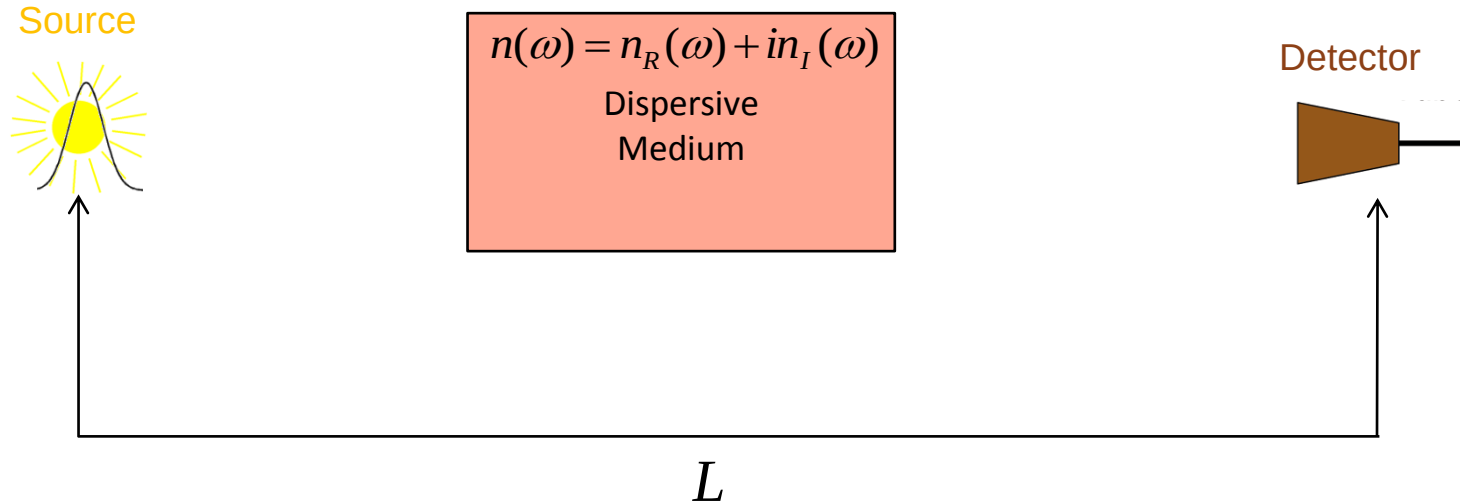
Outline

- Experiment: superluminal ($v > c$) propagation.
- Reshaping due to gain/absorption
- A theoretical method to test if velocity is reliable?
- Answer: is superluminal?
- Acknowledgements.

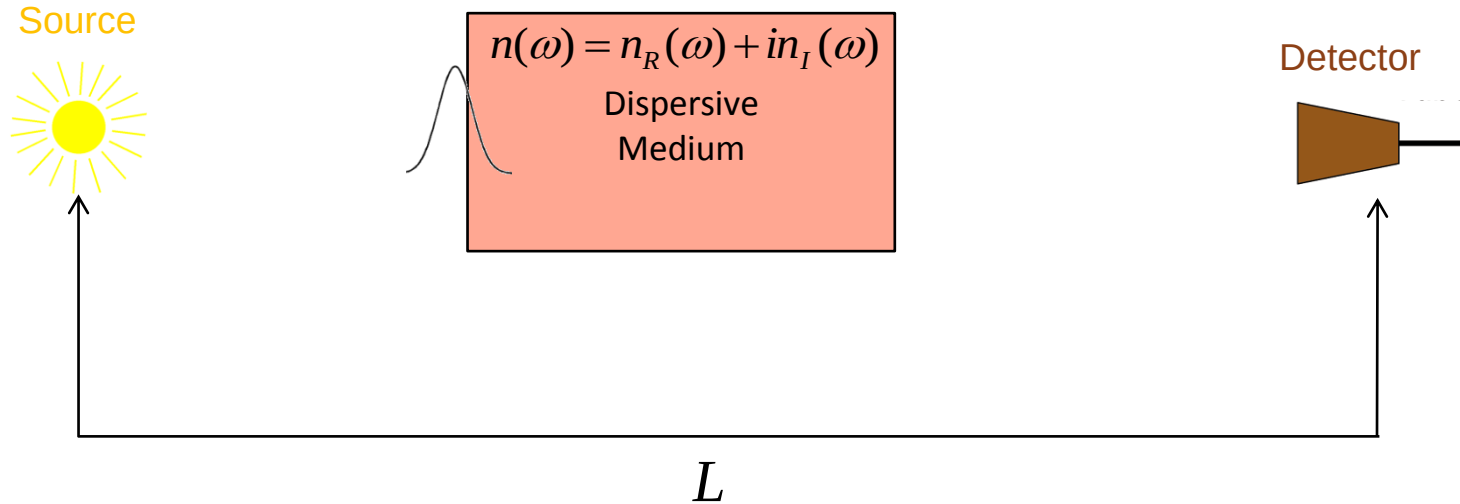
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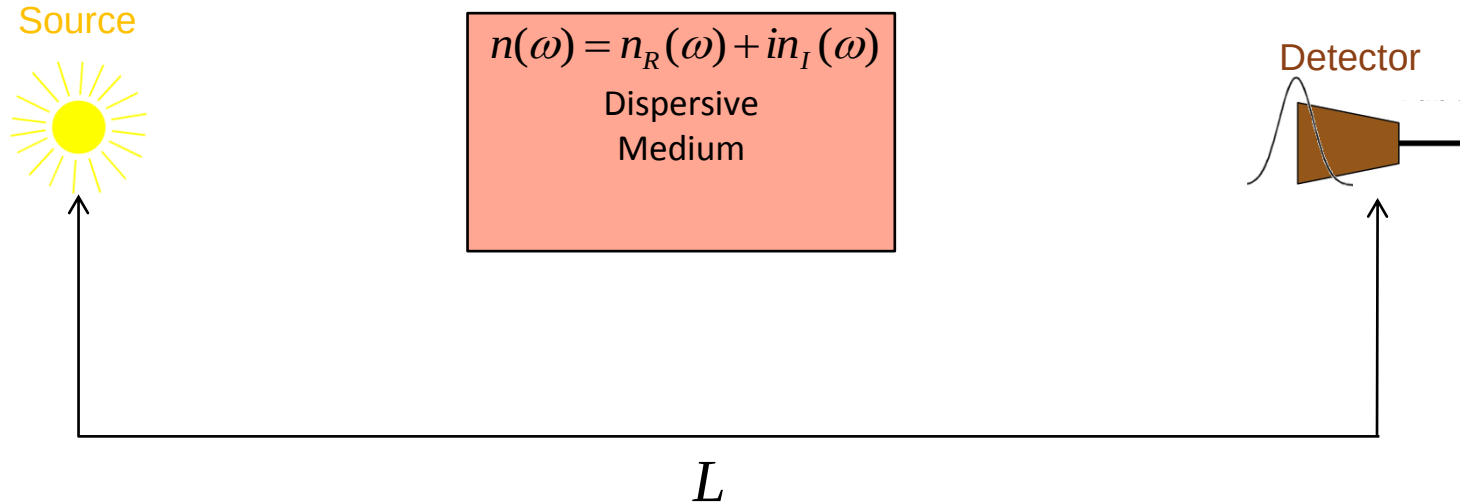
Experiment



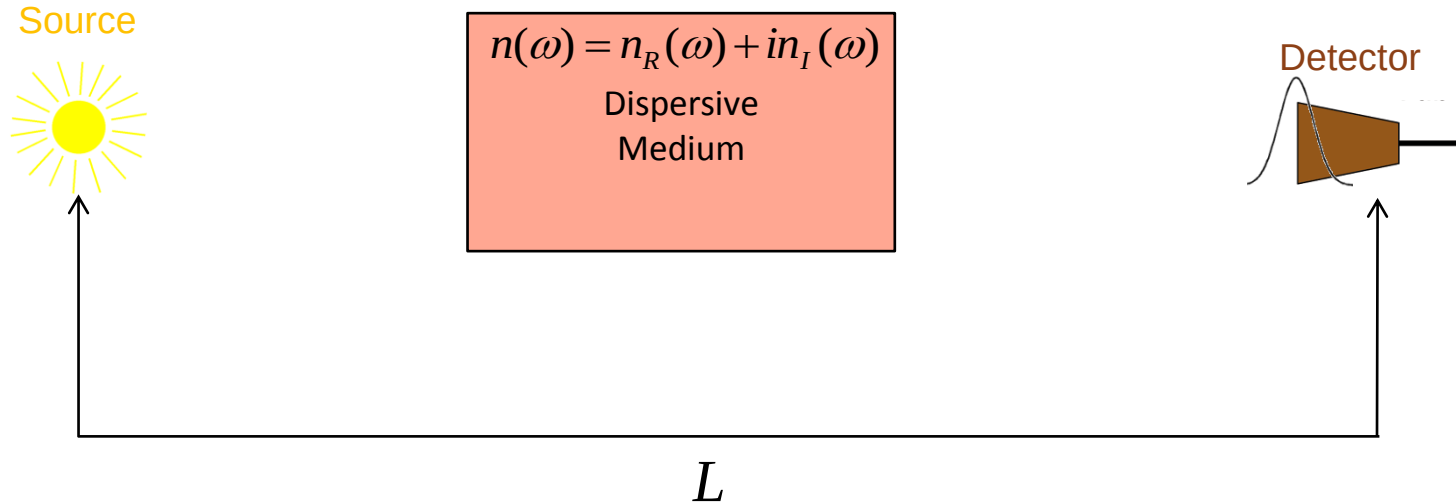
Experiment



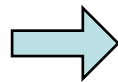
Experiment



Experiment

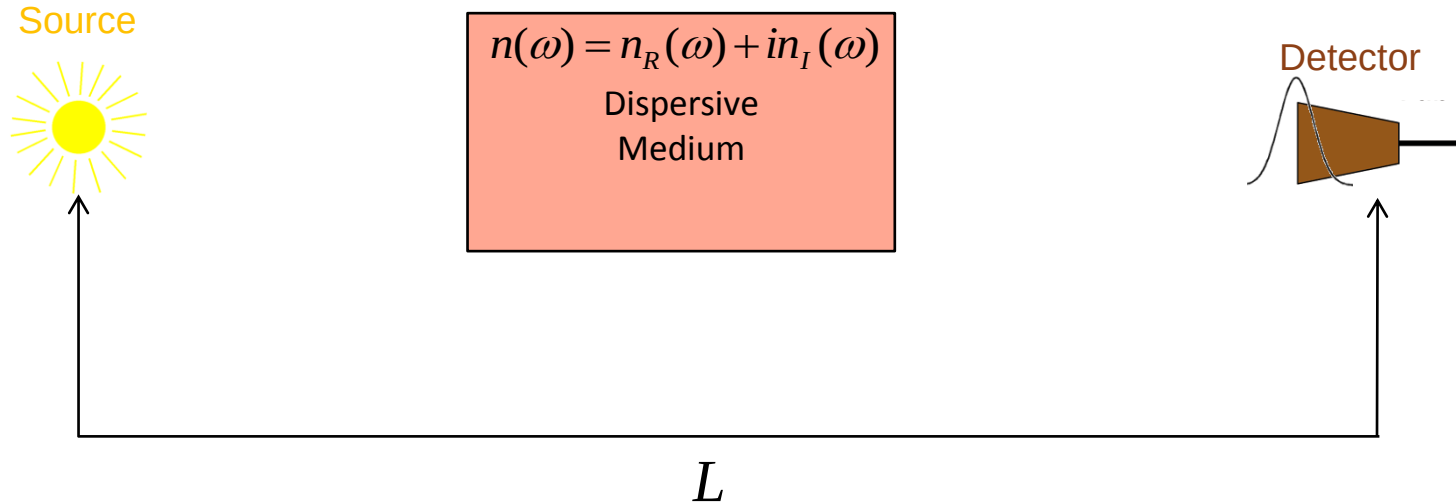


if travels
with speed of light

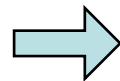


$$\Delta t_0 = L / c$$

Experiment

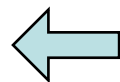


if travels
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$$\Delta t_0 = L / c$$

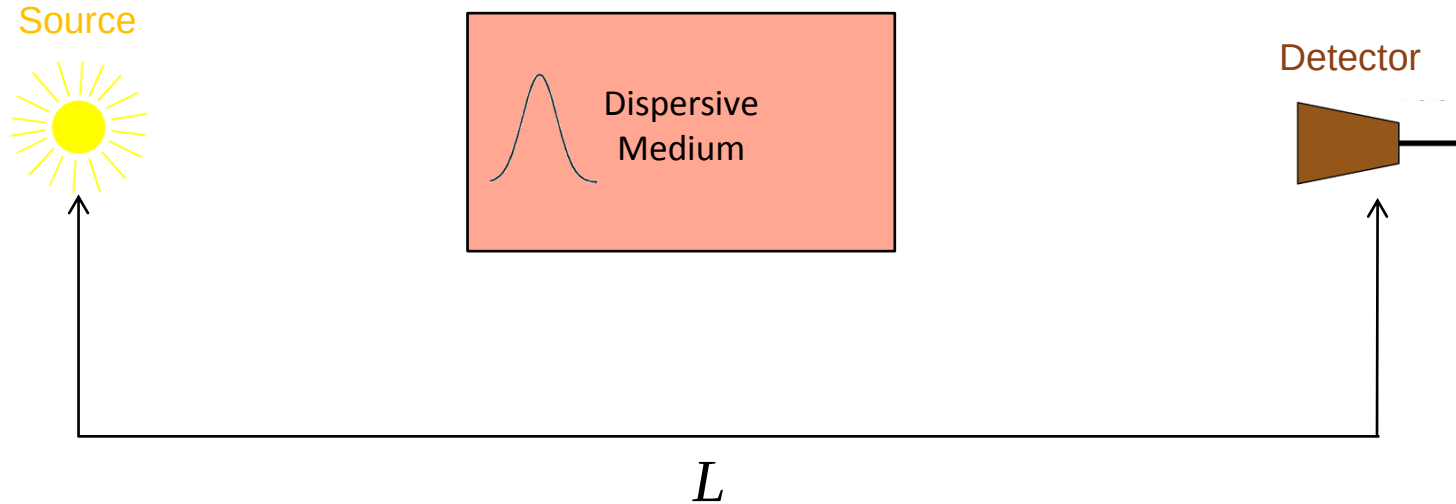
superluminal
propagation



if $\Delta t < \Delta t_0$

[1]

Problem!



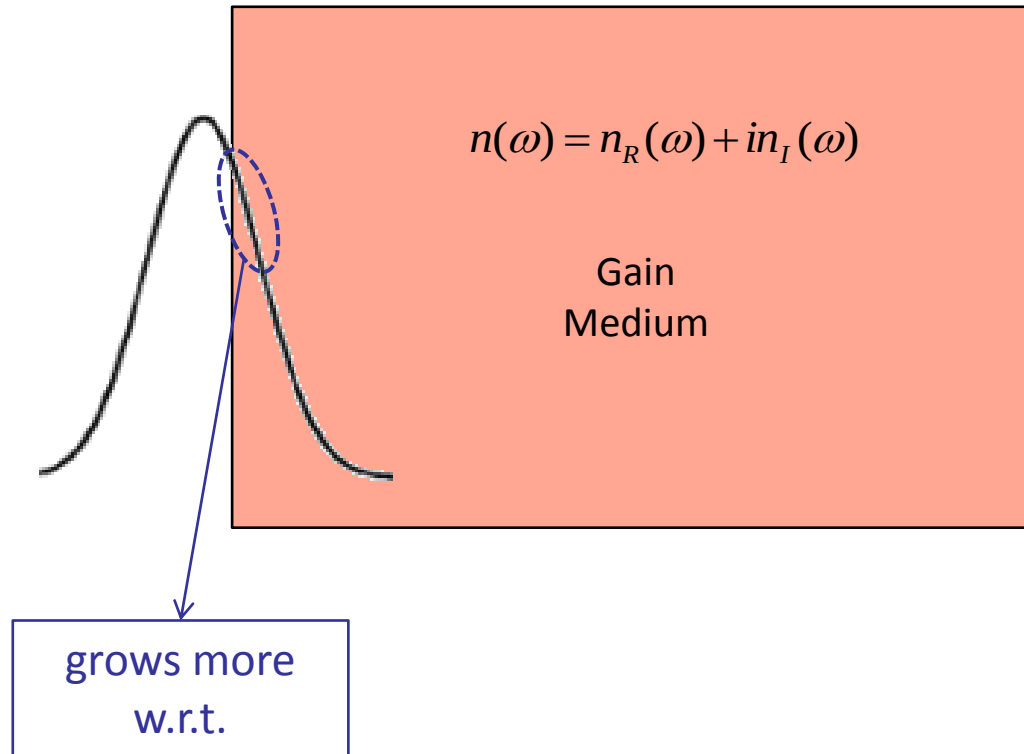
Pulse displaces:

- ☐ Where to choose the reference point for displacement?
- ☐ Pulse also reshapes due to gain/absorption.

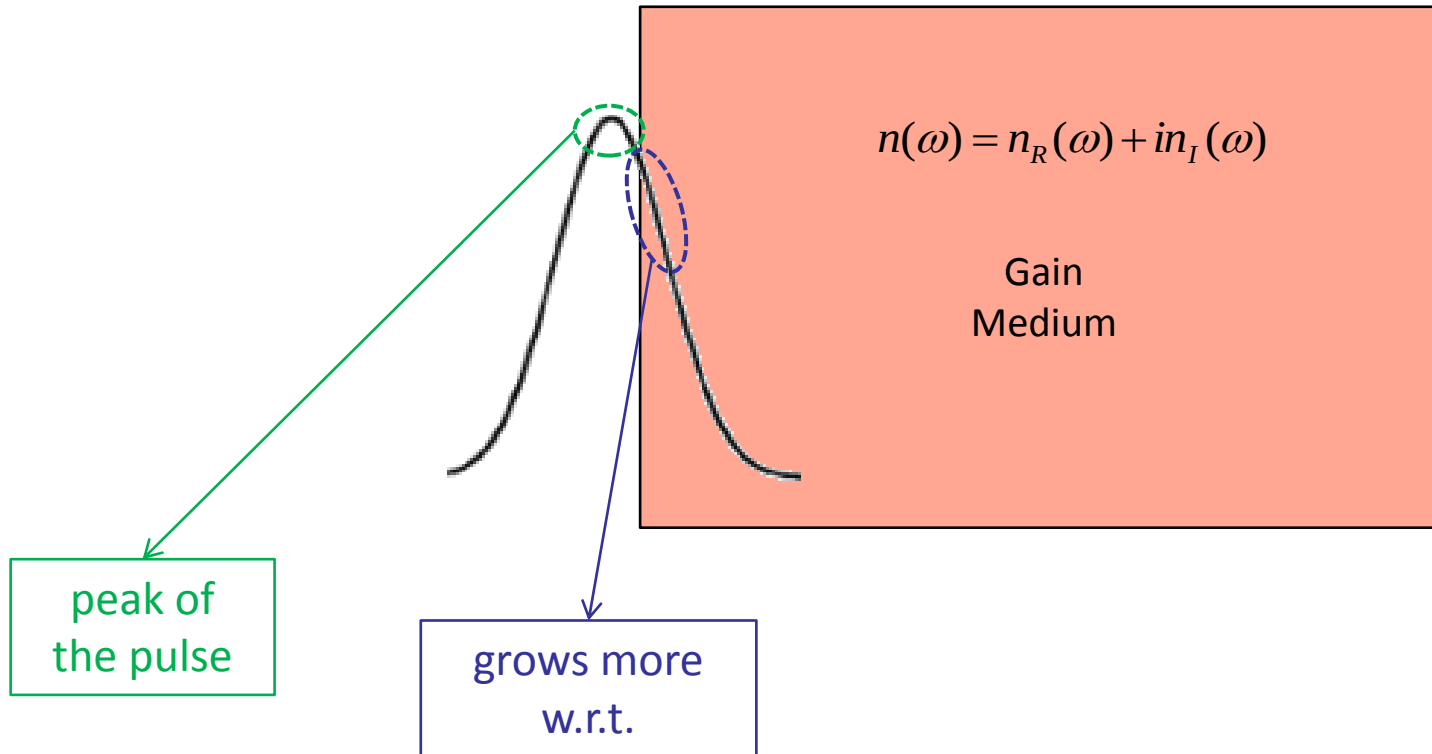
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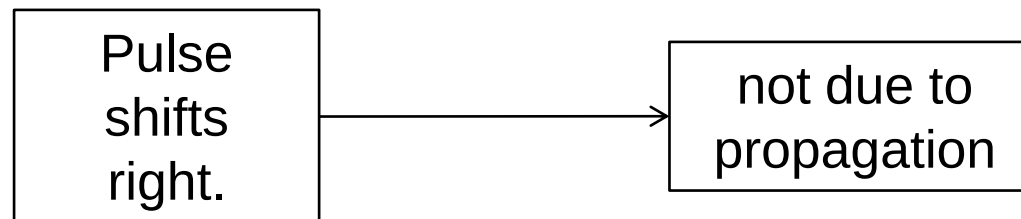
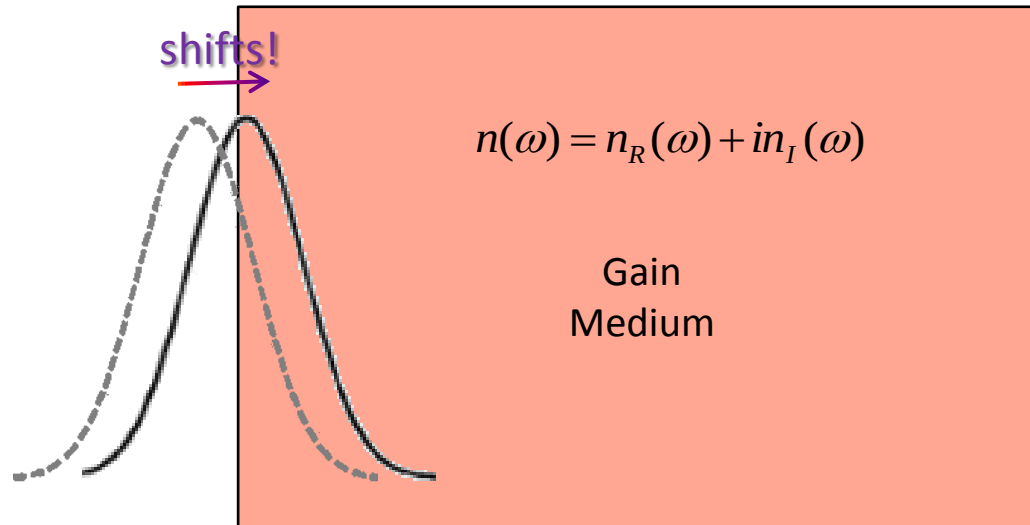
example for reshaping



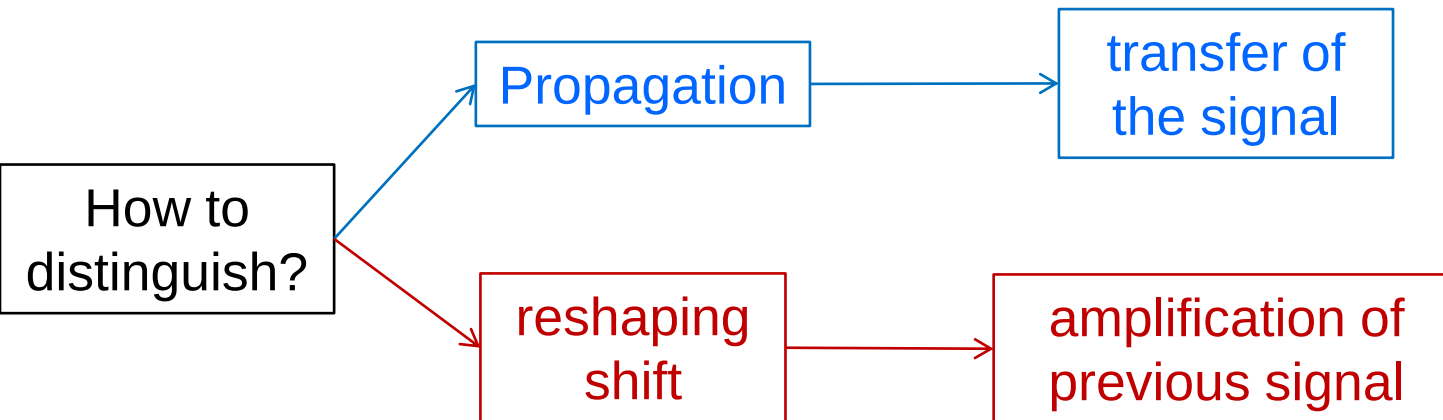
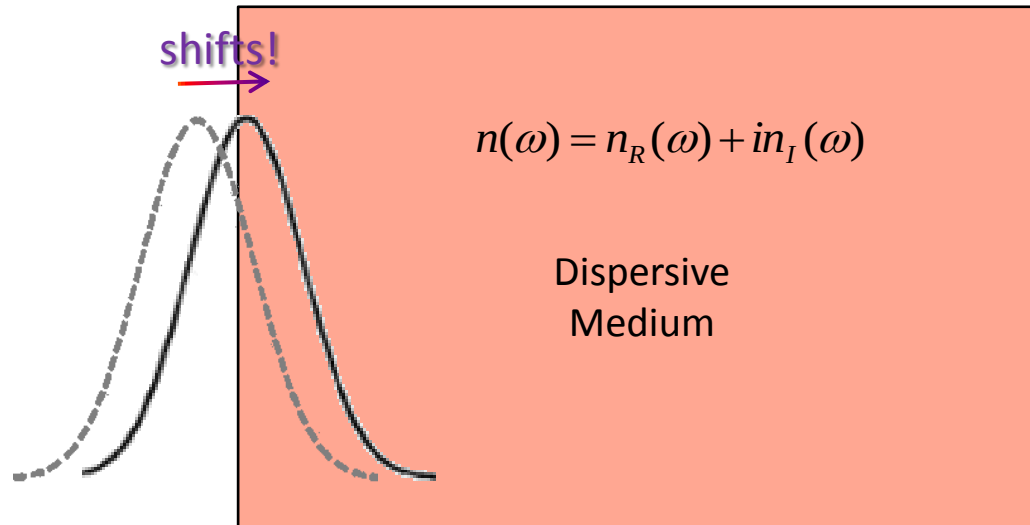
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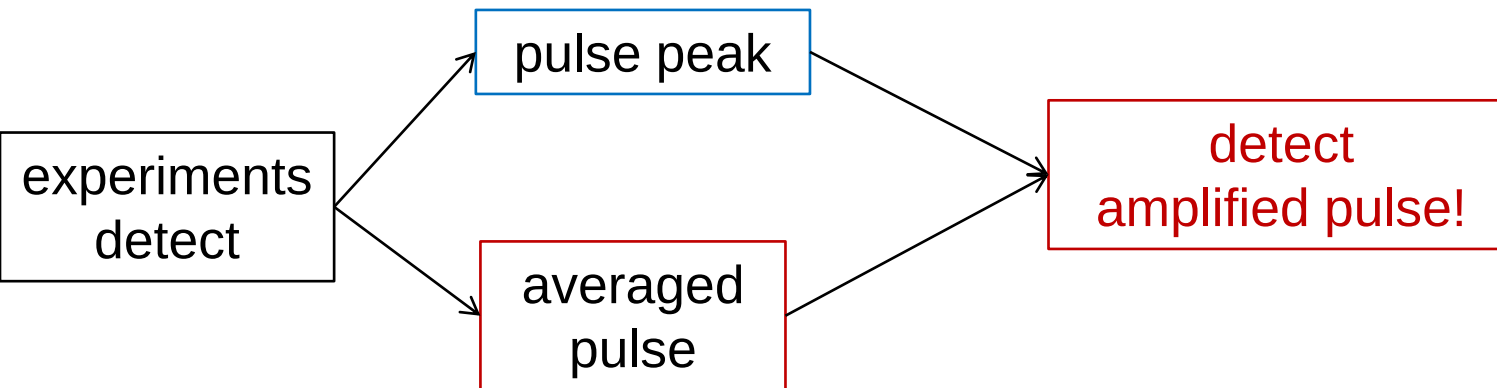
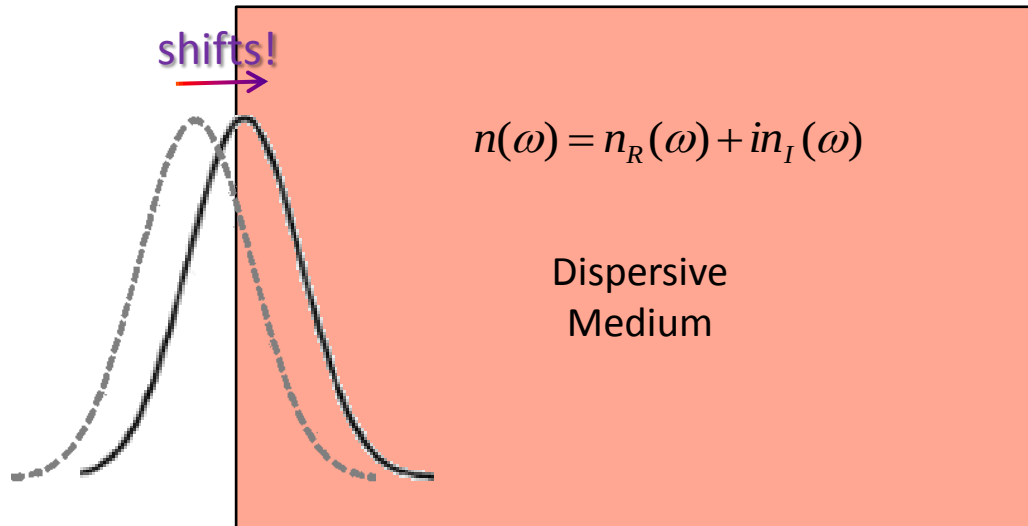
example for reshaping



Problem: to distinguish

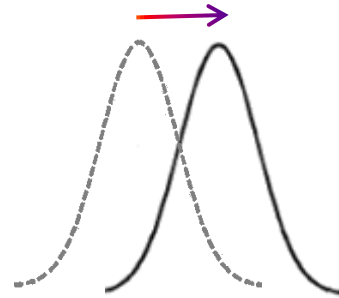


experiments



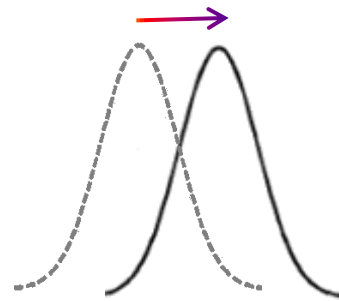
Velocity definitions

- Displacement of the pulse peak



Velocity definitions

- Displacement of the pulse peak



Poynting-vector
(could be Energy)

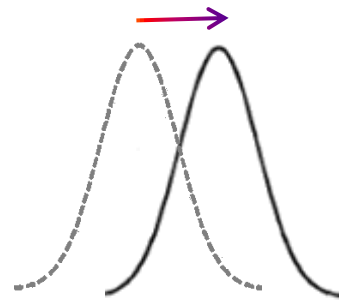
- Energy/Poynting-vector averaged pulse center

$$\langle t \rangle_x = \frac{\int dt t S(x, t)}{\int dt S(x, t)} \quad [2]$$

$$\langle x \rangle_t = \frac{\int dx x S(x, t)}{\int dx S(x, t)}$$

Velocity definitions

- Displacement of the pulse peak



Poynting-vector
(could be Energy)

- Energy/Poynting-vector averaged pulse center

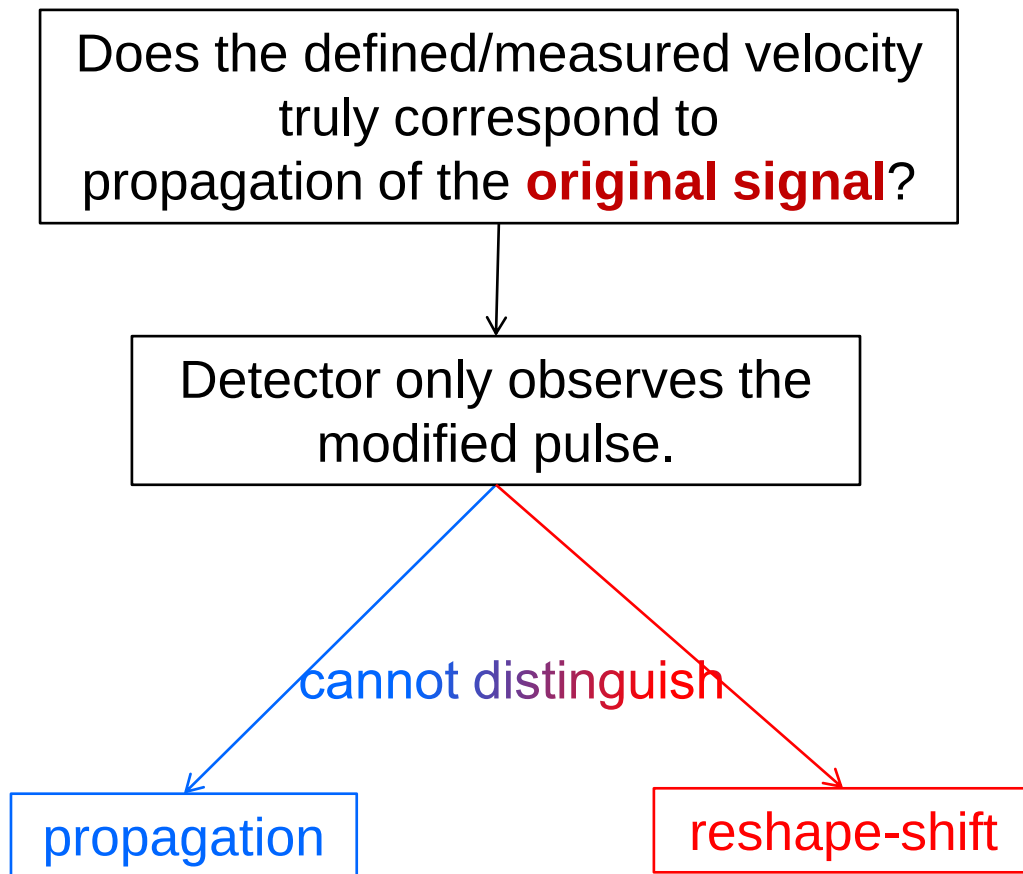
$$\langle t \rangle_x = \frac{\int dt t S(x, t)}{\int dt S(x, t)} \quad [2]$$

$$\langle x \rangle_t = \frac{\int dx x S(x, t)}{\int dx S(x, t)}$$

good
agreement

values at
detectors

Is velocity true?



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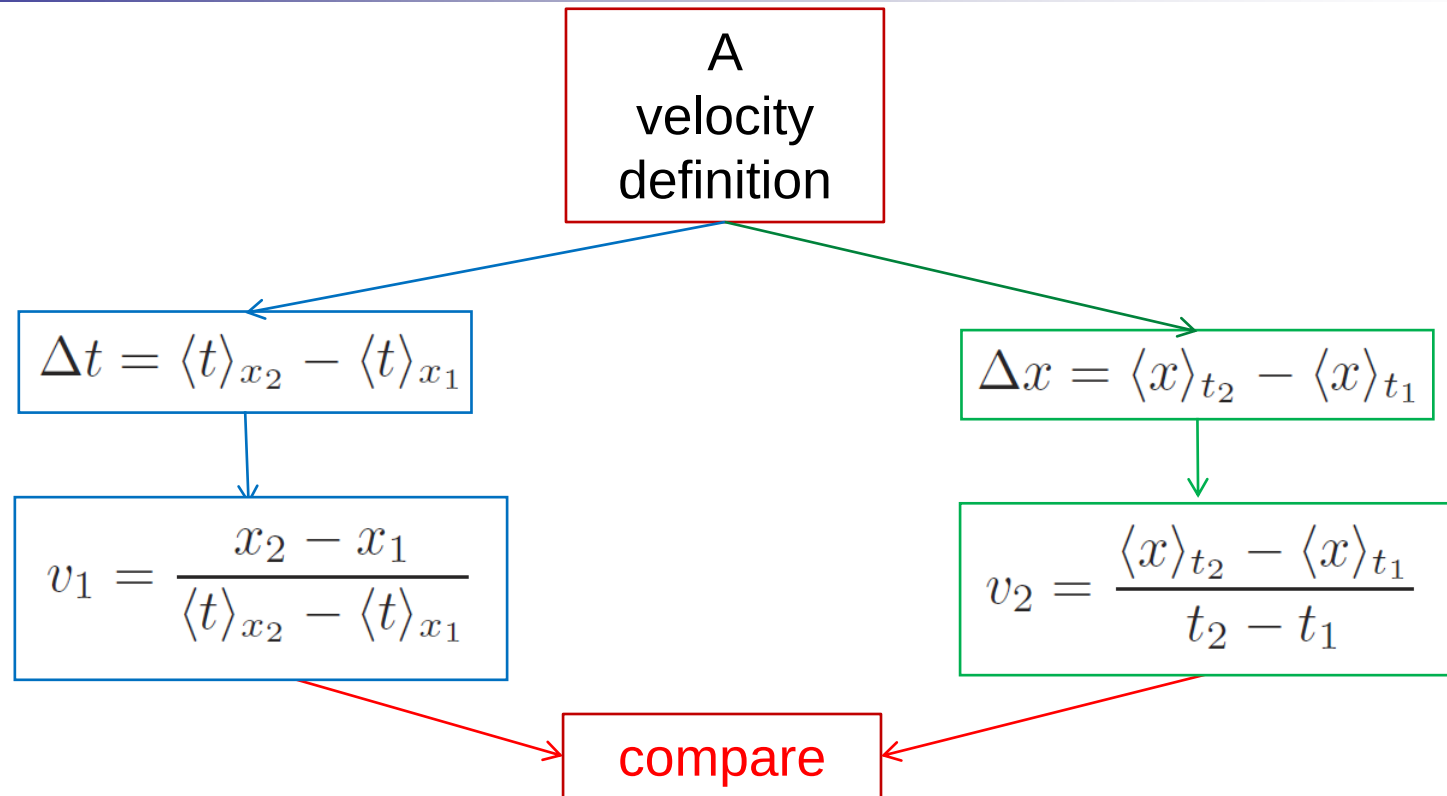
Method to test velocities

A
velocity
definition

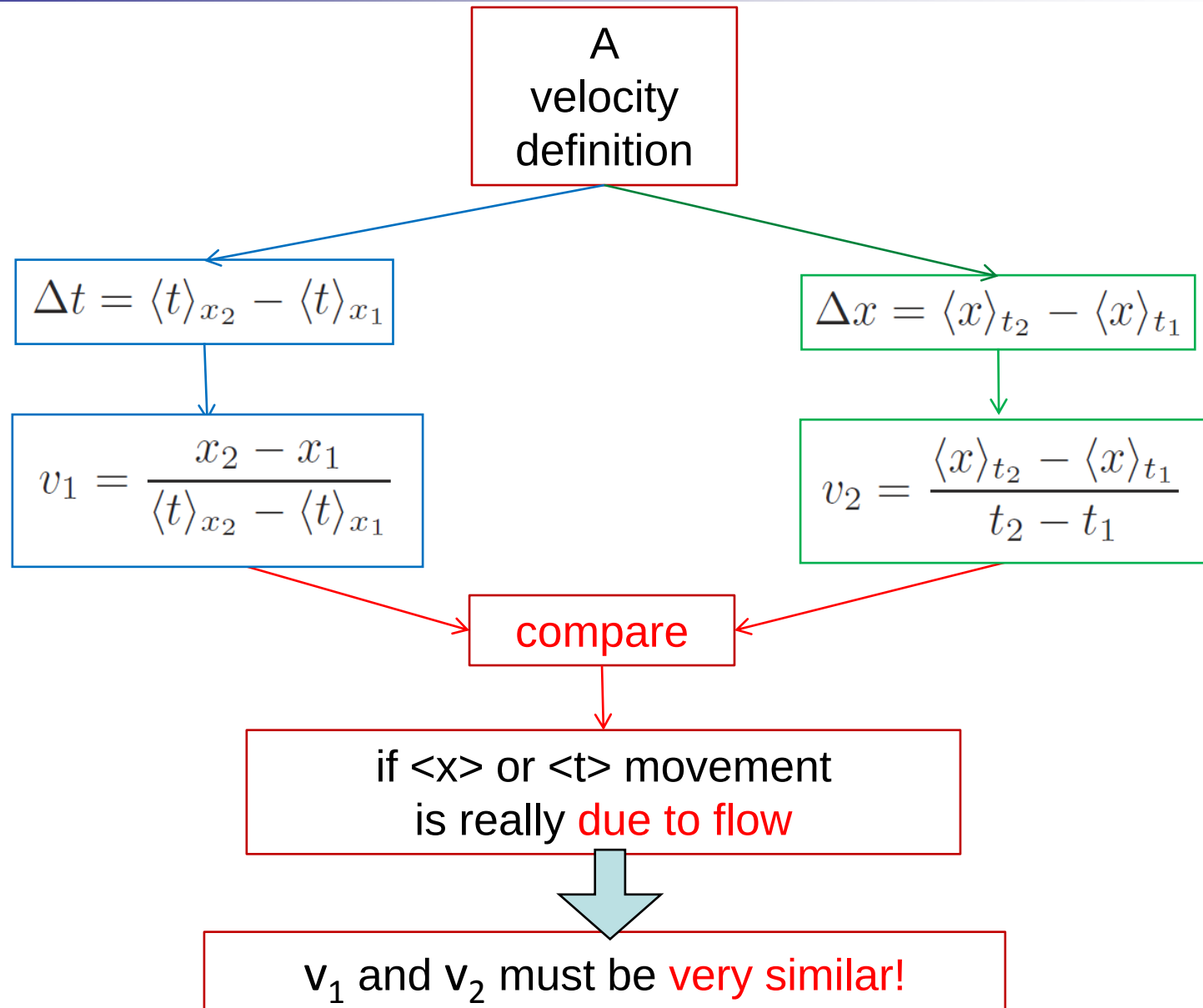
$$\Delta t = \langle t \rangle_{x_2} - \langle t \rangle_{x_1}$$

$$v_1 = \frac{x_2 - x_1}{\langle t \rangle_{x_2} - \langle t \rangle_{x_1}}$$

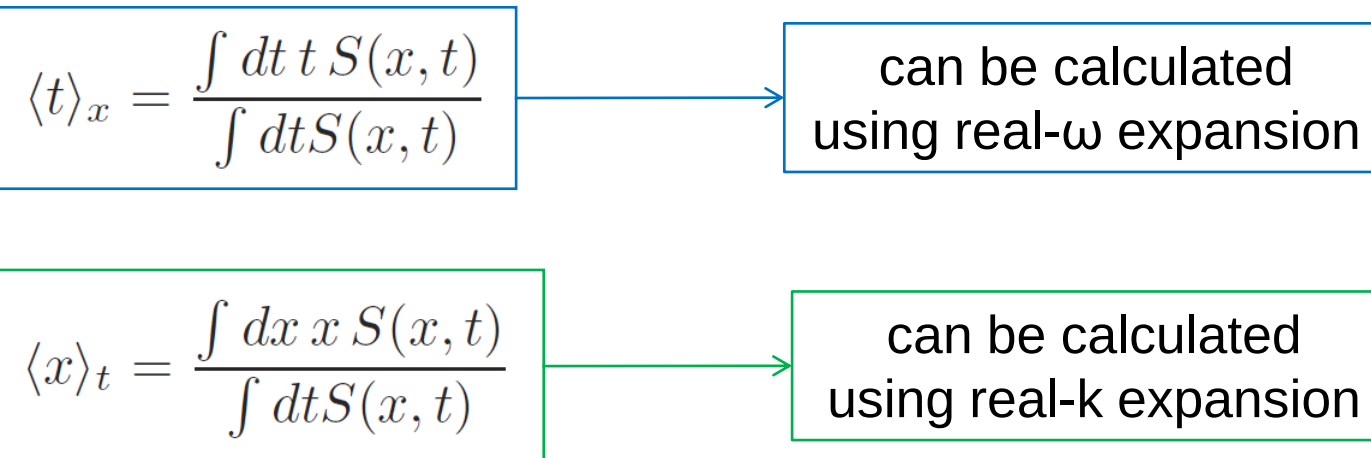
Method to test velocities



Method to test velocities



Fourier space to work within



Fourier space to work within

$$\int dt t S(\Delta x, t) = \Delta x \int_{-\infty}^{+\infty} d\bar{\omega} \frac{dk}{d\omega} e^{-2k_I \Delta x} |D_1(\bar{\omega})|^2 n^*(\bar{\omega}) \\ - i \int_{-\infty}^{+\infty} d\bar{\omega} e^{-2k_I \Delta x} \frac{dD_1}{d\omega} D_1^*(\bar{\omega}) n^*(\bar{\omega})$$

$$\langle t \rangle_x = \frac{\int dt t S(x, t)}{\int dt S(x, t)}$$

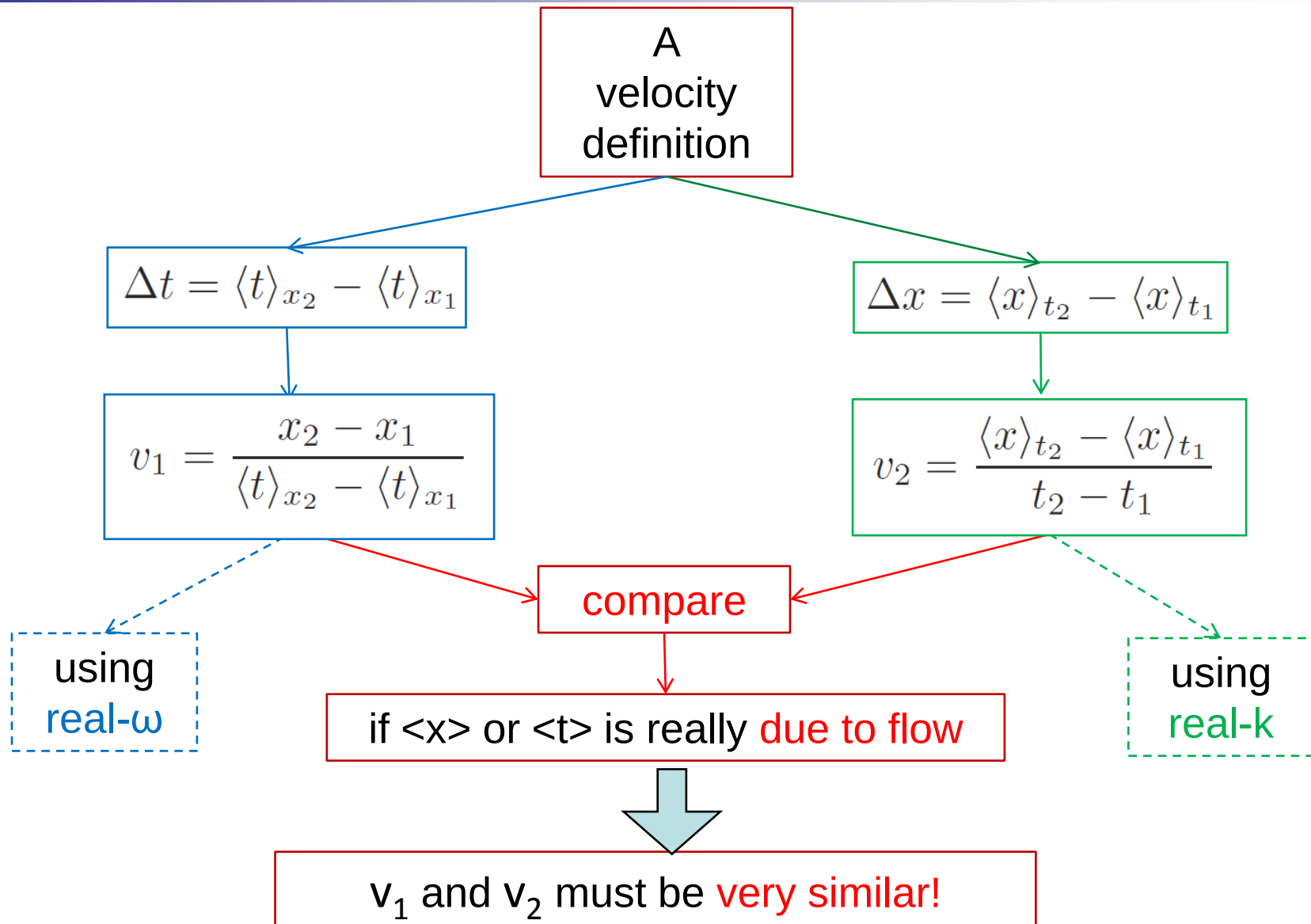
can be calculated
using real- ω expansion

$$\langle x \rangle_t = \frac{\int dx x S(x, t)}{\int dt S(x, t)}$$

can be calculated
using real- k expansion

$$\int dx x S(x, \Delta t) = \Delta t \int_{-\infty}^{+\infty} d\bar{k} \frac{d\omega}{dk} e^{2\omega_I \Delta t} |D_2(\bar{k})|^2 n^*(\bar{k}) \\ + i \int_{-\infty}^{+\infty} d\bar{k} e^{2\omega_I \Delta t} \frac{dD_2}{dk} D_2^*(\bar{k}) n^*(\bar{k})$$

Method



in order to compare

$$\int dt t S(\Delta x, t) = \Delta x \int_{-\infty}^{+\infty} d\bar{\omega} \frac{dk}{d\omega} e^{-2k_I \Delta x} |D_1(\bar{\omega})|^2 n^*(\bar{\omega}) \\ - i \int_{-\infty}^{+\infty} d\bar{\omega} e^{-2k_I \Delta x} \frac{dD_1}{d\omega} D_1^*(\bar{\omega}) n^*(\bar{\omega})$$

$$\langle t \rangle_x = \frac{\int dt t S(x, t)}{\int dt S(x, t)}$$

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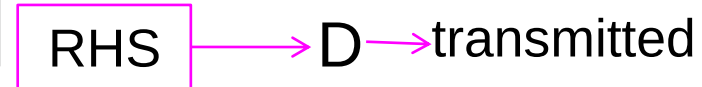
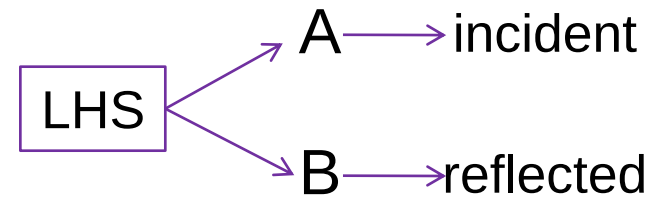
relate

$$D_1(\omega) \leftrightarrow D_2(k)$$

$$\int dx x S(x, \Delta t) = \Delta t \int_{-\infty}^{+\infty} d\bar{k} \frac{d\omega}{dk} e^{2\omega_I \Delta t} |D_2(\bar{k})|^2 n^*(\bar{k}) \\ + i \int_{-\infty}^{+\infty} d\bar{k} e^{2\omega_I \Delta t} \frac{dD_2}{dk} D_2^*(\bar{k}) n^*(\bar{k})$$

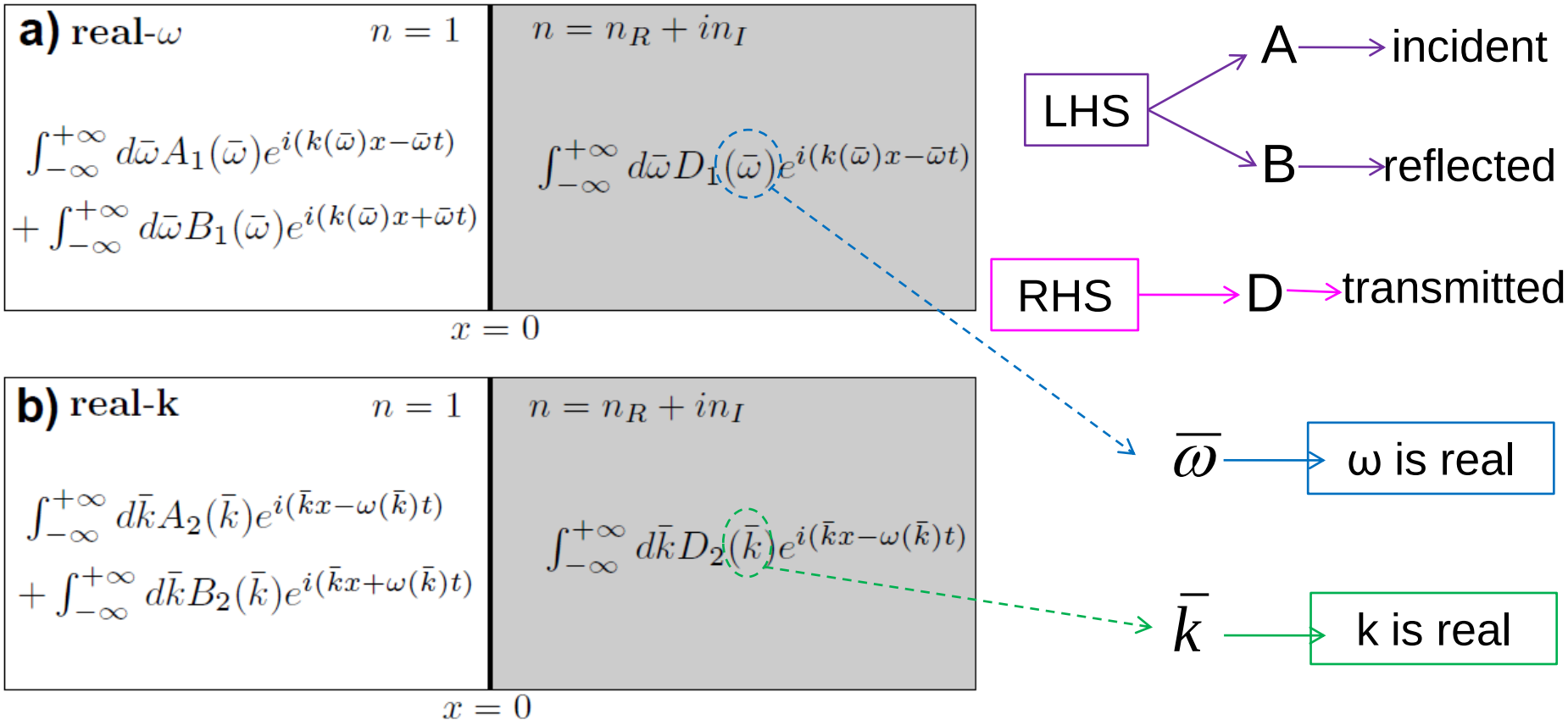
$$D_1(\omega) \leftrightarrow D_2(k)$$

a) real-ω	$n = 1$	$n = n_R + in_I$
$x = 0$		
$\int_{-\infty}^{+\infty} d\bar{\omega} A_1(\bar{\omega}) e^{i(k(\bar{\omega})x - \bar{\omega}t)}$ $+ \int_{-\infty}^{+\infty} d\bar{\omega} B_1(\bar{\omega}) e^{i(k(\bar{\omega})x + \bar{\omega}t)}$		$\int_{-\infty}^{+\infty} d\bar{\omega} D_1(\bar{\omega}) e^{i(k(\bar{\omega})x - \bar{\omega}t)}$

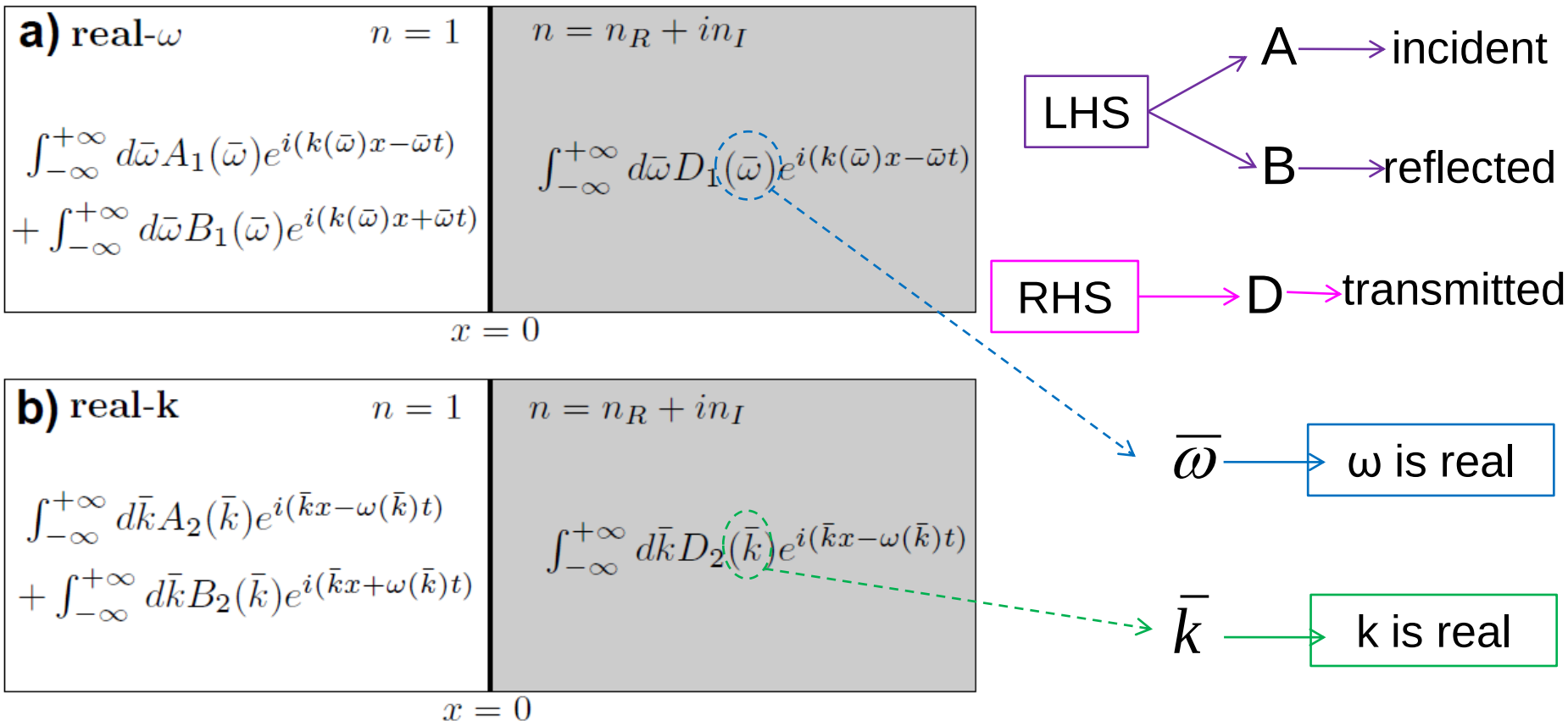


b) real-k	$n = 1$	$n = n_R + in_I$
$x = 0$		
$\int_{-\infty}^{+\infty} d\bar{k} A_2(\bar{k}) e^{i(\bar{k}x - \omega(\bar{k})t)}$ $+ \int_{-\infty}^{+\infty} d\bar{k} B_2(\bar{k}) e^{i(\bar{k}x + \omega(\bar{k})t)}$		$\int_{-\infty}^{+\infty} d\bar{k} D_2(\bar{k}) e^{i(\bar{k}x - \omega(\bar{k})t)}$

$$D_1(\omega) \leftrightarrow D_2(k)$$



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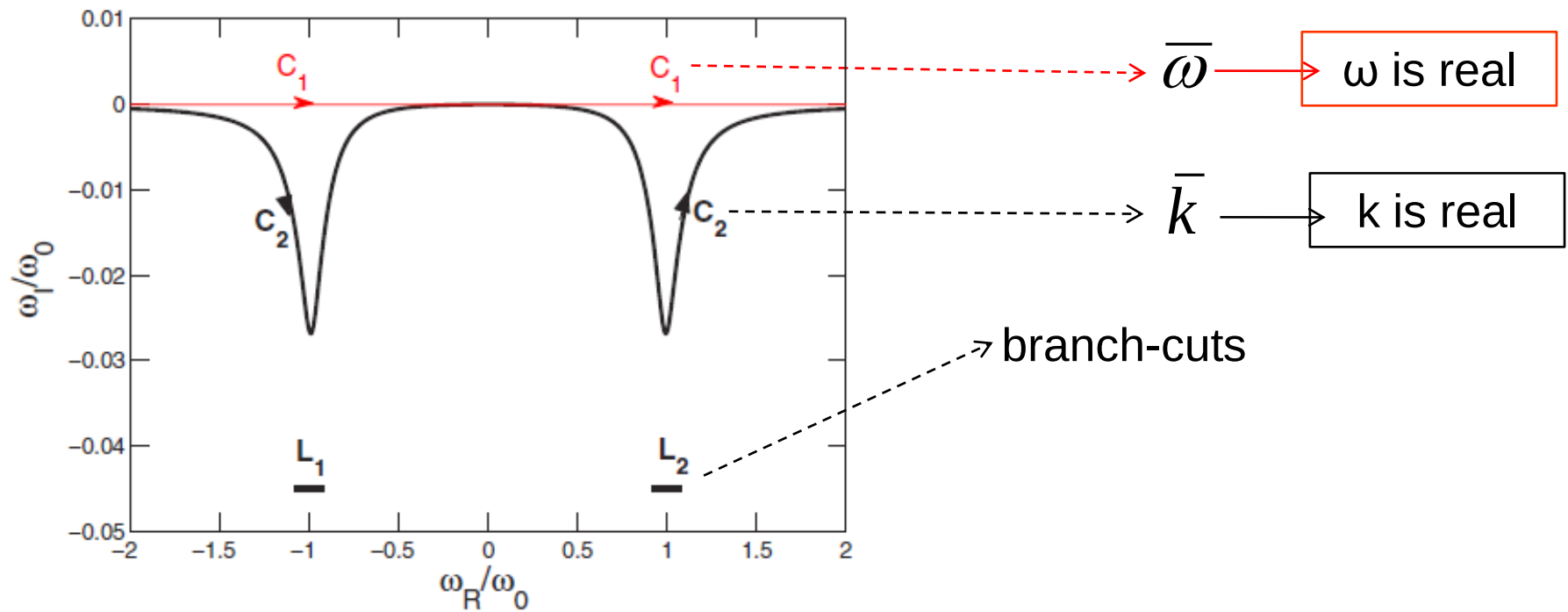


$$\int_{-\infty}^{+\infty} d\bar{\omega} D_1(\bar{\omega}) e^{-i\bar{\omega}t} = \int_{-\infty}^{+\infty} d\bar{k} D_2(\bar{k}) e^{-i\omega(\bar{k})t},$$

RHSs
equal at
 $x=0$

$$D_1(\omega) \leftrightarrow D_2(k)$$

$$\int_{-\infty}^{+\infty} d\bar{\omega} D_1(\bar{\omega}) e^{-i\bar{\omega}t} = \int_{-\infty}^{+\infty} d\bar{k} D_2(\bar{k}) e^{-i\omega(\bar{k})t}, \quad 18$$



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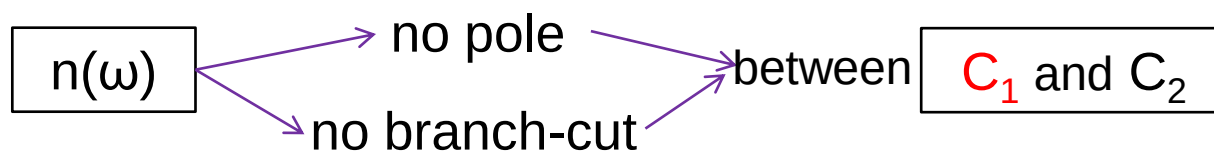
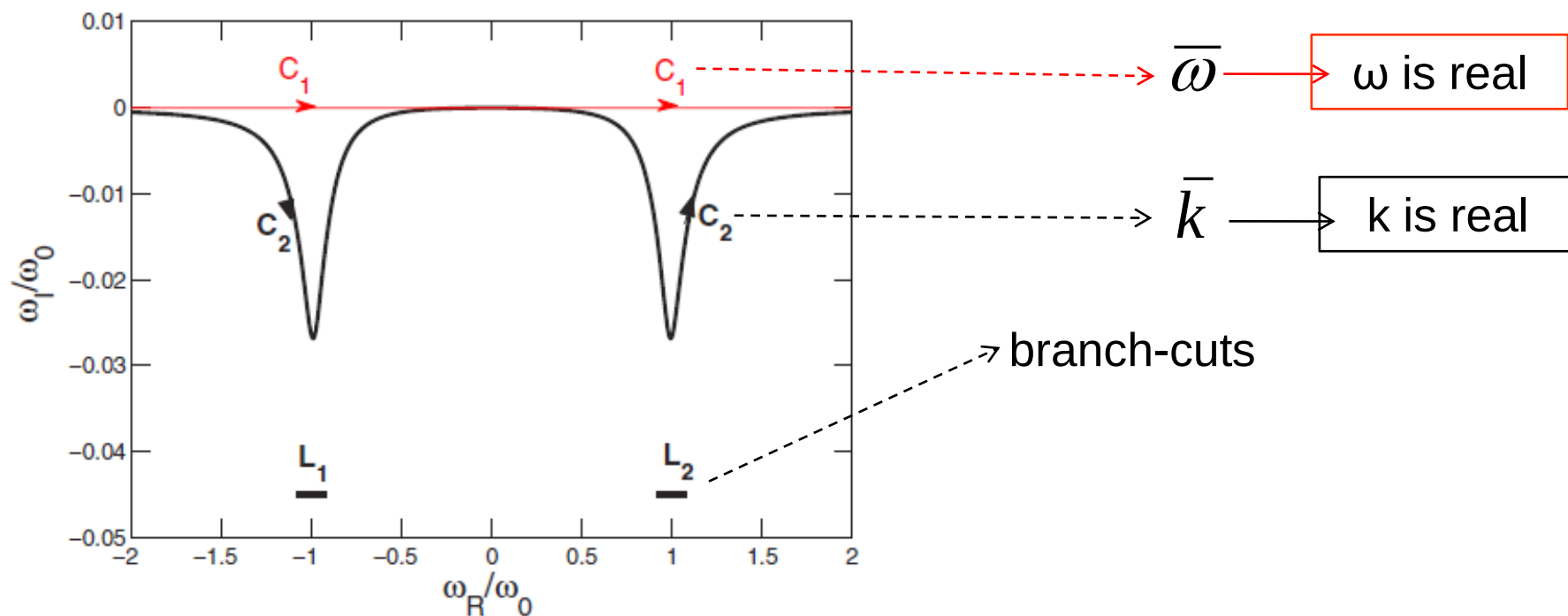


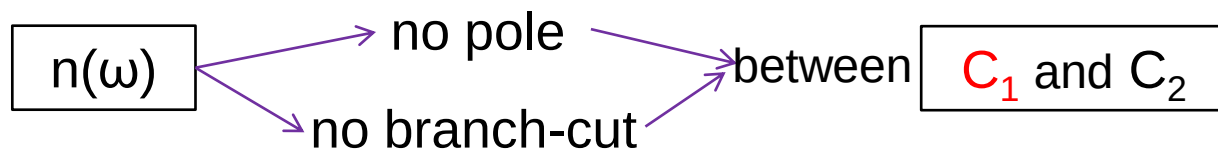
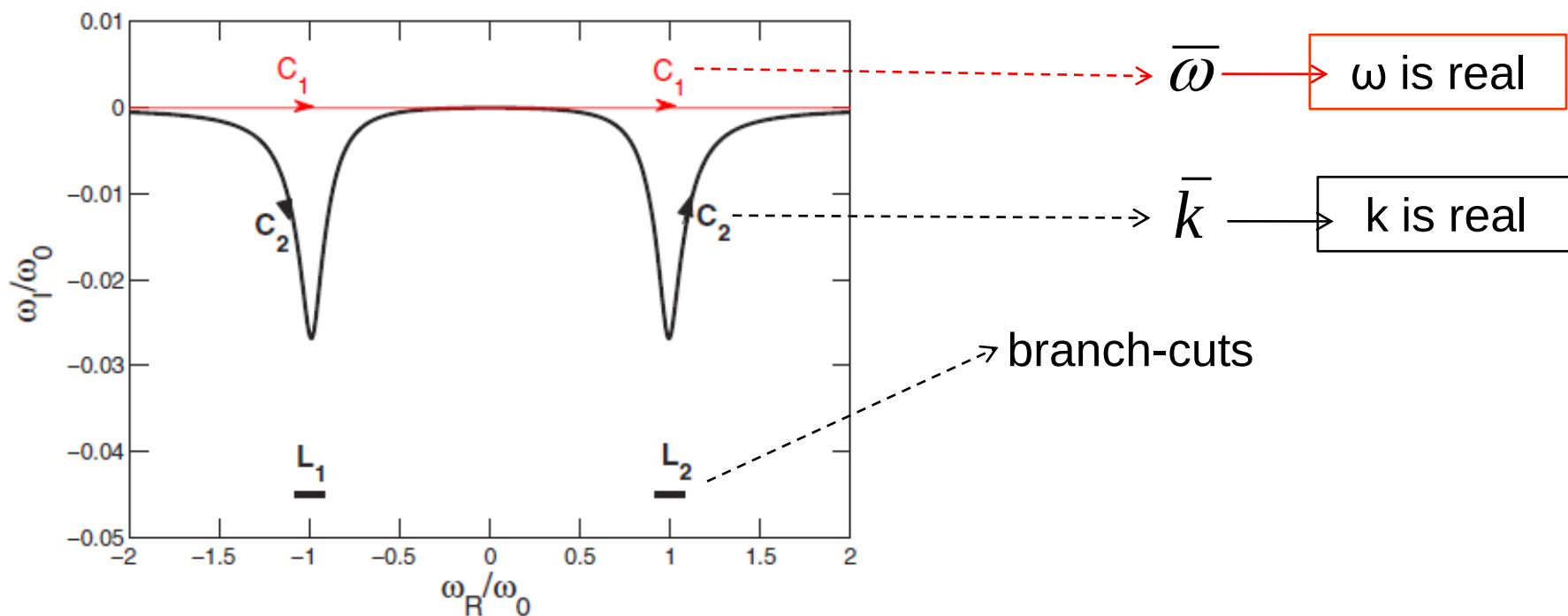
Diagram illustrating the relationship between $D_1(\omega)$ and the branch cuts C_1 and C_2 :

$$D_1(\omega) = \frac{2A(\omega)}{1 + n(\omega)}$$

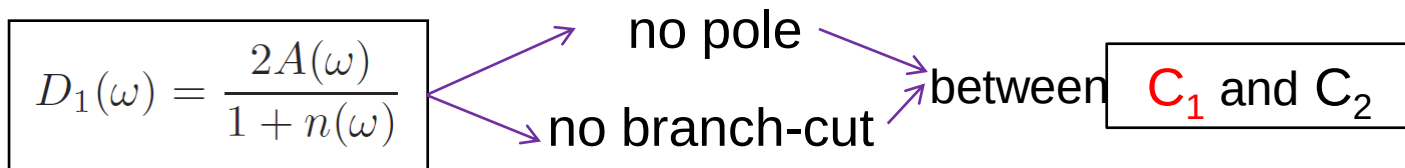
- $D_1(\omega)$ has no pole and no branch-cut between C_1 and C_2 .

$$D_1(\omega) \leftrightarrow D_2(k)$$

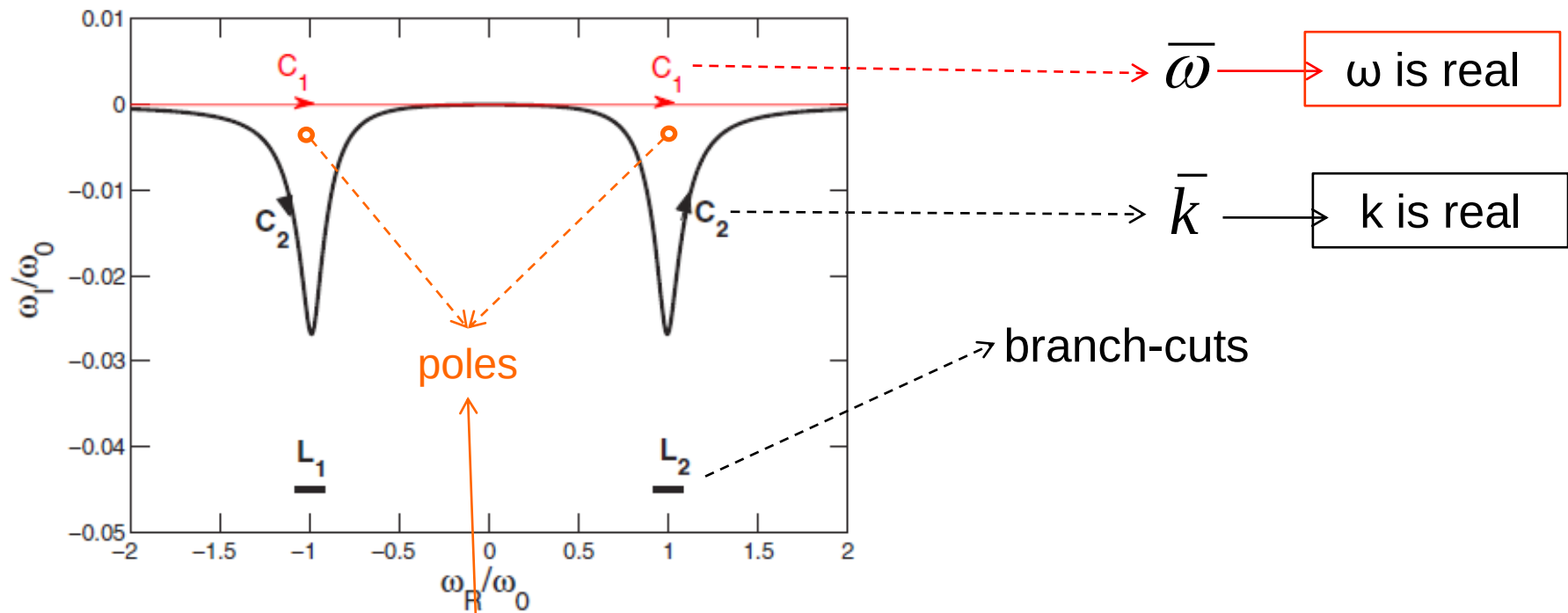
$$\int_{-\infty}^{+\infty} d\bar{\omega} D_1(\bar{\omega}) e^{-i\bar{\omega}t} = \int_{-\infty}^{+\infty} d\bar{k} D_2(\bar{k}) e^{-i\omega(\bar{k})t}, \quad 18$$



$$D_2(\bar{k}) = \frac{d\omega}{dk}(\bar{k}) D_1(\omega(\bar{k})),$$

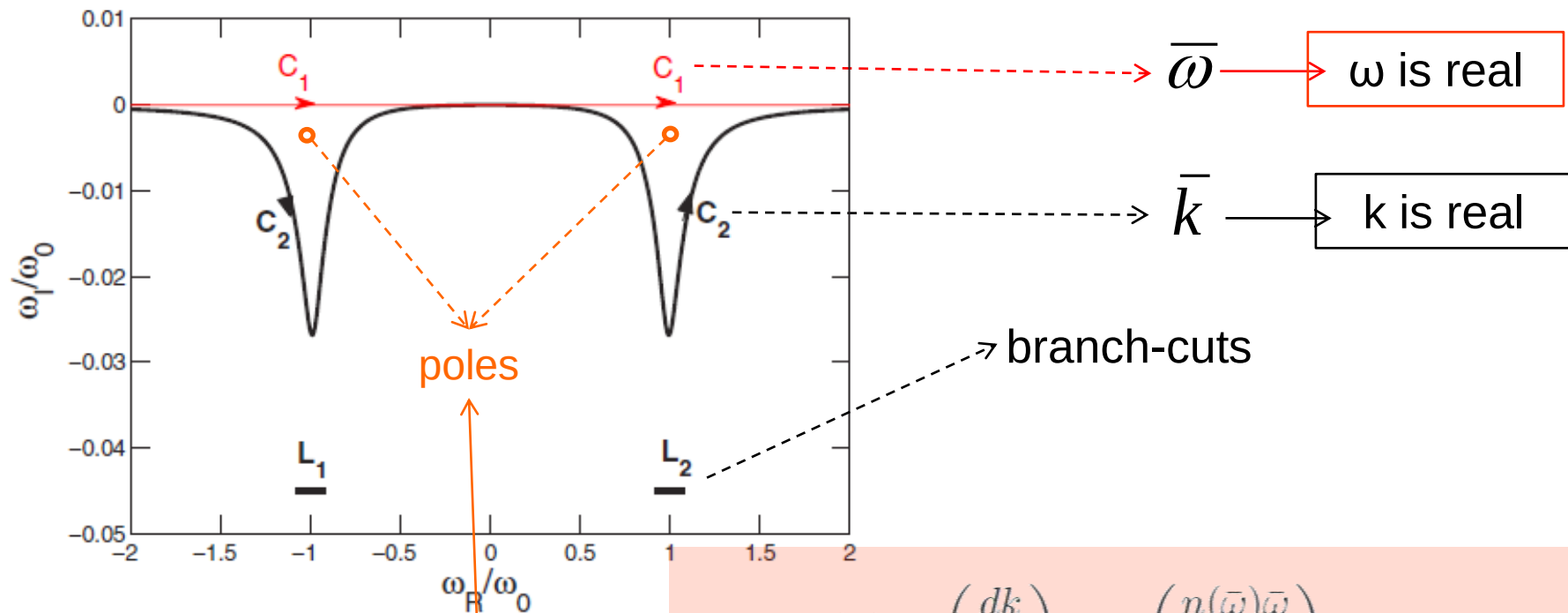


$D_1(\omega) \leftrightarrow D_2(k)$ (if poles)



$$D_1(\omega) = \frac{2A(\omega)}{1 + n(\omega)} \rightarrow \text{has poles}$$

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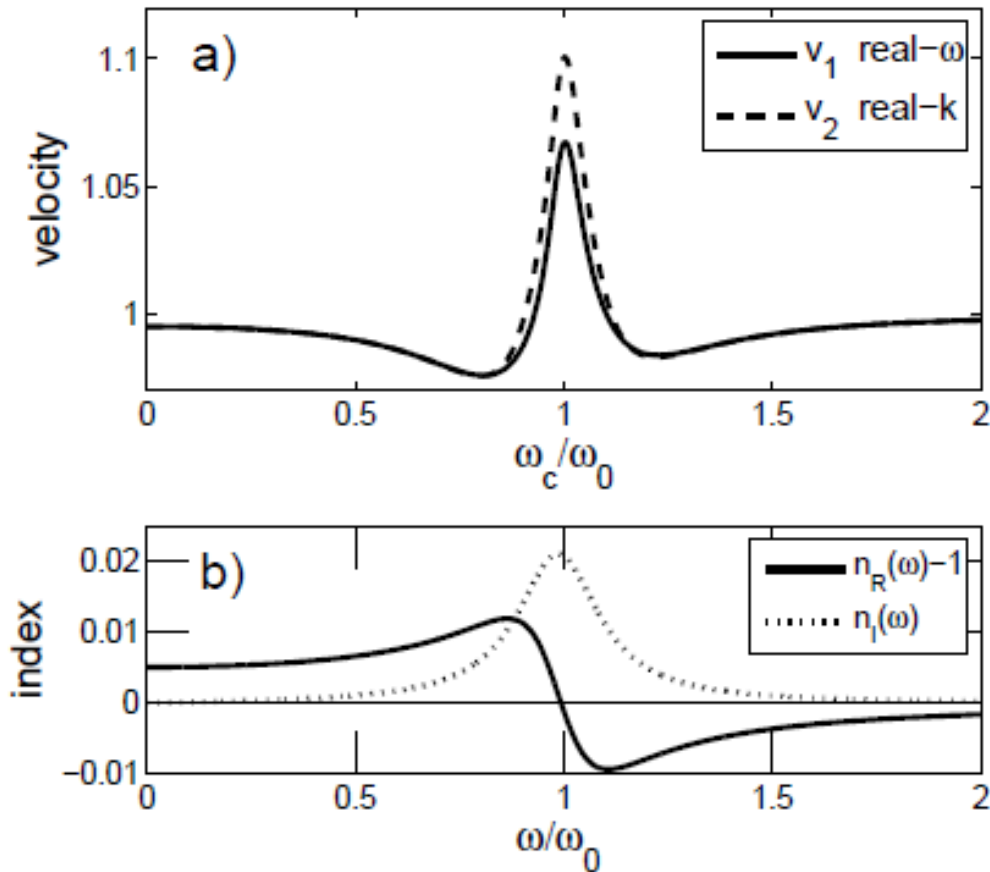
$$D_1(\omega) = \frac{2A(\omega)}{1+n(\omega)} \rightarrow \text{has poles} \rightarrow$$

$$\begin{aligned}
 D_1(\bar{\omega}) &= \left(\frac{dk}{d\omega} \right)_{\bar{\omega}} D_2 \left(\frac{n(\bar{\omega})\bar{\omega}}{c} \right) \\
 &+ 2\pi i \frac{F(\omega_{1R} - i\omega_{1I})}{2\omega_{1R}} \frac{e^{i(\omega_{1R} - \bar{\omega})T} e^{\omega_{1I}T}}{2\pi i(\omega_1 - \bar{\omega})} \\
 &+ 2\pi i \frac{F(-\omega_{2R} - i\omega_{2I})}{-2\omega_{2R}} \frac{e^{i(-\omega_{2R} - \bar{\omega})T} e^{\omega_{2I}T}}{2\pi i(\omega_2 - \bar{\omega})} \\
 &- \lim_{T \rightarrow \infty} \sum_{1,2} \oint_{C_{1,2}} d\omega \left(\frac{dk}{d\omega} \right) D_2 \left(\frac{n(\omega)\omega}{c} \right) \frac{e^{-i(\omega - \bar{\omega})T}}{2\pi i(\omega - \bar{\omega})}
 \end{aligned}$$

Comparison of v_1 and v_2

Gaussian wave-packet

$$\longrightarrow U(0, t) = e^{-t^2/\tau^2} \cos(\omega_c t)$$



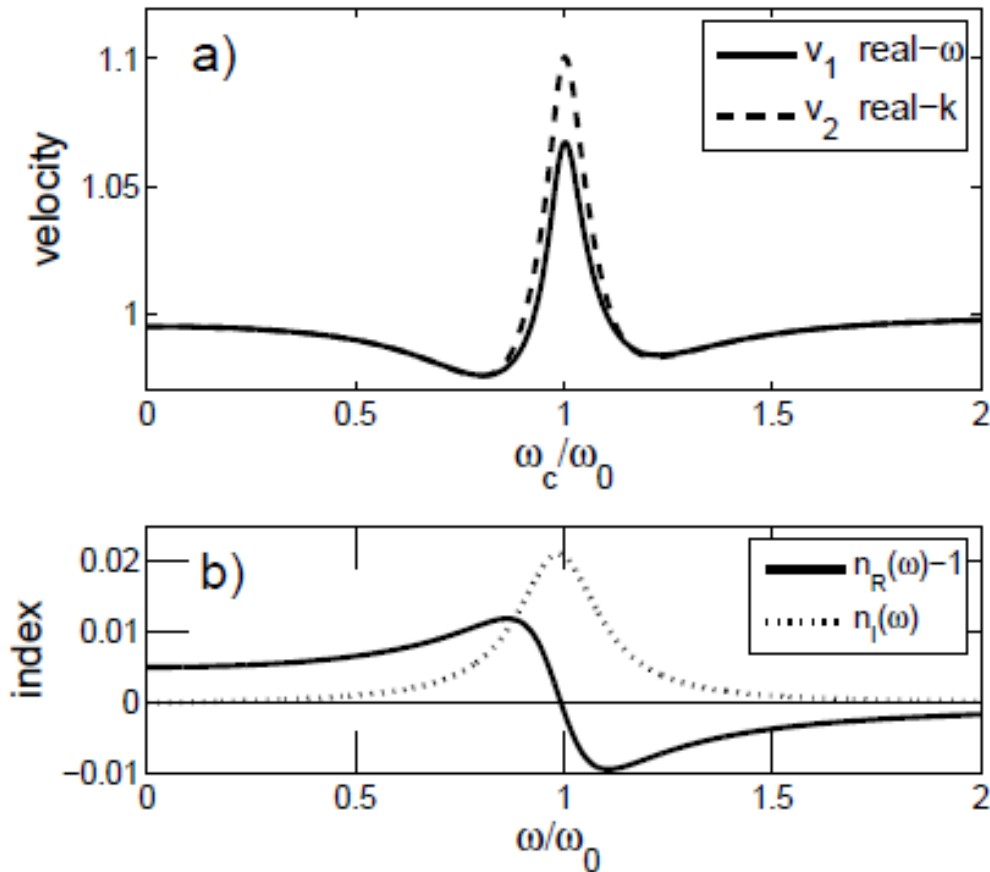
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Luminal regime

$$v_1 \cong v_2$$



Comparison of v_1 and v_2

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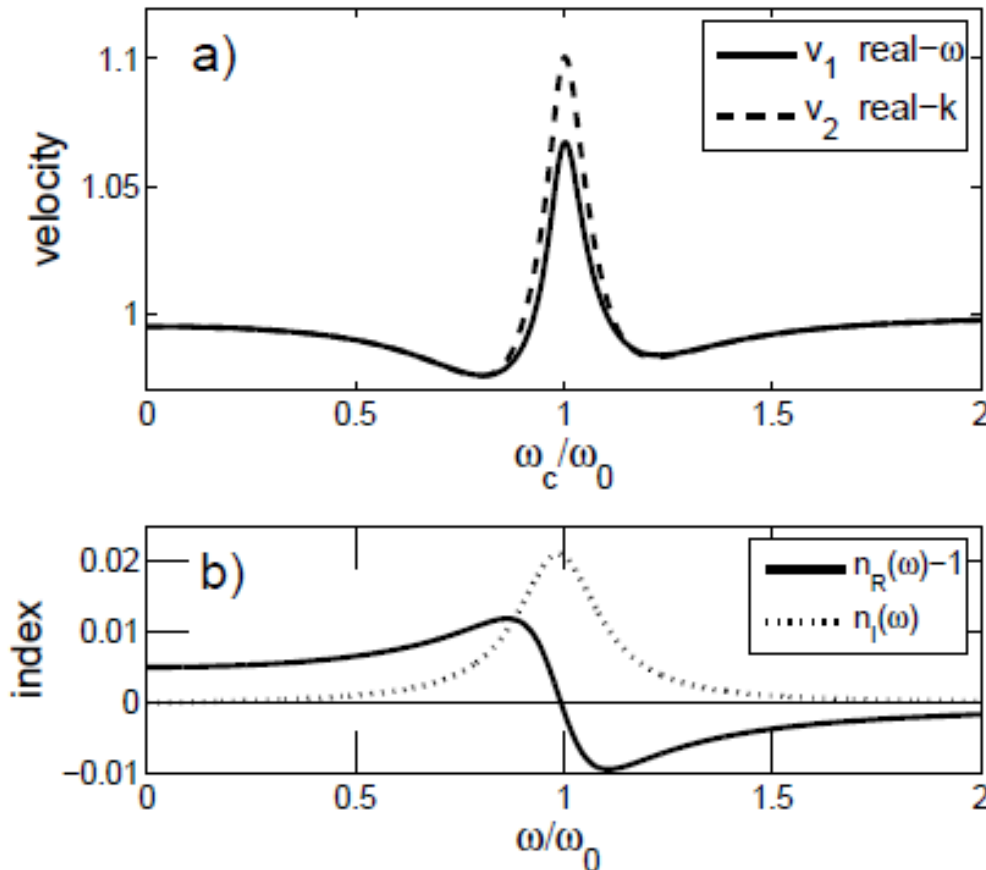
$$U(0, t) = e^{-t^2/\tau^2} \cos(\omega_c t)$$

Luminal regime

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at resonance
($\omega_c \sim \omega_0$)

both
 v_1 and v_2
superluminal



Comparison of v_1 and v_2

Gaussian wave-packet

$$U(0, t) = e^{-t^2/\tau^2} \cos(\omega_c t)$$

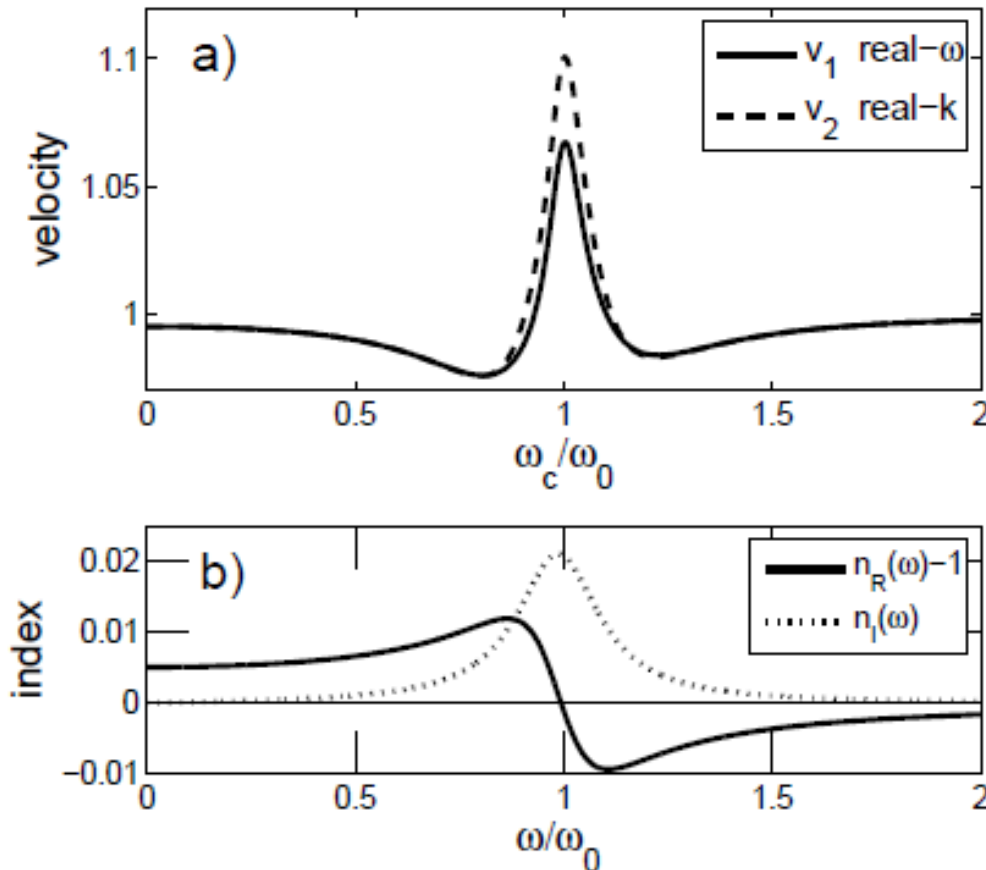
Luminal regime

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however
 v_1, v_2
differs



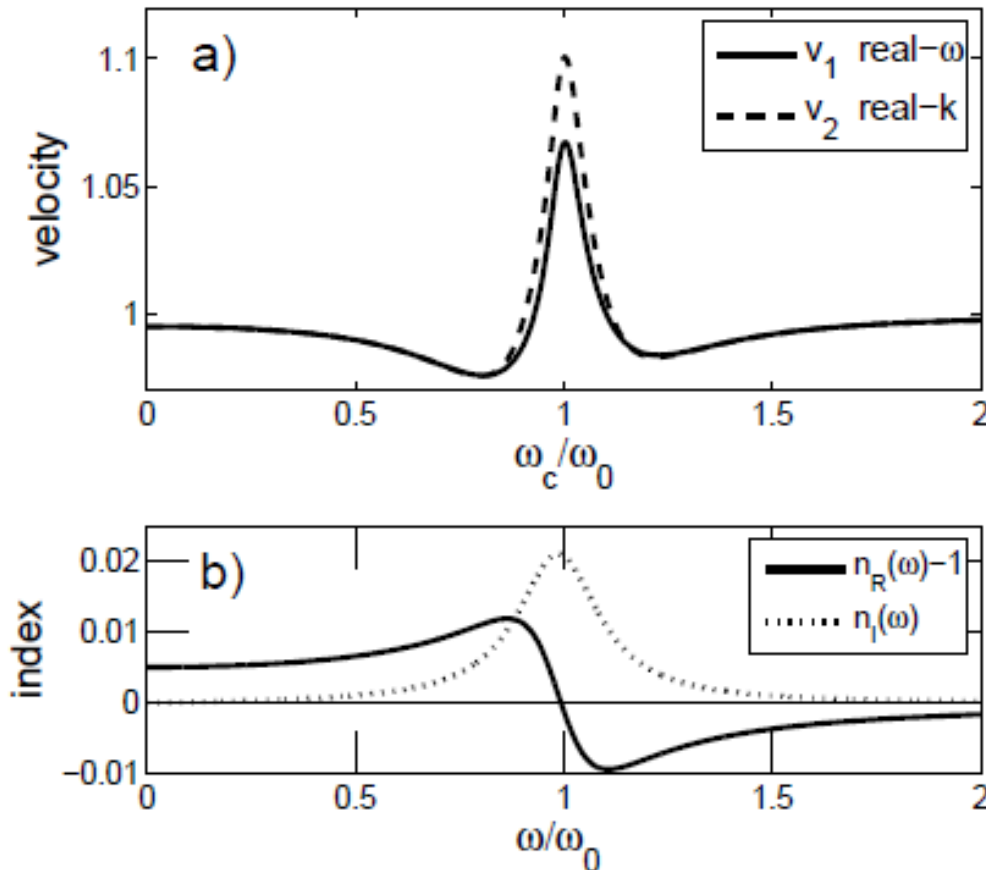
Comparison of v_1 and v_2

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Luminal regime

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at resonance
($\omega_c \sim \omega_0$)

both
 v_1 and v_2
superluminal

however

v_1, v_2
differs

velocity
definition

is inconsistent

not reliable

not
correspond to
a real flow

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Experiment again

Nanda *et al.* [3]

showed

$$\langle t \rangle_x = \frac{\int dt \, t \, S(x, t)}{\int dt \, S(x, t)}$$

corresponds to
detection time

Experiment again

Nanda *et al.* [3]

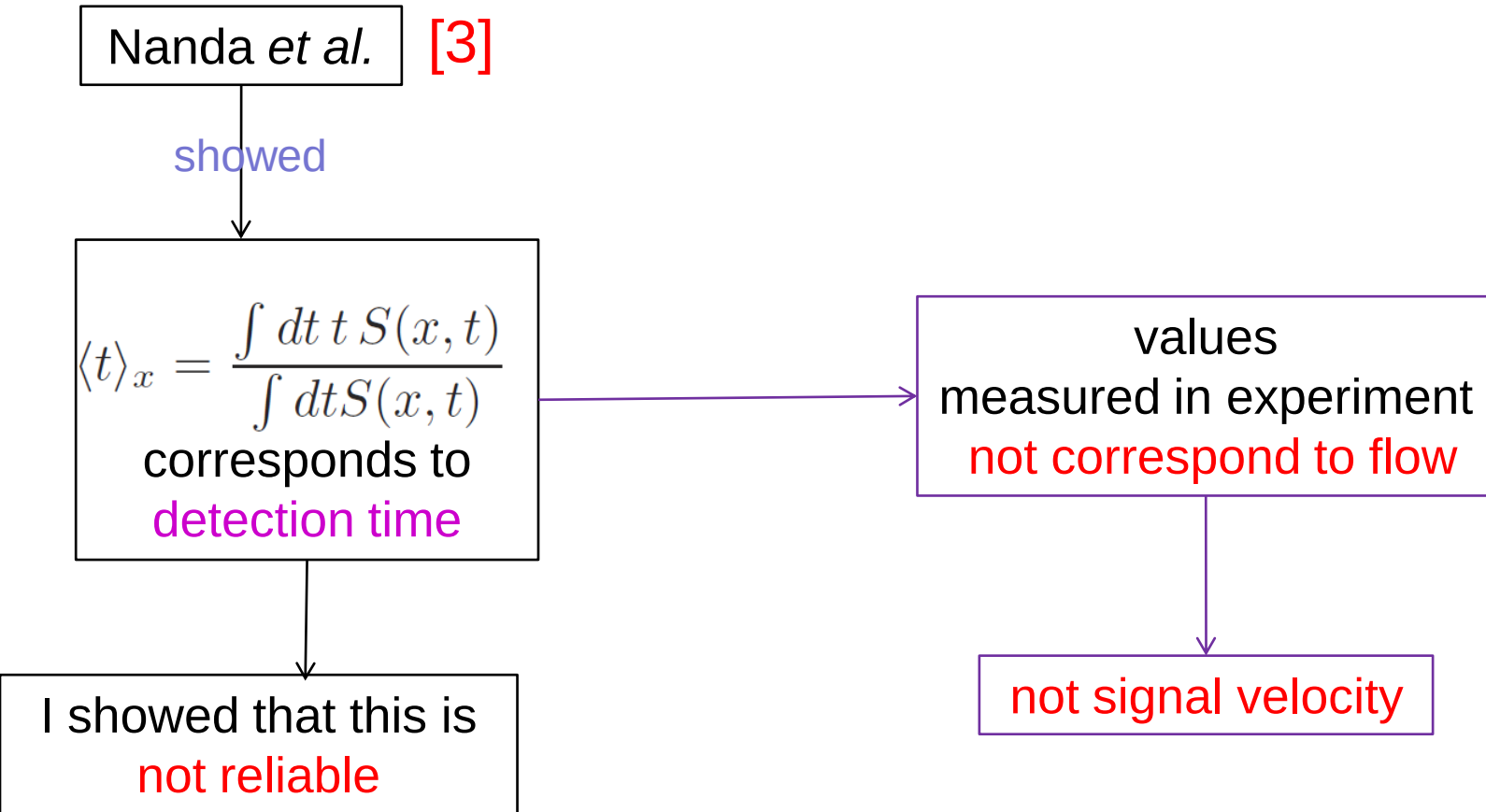
showed

$$\langle t \rangle_x = \frac{\int dt \, t \, S(x, t)}{\int dt \, S(x, t)}$$

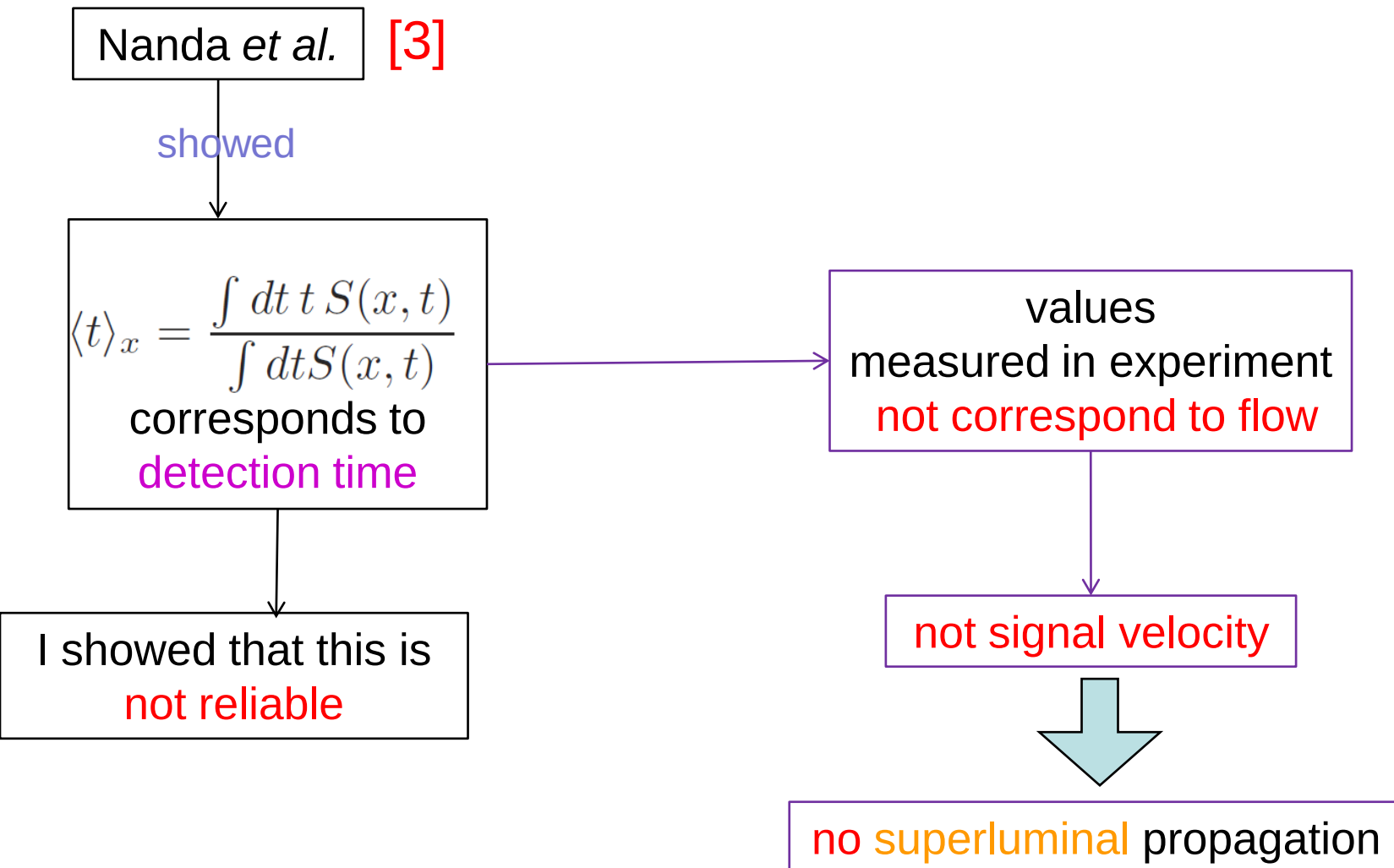
corresponds to
detection time

I showed that this is
not reliable

Experiment again



Experiment again



Summary

- Cannot distinguish between **propagation** and **reshaping**.
- **Signal velocity** and **Pulse-peak** velocity differ.
- Introduced a method to check if a velocity corresponds a physical flow?
- Detectors measure **pulse-peak** velocity.
- Observed is not superluminal propagation; it's **reshaping**.

Acknowledgement

- ❑ Special thanks to Victor Kozlov for illuminating discussions.
- ❑ I thank Gürsoy Akgüç for intensive help in the manuscript.



TUBİTAK-KARİYER No: 112T927

TÜBİTAK-1001 No: 110T876