

## Two-Dimensional Inverse Dynamics

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**Inverse dynamics** is the specialized branch of mechanics that bridges the areas of kinematics and kinetics. It is the process by which forces and moments of force are indirectly determined from the kinematics and inertial properties of moving bodies.

**Direct dynamics**, in contrast, determines the motion of bodies under the influence of applied forces. In principle, inverse dynamics also applies to stationary bodies, but usually it is applied to bodies in motion. It derives from Newton's second law, where the resultant force is partitioned into known and unknown forces. The unknown forces are combined to form a single **net force** that can then be solved. A similar process is done for the moments of force so that a single **net moment of force** is computed. This chapter

- ▶ defines the process of inverse dynamics for planar motion analysis,
- ▶ presents the standard method for numerically computing the internal kinetics of planar human movements,
- ▶ describes the concept of general plane motion,
- ▶ outlines the method of sections for individually analyzing components of a system or segments of a human body,
- ▶ outlines how inverse dynamics aids research of joint mechanics, and
- ▶ examines applications of inverse dynamics in biomechanics research.

Inverse dynamics of human movement dates to the seminal work of Wilhelm Braune and Otto Fischer between 1895 and 1904. These works were later revisited by Herbert Elftman for his research on walking (1939a, 1939b) and running (1940). Little follow-up research

was conducted until Bresler and Frankel (1950) conducted further studies of gait in three dimensions (3-D) and Bresler and Berry (1951) expanded the approach to include the powers produced by the ankle, knee, and hip moments during normal, level walking. Because Bresler and Frankel's 3-D approach measured the moments of force against a Newtonian or absolute frame of reference, they could not determine the contributions made by the flexors or extensors of a joint versus the abductors and adductors (a problem since solved by the 3-D methods described in chapter 7).

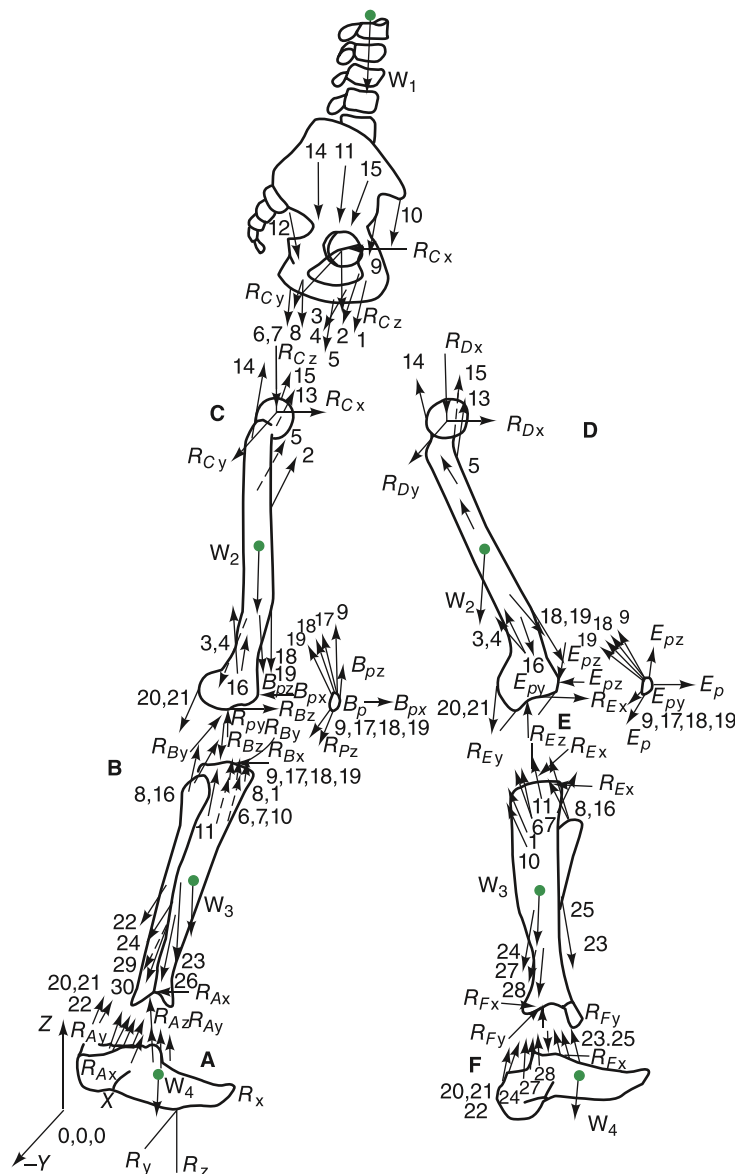
Few inverse dynamics studies of human motion were conducted until the 1970s, when new research was spurred by the advent of commercial force platforms to measure the ground reaction forces (GRFs) during gait and inexpensive computers to provide the necessary processing power. Another important development has been the recent propagation of automated and semiautomated motion-analysis systems based on video or infrared camera technologies, which greatly decrease the time required to process the motion data.

Inverse dynamics studies have since been carried out on such diverse movements as lifting (McGill and Norman 1985), skating (Koning et al. 1991), jogging (Winter 1983a), race walking (White and Winter 1985), sprinting (Lemaire and Robertson 1989), jumping (Stefanyshyn and Nigg 1998), rowing (Robertson and Fortin 1994; Smith 1996), and kicking (Robertson and Mosher 1985), to name a few. Yet inverse dynamics has not been applied to many fundamental movements: swimming and skiing, because of the unknown external forces of water and snow; or batting, puck shooting, and golfing, because of the indeterminacy caused when the two arms and the implement (the bat, stick, or club) form a closed kinematic chain. Future research may be able to overcome these difficulties.

## PLANAR MOTION ANALYSIS

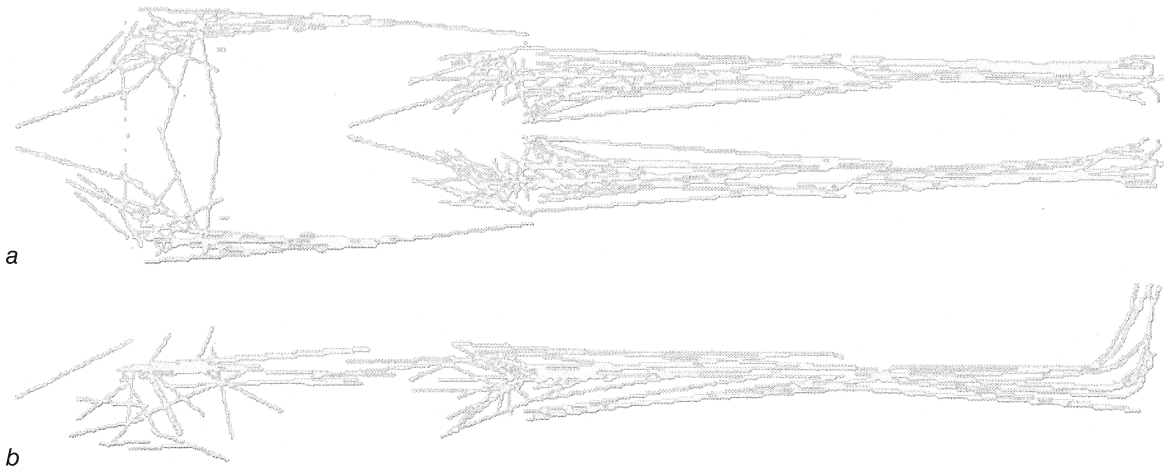
One of the primary goals of biomechanics research is to quantify the patterns of force produced by the muscles, ligaments, and bones. Unfortunately, recording these forces directly (a process called **dynamometry**) requires invasive and potentially hazardous instruments that inevitably disturb the observed motion. Some technologies that measure internal forces include a surgical staple for forces in bones (Rolf et al. 1997) and mercury strain gauges (Brown et al. 1986; Lamontagne et al. 1985) or buckle force transducers (Komi 1990) for forces in muscle tendons or ligaments. Although these

devices enable the direct measurement of internal forces, they have been used only to measure forces in single tissues and are not suitable for analyzing the complex interaction of muscle contractions across several joints simultaneously. Figure 5.1 (Seireg and Arvikar 1975) shows the complexity of forces that a biomechanist must consider when trying to analyze the mechanics of the lower extremity. In figure 5.2, the lines of action of only the major muscles of the lower extremity have been graphically presented by Pierrynowski (1982). It is easy to imagine the difficulty of and the risks associated with attempting to attach a gauge to each of these tendons.



▲ **Figure 5.1** Free-body diagrams of the segments of the lower extremity during walking.

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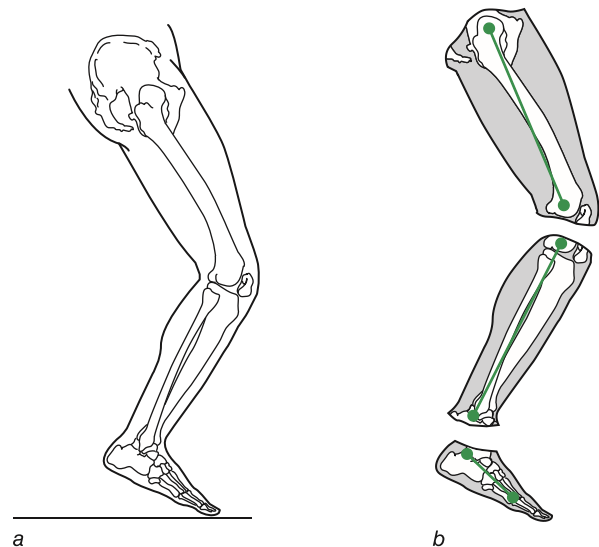


▲ **Figure 5.2** Lines of action of the muscle forces in the lower extremity and trunk: (a) front view, (b) side view. Adapted from data, by permission, from M.R. Pierrynowski, 1982, *A physiological model for the solution of individual muscle forces during normal human walking* (Simon Fraser University).

Inverse dynamics, although incapable of quantifying the forces in specific anatomical structures, is able to measure the *net* effect of all of the internal forces and moments of force acting across several joints. In this way, a researcher can infer what total forces and moments are necessary to create the motion and quantify both the internal and the external work done at each joint. The steps set out next clarify the process for reducing complex anatomical structures to a solvable series of equations that indirectly quantify the kinetics of human or animal movements.

Figure 5.3 shows the space and free-body diagrams of one lower extremity during the push-off phase of running. Three equations of motion can be written for each segment in a two-dimensional (2-D) analysis, so for the foot, three unknowns can be solved. Unfortunately, because there are many more than three unknowns (figure 5.4), the situation is called *indeterminate*. Indeterminacy occurs when there are more unknowns than there are independent equations. To reduce the number of unknowns, each force can be resolved to its equivalent force and moment of force at the segment's endpoint. The process starts at a terminal segment, such as the foot or hand, where the forces at one end of the segment are known or zero. They are zero when the segment is not in contact with the environment or another object. For example, the foot during the swing phase of gait experiences no forces at its distal end; when it contacts the ground, however, the GRF must be measured by, for example, a force platform.

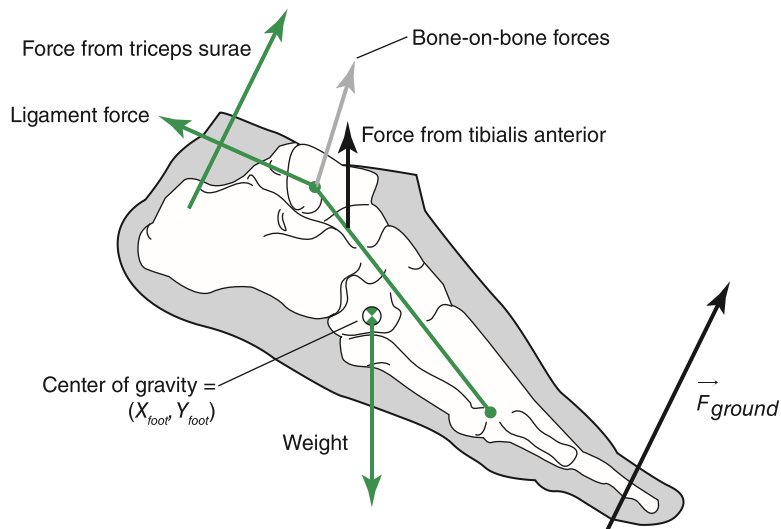
A detailed free-body diagram (FBD) of the foot in contact with the ground is illustrated in figure 5.4. Notice the many types of forces crossing the ankle joint, including muscle and ligament forces and bone-on-bone forces; many others have been left out (e.g., forces from



▲ **Figure 5.3** (a) Space and (b) free-body diagrams of the foot during the push-off phase of running.

skin, bursa, and joint capsule). Furthermore, the foot is assumed to be a “rigid body,” although some researchers have modeled it as having two segments (Cronin and Robertson 2000; Stefanyshyn and Nigg 1998). A rigid body is an object that has no moving parts and cannot be deformed. This state implies that its inertial properties are fixed values (i.e., that its mass, center of gravity, and mass distribution are constant).

Figure 5.5 shows how to replace a single muscle force with an equivalent force and moment of force about a common axis. In this example, the muscle force exerted by the tibialis anterior muscle on the foot segment is replaced by an equivalent force and moment of force at the ankle center of rotation. Assuming that the foot



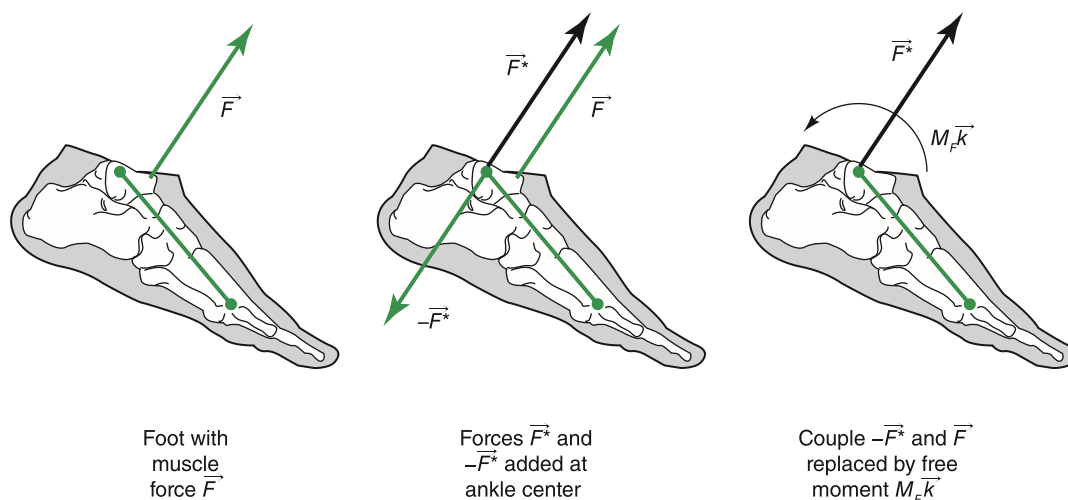
▲ **Figure 5.4** Free-body diagram of the foot showing anatomical forces.

is a “rigid body,” a force ( $\vec{F}^*$ ) equal in magnitude and direction to the muscle force ( $\vec{F}$ ) is placed at the ankle. Because this would unbalance the free body, a second force ( $-\vec{F}^*$ ) is added to maintain equilibrium (figure 5.5b). Next, the force couple ( $\vec{F}$  and  $-\vec{F}^*$ ) is replaced by the moment of force ( $M_F \vec{k}$ ). The resulting force and moment of force in figure 5.5c have the same mechanical effects as the single muscle force in figure 5.5a, assuming that the foot is a rigid body.

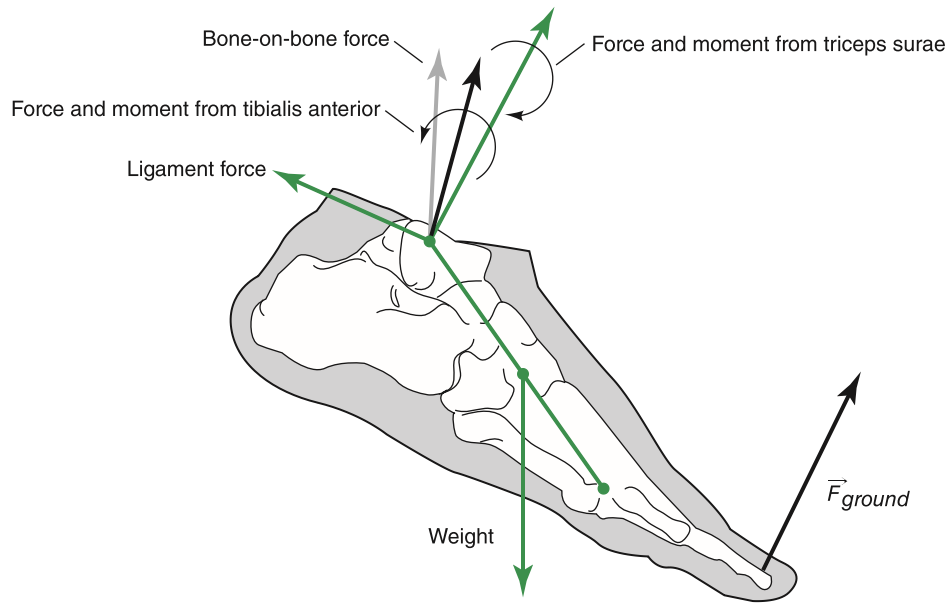
The first step to simplifying the complex situation shown in figure 5.4 is to use the process illustrated in figure 5.5 to replace every force that acts across the ankle with its equivalent force and moment of force about a common axis. Figure 5.6 shows this situation. Note that forces with lines of action that pass through the ankle

joint center produce no moment of force around the joint. Thus, the major structures that contribute to the net moments of force are the muscle forces. The ligament and bone-on-bone forces contribute mainly to the net force experienced by the ankle and only affect the ankle moment of force when the ankle is at the ends of its range of motion.

Muscles attach in such a way that their turning effects about a joint are enhanced, and most have third-class leverage to promote speed of movement. Thus, muscles rarely attach so that they cross directly over a joint axis of rotation because that would eliminate their ability to create a moment about the joint. Ligaments, in contrast, often cross joint axes, because their primary role is to hold joints together rather than to create rotations of the



▲ **Figure 5.5** Replacement of a muscle force by its equivalent force and moment of force at the ankle axis of rotation.

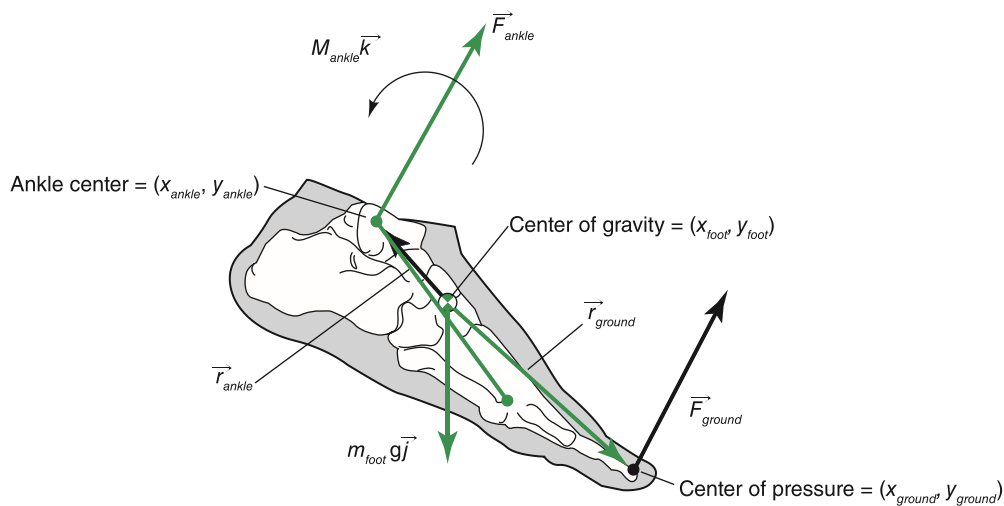


▲ **Figure 5.6** Free-body diagram of the foot showing the muscle forces replaced by their equivalent force and moment about the ankle.

segments that they connect. They do, however, have a role in producing moments of force when the joint nears or reaches its range-of-motion limits. For example, at the knee, the collateral ligaments prevent varus and valgus rotations and the cruciate ligaments restrict hyperextension. Often, the ligaments and bony prominences produce force couples that prevent excessive rotation, such as when the olecranon process and the ligaments of the elbow prevent hyperextension of the elbow.

To complete the inverse dynamics process for the foot, every anatomical force, including ligament and bone-on-bone (actually, cartilaginous) forces, must be transferred to the common axis at the ankle. Note that

only forces that act across the ankle are included in this process. Internal forces that originate and terminate within the foot are excluded, as are external forces in contact with the sole of the foot. Figure 5.7 represents the situation after all of the ankle forces have been resolved. In this figure, the ankle forces and moments of force are summed to produce a single force and a moment of force, called the *net force* and *net moment of force*, respectively. They are also sometimes called the *joint force* and *joint moment of force*, but these are confusing because many different joint forces are included in this sum, such as those caused by the joint capsule, the ligaments, and the articular surfaces (cartilage). Other confusing terms



▲ **Figure 5.7** Reduced free-body diagram showing net force and moment of force.

are *resultant joint force* and *resultant moment of force*, because these terms may be confused with the resultant force and moment of force of the foot segment itself. Recall that the resultant force and moment of force of a rigid body are the sums of all forces and moments acting on the body. These sums are *not* the same as the net force and moment of force just defined. The resultant force and moment of force concern Newton's first and second laws.

The moment of force is often called *torque* in the scientific literature. In engineering, torque is usually considered a moment of force that causes rotation about the longitudinal axis of an object. For example, a torque wrench measures the axial moment of force when one is tightening nuts or bolts, and a torque motor generates spin about an engine's spin axis. In the biomechanics literature, however, as stated in chapter 4, torque and moment of force are used interchangeably.

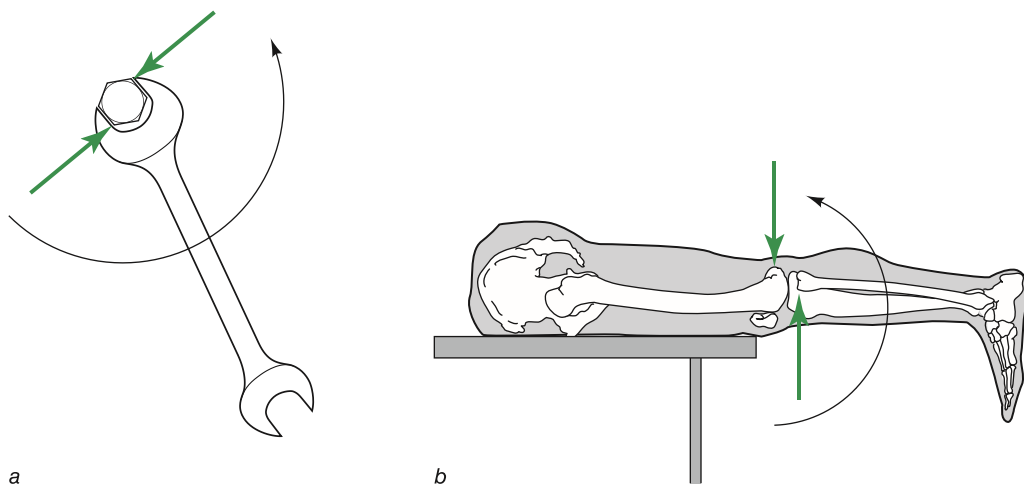
Another term related to moment of force is the *force couple*. A force couple occurs when two parallel, noncollinear forces of equal magnitude but opposite direction act on a body. The effect of a force couple is special because the forces, being equal but opposite in direction, effect no translation on the body when they act. They do, however, attempt to produce a pure rotation, or torque, of the body. For example, a wrench (figure 5.8a) causes two parallel forces when applied to the head of a nut. The nut translates because of the threads of the screw but turns around the bolt because of the rotational forces (i.e., moment of force, force couple, or torque) created by the wrench.

Another interesting characteristic of a force couple, or *couple* for short, is that when the couple is applied to a rigid body, the effect of the couple is independent of its point of application. This makes it a *free moment*, which means that the body experiencing the couple will react in the same way wherever the couple is applied as long

as the lines of axis of the force are parallel. For example, a piece of wood that is being drilled will react the same no matter where the drill contacts the wood as long as the drill bit enters the wood from the same parallel direction. Of course, how the wood actually reacts will depend on friction, clamping, and other forces, but the drill will inflict the same rotational motion on the wood no matter where it enters.

The work done by the net moments of force quantifies the mechanical work done by only the various tissues that act across and contribute a turning effect at a particular joint. All other forces, including gravity, are excluded from contributing to the net force and moment of force. More details about how the work of the moment of force is calculated are delineated in chapter 6.

Net forces and moments are not real entities; they are mathematical concepts and therefore can never be measured directly. They do, however, represent the summed or net effect of all the structures that produce forces or moments of force across a joint. Some researchers (e.g., Miller and Nelson 1973) have called the source of the net moment of force a "single equivalent muscle." They contend that each joint can be thought of as having two single, equivalent muscles that produce the net moments of force about each joint—for example, one for flexion and the other for extension—depending on the joint's anatomy. Others have called the net moments of force "muscle moments," but this nomenclature should be avoided, because even though muscles are the main contributors to the net moment, other structures also contribute, especially at the ends of the range of motion. An illustration of this situation is when the knee reaches maximal flexion during the swing phase of sprinting. Lemaire and Robertson (1989) and others showed that although a very large moment of force occurs, the likely cause is not an eccentric contraction of the extensors;



▲ **Figure 5.8** Force couples produced by a wrench and the ligaments of the knee.

instead it is the result of the calf and thigh bumping together. However, the same cannot be said to occur for the negative work done by the knee extensors during the swing phase of walking, because the joint does not fully flex and therefore muscles must be recruited to limit knee flexion (Winter and Robertson 1979).

## NUMERICAL FORMULATION

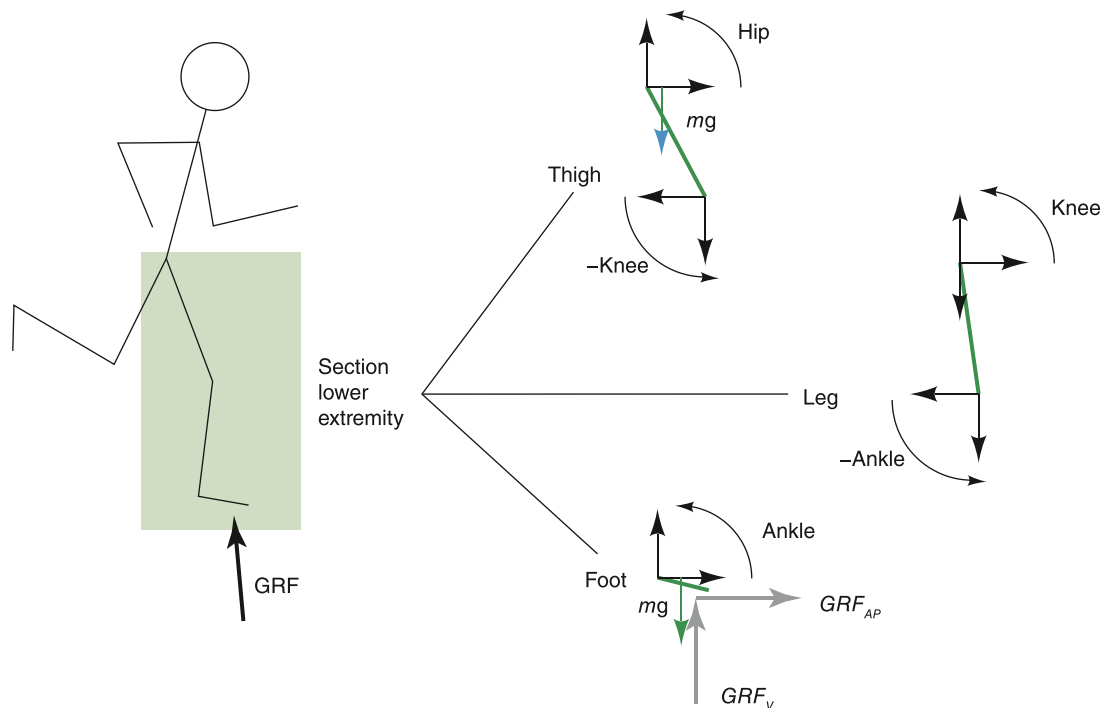
This section presents the standard method in biomechanics for numerically computing the internal kinetics of planar human movements. In this process, we use body kinematics and anthropometric parameters to calculate the net forces and moments at the joints. This process uses three important principles: Newton's second law ( $\Sigma \vec{F} = m\vec{a}$ ), the principle of *superposition*, and an engineering technique known as the *method of sections*. The principle of superposition holds that in a system with multiple factors (i.e., forces and moments), given certain conditions, we can either sum the effects of multiple factors or treat them independently. In the method of sections, the basic idea is to imagine cutting a mechanical system into components and determining the interactions between them. For example, we usually section the human lower extremity into a thigh, leg, and foot. Then, via Newton's second law, we can determine

the forces acting at the joints by using measured values for the GRFs and the acceleration and mass of each segment. This process, called the *linked-segment* or *iterative Newton-Euler method*, is diagrammed in figure 5.9. The majority of this chapter explains how this method works. We will begin with kinetic analysis of single objects in 2-D, then demonstrate how to analyze the kinetics of a joint via the method of sections, and finally explain the general procedure diagrammed in figure 5.9 for the entire lower extremity.

Note the diagram conventions used in this chapter: Linear parameters are drawn with straight arrows, and angular parameters are drawn with curved arrows. Known kinematic data (linear and angular accelerations) are drawn with green lines. Known forces and moments are drawn with black arrows. Unknown forces and moments are drawn with gray arrows. These conventions will assist you in visualizing the solution process.

## General Plane Motion

*General plane motion* is an engineering term for 2-D movement. In this case, an object has three degrees of freedom (DOF): two linear positions and an angular position. Often, we draw these as translations along the  $x$ - and  $y$ -axes and a rotation about the  $z$ -axis. As discussed in chapter 2, many lower-extremity movements can be analyzed using this simplified representation, including

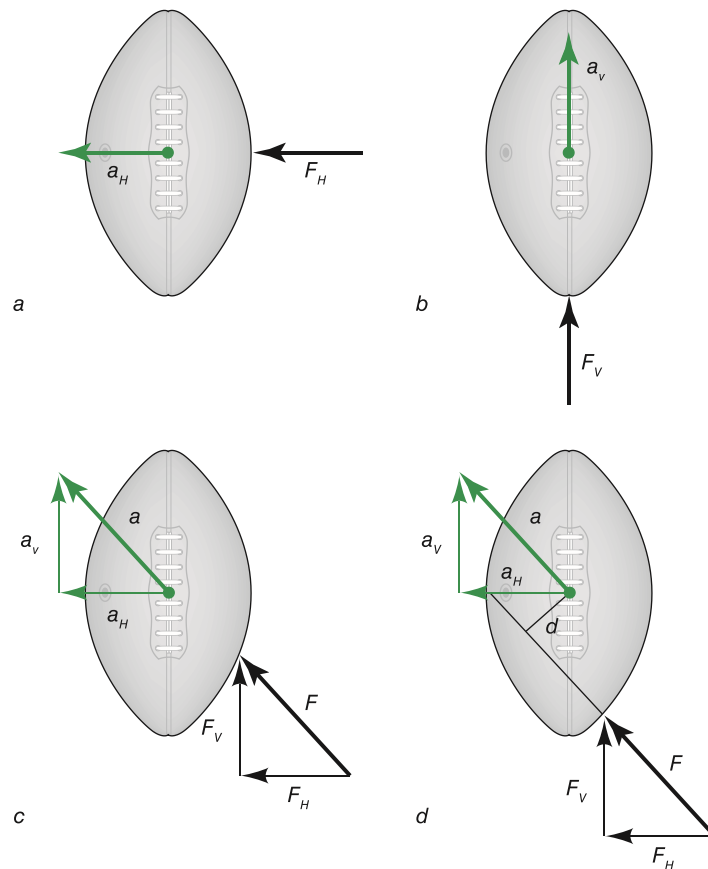


▲ **Figure 5.9** Space diagram of a runner's lower extremity in the stance phase, with three free-body diagrams of the segments.

walking, running, cycling, rowing, jumping, kicking, and lifting. However, despite the simplification to 2-D analysis, the resulting mechanics can still be complicated. For example, a football punter has three lower-extremity segments that swing forward much like a whip, kick the ball, and elevate. Even the ball has a somewhat complicated movement, translating both horizontally and vertically and rotating. To determine the kinetics of such situations, we treat the three DOF independently. That is, we exploit the fact that an object accelerates in the vertical direction only when acted on by a vertical force and accelerates in the horizontal direction only when acted on by a horizontal force. Similarly, the body does not rotate unless a moment (torque) is applied to it. The principle of superposition states that when one or more of these actions occur, we can analyze them separately. We therefore separate all forces and moments into three coordinates and solve them separately.

To illustrate, let us consider the football example. In figure 5.10*d*, a football being kicked is subjected to the force of the punter's foot. The ball moves horizontally and vertically and also rotates. Our goal is to determine the force with which the ball was kicked. We cannot measure the force directly with an instrumented ball

or shoe. However, we can film the ball's movement and measure its mass and moment of inertia. Given these data, we are left with an apparently confusing situation to analyze: The single force of the foot has caused all three coordinates to change. However, the situation becomes simpler when we use superposition. The horizontal and vertical accelerations of the ball's mass center must be proportional to their respective forces, and the angular acceleration must be proportional to the applied moment. Consider the examples in figure 5.10. Figure 5.10*a* and *b* are rather obvious but are presented for the sake of demonstration. In figure 5.10*a*, a horizontal force is applied to the ball through its center of mass, and the ball will accelerate horizontally; it will not accelerate vertically because there is no vertical force. Similarly, in figure 5.10*b*, the ball will accelerate only vertically because there is no horizontal force. In figure 5.10*c*, the force acts at a 45° angle, and therefore the ball accelerates at a 45° angle. This is just a superposition of the situations in figure 5.10*a* and *b*. We do not deal with the force at this angle; rather, we measure the accelerations in the horizontal and vertical directions, and therefore we can determine the forces in the horizontal and vertical directions. In Figure 5.10*d*, the applied force is not



▲ **Figure 5.10** Four free-body diagrams of a football experiencing four different external forces.



directed through the center of mass. In this case, the force is the same as it is in figure 5.10c, so the ball's center of mass has the same acceleration. However, there is also an angular acceleration proportional to the product of the force,  $F$ , and the distance,  $d$ , between its line of action and the center of mass. The acceleration,  $a$ , in this case is the same as in figure 5.10c. However, the ball will also rotate.

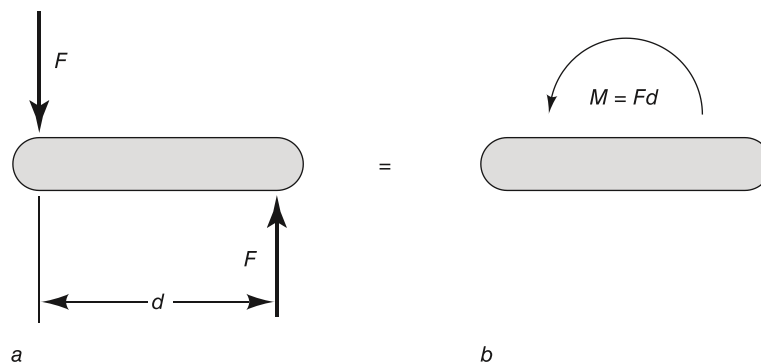
To reiterate, a force causes a body's center of mass to accelerate in the same direction as that force. A force does not cause a body to rotate; only the moment of a force causes a body to rotate. These principles derive from Newton's first and second laws. If a ball is kicked, a resulting force is imparted on the ball. This force is the reaction of the ball being accelerated. Whether the ball was kicked through its center of mass, causing it to rotate, is irrelevant. Rotation is affected in proportion to the distance by which the applied force and mass center are out of line.

Let us explore moments further. Referring to figure 5.11, a moment can be defined as the effect of a force-couple system, that is, two forces of equal magnitude and opposite direction that are not collinear (figure 5.11a). In this system, the sum of the forces is zero. However, because the two forces are noncollinear, they cause the body to rotate. This is drawn diagrammatically as a curved arrow (figure 5.11b).

To return to the football in figure 5.10, there is a force couple in all four diagrams, the applied force,  $F$ , and the reaction,  $ma$ . In figure 5.10,  $a$  through  $c$ , this force couple is collinear, so there is no moment. However, in figure 5.10d, the force and reaction are noncollinear. There is a perpendicular distance between the line of action of the force and the reaction of the mass center, and the result is a moment that rotates the ball.

Let us formalize the present discussion. Given a 2-D FBD, the process is to use Newton's second law in the horizontal, vertical, and rotational directions:

$$\sum F_x = ma_x \quad \sum F_y = ma_y \quad \sum M = I\alpha \quad (5.1)$$



▲ **Figure 5.11** (a) A force couple and (b) its equivalent free moment.

The mass,  $m$ , and moment of inertia,  $I$ , for the object in question are determined beforehand. The linear and angular accelerations are determined from camera data. The sum of the forces or moments on the left side of each equation (i.e., each  $\Sigma$  term) can combine many forces or moments, but it should contain only one unknown—a net force or moment—to solve for. Because of this, we usually must solve for the two forces before we solve for the unknown moment.

When putting together the sums of forces and moments, we must adhere to the sign conventions established by the FBD. Inverse dynamics problems often require careful bookkeeping of positive and negative signs. As the examples in this chapter show, the FBD takes care of this as long as we honor the sign conventions that have been drawn. For example, in many cases we solve for a force or moment even though we are uncertain of its direction. This is not a problem with FBDs: We merely draw the force or moment with an assumed direction. If, in fact, the force points in the opposite direction, our calculations simply return a negative numerical value.

A sum of moments must be calculated about a single point on the object in question. There is no right or wrong point about which to calculate; however, some points are simpler than others. If we sum moments about a point where one or more forces act, then the moments of those forces will be zero because their moment arms are zero. Therefore, sometimes in human movement, it is convenient to calculate about a joint center. However, in most cases we calculate moments about the mass center; there, we can neglect the reaction force ( $ma$ ) and gravity terms because their moment arms are zero.

This text uses the convention that a counterclockwise moment is positive, also called the *right-hand rule*. Coordinate system axes on FBDs establish their positive force directions. When solving a problem from an FBD, the proper technique is to first write the equations in algebraic form, honoring the sign conventions, and then to substitute known numerical values with their signs once

all of the algebraic manipulations have been carried out. These procedures are only learned by example, so we now present a few of them.

### EXAMPLE 5.1

5.1a—Suppose that in our football example, the ball was kicked and its mass center moved horizontally. The horizontal acceleration was  $-64 \text{ m/s}^2$ , the angular acceleration was  $-28 \text{ rad/s}^2$ , the mass of the ball was  $0.25 \text{ kg}$ , and its moment of inertia was  $0.05 \text{ kg}\cdot\text{m}^2$ . What was the kicking force? How far from the center of the ball was the force's line of action?

See answer 5.1a on page 377.

5.1b—Now solve the same problem assuming that there was a force plate under the football and that the tee on which the football rested resisted the kicking force. The force of the tee was  $4 \text{ N}$  in the horizontal direction; its **center of pressure** (COP) was  $15 \text{ cm}$  below the mass center of the ball.

See answer 5.1b on page 378.

### EXAMPLE 5.2

A commuter is standing inside a subway car as it accelerates away from the station at  $3 \text{ m/s}^2$ . She maintains a perfectly rigid, upright posture. Her body mass of  $60 \text{ kg}$  is centered  $1.2 \text{ m}$  off the floor, and the moment of inertia about her ankles is  $130 \text{ kg}\cdot\text{m}^2$ . What floor forces and ankle moment must the commuter exert to maintain her stance?

See answer 5.2 on page 378.

### EXAMPLE 5.3

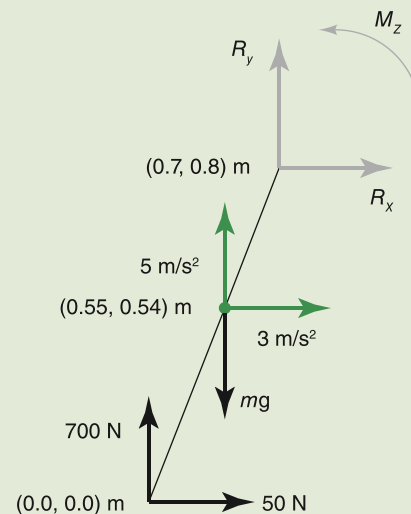
A tennis racket is swung in the horizontal plane ( $X$  and  $Y$  axis are both horizontal). The racket's mass center is accelerating at  $32 \text{ m/s}^2$  in the  $Y$  direction and its angular acceleration is  $10 \text{ rad/s}^2$ . Its mass is  $0.5 \text{ kg}$  and its moment of inertia is  $0.1 \text{ kg}\cdot\text{m}^2$ . From the base of the racket, the locations of the hand and racket mass center are  $7$  and  $35 \text{ cm}$ , respectively. If we ignore the force of gravity, what are the net force and moment exerted on the racket? Given that the hand is about  $6 \text{ cm}$  wide, interpret the meaning of the net moment (i.e., what is the actual force couple at the hand?).

See answer 5.3 on page 379.

In these examples, we provided distances between various points on the FBDs. These distances are not the data we measure with camera systems and force plates. Rather, these instruments measure the locations of points, specifically the locations of joint centers and the GRF (the COP). We need these points to calculate the moment arms of the various forces in the FBD. This is simply a matter of subtracting corresponding positions in the global coordinate system (GCS) directions. However, a commonly made error is that the moment arm of a force in the  $x$  direction is a distance in the perpendicular  $y$  direction, and vice versa. This is a very important point that we again illustrate with examples.

### EXAMPLE 5.4

Given the FBD of the arbitrary object following, calculate the reactions  $R_x$ ,  $R_y$ ,  $M_z$  at the unknown end. Its mass is  $8.0 \text{ kg}$  and its moment of inertia is  $0.2 \text{ kg}\cdot\text{m}^2$  about the mass center.



See answer 5.4 on page 379.

### EXAMPLE 5.5

Given the following data for a bicycle crank, draw the FBD and calculate the forces and moments at the crank axle: pedal force  $x = -200 \text{ N}$ ; pedal force  $y = -800 \text{ N}$ ; pedal axle at  $(0.625, 0.310) \text{ m}$ ; crank axle at  $(0.473, 0.398) \text{ m}$ ; crank mass center at  $(0.542, 0.358) \text{ m}$ . The crank mass is  $0.1 \text{ kg}$  and its moment of inertia is  $0.003 \text{ kg}\cdot\text{m}^2$ . Its accelerations are  $-0.4 \text{ m/s}^2$  in the  $x$  direction and  $-0.7 \text{ m/s}^2$  in the  $y$  direction; its angular acceleration is  $10 \text{ rad/s}^2$ .

See answer 5.5 on page 380.

Having discussed general plane motion for a single object, we now turn to the solution technique for multiobject systems like arms and legs. The procedure for each body segment is exactly the same as for single-body systems. The only difference is that the method of sections is applied to manage the interactive forces between segments. In fact, this technique was applied in the previous example of the bicycle crank: We had to “section” the crank, that is, we imagined cutting it off at the axle, to determine the forces and moment at the axle. Let us explore this technique in detail.

## Method of Sections

Engineering analysis of a mechanical device usually focuses on a limited number of key points of the structure. For example, on a railroad bridge truss, we normally study the points where its various pieces are riveted together. The same is true when we analyze the kinetics of human movement. We generally do not concern ourselves with a complete map of the forces and moments within the body. Rather, we study specific points of the body—most commonly the joints. We therefore “section” the body at the joints and calculate the reactions between adjacent segments that keep them from flying apart. These forces and moments at a sectioned joint are unknown. Therefore, when constructing our FBD, we must draw a reaction for each DOF—that is, a horizontal force, a vertical force, and a moment. In some calculations, it is possible for one or two of these reactions to be zero, but the method of sections requires that each be drawn and solved for because they are unknowns.

The method of sections is straightforward:

- ▶ Imagine cutting the body at the joint of interest.
- ▶ Draw FBDs of the sectioned pieces.
- ▶ At the sectioned point of one piece, draw the unknown horizontal and vertical reactions and the net moment, honoring the positive directions of the GCS.
- ▶ At the sectioned point of the other piece, draw the unknown forces and moment in the negative directions of the GCS. This is Newton’s third law.
- ▶ Solve the three equations of motion for one of the sections.

## Single Segment Analysis

In many cases, we may be interested in only one of the sectioned pieces. Let us start with three such examples and then progress to multisegmented analysis.

## Multisegmented Analysis

Complete analysis of a human limb follows the procedure used in the previous examples. We simply have to for-

### EXAMPLE 5.6

Consider an arm being held horizontally. What are the shoulder reaction forces and joint moment? Assume that the arm is stationary and rigid. The weights of the upper arm, forearm, and hand are 4, 3, and 1 kg, respectively, and their mass centers are, respectively, 10, 30, and 42 cm from the shoulder.

*See answer 5.6 on page 381.*

### EXAMPLE 5.7

Suppose the hand is holding a 2 kg weight. What are the shoulder reaction forces and joint moment now?

*See answer 5.7 on page 381.*

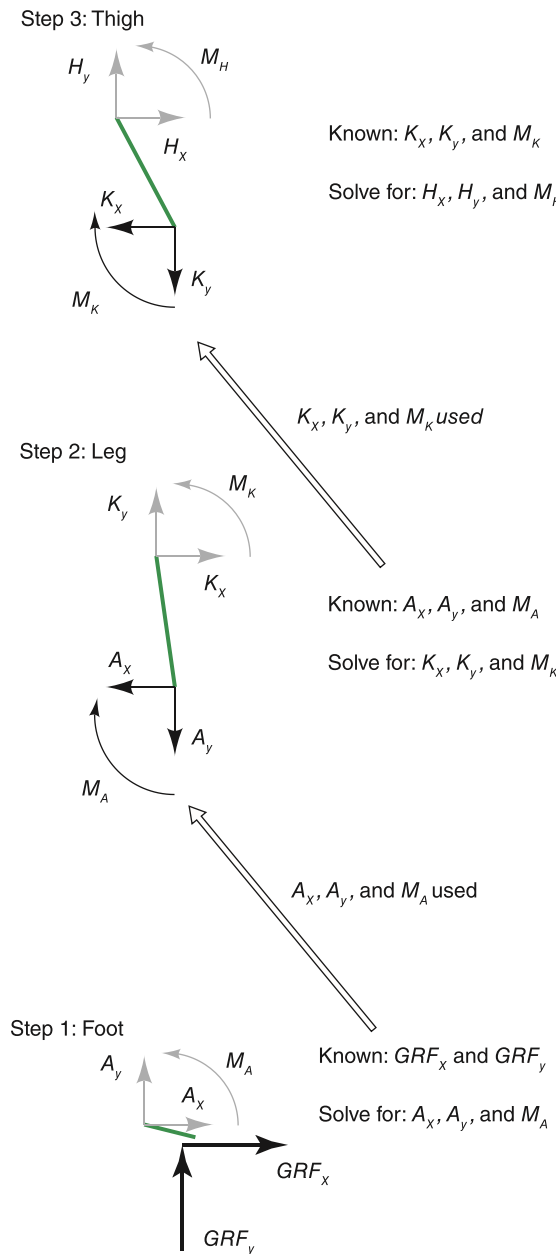
### EXAMPLE 5.8

The elbow is 22 cm from the shoulder. What are the reactions at the elbow on the forearm? On the upper arm? Solve again for the reactions at the shoulder using the FBD of the upper arm.

*See answer 5.8 on page 382.*

malize our solution process. This process has a specific order: We start at the most distal segment and continue proximally. The reason for this is that we have only three equations to apply to each segment, which means that we can have only three unknowns for each segment—one horizontal force, one vertical force, and one moment. However, we can see (return to figure 5.9, if necessary) that if we were to section and analyze either the thigh or the leg, we would have six unknowns—two forces and one moment at each joint. The solution is to start with a segment that has only one joint (i.e., the most distal) and from there proceed to the adjacent segment. For this we use Newton’s third law, applying the negative of the reactions of the segment that we just solved, as shown for the lower extremity in figure 5.12. Of the lower-extremity segments, only the foot has the requisite three unknowns, so we solve this segment first. Note how the actions on the ankle of the foot have corresponding equal and opposite reactions on the ankle of the leg. Then we can calculate the unknown reactions at the knee. These are drawn in reverse in the FBD for the thigh, and then we solve for the hip reactions.

One final, but important, detail about this process is that the sign of each numerical value does not change from one segment to the next. From Newton’s third



▲ **Figure 5.12** Free-body diagrams of the segments of the lower extremity.

law, every action has an equal but opposite reaction. At the joints, therefore, the forces on the distal end of one segment must be equal to but opposite of those on the proximal end of the adjacent segment. However, we never change the signs of numerical values. The FBDs take care of this. Note that in figure 5.12, the knee forces and moments are drawn in opposing directions. Following the procedures shown previously, first construct the equations for a segment from the FBD without considering the numeric values. Once the equations are constructed, the numeric values with their signs are

substituted into these equations. Note how this process is carried out in data tables for examples 5.9 and 5.10.

We provide example calculations there for each of the two distinct phases of human locomotion, swing and stance. When we are solving the kinetics of a swing limb, the process is almost exactly the same as for stance. The only difference is that the GRFs are zero, and thus we can neglect their terms in the equations of motion. The procedure is virtually identical to the calculations that would be carried out in computer code for one frame of data.

## HUMAN JOINT KINETICS

What exactly are these forces and moments that we have just calculated? The answer, which was given earlier in this chapter, deserves revisiting: The net forces and moments represent the sum of the actions of all the joint structures. A common error is thinking of a joint reaction force as the force on the articular surfaces of the bone and a joint moment as the effect of a particular muscle. These interpretations are incorrect because joint reaction forces and moments are more abstract than that; they are sums, net effects. In examples 5.9 and 5.10, we calculate relative measures to compare the efforts of the three lower-extremity joints in producing the movement and forces that were measured. We do not even have estimates of the activities of the quadriceps, triceps surae, or any other muscle. We do not have estimates of the forces on the articular surfaces of the joints or any other anatomical structure. Let us explain this further.

As depicted in figure 5.2, many muscle-tendon complexes, ligaments, and other joint structures bridge each joint. Each of these structures exerts a particular force depending on the specific movement. In figure 5.2, we neglected all friction between the articular surfaces as well as between all other adjacent structures. It cannot be overemphasized that joint reaction forces and moments should be discussed without reference to specific anatomical structures. There are several reasons for this. Equal joint reactions may be carried by entirely different structures. Consider, for example, the elbow joint of a gymnast: When the gymnast is hanging from the rings, the elbow is subjected to a tensile force that must be borne by the various tendons, ligaments, and other structures crossing the elbow. In contrast, when the gymnast performs a handstand, many of these same structures can be lax, because much of the load is shifted to the articular cartilage. From the sectional analysis presented in this chapter, we would calculate equal but opposite joint reaction forces in each case; both would equal one-half of body weight minus the weight of the forearms. However, the distribution of the force among the joint structures is completely different in these two cases.

**EXAMPLE 5.9**

Determine the joint reaction forces and moments at the ankle, knee, and hip given the following data. These occurred during the swing phase of walking, so the GRFs are zero.

The ankle is at (0.303, 0.189) m, the knee is at (0.539, 0.420) m, and the hip is at (0.600, 0.765) m.

See answer 5.9 on page 383.

	Mass (kg)	$I$ (kg·m <sup>2</sup> )	$a_x$ (m/s <sup>2</sup> )	$a_y$ (m/s <sup>2</sup> )	$\alpha$ (rad/s <sup>2</sup> )	Center of mass (m)
Foot	1.2	0.011	-4.39	6.77	5.12	(0.373, 0.117)
Leg	2.4	0.064	-4.01	2.75	-3.08	(0.437, 0.320)
Thigh	6.0	0.130	6.58	-1.21	8.62	(0.573, 0.616)

**EXAMPLE 5.10**

Determine the joint reaction forces and moments at the ankle, knee, and hip given the following data. These occurred during the stance phase of walking, so the ground reaction forces are nonzero. The solution process is almost identical.

The ankle is at (0.637, 0.063) m, the knee at (0.541, 0.379) m, and the hip at (0.421, 0.708) m. The horizontal GRF is -110 N, the vertical GRF is 720 N, and its center of pressure is (0.677, 0.0) m.

See answer 5.10 on page 384.

	Mass (kg)	$I$ (kg·m <sup>2</sup> )	$a_x$ (m/s <sup>2</sup> )	$a_y$ (m/s <sup>2</sup> )	$\alpha$ (rad/s <sup>2</sup> )	Center of mass (m)
Foot	1.2	0.011	-5.33	-1.71	-20.2	(0.734, 0.089)
Leg	2.4	0.064	-1.82	-0.56	-22.4	(0.583, 0.242)
Thigh	6.0	0.130	1.01	0.37	8.6	(0.473, 0.566)

Even if we are not analyzing an agile gymnast, the fact remains that we do not know from net joint forces and moments how loading is shared between various structures. This situation, in which there are more forces than equations, is said to be *statically indeterminate*. In plain language, it is a case in which we know the total load on a system, but we are not able to determine the distribution of the load without considering the specific properties of the load-bearing structures. This is analogous to a group of people moving a heavy object such as a piano: We know that they are carrying the total weight of the piano, but without a force plate under each individual, we cannot know how much weight each person bears.

Consider another specific example, a human standing quietly with straight lower extremities. We could calculate a joint reaction force at the knee. Assuming that the limbs share body weight evenly, the knee reaction force would equal one-half of the body weight minus the weight of the leg and foot. If we asked the subject to clench his muscles as much as possible, the joint reaction forces would not change. However, the tensions in the tendons would increase, as would the compressive

forces on the bones. The changes are equal but opposite and thus do not express themselves as an external, joint reaction force change.

Before discussing patterns of joint moments during human movements, we detail the limitations of these measures. As stated earlier, they are somewhat abstract. However, the purpose of the previous discussion was simply to delineate the limits of joint moments. This being done, these data can be discussed appropriately.

**Limitations**

Aside from the intrinsic limitations of 2-D kinematics discussed in chapter 1, there are several other important limitations in our foregoing analysis of 2-D kinetics.

Effects of friction and joint structures are not considered. The tensions of various ligaments become high near the limits of joints, and thus moments can occur when muscles are inactive. Also, although the frictional forces in joints are very small in young subjects, this is often not the case for individuals with diseased joints. Interested readers should refer to Mansour and Audu (1986),

Audu and Davy (1988), or McFaull and Lamontagne (1993, 1998). Segments are assumed to be rigid. When a segment is not rigid, it attenuates the forces that are applied to it. This is the basis for car suspension systems: The forces felt by the occupant are less than those felt by the tires on the road. Human body segments with at least one full-length bone such as the thigh and leg are reasonably rigid and transmit forces well. However, the foot and trunk are flexible, and it is well documented that the present joint moment calculations are slightly inaccurate for these structures (see, for example, Robertson and Winter 1980). In the foot, for example, various ligaments stretch to attenuate the GRFs. It is for this reason that individuals walking barefoot on a hard floor tend to walk on their toes: The calcaneus-talus bone structures are much more rigid than the forefoot.

The present model is sensitive to its input data. Errors in GRFs, COPs, marker locations, segment inertial properties, joint center estimates, and segment accelerations all affect the joint moment data. Some of these problems are less significant than others. For example, GRFs during locomotion tend to dominate stance-phase kinetics; measuring them accurately prevents the majority of accuracy problems. However, during the swing phase of locomotion, data-treatment techniques and anthropometric estimates are critical. Interested readers can refer to Pezzack and colleagues (1977), Wood (1982), or Whittlesey and Hamill (1996) for more information. Exact comparisons of moments calculated in different studies are not appropriate; we recommend allowing at least a 10% margin of error.

Individual muscle activity cannot be determined from the present model. We do not know the tension in any given muscle simply because muscle actions are represented by moments. Moreover, we do not even know the moment of a single muscle because multiple muscles, ligaments, and other structures cross each joint. Muscle forces are estimated using the musculoskeletal techniques discussed in chapter 9. An important aspect of this is that *cocontraction* of muscles occurs in essentially all human movements. Thus, for example, if knee extensor moments decrease under a certain condition, we do not know whether the decrease occurs because of decreased quadriceps activity or increased hamstring activity. As another example, a subject asked to stand straight and clench her lower-extremity muscles would have joint moments close to zero even though her muscles were fully activated; their actions would work against each other.

*Two-joint muscles* are not well represented by the present model. Although the moments of two-joint muscles are included in joint moment calculations, the segment calculations effectively assume that muscles only cross one joint. Again, this is a problem that is addressed by musculoskeletal models (see chapter 9).

People with amputations require different interpretations than other individuals. For example, an ankle moment can be measured for prostheses even when the leg and foot are a single, semirigid piece. Prosthetic knees have stops to prevent hyperextension and have other components, such as springs and frictional elements, to control their movement. These cause knee moments that require completely different interpretations from those seen in intact subjects. Similar considerations also apply to subjects with braces such as ankle-foot orthoses.

It was a conscious choice on the authors' part to present the interpretation of joint moments after this discussion of their limitations. Limitations do not invalidate model data, but they do limit the extent of interpretation. Note in the following discussion that no reference is made to specific muscles or muscle groups and that the magnitudes of different peaks are not referenced to the nearest 0.1 of a newton meter.

## Relative Motion Method Versus Absolute Motion Method

The approach presented so far is just one way of computing the net forces and moments at the joints. Plagenhoef (1968, 1971) called this approach the *absolute motion method* of inverse dynamics because the segmental kinematic data are computed based on an absolute or fixed frame of reference. An alternative approach outlined by Plagenhoef is the *relative motion method*. This method quantifies the motion of the first segment in a kinematic chain from an absolute frame of reference, but all other segments are referenced to moving axes that rotate with the segment. Thus, each segment's axis, except that of the first, moves relative to the preceding segment. This method has the advantage of showing how one joint's moment of force contributes to the moments of force of the other joints in the kinematic chain. The drawback is that the level of complexity of the analysis increases with every link added to the kinematic chain. Furthermore, the method requires the inclusion of Coriolis forces, which are forces that appear when an object rotates within a rotating frame of reference. These fictitious forces—sometimes called pseudoforces—only exist because of their rotating frames of reference. From an inertial (i.e., fixed or absolute) frame of reference, they do not exist.

Rarely have researchers tried to compare the two methods. However, Pezzack (1976) did compare both methods using the same coordinate data and found that the relative motion method was less accurate, especially as the kinematic chain got longer (had more segments). For short kinematic chains (two or three segments), both methods yielded similar results. Most researchers have

adopted the absolute motion method, because most data-collection systems measure segmental kinematics with respect to axes affixed to the ground or laboratory floor.

## APPLICATIONS

There are many uses for the results of an inverse dynamics analysis. One application sums the extensor moments of a lower extremity during the stance phase of walking and jogging to find characteristic patterns and predict

whether people with artificial joints or pathological conditions have a sufficient support moment to prevent collapse (Winter 1980, 1983a; see From the Scientific Literature). Others have used the net forces and moments in musculoskeletal models to compute the compressive loading of the base of the spine for research on lifting and low back pain (e.g., McGill and Norman 1985). An extension of this approach is to calculate the compression and shear force at a joint. To do this, the researcher must know the insertion point of the active muscle acting

## FROM THE SCIENTIFIC LITERATURE

Winter, D.A. 1980. Overall principle of lower limb support during stance phase of gait. *Journal of Biomechanics* 13:923-7.

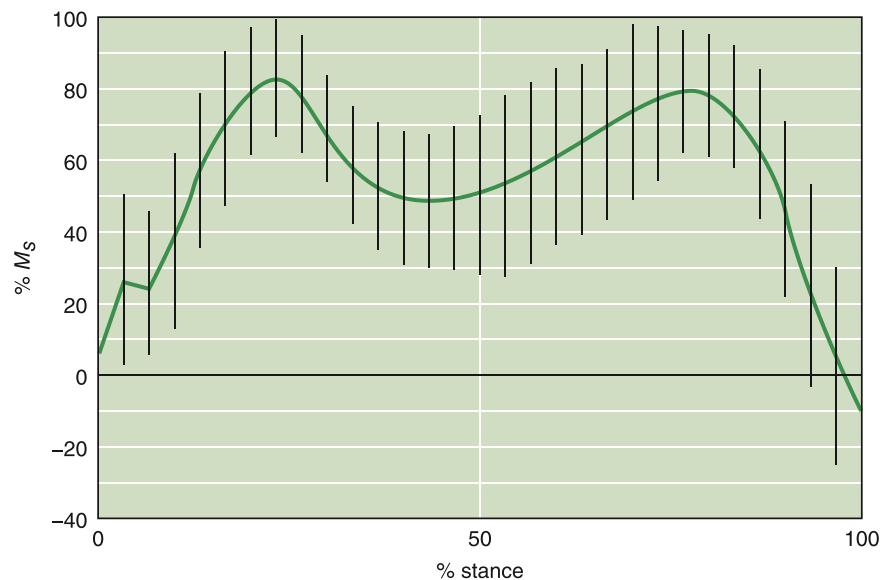
This paper presents a special way of combining the moments of force of the lower extremity during the stance phase of gait. During stance, the three moments of force—the ankle, knee, and hip—combine to support the body and prevent collapse. The author found that by adding the three moments in such a way that the extensor moments had a positive value, the resulting “support moment” followed a particular shape. The support moment ( $M_{support}$ ) was defined mathematically as

$$M_{support} = -M_{hip} + M_{knee} - M_{ankle} \quad (5.2)$$

Notice that the negative signs for the hip and ankle moments change their directions so that an extensor

moment from these joints makes a positive contribution to the support moment. A flexor moment at any joint reduces the amplitude of the support moment. Figure 5.13 shows the average support moment of normal subjects and the support moment and hip, knee, and ankle moments of a 73-year-old male with a hip replacement.

This useful tool allows a clinical researcher to monitor a patient’s progress during gait rehabilitation. As the patient becomes stronger or coordinates the three joints more effectively, the support moment gets larger. People with one or even two limb joints that cannot adequately contribute to the support moment will still be supported by that limb if the remaining joints’ moments are large enough to produce a positive support moment.

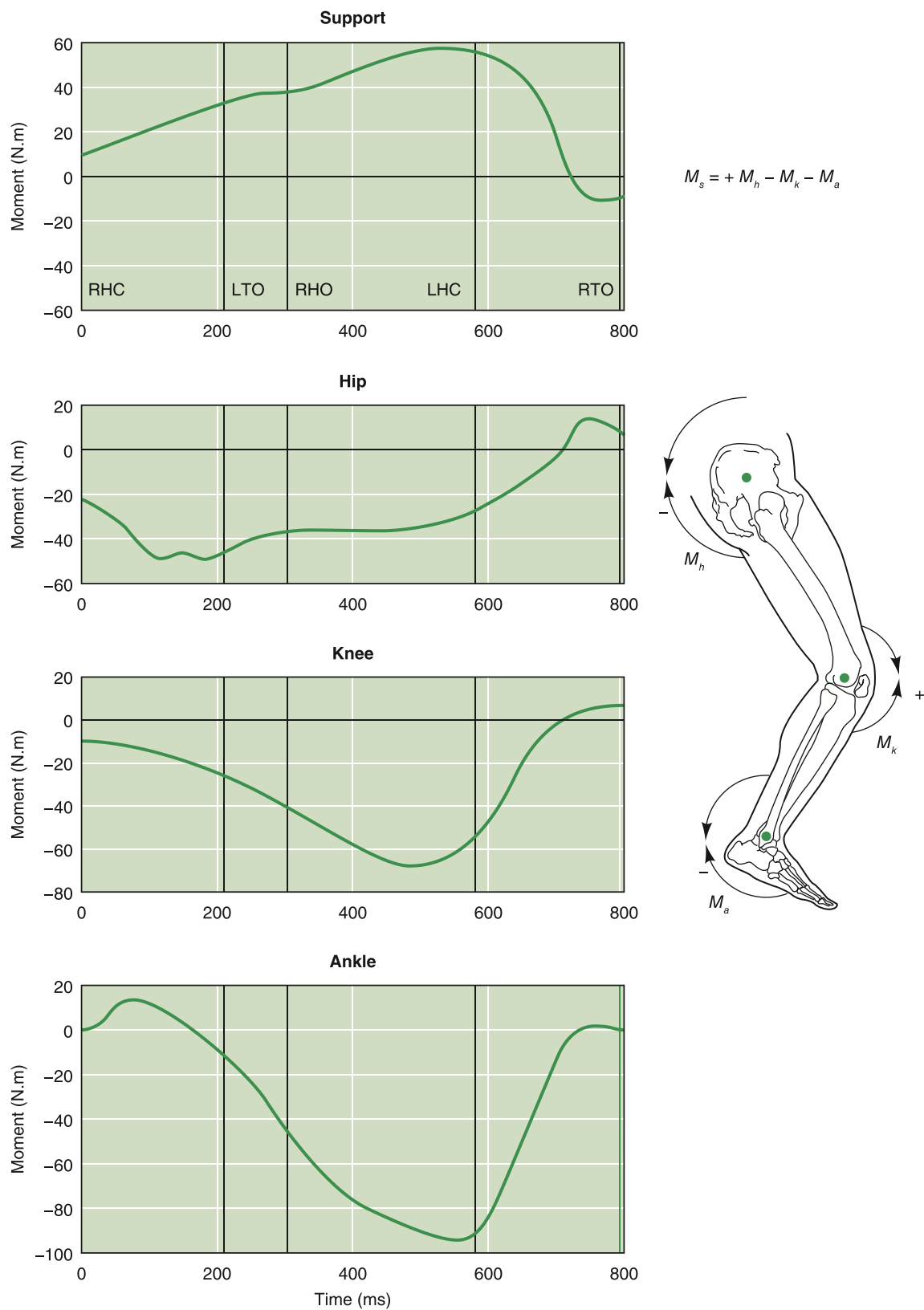


▲ **Figure 5.13a** Averaged support moment of subjects with intact joints.

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(continued)

(continued)



▲ **Figure 5.13b** The support moment and hip, knee, and ankle moments of a 73-year-old male with a hip replacement during the stance phase of walking.

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across a joint and assume that there are no other active muscles.

Computing the force in a muscle requires several assumptions to prevent indeterminacy (i.e., too few equations and too many unknowns). For example, if a single muscle can be assumed to act across a joint and no other structure contributes to the net moment of force, then, if the muscle's insertion point and line of action are known (from radiographs or estimation), the muscle force ( $F_{muscle}$ ) is defined:

$$F_{muscle} = M/(r \sin \theta) \quad (5.3)$$

where  $M$  is the net moment of force at the joint,  $r$  is the distance from the joint center to the insertion point of the muscle, and  $\theta$  is the angle of the muscle's line of action and the position vector between the joint center and the muscle's insertion point. Of course, such a situation rarely occurs because most joints have multiple synergistic muscles with different insertions and lines of action as well as antagonistic muscles that often act in cocontraction. By monitoring the electrical activity (with an electromyograph) of both the agonists and antagonists, one can reduce these problems, but even an inactive muscle can create forces, especially if it is stretched beyond its resting length. The contributions

of other tissues to the net moment of force can be minimized also as long as the motion being analyzed does not include the ends of the range of motion, when these structures become significant.

Once the researcher has estimated the force in the muscle, the muscle stress can be computed by determining the cross-sectional area. The cross-sectional areas of particular muscles can be derived from published reports or measured directly from MRI scans or radiographs. Axial stress ( $\sigma$ ) is defined as axial force divided by cross-sectional area. For pennate types of muscles in which the force is assumed to act along the line of the muscle, the stress is defined as  $\sigma = F_{muscle}/A$ , where  $F_{muscle}$  is the muscle force in newtons and  $A$  is the cross-sectional area in square meters. The units of stress are called pascals (Pa), but because of the large magnitudes, kilopascals (kPa) are usually used. Of course, true stress on the muscle cannot be quantified because of the difficulty of directly measuring the actual muscle force.

The following sections present and discuss the patterns of planar lower-extremity joint moments during walking and running. These movements and others such as stair ascent and descent often can be analyzed two-dimensionally because their principal motions occur primarily in the sagittal plane. The convention

## FROM THE SCIENTIFIC LITERATURE

McGill, S., and R.W. Norman. 1985. Dynamically and statically determined low-back moments during lifting. *Journal of Biomechanics* 18:877-85.

This study presents a method for computing the compressive load at the L4-L5 joint from data collected on the motion of the upper body during a manual lifting task. Three different methods for computing the net moments of force at L4-L5 were compared. A conventional dynamic analysis was performed using planar inverse dynamics to compute the net forces and moments at the shoulder and neck, from which the net forces and moments were calculated for the lumbar end of the trunk (L4-L5). Second, a static analysis was done by zeroing the accelerations of the segments. A third, quasi-dynamic approach assumed a static model but used dynamic information about the load. Once the lumbar net forces and moments were calculated, it was assumed that a "single equivalent muscle" was responsible for producing the moments of force and that the effective moment arm of this muscle was 5 cm. The magnitude of the compressive force ( $F_{compress}$ ) on the L4-L5 joint was then computed by measuring the angle of the lumbar spine (actually, the trunk),  $\theta$ . The equation used was

$$F_{compress} = \frac{M}{r} + F_y \cos \theta + F_x \sin \theta \quad (5.4)$$

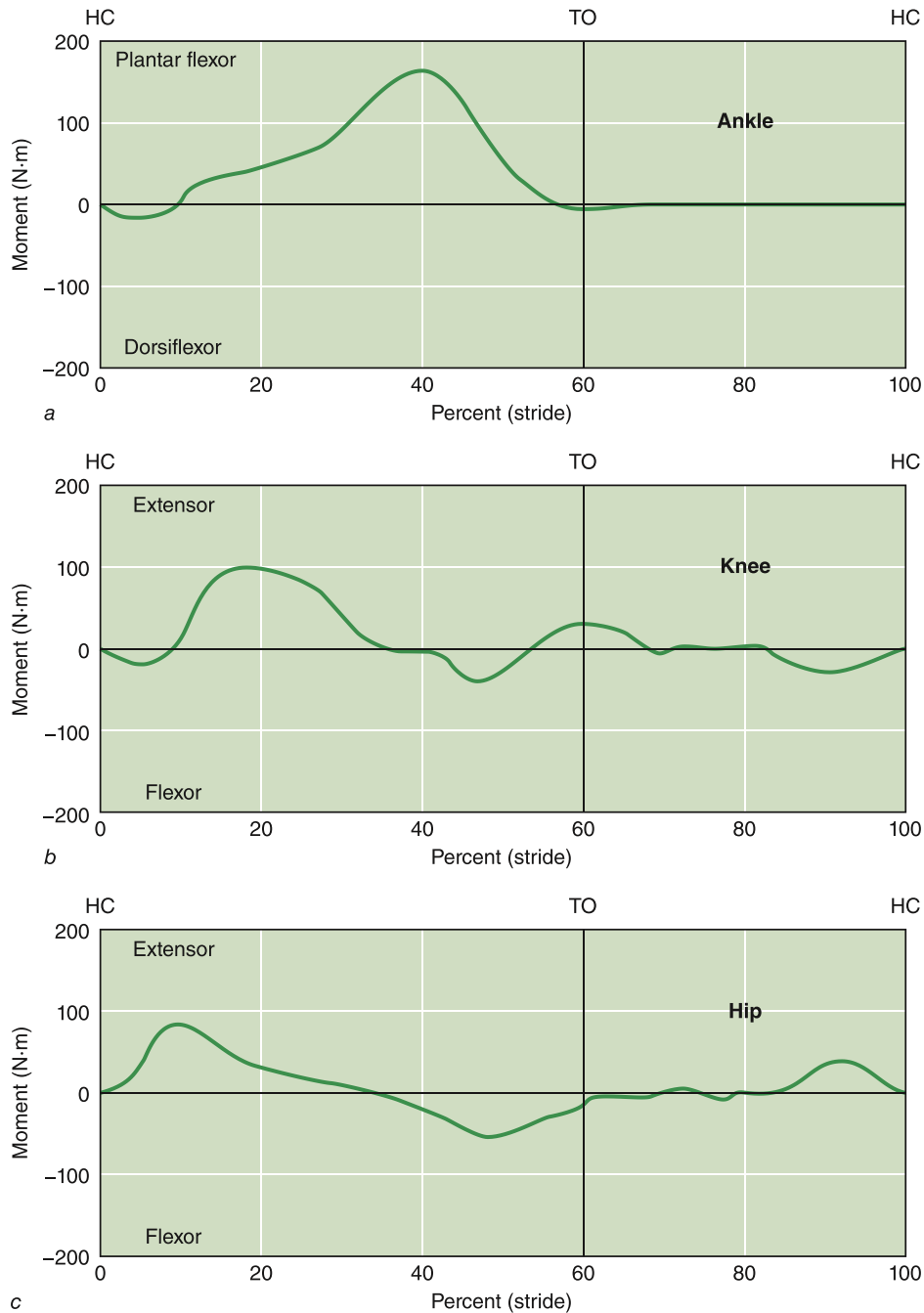
where  $M$  is the net moment of force at the L4-L5 joint,  $r$  is the moment arm of the single equivalent extensor muscle across L4-L5 (5 cm),  $(F_x, F_y)$  is the net force at the L4/L5 joint, and  $\theta$  is the angle of the trunk (the line between L4-L5 and C7-T1).

The researchers showed that there were "statistically significant and appreciable difference(s)" among the results of the three methods for the pattern and peak values of the net moment of force at L4-L5. In general, the dynamic method yielded larger peak moments than the static approach but smaller values than the quasi-dynamic method. Comparisons of a single subject's lumbar moment histories and the averaged histories of all subjects showed that each approach produced very different patterns of activity. These comparisons also showed that although the static approach produced compressive loads that were less than the 1981 National Institute for Occupational Health and Safety (NIOSH) lifting standard, the more accurate dynamic model produced loads greater than the "maximum permissible load." The quasi-dynamic approach produced even higher compressive loads. Clearly, then, one should apply the most accurate method to obtain realistic conclusions.

for presenting these figures is that extensor moments are presented as positive and flexor moments as negative. This is in agreement with the engineering standard that mechanical actions that lengthen a system are positive (positive strain) and actions that shorten the system are negative (negative strain). In figure 5.9, hip flexor moments and dorsiflexor moments were calculated as positive. Therefore, these two moments are usually presented as the negative of what is calculated.

## Walking

Joint moments during walking have many typical features. Sample data are presented in figure 5.14 for the ankle, knee, and hip joint moments. These data are presented as percentages of the gait cycle; the vertical line at 60% of the cycle represents toe-off, and the 0% and 100% points of the cycle reflect heel-contact. On the vertical axes, the joint moments are presented in newton meters.



▲ **Figure 5.14** Moments of force at the (a) ankle, (b) knee, and (c) hip during normal level walking. HC = heel-contact; TO = toe-off.

Sometimes these values are scaled to the percentage of body mass or percentage of body mass times leg length to assist in comparing different subjects. We keep these data in newton meters for continuity with the preceding examples. In general, joint moments do scale up and down with body size. They also change magnitude with the speed of movement.

The ankle moment depicted in figure 5.14a shows that there was a dorsiflexor moment after heel-contact that peaked at about 15 N·m. This moment prevents the foot from rotating too quickly from heel-contact to foot-flat, a condition known clinically as foot-slap. Although 15 N·m is a relatively small moment on the scale of this graph, this peak is nonetheless a very common feature of normal walking. Thereafter, we see a large plantar flexor moment that peaked at about 160 N·m near 40% of the stride duration. This reflects the effort necessary to effect push-off. As this peak diminishes, the limb becomes unloaded. As toe-off occurs, we again see a small dorsiflexor moment of about 10 N·m. This action, although small, is important because it lifts the toes out of the way of the ground. Individuals with dorsiflexor dysfunction have a problem with their toes catching the ground at this part of the gait cycle. For the rest of the swing phase, the ankle moment is near zero.

In figure 5.14b, there are four distinct peaks of the knee moment. The largest peak during stance is extensor, typically peaking around 100 N·m. During this peak, the limb is loaded and thus the extensor moment acts to prevent collapse of the limb. Note that this peak occurs slightly earlier than the peak of the ankle moment. Often we see a smaller knee flexor peak before toe-off as the leg is pulled through the remainder of the stance. During swing, the first peak is extensor; it limits the flexion of the knee that occurs because the lower extremity is being swung forward from the hip. Without this peak, the knee would reach a highly flexed position, especially at faster walking velocities. The second swing-phase peak is a flexor peak of about 30 N·m; it slows the leg before the knee reaches full extension.

Figure 5.14c shows that the hip moment tends to have an 80 N·m extensor peak during the first half of stance. At toe-off, there is a flexor moment that is needed to swing the lower extremity forward. Then, similar to the knee moment, the hip moment has an extensor peak of about 40 N·m at the end swing to slow the lower extremity before heel-contact.

The hip moment is the most variable of the three lower-extremity joint moments. The foot moment tends to be the least variable because the segment is constrained by the ground. The hip, in contrast, not only is responsible for lower-extremity control but also has to control the balance of the torso. This is a significant task, given that the torso accounts for at least two-thirds of the body weight of an average individual. Winter and

Sienko (1988) showed that the movement of the torso reflects a majority of the variability in hip moment. In this regard, the hip moment becomes somewhat hard to interpret during stance.

## Running

Lower-extremity joint moments during running are shown in figure 5.15. These moments have patterns similar to their analogues in walking. The most prominent differences are their magnitudes. Running is a more forceful activity than walking; thus, just as GRFs are larger during running, so, too, are the joint moments. Furthermore, the stance phase (HC to TO) is a smaller percentage of the running cycle than during walking. During stance, the ankle moment (figure 5.15a) has a larger plantar flexor peak of about 200 N·m. Thereafter, the swing-phase (TO to second HC) ankle moment is near zero. The ankle moment at heel-contact varies slightly depending on running style. Runners with a heel-toe footfall pattern exhibit a small dorsiflexor peak at heel-contact, much like in walking (as shown in figure 5.14). Individuals who run foot-flat or on their toes do not have this peak because there is no need to control foot-slap.

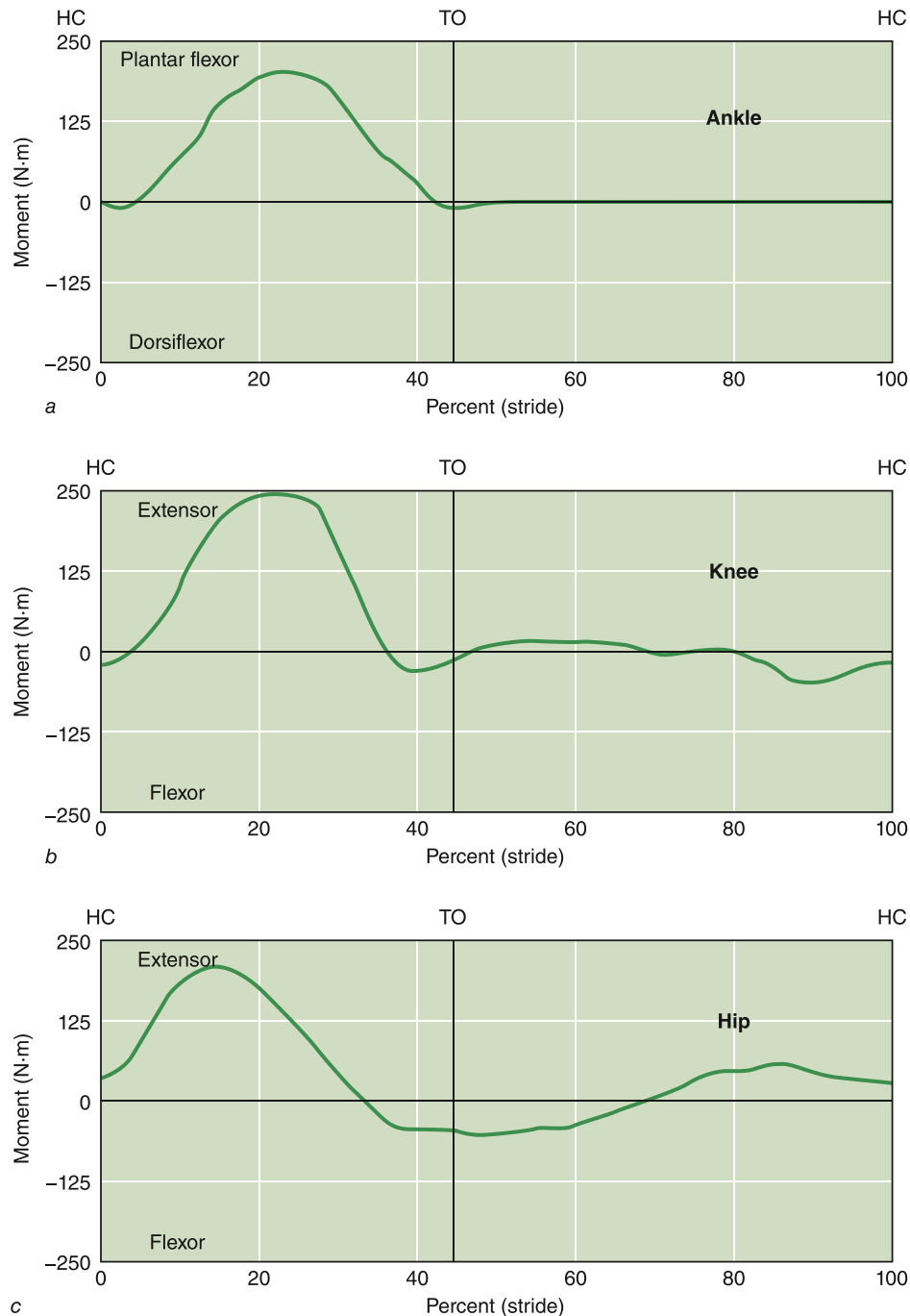
The stance-phase knee moment (Figure 5.15b), like the ankle moment, consists primarily of a single extensor peak of about 250 N·m. Prior to toe-off, there is usually a flexor moment of about 30 N·m. This action flexes the knee rapidly before the lower extremity is swung forward. During the swing, there is an extensor phase in which the leg is swung forward. Finally, there is a flexor phase before heel-contact to slow the leg.

The hip moment (figure 5.15c) is extensor for much of stance, peaking at more than 200 N·m. We then see a shift to net flexor activity around toe-off to swing the lower extremity forward. Then there is an extensor action to slow the thigh before heel-contact.

## SUMMARY

This chapter focused heavily on proper technique for inverse dynamics problems. This focus is necessary simply because the technique clearly has many steps and potential pitfalls (Hatze 2002). Students are encouraged to practice such problems until they can solve them without referring to this book. Students should be able to draw an FBD for any segment, construct the three equations of motion, and solve them.

Students are also encouraged to be mindful of the limitations of joint moments. Joint moments are only a summary representation of human effort, one step more advanced than the information obtained from a force plate. Joint moments are not an end-all statement of human kinetics; rather, they are convenient standards



▲ **Figure 5.15** Moments of force at the (a) ankle and (b) knee during normal level running. HC = heel-contact; TO = toe-off.

for evaluating the relative efforts of different joints and movements. Researchers interested in specific muscle actions must use either electromyography (chapter 8), musculoskeletal models (chapter 9), or both. It is noteworthy that the great Russian scientist Nikolai Bernstein (see Bernstein 1967) refers to moments but in fact preferred to use segment accelerations as overall

representations of segmental efforts. It is hard to argue that joint moments offer much more information. A 200 or 20 N·m joint moment is in itself fairly meaningless. It is only by comparing specific cases like the walking and running examples in figures 5.14 and 5.15 that we can develop a relative basis for the magnitudes of joint moments.

The problem with analyzing limbs segmentally is that it can promote the misconception that each joint is controlled independently. We stated that two-joint muscles are poorly treated with this method. In addition, the individual segments of an extremity interact. For example, we discussed the fact that thigh moments affect the movement of the leg. Therefore, in terms of joint moments, we know that a hip moment will affect the thigh; however, we do not establish the resulting effect

on the leg. Many studies have noted the importance of intersegmental coordination (see, for example, Putnam 1991); in fact, Bernstein cited the use of segment interactions as the final step in learning to coordinate a movement. Clinicians are also beginning to recognize such effects in various populations, particularly in people with amputation. Systemic analyses such as the Lagrangian approach, outlined in chapter 10, may be preferable for these situations.

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## SUGGESTED READINGS

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