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PROCESS CONTROL

1. INTRODUCTION

This experiment is an illustrative example of a closed control system. The purpose of the experiment is to introduce some of the basic principles involved in process control by using "Computer Controlled System". In the experiment, by using various control mechanisms (on-off proportional, derivative, integral and their combinations) control of temperature, flow rate or pressure are observed and discussed.

2. THEORY

2.1. Model Control System

The model control system consists of a hydraulic circuit, with a bottom tank and a superior process tank. By use of appropriate sensors; flow rate, temperature, pressure and pH can be measured.

The transfer function of any system is given as;

$$G(s) = \frac{A}{(\tau_1 s + 1)(\tau_2 s + 1)(\tau_3 s + 1)}$$

Eq.1[1]

Where τ_1, τ_2 and τ_3 are the time constants for the system of interest such as the heater, metal block and thermometer, respectively for the case of heating a metal block by a heater where its temperature is measured continuously.

The conditioning unit (the interface) in the experiment system incorporates the connection between the measuring element and the controller. The required process property (temperature, pressure or flow rate) also called the set-point is adjusted by the computer. The set point is specified as an input to the software. The interface works as a transmission unit where corresponding signals are transferred to the process unit. By choosing the option on the software; on-off, proportional, PD, PI and PID control mechanisms can be examined.

For a better understanding of the experimental apparatus, it will be convenient to present the control system by means of a block diagram shown in Fig.1.

1. Process
2. Measuring element
3. Controller
4. Final control element

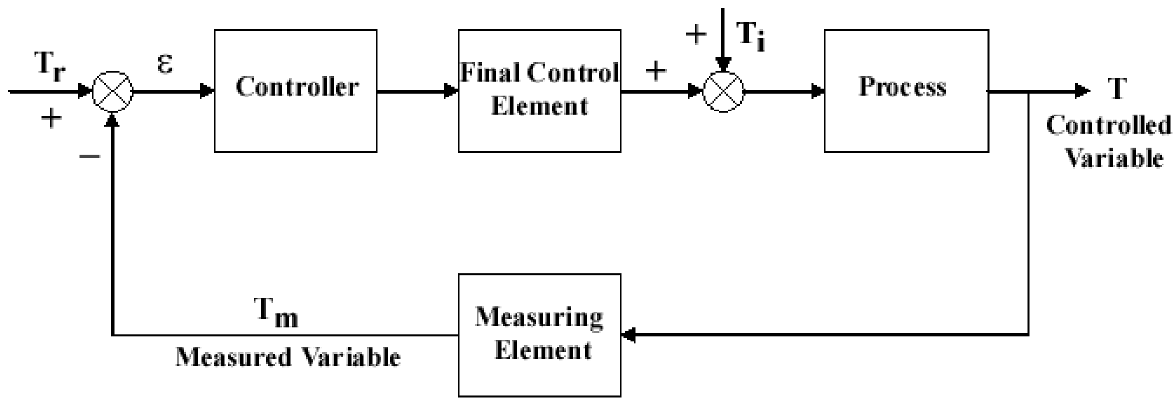


Figure 1. Block diagram for the control system.

Set-point, T_r, is the desired value of controlled variable, where T_i, **load**, is the change in any variable that may cause a variation in controlled variable (T) of process where temperature control is aimed.

ε is the **error**, which is the difference between the set-point and the signal from the measuring element.

As shown in Fig. 1, the input signal to the controller is the error and the output signal of the controller is fed to the final control element. The final control element may be regarded as a device which produces corrective action on the process related to the output signal of the controller.[1]

2.2. Control Mechanisms

2.2.1. On-off Control

This is a special case of proportional control in which K_c is very high, the final control element acts like a switch where it is either fully open or fully closed. This is a very simple controller. The thermostat used in a home-heating system is an example of this kind of controllers.

On-off controller causes the oscillation of the controlled variable around the set value. The magnitude and period of these oscillations depend on process delay time. If we consider the case of controlling the flow rate of the system, set point is specified and by changing K_c system response is observed. As system tries to reach the set point, the pump works in the form of pulses and the valves are either fully closed or open. Therefore the power of the pump in the form of pulses can be explained as illustrated in Fig. 2. As for the case of heating the process fluid (water), in other words temperature control, the temperature of the heater (which in this experiment is serpentine with electric heating) rises and falls along the hard line in Fig. 3. The temperature of measuring element would follow the dashed line of Fig. 3, i.e., similar to the heating medium temperature near the heater but delayed by a time (t) theoretical. The actual temperature of measuring element follows the characteristics of Fig 4. [2]

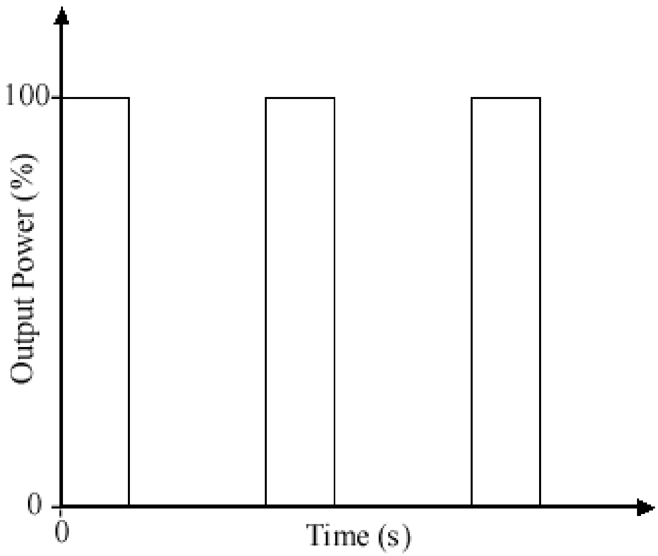


Figure 2. Power of heater versus time[2]

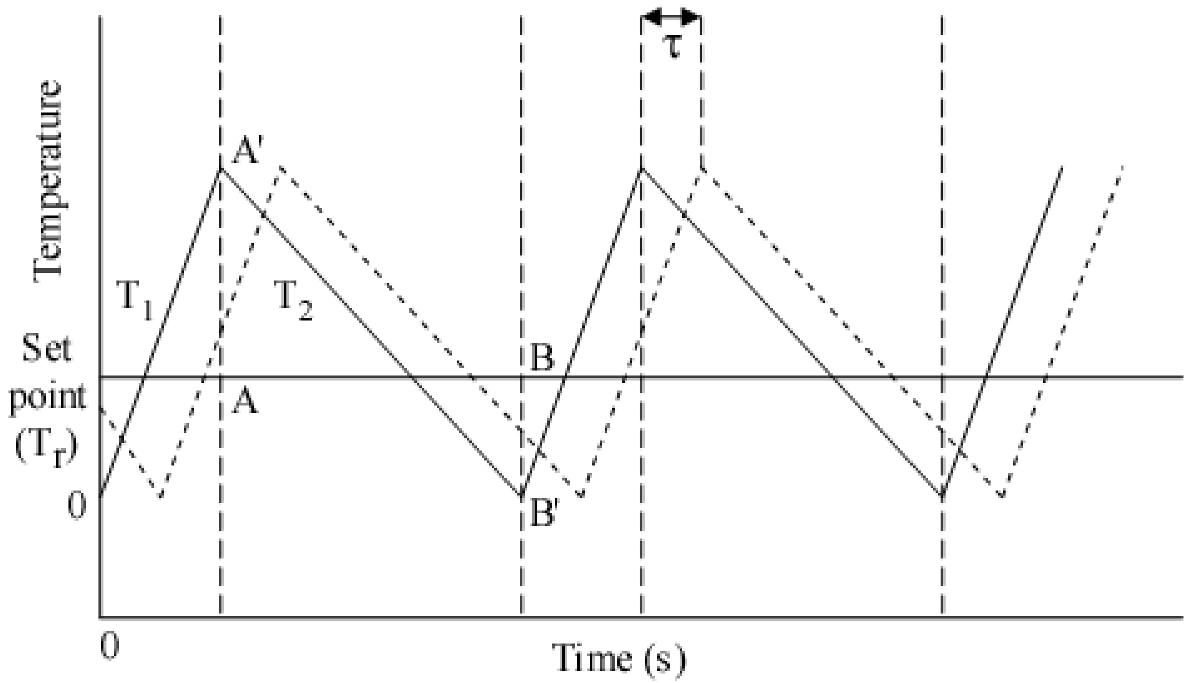


Figure 3. Temperature of heating medium versus time[2]

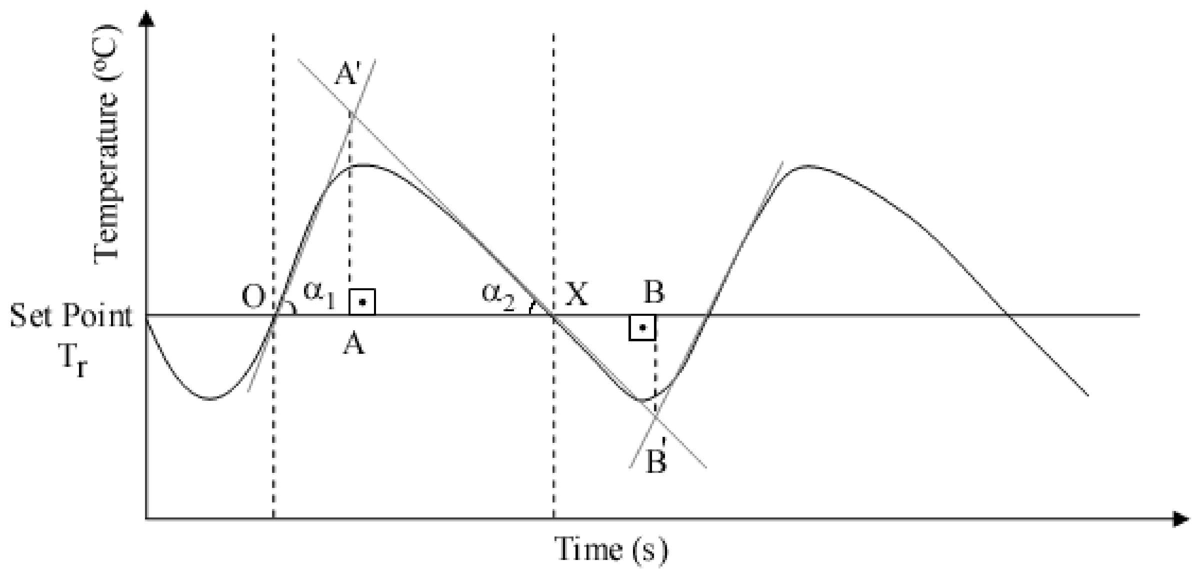


Figure 4. The actual temperature of the measuring element[2]

The temperature of process fluid (water) increases along the curve OA at the rate T_1 (°C/sec). The power fed to the heater is not turned-off when the water temperature reaches the set point T_r, because of the process **lag time** between the heater and the measuring element, and continues to rise at the rate of T_1 until the measuring temperature is equal to T_r.

The **overshoot amplitude** AA' is therefore given by;

$$AA' = \tau \times T_1 \qquad (1)$$

Where τ is the process delay time,

Similarly, the **undershoot** BB' is,

$$BB' = \tau \times T_2 \qquad (2)$$

Adding, the **total amplitude of the overshoot** is obtained;

$$AA' + BB' = \tau\,(T_1 + T_2) \qquad (3)$$

The **period of oscillation** from Fig. 3;

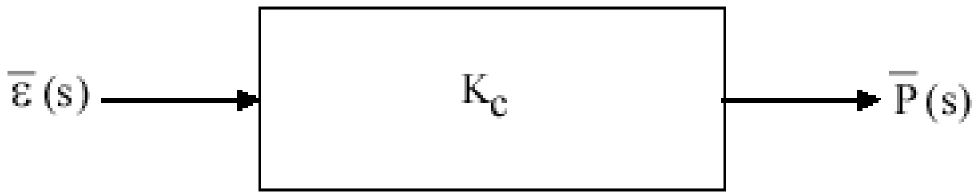
$$\boldsymbol{T} = \frac{AA' + BB'}{T_1} + \frac{AA' + BB'}{T_2} \qquad (4)$$

Or from equation (3)

$$\boldsymbol{T} = \tau\left(2 + \frac{T_1}{T_2} + \frac{T_2}{T_1}\right) \qquad (5)$$

2.2.2. Proportional Control

The proportional controller is a device which receives the error signal and puts out a signal proportional to it.



$$p = K_c \varepsilon + p_s \qquad (6)$$

p : output signal from controller

ε : error (difference between set point and measured value)

p_s : constant (the value is necessary when $\varepsilon = 0$).

Here K_c is the **gain** or the proportional band inversely which is defined as the error (expressed as a percentage of the range of measured variable) required to the final control element from fully closed to the fully open.

Proportional band (PB) = $(1/K_c)$ 100%

To obtain the transfer function of Eq. 6, the deviation variable (P) is described as:

$$P = p - p_s \qquad (7)$$

$P(t) = K_c\, \varepsilon(t)$ by taking the Laplace transform (8)

$$\bar{P}(s) = K_c \bar{\varepsilon}(s) \qquad (9)$$

$$\frac{\bar{P}(s)}{\bar{\varepsilon}(s)} = K_c \qquad (10)$$

This equation gives the transfer function of an ideal proportional controller.

2.2.3. Proportional + Integral Control (PI)

The integral mechanism produces a signal which is proportional with the time integral of the error. PI control mode is described by the following relationship;

$$p(t) = K_c \varepsilon(t) + \frac{K_c}{\tau_I} \int_0^t \varepsilon(t) dt + p_s \tag{11}$$

where τ_I is the integral time constant.

$$p(t) - p_s = P(t) = K_c \varepsilon(t) + \frac{K_c}{\tau_I} \int_0^t \varepsilon(t) dt \tag{12}$$

$$\bar{P}(s) = K_c \bar{\varepsilon}(s) + \frac{K_c}{\tau_I} \frac{1}{s} \bar{\varepsilon}(s) \tag{13}$$

The transfer function is given as;

$$\frac{\bar{P}(s)}{\bar{\varepsilon}(s)} = K_c \left(1 + \frac{1}{\tau_I s}\right) \tag{14}$$

2.2.4. Proportional + Derivative Control (PD)

Derivative control mechanism produces a signal proportional to the derivative of the error. PD control may be represented by;

$$p(t) = K_c \varepsilon(t) + K_c \tau_D \frac{d\varepsilon(t)}{dt} + p_s \tag{15}$$

where τ_D is the derivative time constant.

$$P(t) = K_c \varepsilon(t) + K_c \tau_D \frac{d\varepsilon(t)}{dt} \tag{16}$$

$$\bar{P}(s) = K_c \bar{\varepsilon}(s) + K_c \tau_D \bar{\varepsilon}(s) \tag{17}$$

The transfer function is,

$$\frac{\bar{P}(s)}{\bar{\varepsilon}(s)} = K_c (1 + \tau_D s) \tag{18}$$

2.2.5. Proportional + Integral + Derivative Control (PID)

This mode of control is a combination of the previous modes and is given by the expression;

$$p(t) = K_c \varepsilon(t) + \frac{K_c}{\tau_I}\int_0^t \varepsilon(t)dt + K_c \tau_D \frac{d\varepsilon(t)}{dt} + p_s \tag{19}$$

$$\bar{P}(s) = K_c \bar{\varepsilon}(s) + \frac{K_c}{\tau_I}\frac{1}{s}\bar{\varepsilon}(s) + K_c \tau_D s \bar{\varepsilon}(s) \tag{20}$$

The transfer function is

$$\frac{\bar{P}(s)}{\bar{\varepsilon}(s)} = K_c\left(1 + \frac{1}{\tau_I s} + \tau_D s\right) \tag{21}$$

where τ_I and τ_D are the integral and derivative time constants respectively.

2.3. Comparison of Various Control Modes

Response of the control system showing the effect of various modes of control can be shown by the graph given below (Figure 5).

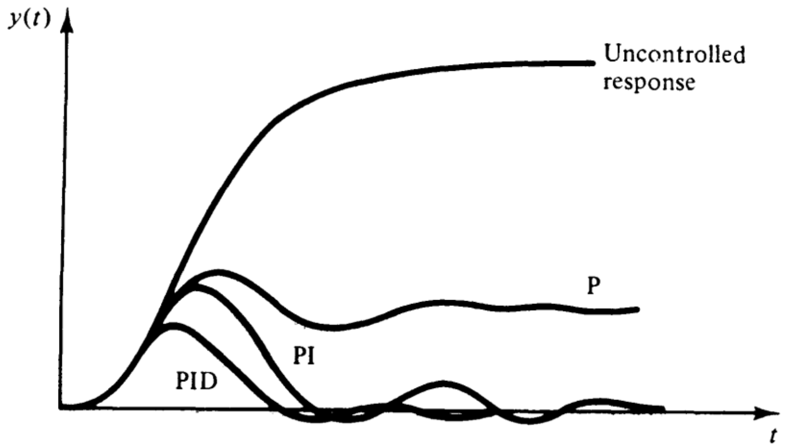


Figure 5. Response of a typical control system showing the effect of various modes of control. [3]

Offset is the difference between the new set point value and the ultimate value of controlled variable.[3]

3. EXPERIMENTAL SECTION

3.1 GENERAL DESCRIPTION OF THE SYSTEM

This unit consists of a hydraulic circuit, with a bottom tank (1) and a superior process tank (2), both dual ones, two pumps of centrifugal circulation (3), two flow meters with a manual control valve (4), three on/off solenoid valves (5) and a motorized proportional valve (infinitely variable) (6). Together with the tubes, the system also has union elbows, connections, a main valve and appropriate drainage for the circuit operation. All the above-mentioned is set on a designed support structure (7).

As additional fixed elements, there is also a turbine flow sensor that is installed in one of the upward lines of flow (8), and a temperature sensor located in a lateral bottom of the process tank (9) together with a serpentine with electric heating (11).

The interchangeable additional elements are an agitator (10), the immersion level sensor located in the process tank (12) and the pH sensor (solenoid) (13), to study the effect of the time out. [4]

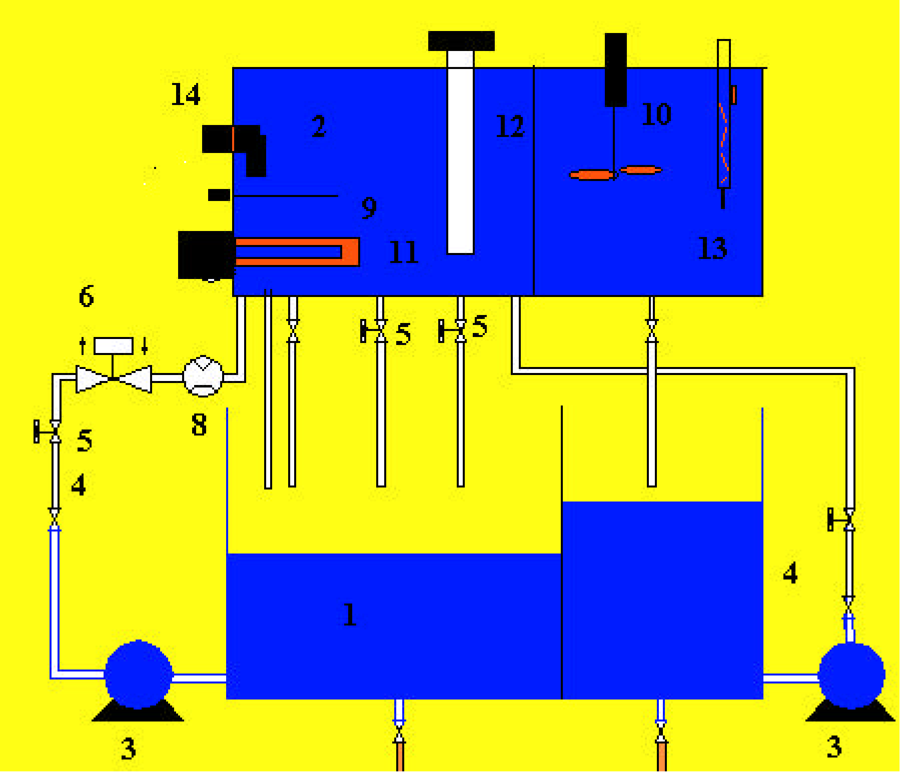


Figure 6. Main diagram of the equipment[4]

3.2 EXPERIMENTAL PROCEDURE

3.2.1. MANUAL CONTROL

A. Level control loops

1.- Connect the interface of the equipment and execute the program SACED UCP-L.

2.- Inside the program, select the option Manual.

3.- In the manual regulation (no controller) the flow can be regulated by the manual adjustable valve VR1, placed in the inferior part of the flow meter. Change its position and observe the adjustment of the level in function of their position.

5.- Connect pump 1 and vary the position of the motorized valve in the slip bar or the command associated to this action. Open AVS-1 or AVS-2 and check how, for a given position, the level of water in the tank fixes.

6.- Change the position of the valve and repeat the values to observe the reproducibility of the level control.

7.- Use the controls prepared in the software for the control of the solenoid valves AVS-1, AVS-2 and AVS-3 and the switch on/off button of the pump. Observe how an on/off of it also produces a level control of the liquid.

B. Flow control loops

1.- Connect the interface of the equipment and execute the program SACED UCP-F.

2.- Inside the program, select the option Configuration and connect pump 1 (AB-1).

3.- In the manual regulation (no controller) the flow can be regulated by the manual adjustable valve VR1, placed in the inferior part of the flow meter. Vary its position and observe the adjustment of the flow in function of its position.

4.- Select the option “Manual Control” of the software supplied with the equipment.

5.- Connect pump 1 (AB-1) and vary the position of the motorized valve by the Slip bar or the command associated to this action. Check a fixed position, the flow is regulated.

6.- Vary the position of the valve and repeat the values to observe the reproduction of the flow control.

7.- Use the controls prepared in the software for controlling the solenoid valves AVS-1 and the on/off button of the pump. Observe how an on and off button also produces a flow control of the liquid. [4]

C. Temperature control loops

1.- Connect the interface of the equipment and execute the program SACED UCP-T.

2.- Inside the program, select the option *Configuration* and connect pump 1.

3.- The temperature regulation of a tank of water can be carried out by two different procedures that we will identify as:

a. - **Static**; it consists on filling the left superior tank above the level alarm.

b. - **Continuous or Dynamic**; it consists on fixing a water level in the left superior tank but with an inlet and outlet of constant water. In this second procedure, it is required that the incoming and outcoming water flows are small in order to establish a thermal balance in the tank.

4.- In the manual regulation (no controller) the temperature can be regulated by the on off immersion resistor placed in the tank.

5.- Select the option “Manual Control” of the software supplied with the equipment.

6.- Connect pump 1

a) Fill the tank above the level alarm. Disconnect pump 1 and close the valve VR1 manually.

b) Connect pump 1 and fill the tank until getting the level alarm.

Once surpassed this, AVS-1 opens, and using VR1 fix a constant inlet and outlet water flow in the tank.

7.- Connect the agitator given with the equipment.

8.- By the connection and disconnection of the resistor, fix a temperature for the water. In the case *b*, fix the temperature varying the inlet and outlet of flow. So, open or close valve VR1 or the AVS-1. [4]

3.2.2. ON/OFF CONTROL

A. Level control loops

1.- Connect the interface of the equipment and the control software.

2.- Select the control on/off option.

3.- Make a double click on the on/off control, select the wanted flow. By defect, there is a certain flow, tolerance and performance time. It allows you to play with these parameters and see the influences of each one.

4.- The level control can be carried out by the activation of a single actuator, or of several ones, to which different tolerances are allowed. These controllers work as security system measures when the controlled variable exceeds in a tolerance the set value. To activate or to disable each one of these controllers you have to make a double click on each one of them and press the button “PAUSE”.

5.-Calculate the inertia of the system before an on/off response and determine the limit time for an exact control.

B. Flow control loops

1.- Connect the interface of the equipment and the control software.

2.- Select the control option on/off.

3.- By a double click on the on/off control, select the flow wanted. By defect there is certain flow, a tolerance and a performance time. It allows you to play with these parameters and they can see the influences of each one of them.

4.- It calculates the inertia of the system before an on/off response and determines the limit time for an exact control.

C. Temperature control loops

1.- Connect the interface of the equipment and the control software.

2.- Select the control option on/off.

3.- Select the wanted temperature (Set point). There is a set point, tolerance and performance time determined. It allows you to play with these parameters and to see the influences of each of them.

4.- It calculates the inertia of the system before an on/off response and determines the limit time for an exact control. [4]

3.2.3. PROPORTIONAL (P) CONTROL

A. Level control loops

1.- Connect the Interface and execute the control software.

2.- Select the Option “Control PID” on the capture screen.

3.- Select a set point, PID controller and a proportional constant.

4.- Indicate a value of 0 for the integral and derivative performance. In this experiment, we want to observe the effects of a proportional action.

5.- Activate the PID controller, go out and save the values. You will observe that the motorized valve begins to work.

6.- Connect pump 1 (AB-1).

7.- Activate the solenoid valve AVS-2.

8.- The controller will modify the position of the AVP-1 (Proportional Valve) to adjust the flow that controls the level from the water tank to the set value.

B. Flow control loops

1.- Connect the Interface and execute the control software.

2.- Select the Option “Control PID” on the capture screen.

3.- Select a set point, PID controller and a proportional constant.

4.- Indicate a value of 0 for the integral and derivative performance. In this experiment we want to observe the effects of a proportional action.

5.- Activate the PID controller and start and go out and save the values. You will observe that the motorized valve begins to act.

6.- Connect pump 1 (AB-1).

7.- The controller will modify the position of the AVP (Proportional Valve) to adjust the flow to the set value.

C. Temperature control loops

1.- Connect the Interface and execute the control software.

2.- Select the option “Control PID” on the capture screen.

3.- Indicate a value of 0 for the integral and derivative performance. In this experiment we want to observe the effects of a proportional action.

4.- Activate the PID controller, go out and save the values. You will observe that the motorized valve begins to work. [4]

3.2.4. PROPORTIONAL + INTEGRAL (PI) CONTROL

A. Level control loops

1.- Connect the Interface and execute the control software.

2.- Select the option "Control PID" on the capture screen.

3.- Select a set point, PID controller and a proportional constant and an integral value. The value for the integral constant should be big so that the error accumulation is carried out smoothly and it doesn't generate an on/off performance in the actuator.

4.- Indicate a value of 0 for the derivative performance. In this experiment we want to observe the effects of a proportional action plus an integral action.

5.- Activate the PID controller, go out, and save the values. You will observe that the motorized valve begins to act.

6.- Connect pump 1.

7.- Open the solenoid valve AVS-1.

8.- The controller will modify the position of the AVP-1 (Proportional Valve) to adjust the flow that controls the set value.

B. Flow control loops (Proportional + Integral)

1.- Connect the Interface and execute the control software.

2.- Select the Option "Control PID" on the capture screen.

3.- Select a set point, PID controller and a proportional constant and an integral value. The value for the integral constant should be big so that the error accumulation is carried out smoothly and it doesn't generate an on/off performance in the actuator.

4.- Indicate a value of 0 for the derivative performance. In this experiment, we want to observe the effects of a proportional action plus an integral action.

5.- Activate the PID controller, go out, and save the values. You will observe that the motorized valve begins to work.

6.- Connect pump 1 (AB-1).

7.- The controller will modify the position of the AVP-1 (Proportional Valve) to adjust the flow to the set value. [4]

C. Temperature control loops

1.- Connect the Interface and execute the control software.

2.- Select the option “Control PID” on the capture screen.

3.- Select a set point, PID controller and a proportional constant and an integral value. The value for the integral constant should be big so that the error accumulation is carried out smoothly and doesn’t generate an on/off performance in the actuator.

4.- Indicate a value of 0 for the derivative performance. In this experiment we want to observe the effects of a proportional action plus an integral action.

5.- Activate the PID controller, go out, and save the values. You will observe that the resistor begins to work.

3.2.5. PROPORTIONAL DERIVATIVE INTEGRAL (PID) CONTROL

A. Level control loops

1.- Connect the Interface and execute the control software.

2.- Select the option “Control PID” on the capture screen.

3.- Select a set point, PID controller and a proportional constant, derivative and integral. The value for the derivative constant should be small and the integral constant should be big so that the performance is small and doesn’t generate an on/off performance in the actuator.

4.- Activate the PID controller, go out and save the values. You will observe that the motorized valve begins to act.

5.- Connect pump 1.

6.- Open the solenoid valve AVS-2.

7.- The controller will modify the position of the AVP-1 (Proportional Valve) to vary the flow to adjust the level to the set value. [4]

B. Flow control loops

1.- Connect the Interface and execute the control software.

2.- Select the option “Control PID” on the capture screen.

3.- Select a set point, PID controller and a proportional constant, derivative and integral. The value for the derivative constant should be small and the integral constant should be big so that the performance is small and it does not generate an on/off performance in the actuator.

4.- Activate the PID controller, go about, and save the values. You will observe that the motorized valve begins to work.

5.- Connect pump 1 (AB-1).

6.- The controller will modify the position of the AVP-1 (Proportional Valve) to adjust the flow to the set value.

C. Temperature control loops

1.- Connect the Interface and execute the control software.

2.- Select the Option “Control PID” on the capture screen.

3.- Select a set point, PID controller and a proportional, derivative and integral constant. The value for the derivative constant should be small and the integral constant should be big so that the performance is small and it doesn’t generate an on/off performance in the actuator.

4.- Activate the PID controller, go out and save the values. You will observe that the motorized valve begins to work.[4]

3.3 ADJUSTMENT OF THE CONSTANTS OF A FLOW CONTROLLER (ZIEGLER-NICHOLS METHOD)

3.3.1 Experimental procedure

The data to be analyzed will be obtained configuring only the controller with the Proportional Band or the proportional action. The integral and derivative actions should be at zero. The objective of the experience is to maintain the system with a constant level using a controller *P* for the control of the motorized valve. With the motorized valve at the 50% of its way, regulate the needle valve manually VR-1, until getting that the level of the tank is constant.

3.3.2 Method of the minimum period (Ziegler-Nichols)

Pass now to an automatic control and observe how the level stays constant at the 50% of the process variable. Change the variables of the process for partial opening of the needle valve, VR-1. As the process will become stable, increase the value of the proportional constant and close the needle valve, VR-1, partially observing the behavior of the process. Continue increasing the value of the proportional constant, applying each time an interference in step (closing or opening VR-1), until the variable of the process oscillates continually. Note down the value of the proportional constant (Limit Proportional Band, L.P.B.) when this happens, measure the oscillation time of the process (O.T.).

The optimum values, depending on the control type we will make on our process are:

Type of Control	B.P.	I.T.	D.T.
P	2 (L.P.B.)	--	--
P+I	2.2 (L.P.B.)	T.O/1.2	--
P+I+D	1.7 (B.P.L)	O.T. / 2.0	T.O/8.0

A variant of the gain limit method is the method of the minimum overflow of the set point. Once the self-maintained oscillation of the Time of Oscillation O.T. is obtained for a Limit Proportional Band L.P.B., the values of the control actions are the following ones:

B.P (%) = 1.25 L.P.B.

I.T. (MIN/REP) = 0.6 O.T.

D.T. (MIN) = 0.19 O.T.s[4]

4. CALCULATIONS

a. Manual Mode

Explain how manual control is done for controlling the flow rate, level and temperature.

b. On-off Control

The flow rate, level and the temperature of the system will be controlled. The system response is observed from the monitor. Try set point values of 1L/min and 1.50 L/min.

Draw the graphs of flow rate versus time and compare the results to each other.

c. Proportional Control

The flow rate of the system will be controlled. Here Kc values will be changed to see the differences in the system's responses. Calculate the offset values and determine the optimum K_c by using Ziegler-Nichols method.

Draw the graphs of flow rate versus time and compare the results to each other.

d. Proportional + Integral Control

The flow rate of the system will be controlled. Here τ_I values will be changed to see the differences in the system's responses. Calculate the offset values and determine the optimum τ_I by using Ziegler-Nichols method.

Draw the graphs of flow rate versus time and compare the results to each other.

e. Proportional + Integral + Derivative Control

The flow rate of the system will be controlled. Here τ_D values will be changed to see the differences in the system's responses. Calculate the offset values and determine the optimum τ_D by using Ziegler-Nichols method.

Draw the graphs of flow rate versus time and compare the results to each other.

5. NOMENCLATURE

AA'	: Overshoot amplitude
BB'	: Undershoot amplitude
E	: Error (difference between set point and measured value)
G(s)	: Transfer function of any system
K_c	: Process Gain
P	: Output signal from controller
PB	: Proportional Band
T	: Period of oscillation (1/s)
O.T.	: Oscillation time of the process (s)
τ_1	: Integral time constant
τ_D	: Derivative time constant

6. REFERENCES

1. Donald R. Coughanowr, Lowel B. Koppel, “Process System Analysis and Control”, McGraw-Hill Book Kogakujha Ltd., New York, 1965.

2. Operation Manual of Process Controller CE.3 and Model Process, Tecquipment Limited, London, 1984.

3. Shinskey F. G., “Process Control Systems (Application, Design and Tuning)“ McGraw-Hill Book Company, 1988.

4. EDIBON Practices Manual

DATA SHEET

Name Surname: **Date:**

Group No:

Assistant:

Sign below to state that you have received a CD where original data of your experiment consisting of temperature, level, pressure and flow rate values as a function of time have been recorded.

Research Assistants:

Dr. Hande GÜNAN YÜCEL

Anıl Kuban *2019-2020 SPRING SEMESTER*

DYNAMIC BEHAVIOUR OF U-MANOMETERS

1. INTRODUCTION

Transfer Function

In order to determine the process control characteristics of a given physical system transfer function of the system should be obtained. In general a transfer function relates two variables in a physical process; one of them is the cause (forcing function or input variable) and the other is the effect (response or output variable). As in the example of a mercury thermometer; the surrounding temperature was the cause or input, whereas the thermometer reading was effect or output.

We shall develop the transfer function for a first-order system by considering the unsteady-state behavior of an ordinary mercury-in-glass thermometer. A cross sectional view of the bulb is shown in Fig. 1.

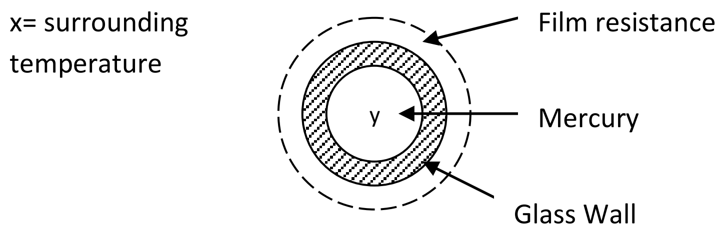


Fig. 1. Cross-sectional view of thermometer.

Consider the thermometer to be located in a flowing stream of fluid for which the temperature x varies with time. Our problem is to calculate the response or the time variation of the thermometer reading y for a particular change in x.

The following assumptions will be used in this analysis:

1. All the resistance to heat transfer resides in the film surrounding the bulb (i.e.,the resistance offered by the glass and mercury is neglected).

2. All the thermal capacity is in the mercury. Furthermore, at any instant the mercury assumes a uniform temperature throughout.

3. The glass wall containing the mercury does not expand or contract during the transient response. (In an actual thermometer, the expansion of the wall has an additional effect on the response of the thermometer reading.)

It is assumed that the thermometer is initially at steady state. This means that, before time zero, there is no change in temperature with time. At time zero the termometer will be subjected to some change in the surrounding temperature x(t).

By applying the unsteady state energy balance

Input rate – output rate = rate of accumulation

We get the result

$$hA_t(x-y)-0 = mC\frac{dy}{dt} \qquad (1)$$

Equation 1 states that the rate of flow of heat through the film resistance surrounding the bulb causes the internal energy of the mercury to increase at the same rate. The increase in internal energy is manifested by a change in temperature and a corresponding expansion of mercury which causes the mercury column, or "reading" of the thermometer to rise.

The coefficient h will depend on the flow rate and properties of the surrounding fluid and the dimensions of the bulb. We shall assume that h is constant for a particular installation of the thermometer.

Our analysis has resulted in Eq. 1, which is a first order differential equation. Prior to the change in x, the thermometer is at steady state and the derivative dy/dt is zero. For the steady-state condition, Eq. 1 may be written

$$hA_t\,(x_s-y_s) = 0 \qquad t<0 \qquad (2)$$

The subscript s is used to indicate that the variable is the steady-state value. Eq. 2 simply states that $x_s = y_s$, or the thermometer reads the true bath temperature. Subtracting Eq. 2 from Eq. 1 gives

$$hA_t\,[(x - x_s\,) - (y - y_s\,)] = mC\,\frac{d(y - y_s)}{dt} \qquad (3)$$

Notice that $d(y - y_s\,)/dt = dy/dt$, because y_s is a constant.

If we define the deviation variables to be the differences between the variables and their steady-state values

$$X = x - x_s$$

$$Y = y - y_s$$

Eq. 3 becomes

$$hA_t\,(X - Y\,) = mC\,\frac{dY}{dt} \qquad (4)$$

If we let $\frac{mC}{hA_t} = \tau_P$, Eq. 4 becomes

$$X - Y = \tau_P\,\frac{dY}{dt} \qquad (5)$$

Taking the Laplace transform of Eq. 5 gives

$$\overline{X}(s) - \overline{Y}(s) = \tau_P\, s\, \overline{Y}(s) - \underbrace{\tau Y(0)}_{0} \qquad (6)$$

Rearranging Eq. 6 as a ratio of $\overline{Y}(s)$ to $\overline{\mathrm{X}}(\mathrm{s})$ gives

$$\frac{\overline{\mathrm{Y}}(\mathrm{s})}{\overline{\mathrm{X}}(\mathrm{s})} = \frac{1}{\tau_P s + 1} \qquad (7)$$

For a first order system; the general form of the transfer function is

$$\frac{\overline{\mathrm{Y}}(\mathrm{s})}{\overline{\mathrm{X}}(\mathrm{s})} = \frac{1}{\tau_P s + 1}$$

The parameter τ_P is called the time constant of the system and has the units of time. The expression on the right side of Eq. 7 is called the *transfer function* of the system. It is the ratio of the Laplace transform of the deviation in thermometer reading to the Laplace transform of the deviation in the surrounding temperature.

In examining other physical systems, we shall usually attempt to obtain a transfer function.

Any physical system for which the relation between Laplace transforms of input and output deviation variables is of the form given by Eq. 7 is called a *first order system.* Synonyms for first-order system are first-order lag and single exponential stage. All these terms are motivated by the fact that Eq. 7 results from a first-order linear differential equation, Eq. 5. By reviewing the steps leading to Eq. 7, it can be seen that the introduction of deviation variables prior to taking the Laplace transform of the differential equation results in a transfer function that is free of initial conditions because the initial values of X and Y are zero. In control system engineering, we are primarily concerned with the deviations of system variables from their steady-state values. The use of deviation variables is, therefore, natural as well as convenient.

Properties of Transfer Functions In general, a transfer function relates two variables in a physical process; one of them is the cause (forcing function or input variable) and the other is the effect (response or output variable). We may write

$$\text{Transfer function} = G(s) = \frac{\overline{Y}(s)}{\overline{X}(s)}$$

where G(s) = symbol for transfer function,

$\overline{X}(s)$ = Laplace transform of forcing function or input, in deviation form,

$\overline{Y}(s)$ = Laplace transform of response or output, in deviation form.

The transfer function completely describes the dynamic characteristics of the system. If we select a particular input variation X(t) for which the transform is $\overline{X}(s)$, the response of the system is simply

$$\overline{Y}(s) = G(s)\ \overline{X}(s)$$

By taking the inverse of $\overline{Y}(s)$, we get Y(t), the response of the system.

The transfer function results from a linear differential equation; therefore, the principle of superposition is applicable. This means that the transformed response of a system with transfer function G(s) to a forcing function

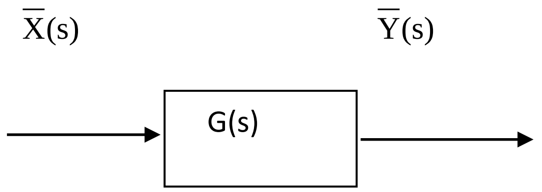


Fig. 2. Block diagram

$$\overline{X}(s) = a_1\overline{X_1}(s) + a_2\overline{X_2}(s)$$

where X_1 and X_2 are particular forcing functions and a_1 and a_2 are constants, is

$$\begin{aligned}\overline{Y}(s) &= G(s)\ \overline{X}(s)\\ &= a_1G(s)\ \overline{X_1}(s) + a_2G(s)\ \overline{X_2}(s)\\ &= a_1\overline{Y}_1(s) + a_2\overline{Y}_2(s)\end{aligned}$$

$\overline{Y}_1(s)$ and $\overline{Y}_2(s)$ are the responses to X_1 and X_2 alone, respectively.

The functional relationship contained in a transfer function is often expressed by a *block-diagram* representation, as shown in Fig. 2. The arrow entering the box is the forcing function or input variable, and the arrow leaving the box is the response or output variable. Inside the box is placed the transfer function. We state that the transfer function G(s) in the box "operates" on the input function $\overline{X}(s)$ to produce an output function $\overline{Y}(s)$.

Transient Response : Now that the transfer function of a first order system has been established, we can easily obtain its transient response to any forcing function. Since this type of system occurs so frequently in practice, it is worthwhile to study its response to several common forcing function: step, impulse and sinusoidal. Two of them used frequently in order to determine the process control characteristics of the physical systems are as follows:

Step Function: Mathematically, the step function of magnitude A can be expressed as,

$X(t) = Au(t)$

where, $u(t)$ is the step function. A graphical representation is shown in Fig. 3.

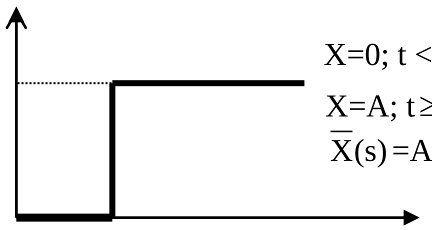


Fig. 3. Step function

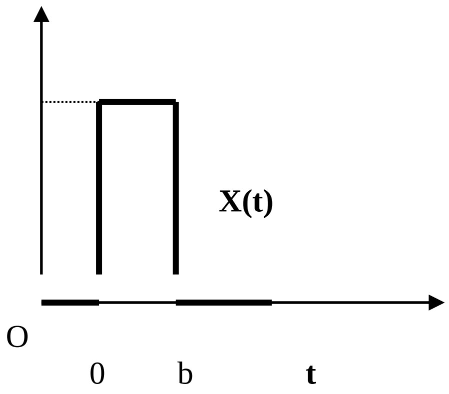
The transform of this function is $\overline{X}(s) =$ A/s. A step function can be approximated very closely in practice. For example, a step change in flow rate can be obtained by the sudden opening of a valve.

Impulse function: Mathematically, the impulse function of magnitude A is defined as,

$X(t) = A\delta(t)$

where $\delta(t)$ is the unit impulse function. A graphical representation of this function, before the limit is taken, is shown in Fig. 4.

The true impulse function, obtained by letting $b \rightarrow 0$ in Fig. 4, has Laplace transform A.



A/b

X=0; t<0

X=A/b; $0 \le t \le b$

X=0; t>b

$\lim_{b \to 0} X(t) = A\delta(t)$

$L\{A\delta(t)\} = A$

Fig. 4. Impulse function

A first-order system, the time constant has been expressed in terms of system parameters; thus

$$\tau_P = \frac{mC}{hA_t}$$

We may consider the time constant to be the product of a resistance and a capacitance; thus

$$\tau_P = (\text{resistance})(\text{capacitance})$$

The time constant has the units of time and the resistance and capacitance must have units consistent with this requirement.

In general, capacitance is defined as the change in storage of a quantity per unit change in driving force or potential; thus

$$\text{Capacitance} = \frac{\Delta\ \text{storage}}{\Delta\ \text{drivingforce}}$$

The driving force or potential is said to create the flow of the quantity under consideration.

The resistance is the ratio of the change in driving force to the change in flow:

$$\text{Resistance} = \frac{\Delta\ \text{drivingforce}}{\Delta\ \text{flow}}$$

A second order system is one whose output is described by the solution of a second order differantial equation.

U-manometers are typical second order systems. A second order linear differential equation describes the dynamic behaviour of a U-manometer. A typical U-tube manometer is shown in Fig. 5.

The pressure force acting on the liquid causing a level change of x will equal to the sum of the gravity (F_g), friction (F_f) and inertia (F_i) forces.

By performing a force balance;

$$(P_1\text{-}P_2)\left[\frac{\pi D_t^{\,2}}{4}\right] = F_g + F_f + F_i \qquad (8)$$

The gravity force for the system is given by,

$$F_g = \frac{[\pi D_t^{\,2}]}{4}\rho g r \tag{9}$$

Similarly the inertia force may be expressed by using Newton's Law;

$$F_i = ma = \frac{\left[\pi D_t^2\right]}{4} L\rho \frac{d^2x}{dt^2} \tag{10}$$

Friction force for laminar flow can be defined by Hagen-Pouseuille Equation.

$$F_f = \frac{32\mu\ LQ}{D_t^2} \tag{11}$$

where Q is volumetric flow rate given by

$$Q = \frac{[\pi D_t^2]}{4}\frac{dx}{dt} \tag{12}$$

By applying the following transformations

$$r = 2x \tag{13}$$

$$\frac{dr}{dt} = 2\frac{dx}{dt} \quad ; \quad \frac{d^2r}{dt^2} = 2\frac{d^2x}{dt^2}$$

on Eqs. 9, 10 and 12 substituting the resulting expressions into Eq. 8 yields;

$$A\frac{d^2r}{dt^2}+2B\frac{dr}{dt}+r=h \tag{14}$$

where;

$$A=\frac{L}{2g},\quad B=\frac{16\mu L}{\rho g D_t^2},\quad h=\frac{P_1\text{-}P_2}{\rho g} \tag{15}$$

By defining following deviation variables;

$$R = r\text{-}r_s,\quad H = h - h_s \tag{16}$$

where, r_s and h_s are the steady state values. Applying the Laplace transformations on the resulting equation in terms of deviation variables and rearranging the transformed equation yields the transfer function describing the dynamic behaviour of the U-manometer. Eq. 17 shows that U-manometers are the second order systems.

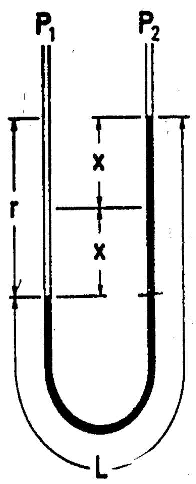


Fig. 5. Schematic representation of U-manometer

$$\frac{\overline{R}(s)}{\overline{H}(s)} = \frac{1}{A\,s^2 + B\,s + 1} \qquad (17)$$

For a second order system ; the general form of the transfer function is

$$\frac{\overline{Y}(s)}{\overline{X}(s)} = \frac{K_p}{\tau^2 s^2 + 2\xi\tau s+1} \qquad (18)$$

where, ξ is the damping factor of the system. The comparison of Eq. 17 with Eq. 18 gives $\tau^2 = A = L/2g$ and $2\xi\tau = B = (16\mu L)/(\rho g D_t^2)$ in terms of the system properties. The transfer function given by Eq. 17 is written in standart form, and we shall show later that other physical systems can be represented by a transfer function having the denominator $\tau^2s^2+2\xi\tau s+1$. All such systems are defined as second order. Note that it requires two parameters, τ and ξ, to characterize the dynamics of a second order system.

Table 1. The response of a second-order system to a unit-step forcing function depending on the parameter

ξ	Nature of roots	Description of response
<1	Complex	Underdamped
=1	Real and equal	Critically damped
>1	Real	Overdamped

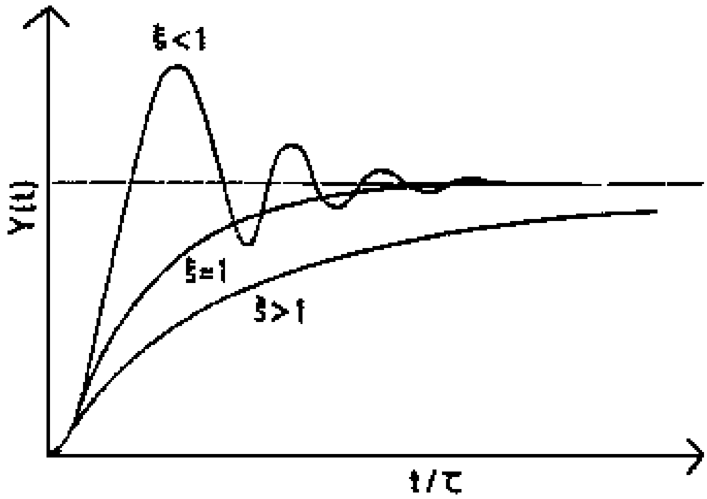


Fig. 6. Response of a second-order system to a unit-step forcing function

Case A: Overdamped Response, when ξ > 1 (Response equation of a second-order system to a unit step forcing function when ξ>1).

$$y(t)=K_p[1-e^{\xi t/\tau}(\cosh\sqrt{\xi^2-1}\,\frac{t}{\tau}\;\frac{\xi}{\sqrt{\xi^2-1}}\sinh\sqrt{\xi^2-1}\,\frac{t}{\tau})] \qquad (19)$$

$$\sinh\alpha=\frac{e^{\alpha}-e^{-\alpha}}{2} \quad \text{and} \quad \cosh\alpha=\frac{e^{\alpha}+e^{-\alpha}}{2}$$

Case B: Critically damped response, when ξ = 1 (Response equation of a second-order system to a unit step forcing function when ξ = 1).

$$y(t)=K_p\left[1-\left(1+\frac{t}{\tau}\right)e^{-t/\tau}\right] \qquad (20)$$

Case C: Underdamped response, when ξ<1 (Response equation of a second-order system to a unit step forcing function when ξ<1).

$$y(t) = K_p \left[1 - \frac{1}{\sqrt{1-\ \xi}} e^{-\xi t/\tau} \sin\left(\omega t + \phi\right) \right] \tag{21}$$

where;

$$\omega = \frac{\sqrt{1-\xi^2}}{\tau} \quad \text{and} \quad \phi = \tan^{-1}\left[\frac{\sqrt{1-\xi^2}}{\xi} \right]$$

$$t_{theoretic} = t_{rise} + (2n-1)\frac{T}{4} \qquad \text{n = Oscillation number} \tag{22}$$

Case D: Underdamped response, when ξ<1 (Response equation of a second-order system to a unit impulse forcing function when ξ<1).

$$y(t) = K_p \left[\frac{1}{\tau\sqrt{1-\xi^2}} e^{\xi t/\tau} \sin\sqrt{1-\xi^2}\,\frac{t}{\tau} \right] \tag{23}$$

$$t_{theo} = (2n-1)\frac{T}{4} \tag{24}$$

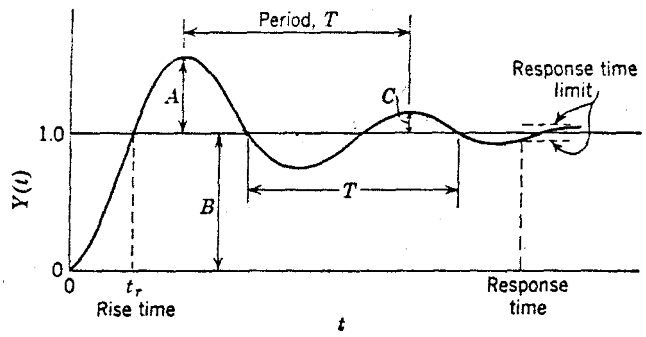


Fig. 7. Terms used to describe an underdamped second-order response

Terms Used to Describe an Underdamped System: Of these three cases, the underdamped response occurs most frequently in control systems. Hence a number of terms are used to describe the underdamped response quantitatively. In general, the terms depend on ξ and/or τ.

1. Overshoot Overshoot is a measure of how much the response exceeds the ultimate value following a step change and is expressed as the ratio A/B in Fig. 7.

The overshoot for a unit step is related to ξ by the expression

Overshoot = exp ($\frac{-\pi\xi}{\sqrt{1-\xi^2}}$) . The overshoot increases for decreasing ξ.

2. Decay ratio. The decay ratio is defined as the ratio of the sizes of successive peaks and is given by C/A in Fig. 7. The decay ratio is related to ξ by the expression

$$\text{Decay ratio} = \exp\left(-2\pi\xi/\sqrt{1-\xi^2}\right) = (\text{overshoot})^2 \qquad (26)$$

Notice that larger ξ means greater damping, hence greater decay.

3. Rise time. This is the time required for the response to first reach its ultimate value and is labeled t_r, in Fig.7. t_r increases with increasing ξ.

4. Response time. This is the time required for the response to come within ± 5 percent of its ultimate value and remain there. The response time is indicated in Fig. 7.

5. Period of oscillation. The radian frequency (radians/time) is the coefficient of t in the sine term; thus,

$$\omega\text{, radian frequency} = \frac{\sqrt{1-\xi^2}}{\tau} \qquad (27)$$

Since the radian frequency ω is releated to the cyclical frequency f by $\omega = 2\pi f$, it follows that

$$f = \frac{1}{T} = \frac{1}{2\pi} \; \frac{\sqrt{1-\xi^2}}{\tau} \qquad (28)$$

where T is the period of oscillation (time/cycle). In terms of Fig. 7, T is the time elapsed between peaks.

The smaller the value of the time constant τ, the steeper the initial response of the system.

Equivalently; the time constant τ of a process is a measure of the time necessary for the process to adjust to a change in its input.

Characteristic explains the name steady state or static gain given to parameter K_p, since for any step change Δ (input) in the input, the resulting change in the output steady state is given by

$$K_p = \frac{\Delta\,(\text{outputsteadystate})}{\Delta\,(\text{inputsteadystate})} \qquad (29)$$

A small change in the input if K_p is large (very sensitive systems)

A large change in the input if K_p is small.

2. EXPERIMENTAL SET-UP

The experimental set-up consist of the various U-manometers in different diameters and length that contain several kind of liquids with the different physicochemical properties such as water, glycerol and their mixtures.

The pressure difference in the U-manometer is created by a vacuum pump.

3. EXPERIMENTAL PROCEDURE

Apply a pressure difference on the U-manometer by vacuum pump and determine the variation of the liquid level with time until the steady state is reached. Stop the vacuum pump when the constant liquid level is observed in U-manometer, and determine again the variation of the liquid level with time.

After the constant liquid level is reached in the U-manometer, compress the silicone tube between the connection points of U-manometer and give up the silicone tube suddenly and then record the variation of the liquid level with time. Determine the variation of oscillation amplitude according to initial liquid level with time. If oscillation is observed on the U-manometer liquid.

Repeat these steps at least twice for each U-manometer.

4. CALCULATIONS

a. Sketch the graph of experimental liquid level difference vs. time for rising and also falling cases of the liquid.

b. Obtain theoretically the same graphs by using the proper response equations.

c. For oscillatory U-manometers, sketch the variation of the oscillation amplitude according to initial liquid level with time.

d. Calculate the frequency and period values of all oscillations.

e. Discuss the effects of tube internal diameter, kinematic viscosity of the liquid and the total length of the liquid on the dynamic behaviour of the U-manometer.

SYMBOLS

a	Acceleration of the manometer liquid.
A, B	Constants in the transfer function
A_t	Surface area of bulb for heat transfer
C	Heat capacity of mercury
D_t	Internal diameter of the tube
f	Cyclical frequency
F	Force
g	Acceleration of gravity
h	Applied pressure difference as liquid level
h	Film heat transfer coefficient
K_p	Static gain or gain
L	Total length of the liquid in U-manometer
m	Mass of liquid in the monometer
r	Liquid level difference at any time in U-manometer
t	Time
t_r	Rise time
T	Period of oscillation
Q	Volumetric flow rate of the liquid
τ	Natural period of oscillation
τ_p	Time constant
μ	Viscosity of the liquid
ρ	Density of the liquid
ω	Radian frequency
ξ	Damping factor

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DYNAMIC BEHAVIOUR OF U-MANOMETERS
Data Sheet

Name, Surname:
Group No:
Assistant:

Table 1. Properties of U-manometers

Properties	Manometer 1	Manometer 2	Manometer 3	Manometer 4	Manometer 5	Manometer 6
D (cm)	0,6	1,1	0,6	1,1	1,1	1,1
L (cm)	88	95	102	95	85	116

Manometer 1: Engine oil **Manometer 2:** Water **Manometer 3:** Glycerol
Manometer 4: Engine oil **Manometer 5:**15 % glycerol solution **Manometer 6:** Glycerol

Table 2. Data of Overdamped U-manometers

Manometer 1			Manometer 3			Manometer 4			Manometer 6		
t(s)	hr(cm)	hf(cm)	t(s)	hr(cm)	hf(cm)	t(s)	hr(cm)	hf(cm)	t (s)	hr(cm)	hf(cm)

Table 3. Data of Underdamped U-manometers to step change

	Manometer 2		Manometer 5	
Oscillation	**t(s)**	**h(cm)**	**t(s)**	**h(cm)**
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

Table 4. Data of Underdamped U-manometers to impulse change

	Manometer 2		Manometer 5	
Oscillation	**t(s)**	**h(cm)**	**t(s)**	**h(cm)**
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				