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FLOW IN CIRCULAR PIPES

1. INTRODUCTION

In the process industries it is often necessary to pump fluids over long distances from storage to reactor units, and there may be a substantial drop in pressure in both the pipeline and in the individual units themselves. Many intermediate products are pumped from one factory site to another, and raw materials such as natural gas and petroleum products may be pumped very long distances to domestic or industrial consumers. It is necessary, therefore, to consider the problems concerned with calculating the power requirements for pumping, with designing the most suitable flow system, with estimating the most economical sizes of pipes, with measuring the rate of flow, and frequently with controlling this flow at a steady rate [1].

Chemical engineering design frequently concerns equipment for the transfer of material or heat from one phase to another, and, in order to understand the mechanism of the transport process, the flow pattern of the fluid, and particularly the distribution of velocity near a surface, must be studied.

When a fluid is flowing through a tube or over a surface, the pattern of the flow will vary with the velocity, the physical properties of the fluid, and the geometry of the surface. This problem was first examined by Reynolds in 1883.

When a fluid flows over a surface the elements in contact with the surface will be brought to rest and the adjacent layers retarded by the viscous drag of the fluid. Thus the velocity in the neighbourhood of the surface will change in a direction at right angles to the stream flow. It is important to realise that this change in velocity originates at the walls or surface.

When a fluid with uniform flow over the cross-section enters a pipe, the layers of fluid adjacent to the walls are slowed down as on a plane surface and a boundary layer forms at the entrance. This builds up in thickness as the fluid passes into the pipe. At some distance downstream from the mouth, the boundary layers reach a thickness equal to the pipe radius and join to the axis, after which conditions remain constant and *fully developed flow* exists.

The proportionality factor is known as the *friction factor*. It is evident that the magnitude and nature of the friction factor are directly related to the definitions of the characteristic area and the characteristic kinetic energy. For the steady state and fully developed flow of fluids in circular pipes of uniform cross section, the friction factor is a function of pipe's diameter and the density, viscosity and average velocity of the fluid.

2. THEORY

2.1. Classification of Flow Types

The flow can be considered to be incompressible if (i) the substance flowing is a liquid or (ii) if it is a gas whose density changes within the system by no more than 10 percent. In this event, if the inlet density is employed, the resulting error in computed pressure drop will generally not exceed the uncertainty limits in the friction factor.

Steady flow means steady with respect to time. Thus the flow properties at every point remain constant with respect to time.

Pressure flow implies that flow occurs under pressure gradient. Gases always flow in this manner. When a liquid flows with a free surface, the flow is referred to as gravity flow because gravity is the primary moving force.

At low velocities fluids tends to flow without lateral mixing, and adjacent layers slide past one another like playing cards. There are neither cross currents nor eddies. This regime is called as laminar flow. At higher velocities turbulence appears, and eddies form, which is named turbulent flow.

2.2. Bernoulli Equation

A more useful type of energy balance for flowing fluids, especially liquids, is a modification of the total energy balance to deal with mechanical energy. Engineers are often concerned with this special type of energy, called *mechanical energy*, which includes the work term, kinetic energy, potential energy and the flow work part of the enthalpy term. Mechanical energy is a form of energy that is either work or a form that can be directly converted into work. The heat and the internal energy terms in the energy balance equation do not permit simple conversion into work because of the second law of thermodynamics and the efficiency of conversion, which depends on the temperatures. Mechanical energy terms have no such limitation and can be converted almost completely into work. Energy converted to heat or internal energy is lost work or a loss in mechanical energy which is caused by frictional resistance to flow [2].

An important relation, called the *Bernoulli equation,* is a special form of a mechanical energy balance for turbulent flow without friction and without added mechanical energy, as shown by the fact that all the terms in Eq. [1] are scalar.

For an incompressible fluid flowing through a pipe ***Bernoulli equation*** with friction loss and ***continuity equation*** apply:

$$z_1 + \frac{P_1}{\rho g} + \frac{V_1^2}{2g} = z_2 + \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_f \qquad (1)$$

where z is height above datum plane, P is pressure (Pa), ρ is fluid density (kg/m^3), g is gravitational acceleration (m/s^2), V is fluid velocity (m/s) and the h_f is the friction loss (m).

Bernoulli equation covers many situations of practical importance and is often used in conjunction with the mass-balance equation for steady state, where m^0 is mass flow rate and A is the area:

$$m^o = \rho_1 A_1 V_1 = \rho_2 A_2 V_2 \qquad (2)$$

2.3. Friction Losses in Bernoulli Equation

The friction loss in a pipe circuit falls into two categories: a) that due to viscous resistance extending throughout the total length of the circuit, b) that due to localized effects such as valves, sudden changes in area of flow and bends. The overall friction loss is a combination of both these categories.

2.3.a. Friction Loss in Straight Pipes

The friction loss along a length L of straight pipe (m), of constant diameter *D* (m) is given by the expression:

$$h_{fp} = \frac{4fLV^2}{2gD} \qquad (3)$$

where f is a dimensionless constant which is a function of the Reynolds number of the flow and the roughness of the internal surface of the pipe.

Stanton-Pannell and Moody measured the pressure drop due to friction for a number of fluids flowing in pipes of various diameters and surface roughness. They expressed their results by using the concept of a friction factor, defined as the dimensionless group R/gV^2 which is plotted as a function of the Reynolds number. Friction factor f is independent of surface roughness when the Reynolds number is smaller than 2100. At high values of Reynolds number which is greater than 2100, f becomes a function of surface roughness. For design purposes, the frictional characteristics of various types of pipes are summarised by the friction factor chart, which is a log-log plot of friction factor and Reynolds number [3].

Friction factor for various types of pipes can be calculated by *Colebrook equation* in turbulent flow when $N_{Re}>3000$, where ε is the dimensionless toughness parameter:

$$\frac{1}{\sqrt{f}} = -4\log\left[\frac{\varepsilon}{3.7D} + \frac{1.256}{N_{Re}\sqrt{f}}\right] \tag{4}$$

For smooth pipes, ε/ D = 0, Eq.[4] reduces to *Prandtl 's equation:*

$$\frac{1}{\sqrt{f}} = -4\log\left[\frac{1.256}{N_{Re}\sqrt{f}}\right] \tag{5}$$

whereas for very rough pipes Eq.[4] reduces to *von Karman 's equation:*

$$\frac{1}{\sqrt{f}} = -4\log\left[\frac{\varepsilon}{3.7D}\right] \tag{6}$$

When the flow rate is specified, the friction factor can be conveniently computed from the equation:

$$\frac{1}{\sqrt{f}} = 3.6\log\left[\frac{N_{Re}}{7}\right] \tag{7}$$

or for $2500<N_{Re}<10^5$ by the *Blasius equation:*

$$f = \frac{0.0785}{N_{Re}^{0.25}} \tag{8}$$

2.3.b. Friction Losses in Expansion, Contraction and Pipe Fittings

Sudden Expansion

If the cross section of the conduit is suddenly enlarged, the fluid stream separates from the wall and issues as a jet into the enlarged section. The jet then expands to till the entire cross section of the larger conduit. The space between the expanding jet and the conduit wall is filled with fluid in vortex motion characteristic of boundary-layer separation, and considerable friction is generated within this space. This effect is shown in Fig. 1.

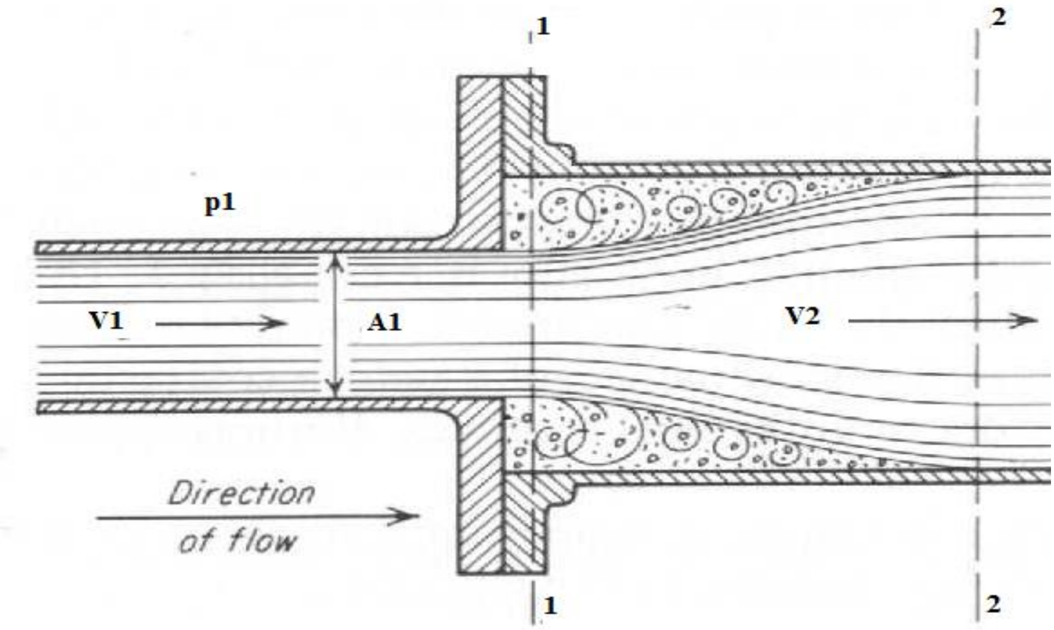

Figure 1. Sudden Expansion

The friction loss, h_{fe}, from a sudden expansion of cross section is proportional to the velocity head of the fluid in the small conduit and can be written:

$$h_{fe} = \frac{(V_1 - V_2)^2}{2g} \tag{9}$$

Where V_1 and V_2 are linear velocities (m/s) at point 1 and 2 respectively.

Sudden Contraction

When the cross section of conduit is suddenly reduced, the fluid stream cannot follow around the sharp corner and the stream breaks contact with the wall of the conduit. A jet is formed, which flows into the stagnant fluid in the smaller section. The jet first contracts and then expands to fill the smaller cross section, and downstream from the point of contraction the normal velocity distribution eventually is reestablished. The cross section of minimum area at which the jet changes from a contraction to an expansion is called the *vena contracta*.

The flow pattern of a sudden contraction is shown in Fig. 2. C-C plane is drawn at the vena contracta. Eddies appear as shown in the figure.

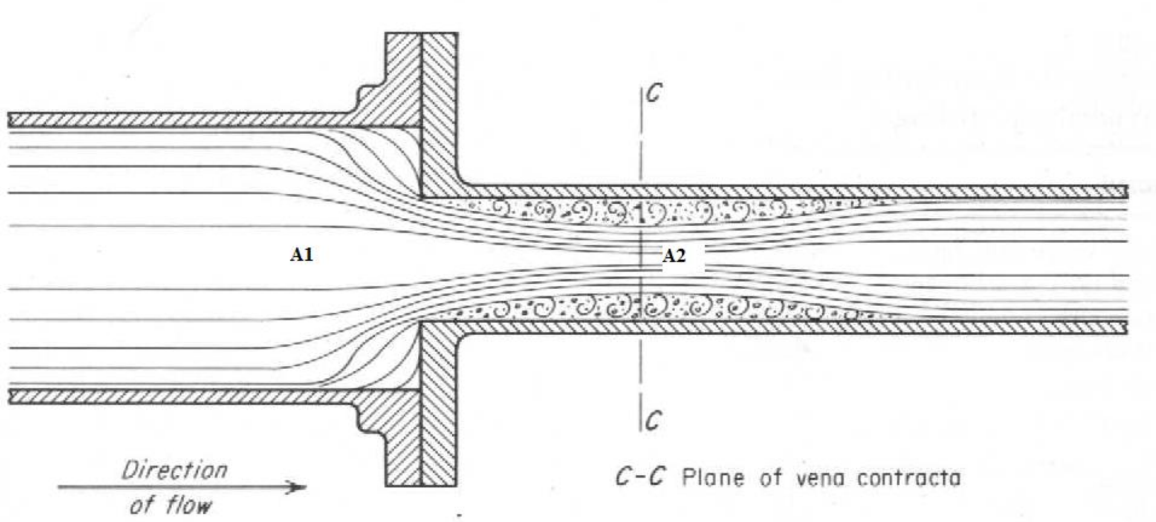

Figure 2. Sudden Contraction

The friction loss, h_{fc}, from sudden contraction is proportional to the velocity head in the smaller conduit and can be calculated by the equation,

$$h_{fc} = K_c \frac{V_2^2}{2g} \tag{10}$$

where the proportionality factor *Kc* is called the *contraction-loss coefficient* and V_2 is the average velocity in the smaller, or downstream, section. Experimentally, for laminar flow, $K_c < 0.1$, and the contraction loss h_{fc} negligible.

Loss coefficients for sudden contractions are presented in Table 1.

Table 1. Loss Coefficient for Sudden Contractions

A_2/A_1	0	0.1	0.2	0.3	0.4	06	0.8	1.0
K_c	0.50	0.46	0.41	0.36	0.30	0.18	0.06	0

Also for turbulent flow, *Kc* is given by the empirical equation:

$$K_c = 0.4\left[1 - \frac{A_2}{A_1}\right] \tag{11}$$

where A_1 and A_2 are the cross-sectional areas of the upstream and downstream conduits(m^2), respectively.

Bends

The friction loss, h_{fb}, due to a bend is given by the expression,

$$h_{fb} = \frac{K_b V^2}{2g} \tag{12}$$

where K_b is a dimensionless coefficient which depends upon the bend radius/pipe radius ratio and the angle of the bend.

Valves

The two most common types of valves, gate valves and globe valves are illustrated in Fig.3. In a gate valve the diameter of the opening through which the fluid passes is nearly the same as that of the pipe, and the direction of flow does not change. As a result, a wide-open gate valve introduces only a small pressure drop. The disk is tapered and fits into a tapered seat; when the valve is opened, the disk rises into the bonnet, completely out of the path of the fluid. Gate valves are not recommended for controlling flow and are usually left fully open or closed.

Globe valves are widely used for controlling flow. The opening increases almost linearly with stem position, and wear is evenly distributed around the disk. The fluid passes through the valve illustrated in Fig. 3b. The pressure drop in this kind of valve is large [4].

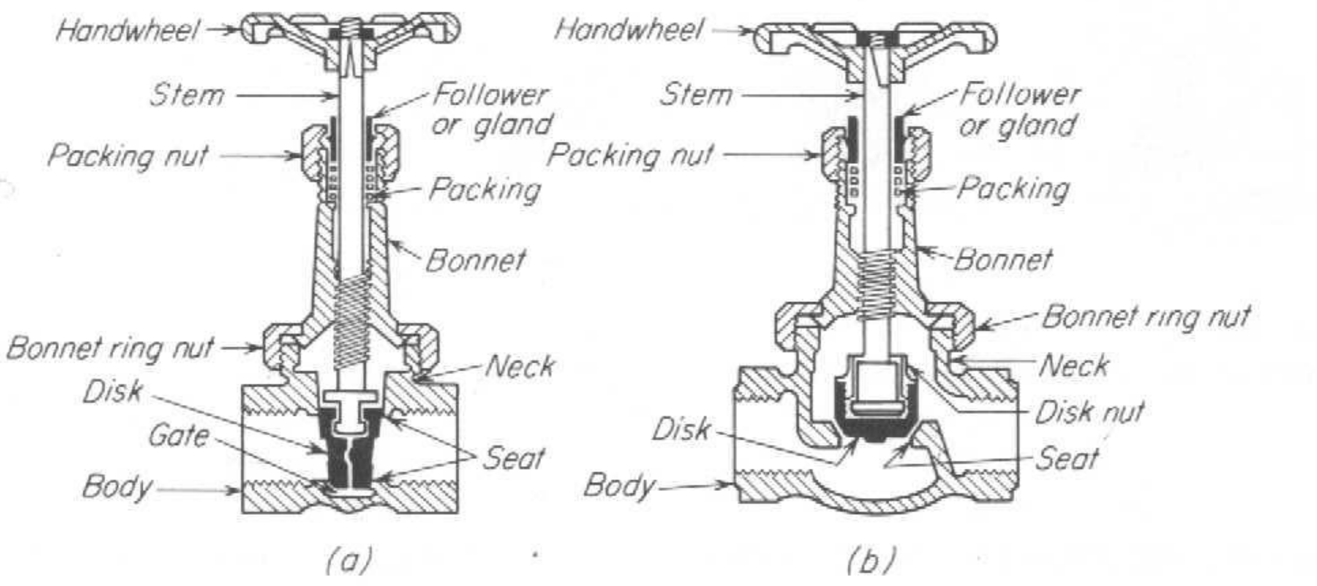

Figure 3. Valves (a) gate valve, (b) globe valve.

The friction loss, h_{fv}, due to a valve is given by the expression,

$$h_{fv} = \frac{K_v V^2}{2g} \tag{13}$$

where the value of K_v depends upon the type of valve and the degrees of opening.

2.4. Principles of Pressure Loss Measurements

Manometers and piezometers are used to measure the pressure differences. The main difference between these two apparatus is; piezometer shows the local pressure at a point, whereas the manometer measures the pressure difference between two points. Also U-tube manometers containing mercury is used to measure big pressure losses[5].

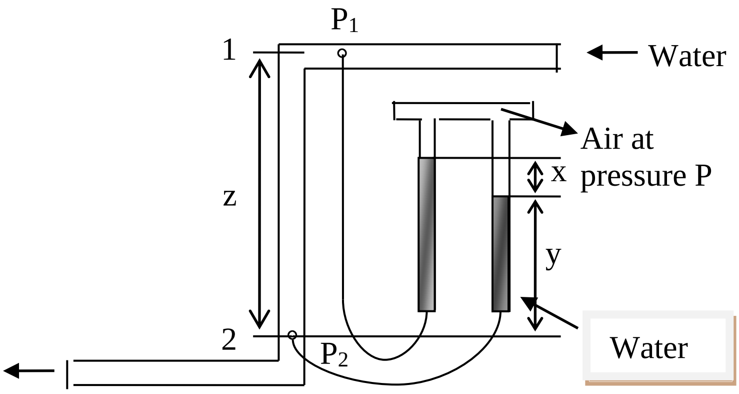

Figure 4. Pressurised Piezometer Tubes

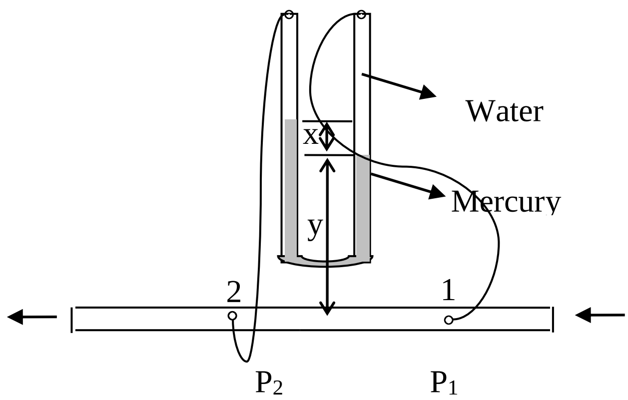

Figure 5. U-Tube Manometer Containing Mercury

Applying Bernoulli's equation between 1 and 2 for the flow system in Fig. 4,

$$z_1 + \frac{P_1}{\rho_{H_2O} g} + \frac{V_1^2}{2g} = z_2 + \frac{P_2}{\rho_{H_2O} g} + \frac{V_2^2}{2g} + h_f \tag{14}$$

where z_1=z, z_2=0 and $V_1 = V_2$

$$h_f = z + \frac{(P_1 - P_2)}{\rho_{H_2O} g} \tag{15}$$

For piezometer tubes in Fig.4;

$$P = P_1 + \rho_{H_2O} g\left[z - (x + y)\right] \tag{16}$$

where z is height above reference point, x and y are the height differences of the liquid in the tubes according to reference point.

$$P = P_2 - \rho_{H_2O} g y \tag{17}$$

$$x = z + \frac{(P_1 - P_2)}{\rho_{H_2O} g} \tag{18}$$

comparing equations [15] and [18] gives h_f=x. Considering Fig. 5, since 1 and 2 have the same elevation and pipe diameter:

$$\frac{P_1 - P_2}{\rho_{H_2O} g} = \Delta h \tag{19}$$

When U-tube is considered, pressures in both limbs of U-tube are equal at 0-0 plane. Therefore equating pressures at 0-0 plane,

$$P_2 + \rho_{Hg} g x = P_1 + \rho_{H_2O} g x \tag{20}$$

$$P_1 - P_2 = xg(\rho_{Hg} - \rho_{H_2O}) \tag{21}$$

$$\frac{P_1 - P_2}{\rho_{H_2O} g} = x(s - 1) \tag{22}$$

with taking the specific gravity (which is s) of mercury as 13.6 and Δh= 12.6x

2.5. Partially Rough Entry Conditions for Turbulent Flow

For partially rough entry conditions for turbulent flow Equatin 23 is defined.

$$\frac{L_e}{D_p} = 0.0288N_{Re} \tag{23}$$

where L_e is entrance length, D *is* the inside diameter of the pipe and N_{Re} is the Reynolds number with respect to pipe diameter, and based on the mean velocity of flow in pipe. This expression is only approximate, and is inaccurate for Reynolds numbers in the region of 2500 because the boundary layer thickness increases very rapidly in this region. At average value of L_e is between 50 - 100 at the Reynolds number of 2500.

2.6. Modelling of Efflux Time for a Cylindrical Tank with an Exit Pipe

A cylindrical tank with R_t Radius and H heightthat is shown in Fig. 6 is to be drained by means of pipe with Dp diameter and L length which is vertically attached to the bottom of the tank. The liquid in the tank is of constant density and viscosity (i.e. an incompressible, Newtonian liquid at isothermal conditions). A quasi steady-state analysis together with the neglect of the entrance loss at the inlet of the pipe and the kinetic energy of the emerging stream relates the efflux time to the dimensions of the system and the properties of the liquid in a simple manner [6].

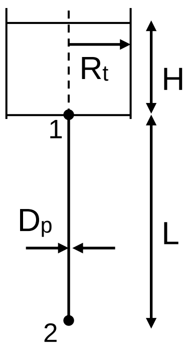

Figure 6. Schematic Description of a Cylindrical Tank with an Exit Pipe

Applying mechanical energy balance between the points 1 and 2 gives:

$$\frac{(P_2 - P_1)}{\rho g} + \frac{(V_2^2 - V_1^2)}{2g} + (z_2 - z_1) + \frac{4fLV^2}{2gD_p} = 0 \tag{24}$$

where $V_1=V_2$, $z_1=L$, $z_2=0$ and $P_1=P_{atm} + \rho gH$. $P_2=P_{atm}$

$$(H+L)g=\frac{4fLV^2}{2D_p} \tag{25}$$

$$V^2=\frac{(H+L)gD_p}{2fL} \tag{26}$$

When laminar flow occurs,

$$f=\frac{16}{N_{Re}} \tag{27}$$

$$V=\frac{(H+L)D_p^2\rho g}{32\mu L} \tag{28}$$

For turbulent flow through smooth tubes the Blassius formula is applicable,

$$f=\frac{0.0785}{N_{Re}^{0.25}} \tag{29}$$

and Eq.[26] becomes,

$$V=\frac{(H+L)^{\frac{4}{7}}D_p^{\frac{5}{7}}\rho^{\frac{1}{7}}g^{\frac{4}{7}}}{0.157^{\frac{4}{7}}\mu^{\frac{1}{7}}L^{\frac{4}{7}}} \tag{30}$$

Rearrangement of the macroscopic mass balance in terms of the geometry of the system under consideration gives,

$$\frac{dH}{dt}=-V\left[\frac{D_p}{D_t}\right]^2 \tag{31}$$

Substitution of Eq.[28] and Eq.[30] into Eq.[31] and subsequent integration gives,

$$t_{efflux}=\frac{32\mu LD_t^2}{\rho gD_p^4}\ln\left[\frac{L+H_1}{L+H_2}\right] \tag{32}$$

where H_1 is the initial liquid depth in the tank and H_2 is the final liquid depth in the lank for laminar flow through the exit pipe and:

$$t_{efflux} = \frac{7D_t^2}{3D_p^2}\frac{0.157^{\frac{4}{7}} L^{\frac{4}{7}} \mu^{\frac{1}{7}}}{D_p^{\frac{5}{7}} g^{\frac{4}{7}} \rho^{\frac{1}{7}}}\left[(L+H_1)^{\frac{3}{7}} - (L+H_2)^{\frac{3}{7}}\right] \qquad (33)$$

for turbulent flow through the exit pipe.

3. EXPERIMENTAL

3.1. Experimental Setup

One of the most common problem in fluid mechanics is the estimation of pressure loss. This apparatus enables pressure loss measurements to be made on several small bore pipe circuit components. In the first part of the experiment, the apparatus shown in Fig.7 will be used.

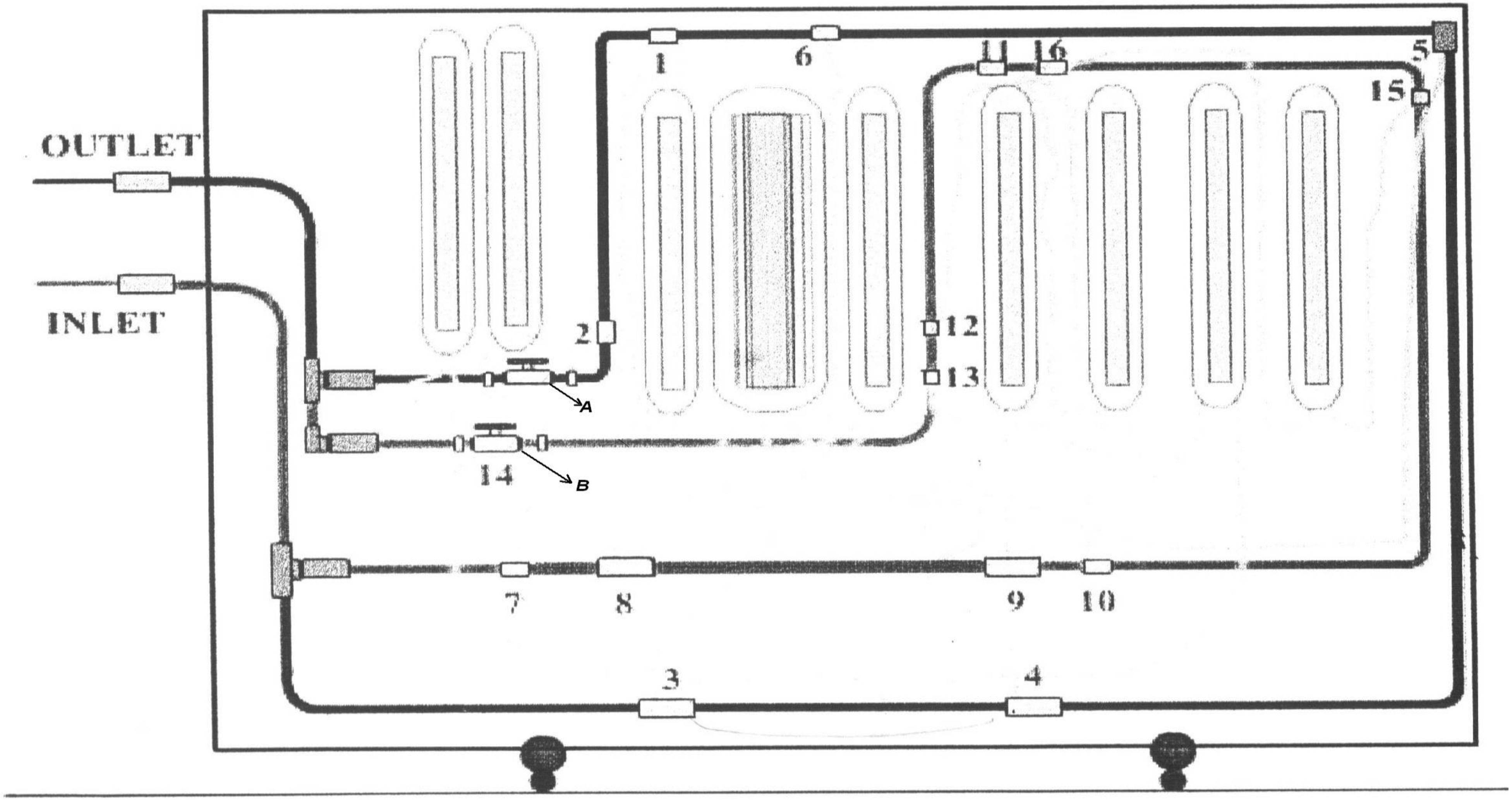

Figure 7. Diagrammatic Arrangement of Apparatus

The apparatus shown diagrammatically in Fig.7, consists of two separate hydraulic circuits, one painted dark blue, one painted light blue, each one containing a number of pipe system components. Both circuits are supplied with water from the hydraulic bench. The components in each of the circuits are as follows:

Dark Blue Circuit

1-2 Standard Elbow Bend

3-4 Straight Pipe (13.6mm)

5-6 90° Mitre Bend

A Gate Valve

Light Blue Circuit

7-8 Sudden Expansion 13.7mm / 26.4mm

9-10 Sudden Contraction 26.4mm/ 13.7mm

11-12 101.6mm 90° Radius Bend

13-14 152.4mm 90° Radius Bend

15-16 50.8mm 90° Radius Bend

B Globe Valve

In all cases (except the gate and globe valves) a pair of pressurised piezometer tubes measures the pressure change across each of the components. In the case of the valves, pressure measurement is made by U-tubes containing mercury.

In the second part, the apparatus consists of a cylindrical tank and interchangeable exit pipes of different lengths and diameters is used. A typical apparatus is shown in Fig.7.

3.2. Experimental Procedure

In the first part of the experiment two separate hydraulic circuits, one painted dark blue and one painted light blue, will be used. Each of the hydraulic circuits contains several piping system components which are mentioned in the "Description of Apparatus" section.

Open fully the water control valve on the hydraulic bench. With globe valve closed, open the gate valve fully to obtain maximum flow through the Dark Blue circuit. Record the readings on the piezometer tubes and the U-tube. Collect a sufficient quantity of water in the weighing tank. Repeat the above procedure for a total of six different flow rates, obtained by closing gate valve.

After taking data at six different flow rates, switch off the pump, close the gate valve and open the globe valve. Repeat the experimental procedure for the Light Blue circuit. Before switching off the pump, close both the globe valve and the gate valve. This procedure prevents air gaining access to the system and so saves time in subsequent setting up.

In the second part of the experiment, fill the cylindrical tank with water. Measure and record the efflux time for each pipe by stopwatch.

4. CALCULATIONS

4.1. First part of the experiment

a. Calculate the pressure loss in piezometer tubes,

b. Calculate pressure loss in U-tubes,

c. Calculate the friction factor in straight pipes using friction loss and also using Blasius's equation,

d. Calculate the loss coefficient values for five bends,

- Draw r/D vs. K_b for five bends,

e. Calculate the loss coefficient values for both the gate valve and the globe valve,

- Draw flow rate % vs. K_v, for the gate valve and the globe valve,

4.2. Second part of the experiment

a. Calculate the initial and final N_{Re} numbers,

b. Calculate the kinetic energy of the fluid,

c. Calculate the potential energy at $h=H_1$, and at $h=H_2$,

d. Calculate the viscous energy losses in the pipe,

e. Calculate the efflux time for each pipe,

- Draw the experimental and calculated efflux time vs. the ratio of diameters,
- Draw the experimental and calculated efflux time vs. pipe length.

6. SYMBOLS

A Area, m^2

D_p Pipe diameter, m

D_t Tank diameter, m

f Friction factor, dimensionless

g Gravitational acceleration, m/s^2

h_f Friction loss, N-m/g

h_{fb} Friction loss in bends, N-m/g

h_{fc} Friction loss in sudden contraction, N-m/g

h_{fe} Friction loss in sudden expansion, N-m/g

h_{fp} Friction loss in straight pipe, N-m/g

h_{fv} Friction loss in valves, N-m/g

H Height of fluid in cylindrical tank, m

K Loss coefficient, dimensionless

L Length of pipe, m

L_e Entrance length, m

$\dot{m}$ Mass flow rate, kg/s

N_{Re} Reynolds number, dimensionless

P Pressure, N/m^2

R_t Radius of the cylindrical tank, m

s Specific gravity, dimensionless

t Time, s

t_{efflux} Efflux time for a cylindrical tank. s

V Fluid velocity, m/s

x Height difference in piezometer tubes and U-manometers, m

y Height difference in piezometer tubes and U-manometers, m

z Height above datum plane, m

Greek Letters

Δh Height difference, m

ε Roughness parameter, dimentionless

μ Absolute viscosity, P

ρ Density, kg/m^3

7. REFERENCES

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DATA SHEET

1. Experimental Results for Dark Blue Circuit

Test Number	Time to Collect 5 L of Water (s)	Piezometer Tube Readings (cm H_2O)						Gate Valve Readings (cm Hg)
		Standard Elbow		Straight Pipe (D=13.7mm)		90^o Mitre		
		1	2	3	4	5	6	
1								
2								
3								
4								
5								
6								

2. Experimental Results for Light Blue Circuit

Test Number	Time to Collect 5 L of Water (s)	Piezometer Tube Readings (cm H_2O)										Globe Valve Readings (cm Hg)
		Expansion		Contraction		Bend (r=101.6mm)		Bend (r=152.4mm)		Bend (r=50.8mm)		
		7	8	9	10	11	12	13	14	15	16	
1												
2												
3												
4												
5												
6												

3. Experimental Results for Cylindrical Tank

D_p= 0.28 cm	$t_{initial}$ (s)	t_{efflux} (s)	L = 61.4 cm	$t_{initial}$ (s)	t_{efflux} (s)
L = 8 cm			D_p = 0.17 cm		
L = 14.8 cm			D_p = 0.22 cm		
L = 30.1 cm			D_p = 0.355 cm		

BASIC DATA

Pipe diameter (internal) (mm)	13.7
Pipe diameter (between sudden expansion and contraction) (mm)	26.4
Pipe material	Copper
Distance between pressure tappings for straight pipe (m)	0.914

Piezometer Tube	z (m)
[1-2]	0.85
[11-12]	0.80

Bend Radii	90º elbow (mitre)	0
	90º proprietory elbow	12.7 mm
	90º smooth bend	50.8 mm
	90º smooth bend	101.6 mm
	90º smooth bend	152.4 mm

Cylindrical tank + pipe volume = $\pi/4.D_t^2.H + \pi/4.D_p^2.L$	
Cylindrical diameter (cm)	15
Volume of water (L)	2
Density of water (kg/m^3)	1000
Viscosity of water (cP)	0.95
Gravity accelaration (m/s^2)	9.81
Roughness parameter for copper pipe	0.00085