1. Wave Nature of Light

1.1. Light waves in a homogenous medium

A. Plane electromagnetic wave

we can treat light as an em-wave with time varying $\vec{E}$ and $\vec{B}$, that is E_x and B_y as it is propagating along z-dir.

phase constant

Travelling wave along z

$$E_x = E_0 \cos(\omega t - kz + \phi_0)$$

$k = 2\pi/\lambda$

k: propagation const. or wave number

The argument $(\omega t - kz + \phi_0)$ is called the PHASE.

E_x is always accompanied with by a travelling magnetic field B_y oscillating at the same freq, ω, and prop. constant, k.

The direction of two fields ($\vec{E}$ and $\vec{B}$) are orthogonal

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A travelling wave can also be represented in the exp-notation:

$$\boxed{\begin{aligned} E_x(z,t) &= Re\left\{ E_0 \exp(j\phi_0) \exp j(\omega t - kz) \right\} \\ E_x(z,t) &= Re\left\{ E_c \exp j(\omega t - kz) \right\} \end{aligned}}$$

E_c $(= E_0 \exp j\phi_0)$ is a complex number representing the amplitude of the wave and includes the const. phase information ϕ_0

"The direction of propagation" is shown by a vector $\vec{k}$, wave vector, whose magnitude is

$$|\vec{k}| = \frac{2\pi}{\lambda}$$

$\vec{k}$ is $\perp$ to the "phase planes".

$\therefore$ The field $\vec{E}$ at a point $\vec{r}$ on a plane perp. to $\vec{k}$ is

$$\boxed{\vec{E}(\vec{r},t) = E_0 \cos(\omega t - \vec{k}\cdot\vec{r} + \phi_0)}$$

The relationship between time and space for a given phase ϕ corresponding to a max. field.

$$\phi = \omega t - kz + \phi_0 = \text{const.}$$

During a time interval δt, this const. phase moves a distance δz,

$\therefore$ The phase velocity $v = \frac{\delta z}{\delta t}$

$$v = \frac{dz}{dt} = \frac{\omega}{k} = \nu\lambda$$

frequency

$\Delta\phi$, phase difference, ~~the~~ between two points separated by Δz is simply $\boxed{\Delta\phi = k\Delta z}$

$\because$ ωt is the same for each point.

If $\Delta\phi$ is 0 or multiples of 2π then two points are in phase

B. Maxwell's Wave Eq. and Diverging Waves

There are many types of possible EM waves all of which must ~~obet~~ obey a special wave eq. describing the time and space dependence of $\vec{E}$.

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} = \varepsilon_0 \varepsilon_r \mu_0 \frac{\partial^2 E}{\partial t^2}$$

$\vec{E}$ must obey Maxwell's EM Wave eq.

ε_0 : Absolute permittivity
μ_0 : Absolute permeability
ε_r : Relative permittivity of the medium.

A spherical wave, emerges from a point EM source whose amplitude decays with the distance r from the source.

$$E = \frac{A}{r} \cos(\omega t - kr)$$

A soln. of Maxwell's EM wave eq.

Optical divergence, refers to the angular separation of wavevectors on a given wavefront.

Many light beams, such as the output from a laser, can be assumed as Gaussian beams.

It slowly diverges. The light intensity distribution across the beam cross section is Gaussian.

- The beam diameter, $2w$, is defined such that πw^2 contains %85 of the beam power

- The finite width $2w_0$ where the wavefronts are // is called waist of the beam.

$w_0 \rightarrow$ waist radius
$2w_0 \rightarrow$ spot size

Far away from the source, the beam diameter $2w$ increases linearly with distance z,
The increase in $2w$ with z makes an angle 2θ at O, which is called beam divergence:

$$2\theta = \frac{4\lambda}{\pi (2w_0)}$$

$\rightarrow$ The greater the waist, the narrower the divergence.

Ex 1.1.1. A diverging laser beam

A HeNe laser beam at 633 nm with a spot size of 10mm. $2\theta = ?$

$$2\theta = \frac{4\lambda}{\pi (2w_0)} = \frac{4 \cdot (633 \times 10^{-9}\,m)}{\pi (10 \times 10^{-3}\,m)} = (8.06 \times 10^{-5}\,rad) = 0.0046^\circ$$

1.2 Refractive Index

When an EM wave is travelling in a dielectric medium, the oscillating electric field polarizes the molecules of the medium at the same frequency.

Indeed, EM-wave propagation can be treated as the propagation of the polarizatio

-> Delay the propagation of EM-wave

=> The stronger the interaction between dipoles and the field, the slower the propagation of the EM wave.

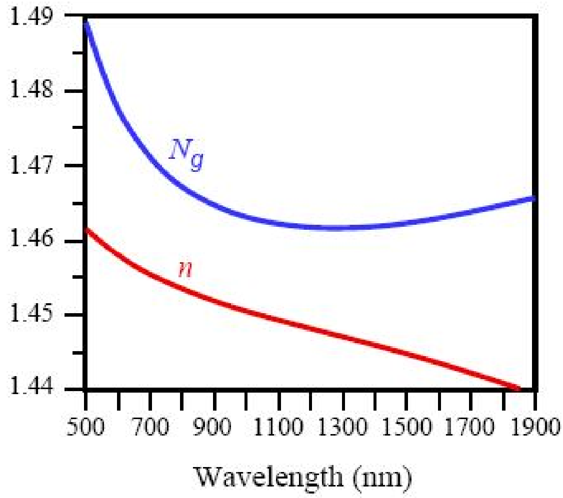
The phase velocity,
$$\boxed{v = \frac{1}{\sqrt{\varepsilon_r \varepsilon_0 \mu_0}}}$$
(non-magnetic medium)

for vacuum, the relative permittivity, $\varepsilon_r = 1$,

$$v_{vacuum} = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = c = 3 \times 10^8 \, m/s$$

⟶ The ratio of the speed of light in free-space to its speed in a medium is called the refractive index of the medium.

Refractive index
$$\boxed{n = \frac{c}{v} = \sqrt{\varepsilon_r}}$$



Refractive index n and the group index N_g of pure SiO_2 (silica) glass as a function of wavelength.

Ex. 1.3.1 Group Velocity

Consider two sinusoidal waves that are close in freq.:

$$\omega_1 = \omega - \delta\omega \qquad k_1 = k - \delta k$$
$$\omega_2 = \omega + \delta\omega \qquad k_2 = k + \delta k$$

The resultant wave

$$E_x(z,t) = E_o \cos[(\omega - \delta\omega)t - (k - \delta k)z] + E_o \cos[(\omega + \delta\omega)t - (k + \delta k)z]$$

$$E_x(z,t) = \underbrace{2E_o \cos[(\delta\omega)t - (\delta k)z]} \cos[\omega t - kz]$$

Amplitude modulated by a very slowly varying sinusoidal freq. of $\delta\omega$

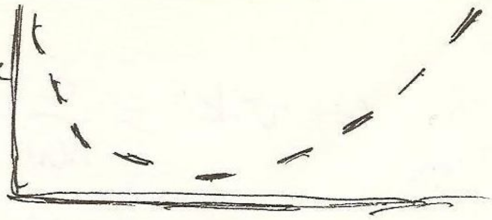
→ The max. of the field occurs when $(\delta\omega)t - (\delta k)z = 2m\pi = \text{const.}$ → m: integer

which travels with a velocity

$$\frac{dz}{dt} = \frac{\delta\omega}{\delta k} \quad \text{or} \quad v_g = \frac{d\omega}{dk}$$

This is group velocity, since it determines the speed of propagation of max. el-field along z.

$$\omega = v k = \frac{c}{n}\frac{2\pi}{\lambda} = \frac{c}{n} k$$



$$\frac{1.7}{Ek}$$

$$\frac{d\omega}{dk} = \frac{d}{dk}\left(\frac{c}{n}k\right)$$

$$= ck\left(\frac{d}{dk}n^{-1}\right) + c\frac{1}{n}\frac{dk}{dk}$$

$$= ck(-1)\frac{1}{n^2}\frac{dn}{dk} + \frac{c}{n}$$

$$= \frac{c}{n}\left\{1 - \frac{k}{n^2}\frac{dn}{dk}\right\}$$

$$\frac{dn}{dk} = \frac{dn}{d\lambda}\cdot\frac{d\lambda}{dk}$$

$$= \frac{dn}{d\lambda}\cdot\left(-\frac{2\pi}{k^2}\right)$$

$$= \frac{dn}{d\lambda}\left(-\frac{\lambda^2}{2\pi}\right)$$

$$=$$

$$\lambda = \frac{2\pi}{k}$$

$$\frac{d\lambda}{dk} = -\frac{2\pi}{k^2}$$

$$\frac{2\pi}{k^2} = \frac{2\pi}{(2\pi)^2}\lambda^2 = \frac{\lambda^2}{2\pi}$$

$$\frac{d\omega}{dk} = \frac{c}{n}\left\{1 - \frac{k}{n^2}\cdot\frac{dn}{d\lambda}\cdot\left(-\frac{2\pi}{k^2}\right)\right\}$$

$$= \frac{c}{n}\left\{1 + \frac{2\pi}{nk}\frac{dn}{d\lambda}\right\}$$

$$= \frac{c}{n}\left\{1 + \frac{\lambda}{n}\frac{dn}{d\lambda}\right\}$$

$$\cancel{= \frac{c}{n^2}\left\{n + \lambda\frac{dn}{d\lambda}\right\}}$$

$$= \frac{c}{n}\frac{1}{1 - \frac{\lambda}{n}\frac{dn}{d\lambda}}$$

Similar to $(1+x)$

$$\frac{1}{1-t} = \sum_{i=0}^{n} t^n, |t|<$$

$$\omega = v \cdot k = \frac{c}{n(\lambda)} \cdot k = \frac{c}{n(\lambda)} \left(\frac{2\pi}{\lambda}\right)$$

$$\frac{d\omega}{dk} = \frac{d\omega}{d\lambda} \cdot \frac{d\lambda}{dk}$$

$$\frac{d\omega}{d\lambda} = 2\pi c \frac{d}{d\lambda} \left\{ \frac{1}{n\lambda} \right\}$$

$$= 2\pi c \left\{ \frac{1}{n} \frac{d}{d\lambda} [\lambda^{-1}] + \frac{1}{\lambda} \frac{d}{d\lambda} [n^{-1}] \right\}$$

$$= 2\pi c \left\{ -\frac{1}{n} \lambda^{-2} \frac{d\lambda}{d\lambda} + \frac{1}{\lambda} (-1) n^{-2} \frac{dn}{d\lambda} \right\}$$

$$= -2\pi c \left\{ \frac{1}{n\lambda^2} + \frac{1}{\lambda n^2} \frac{dn}{d\lambda} \right\}$$

$$= \frac{-2\pi c}{n\lambda} \left\{ \frac{1}{\lambda} + \frac{1}{n} \frac{dn}{d\lambda} \right\}$$

$$\frac{d\lambda}{dk} = \frac{d}{dk} \left(\frac{2\pi}{k}\right) = -\frac{2\pi}{k^2} = -\frac{2\pi}{(2\pi)^2} \cdot \lambda^2 = -\frac{\lambda^2}{2\pi}$$

$$\frac{d\omega}{dk} = \left(-\frac{2\pi c}{n\lambda}\right) \cdot \left\{ \frac{1}{\lambda} + \frac{1}{n} \frac{dn}{d\lambda} \right\} \left(-\frac{\lambda^2}{2\pi}\right)$$

$$= \frac{c}{n} \lambda \left\{ \frac{1}{\lambda} + \frac{1}{n} \frac{dn}{d\lambda} \right\}$$

$$= \frac{c}{n} \left\{ 1 + \frac{\lambda}{n} \frac{dn}{d\lambda} \right\}$$

$$= \frac{c}{n^2} \left\{ n + \lambda \frac{dn}{d\lambda} \right\}$$

1.4 Magnetic field, Irradiance and Poynting Vector

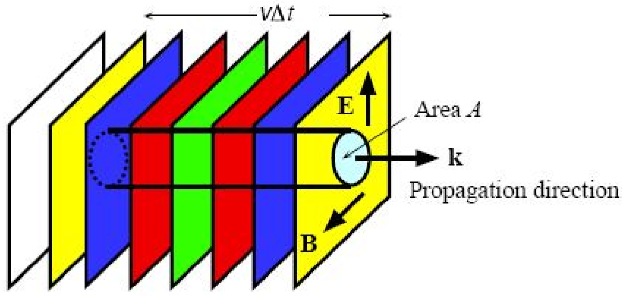
E_x and B_y

v: the phase velocity of an EM-W in an isotropic dielectric medium.

$\therefore$ ALWAYS and ANYWHERE in an EM-W

$$\boxed{E_x = v B_y = \frac{c}{n} B_y} \quad (1)$$

$$\boxed{v = \left[\varepsilon_0 \varepsilon_r \mu_0\right]^{-\frac{1}{2}}} \quad \text{and} \quad \boxed{n = \sqrt{\varepsilon_r}}$$



A plane EM wave travelling along **k** crosses an area A at right angles to the direction of propagation. In time Δt, the energy in the cylindrical volume $Av\Delta t$ (shown dashed) flows through A.

There is an energy flow along $\vec{k}$,

→ A small region of space (with E_x) has an en. density (en. per. unit vol.):

$$\frac{1}{2}\varepsilon_0 \varepsilon_r E_x^2$$

→ Similarly the region with B_y has an en. dens.

$$\frac{1}{2\mu_0} B_y^2$$

Two fields are related by eq.(1) $\quad \frac{1}{2}\varepsilon_0 \varepsilon_r E_x^2 = \frac{1}{2\mu_0} B_y^2$

$\therefore$ Total en. density is $\boxed{\varepsilon_0 \varepsilon_r E_x^2}$

Suppose $\quad S$ = En. flow per. unit time per. Area

volume

Instanteneous Irradiance ← $\quad \boxed{S = \frac{(A v \Delta t)(\varepsilon_0 \varepsilon_r E_x^2)}{A \cdot \Delta t} = v \varepsilon_0 \varepsilon_r E_x^2 = v^2 \varepsilon_0 \varepsilon_r E_x B_y}$

In vector form $\quad \boxed{\bar{S} = v^2 \varepsilon_0 \varepsilon_r \bar{E} \wedge \bar{B}}$ Poynting Vector

$|\bar{S}|$ → Irradiance (or intensity?)

$$\vec{S} = v^2 \varepsilon_0 \varepsilon_r \vec{E} \wedge \vec{B}$$

At the receivers location, E_x varies sinusoidally.

To calculate the average irradiance, we write the field as $\boxed{E_x = E_0 \sin \omega t}$.

Averaging S over one period:

$$I = S_{avg} = \frac{1}{2} v \varepsilon_0 \varepsilon_r E_0^2$$

$v = \frac{c}{n}$ and $\varepsilon_r = n^2$

Average Irradiance (Intensity)

$$\boxed{\begin{aligned} I &= S_{avg} = \frac{1}{2} c \varepsilon_0 n E_0^2 \\ I &= S_{avg} = \left(1.33 \times 10^{-3} \frac{F}{s}\right) n E_0^2 \end{aligned}}$$

Exp. 1.4.1. Electric and Magnetic fields in Light

A red laser beam from a He-Ne laser at a certain position produces an intensity of $1 \text{mW}/\text{cm}^2$

$E_0 = ?$, $B_0 = ?$ if the medium is glass with $n = 1.45$

The irradiance

$$I = \frac{1}{2} c \varepsilon_0 n E_0^2$$

$$E_0 = \sqrt{\frac{2I}{c\varepsilon_0 n}} = \sqrt{\frac{2 \times (1 \times 10^{-3} \times 10^{+4} \, \text{W}/\text{m}^2)}{(3 \times 10^8 \, \text{m}/\text{s})(8.85 \times 10^{-12} \, \text{F}/\text{m})(1.00)}}$$

in vacuum

$$E_0 = 87 \, \text{V}/\text{m}$$

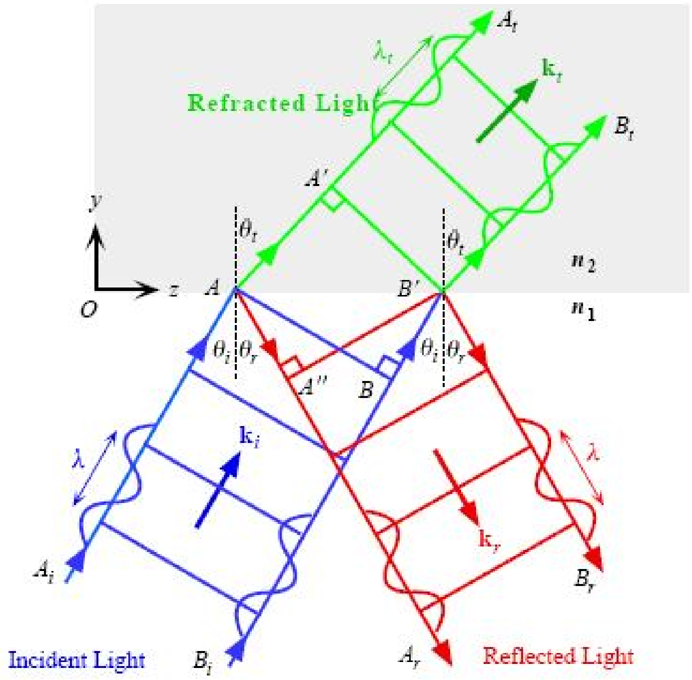
$$B_0 = \frac{E_0}{c} = (87 \, \text{V}/\text{m}) / (3 \times 10^8 \, \text{m}/\text{s}) = 0.29 \, \mu\text{T}$$

If in glass $n = 1.45$

$$E_{0(medium)} = \sqrt{\frac{2(1 \times 10^{-3} \times 10^4)}{(3 \times 10^8)(8.85 \times 10^{-12})(1.45)}} =$$

$$E_{0(medium)} = 72 \, \text{V}/\text{m}$$

$$B_{0(medium)} = n \frac{E_{0(medium)}}{c} = 1.45 \frac{(72 \, \text{V}/\text{m})}{(3 \times 10^8 \, \text{m}/\text{s})} = \boxed{0.35 \, \mu\text{T}}$$



A light wave travelling in a medium with a greater refractive index $(n_1 > n_2)$ suffers reflection and refraction at the boundary.

Figure 1.9

1.5. Snell's Law and Total Internal Reflection (TIR)

EM-W propagating from medium 1 to medium 2

Transmitted wave → refracted light

Fig 1.9.

Reflected waves)

→ Two waves are in phase (A_i and B_i)

A_r and B_r (reflected waves) must be in phase not to interfere destructively.

To do so θ_i must be equal to θ_r

Refracted Waves

The velocities of A_i and B_i are different then A_t and B_t

The wavefront AB becomes the front A'B' in medium 2

Refraction

$$BB' = v_1 t = \frac{c}{n_1} t \quad \text{and} \quad AA' = v_2 t = \frac{c}{n_2} t$$

And

$$AB' = \frac{BB'}{\sin\theta_i} \quad \text{and} \quad AB' = \frac{AA'}{\sin\theta_t}$$

$$\therefore \quad AB' = \frac{v_1 t}{\sin\theta_i} = \frac{v_2 t}{\sin\theta_t}$$

$$\boxed{\frac{\sin\theta_i}{\sin\theta_t} = \frac{v_1}{v_2} = \frac{n_2}{n_1}}$$

Snell's Law

Reflection

$$BB' = AA'' = v_1 t$$

$$AA' = \frac{v_1 t}{\sin\theta_i} = \frac{v_1 t}{\sin\theta_r} \rightarrow \theta_i = \theta_r$$

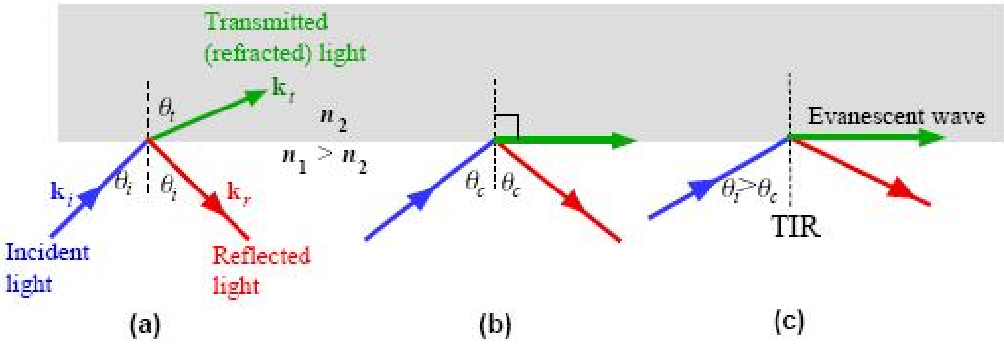
1-11.

When $\theta_t \to 90^\circ$, $\theta_i = \theta_c$ that is critical angle

$$\boxed{\sin\theta_c = \frac{n_2}{n_1}}$$

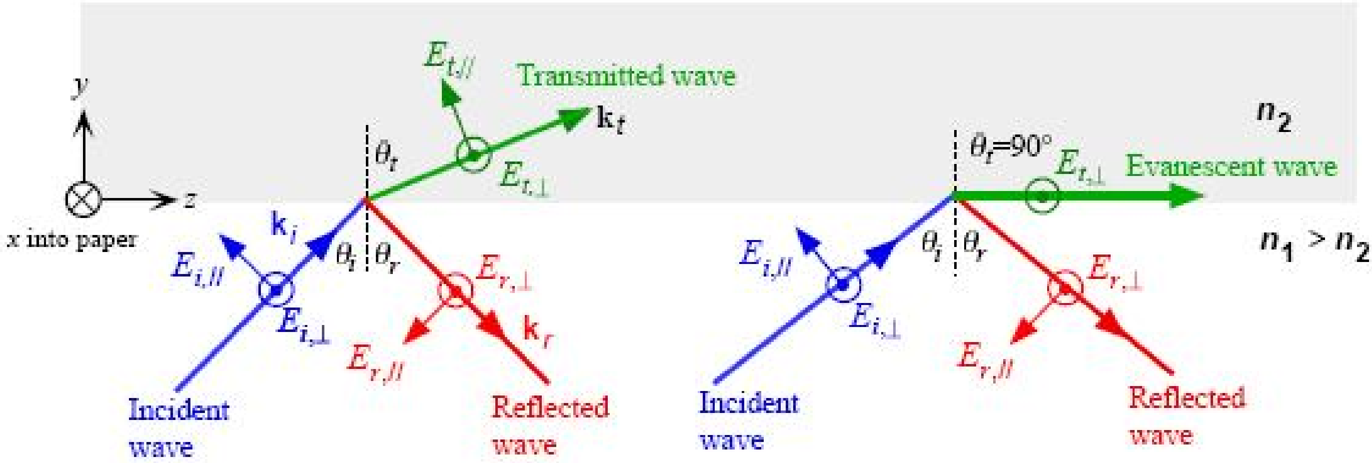
Total internal Reflection (TIR)

When θ_i exceeds θ_c $\longrightarrow$ there is no light transmitting



Light wave travelling in a more dense medium strikes a less dense medium. Depending on the incidence angle with respect to θ_c, which is determined by the ratio of the refractive indices, the wave may be transmitted (refracted) or reflected. (a) $\theta_i < \theta_c$ (b) $\theta_i = \theta_c$ (c) $\theta_i > \theta_c$ and total internal reflection (TIR).

Figure 1.10



(a) $\theta_i < \theta_c$ then some of the wave is transmitted into the less dense medium. Some of the wave is reflected.

(b) $\theta_i > \theta_c$ then the incident wave suffers total internal reflection. However, there is an evanescent wave at the surface of the medium.

Light wave travelling in a more dense medium strikes a less dense medium. The plane of incidence is the plane of the paper and is perpendicular to the flat interface between the two media. The electric field is normal to the direction of propagation . It can be resolved into perpendicular (⊥) and parallel (//) components

Figure 1.11

1.6 Fresnel's Equations

A. Amplitude Reflection and Transmission Coefficients (r and t)

we can resolve E_i into two-components : $E_{i//}$ – in the plane of incidence

$E_{i\perp}$ – perp. to the plane of incidence

Fig. 1.11

The fields $E_{i,\perp}$, $E_{r\perp}$ and $E_{t,\perp}$ $\rightarrow$ Transverse Electric field (TE) waves

waves with $E_{i,//}$, $E_{r,//}$ and $E_{t,//}$ $\longrightarrow$ Transverse Magnetic field (TM) waves

Incident Wave $\quad E_i = E_{i0} \exp j(\omega t - \vec{k}_i \cdot \vec{r})$

Reflected Wave $\quad E_r = E_{r0} \exp j(\omega t - \vec{k}_r \cdot \vec{r})$

Transmitted Wave $\quad E_t = E_{t0} \exp j(\omega t - \vec{k}_t \cdot \vec{r})$

Our objective is to find E_{r0} and E_{t0} with respect to E_{i0}.

Magnetic fields

$$B_\perp = \frac{n}{c} E_{//}$$

$$B_{//} = \frac{n}{c} E_\perp$$

Boundary Conditions at the boundary between media 1 and 2 : $y=0$

$$E_{tangential}(1) = E_{tangential}(2)$$

$$B_{tangential}(1) = B_{tangential}(2)$$

(Provided that two media are NON-magnetic : $\mu_r = 1$)

These can only be satisfied with

$$\theta_i = \theta_r \quad \text{and} \quad \sin\theta_i \, n_1 = \sin\theta_t \, n_2$$

Applying the boundary conditions above to the EM-W going from (1) to (2) in terms of incidence angle θ_i and ~~re~~ ref. index : $n = \frac{n_2}{n_1}$

Fresnel Equations :

→ Reflection and transmission coefficients for $E_\perp$

$$r_\perp = \frac{E_{r0,\perp}}{E_{i0,\perp}} = \frac{\cos\theta_i - [n^2 - \sin^2\theta_i]^{1/2}}{\cos\theta_i + [n^2 - \sin^2\theta_i]^{1/2}}$$

$$t_\perp = \frac{E_{t0,\perp}}{E_{i0,\perp}} = \frac{2\cos\theta_i}{\cos\theta_i + [n^2 - \sin^2\theta_i]^{1/2}}$$

→ Reflection and transmission coefficients for $E_{\parallel}$

$$r_{\parallel} = \frac{E_{r0,\parallel}}{E_{i0,\parallel}} = \frac{[n^2 - \sin^2\theta_i]^{1/2} - n^2\cos\theta_i}{[n^2 - \sin^2\theta_i]^{1/2} + n^2\cos\theta_i}$$

$$t_{\parallel} = \frac{E_{t0,\parallel}}{E_{i0,\parallel}} = \frac{2n\cos\theta_i}{n^2\cos\theta_i + [n^2 - \sin^2\theta_i]^{1/2}}$$

$$r_{\parallel} + n\,t_{\parallel} = 1 \quad \text{and} \quad r_\perp + 1 = t_\perp$$

The significance of these eqs.

Amplitudes and phases of reflected and transmitted waves can be determined from $r_\perp$, $r_{\parallel}$, $t_\perp$ and $t_{\parallel}$.

→ Take E_{i0} to be a real number

so that $r_\perp$ and $t_\perp$ correspond to phase changes (with respect to the incident)

If $r_\perp$ is a complex number, we can write $r_\perp = |r_\perp| \exp(-j\phi_\perp)$

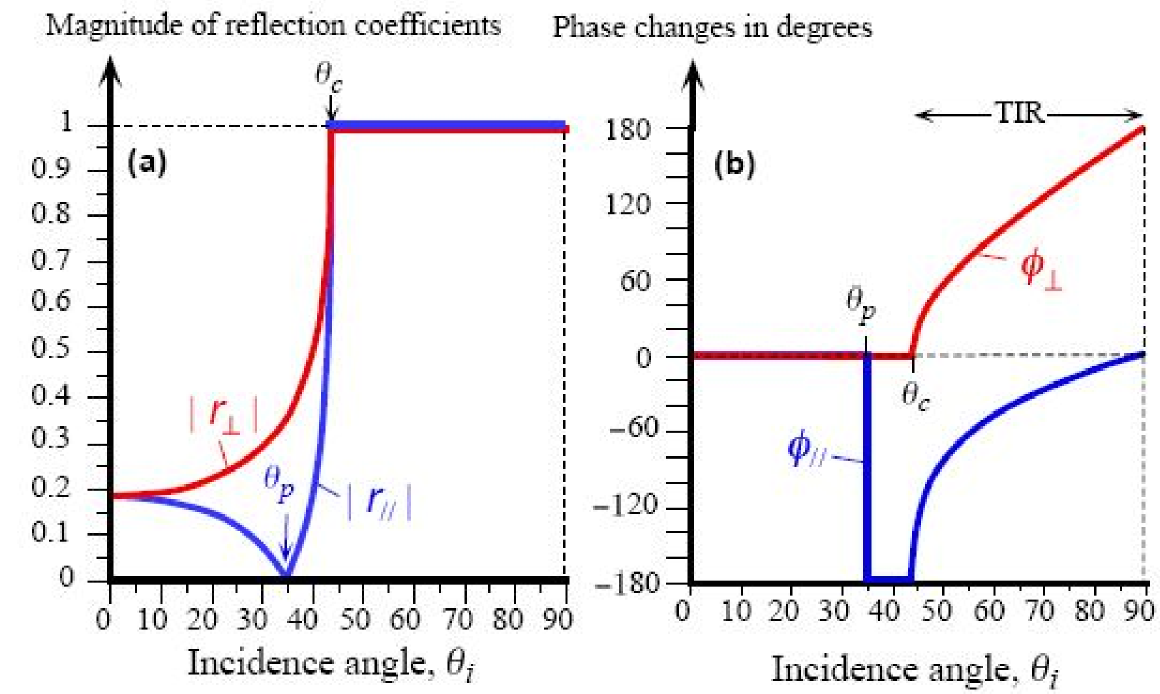
If $r_\perp$ is a real quantity, → (+ve) → no phase shift

→ (−ve) → phase shift of $180°$ (or π rad)

Complex numbers can be obtained only from $(\text{equations})^{1/2}$ of Fresnel's Eq.

∴ when $\boxed{n < 1 \ (n_1 > n_2)}$ and when $\boxed{\theta_i > \theta_c}$ TIR

Thus phase changes other than $0°$ or $180°$ occur, only when there is TIR



Internal reflection: (a) Magnitude of the reflection coefficients $r_{//}$ and $r_\perp$ vs. angle of incidence θ_i for $n_1 = 1.44$ and $n_2 = 1.00$. The critical angle is 44°. (b) The corresponding phase changes $\phi_{//}$ and $\phi_\perp$ vs. incidence angle.

Figure 1.12

- The critical angle $\theta_c \rightarrow \sin\theta_c = n_2/n_1 \rightarrow 44°$
- Incidence close to normal (θ_i is small) no phase change
- Drewster's Polarization angle $\boxed{\tan\theta_p = \frac{n_2}{n_1}}$ By equating $r_{\parallel}$ to 0

- The reflected wave is said to be linearly polarized

→ If $\theta_i > \theta_p$ but $\theta_i < \theta_c$

$r_{//}$ gives a (−ve) number ⇒ $\phi_{//} = 180°$

→ When $\theta_i > \theta_c$ ⇒ $|r_\perp| = |r_{//}| = 1$ in the presence of TIR

- Both (1a) and (2a) are complex quantities ∵ $\sin\theta_i > n$
- $r_\perp$ and $r_{//}$ both have " " :

$$r_\perp = 1 \cdot \exp(-j\phi_\perp)$$
$$r_{//} = 1 \cdot \exp(-j\phi_{//})$$

- The reflected wave has phase changes $\phi_\perp$ (corres. $E_\perp$) and $\phi_{//}$ (corres. $E_{//}$)

From (1a) $\theta_i > \theta_c$, $|r_\perp| = 1$ and $\phi_\perp$?

$$\perp \Rightarrow \quad \tan\left(\frac{1}{2}\phi_\perp\right) = \frac{\left[\sin^2\theta_i - n^2\right]^{1/2}}{\cos\theta_i}$$ Phase change in TIR

$$// \Rightarrow \quad \tan\left(\frac{1}{2}\phi_{//} + \frac{1}{2}\pi\right) = \frac{\left[\sin^2\theta_i - n^2\right]^{1/2}}{n^2\cos\theta_i}$$ Phase change in TIR

In TIR Amp. does NOT change but $\phi_{//}$ has additional π shift that makes $\phi_{//}$ negative

What happens to the transmitted wave when $\theta_i > \theta_c$?

Boundary conditions → still $\vec{E}$ field in medium 2

In medium 2, the wave travels near the SURFACE along z,

This is EVANESCENT WAVE, with decreasing its magnitude as we go into medium 2.

$$\boxed{E_{t,\perp}(y,z,t) = e^{-\alpha_2 y} \exp j(\omega t - \underbrace{k_{iz}}_{} z)}$$

$k_{iz} = k_i \sin\theta_i$ is wavevector along z-axis.

α_2 : Attenuation coefficient for E-field penetrating into medium 2

$$\boxed{\alpha_2 = \frac{2\pi n_2}{\lambda}\left[\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i - 1\right]^{1/2}}$$

Attenuation of evanescent wave

$$e^{-1} \rightarrow \boxed{y = \frac{1}{\alpha_2} = \delta} \rightarrow \boxed{\text{penetration depth}}$$

- If the incident light were coming from lighter (lower index) side ($n_1 < n_2$) then the reflection would be called external reflection.

↳ reflection from denser medium.

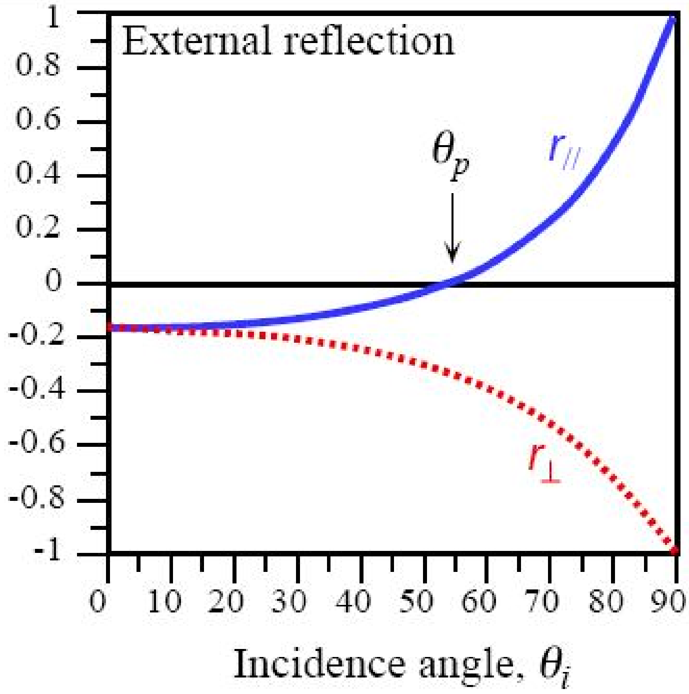
External reflection :

→ $r_\perp$ and $r_{//}$ are dependent of θ_i

→ Normal incidence, both are (−ve) → for ext. refl. there is a phase shift of 180°.

→ $r_{//} \rightarrow 0$ at $\theta_i = \theta_p$

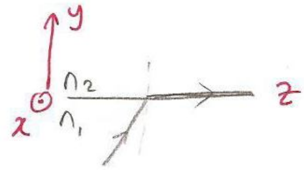
TRANSMITTED LIGHT in both External / Internal Refl. ($\theta_i < \theta_c$) DOES NOT EXPERIENCE A PHASE SHIFT



The reflection coefficients $r_{//}$ and $r_\perp$ vs. angle of incidence θ_i for $n_1 = 1.00$ and $n_2 = 1.44$.

Figure 1.13

Example 1.6.1. Evanescent Wave



$n_1 \longrightarrow n_2 \quad (n_1 > n_2)$

TIR $\rightarrow$ evanescent wave propagating in medium 2 near the boundary

Find the functional form of this wave and discuss how its magnitude varies with the distance into the medium 2.

Transmitted wave $\quad E_{t,\perp} = t_\perp E_{io,\perp} \exp j(\omega t - \vec{k}_t \cdot \vec{r})$ $\quad$ ($t_\perp$: transmission coeff.)

$$\vec{k}_t \cdot \vec{r} = y k_t \cos\theta_t + z k_t \sin\theta_t$$

Snell's Law for $\theta_i > \theta_c$,

$$\sin\theta_t = \frac{n_1}{n_2}\sin\theta_i > 1$$

and $\quad \cos\theta_t = \sqrt{[1-\sin^2\theta_t]} = \pm jA_2$ is purely imaginary number

$\therefore$ taking $\rightarrow \quad \cos\theta_t = -jA_2$

$$E_{t,\perp} = t_\perp E_{io,\perp} \exp j\left[\omega t - z k_t \sin\theta_t + j y k_t A_2\right]$$

$$= t_\perp \underbrace{E_{io,\perp} \exp(-y k_t A_2)} \exp j\left[\omega t - z k_t \sin\theta_t\right]$$

Amp. decays along y as $\boxed{\exp(-\alpha_2 y)}$ in which $\boxed{\alpha_2 = k_t A_2}$.

$\rightarrow$ Note that $+jA_2$ is ignored. $\because$ it implies a growing amp. in medium 2.

Travelling Wave part: $\quad \exp j\left[\omega t - z k_t \sin\theta_t\right]$

Snell's Law $\rightarrow \quad k_t \sin\theta_t = k_i \sin\theta_i$ but $\rightarrow$ $k_i \sin\theta_i = k_{iz}$ along z!

Since $k_{iz} = k_i \sin\theta_i$;

the evanescent wave propagates along z (boundary) at the same speed as the INCIDENT and REFLECTED waves

Furthermore, TIR $\rightarrow \sin\theta_i > n_2/n_1$

$$\therefore \quad t_\perp = \frac{n_i \cos\theta_i}{\cos\theta_i + \left[\left(\frac{n_2}{n_1}\right)^2 - \sin^2\theta_i\right]^{1/2}} = t_{\perp 0} \exp(j\psi_\perp)$$

- $t_{\perp 0} \exp(j\psi_\perp)$ is a complex number, since $t_{\perp 0}$ is a real one.

  and $\psi_\perp$ is phase change

- $t_\perp$ does not change the general behavior of propagation along z and the penetration along y.

B. Intensity, Reflectance and Transmittance

It is frequently necessary to calculate the intensity of the reflected and transmitted waves, when the light is travelling from n_1 to n_2.

Light intensity with a velocity v and an amp. E_0

$$\boxed{I = \frac{1}{2} v \varepsilon_r \varepsilon_0 E_0^2}$$

Since, $\frac{1}{2}\varepsilon_r\varepsilon_0 E_0^2$ is en. per unit vol.

when multiplied with v it gives the rate at which en. is transferred though a unit area.

Since $v = \frac{c}{n}$ and $\varepsilon_r = n^2$

The intensity is proportional to $\rightarrow$ $\boxed{I \propto n E_0^2}$

Reflectance, R measures the intensity of ref. light to that of incident.

$$R_\perp = \frac{|E_{r0,\perp}|^2}{|E_{i0,\perp}|^2} = |r_\perp|^2 \quad \text{and} \quad R_{//} = \frac{|E_{r0,//}|^2}{|E_{i0,//}|^2} = |r_{//}|^2$$

Although the reflection coefficients can be complex numbers, that can represent phase changes,

REFLECTANCES are necessarily REAL numbers representing the intensity changes.

For normal incidence (from Eq. 1 and 2)

$$\boxed{R = R_\perp = R_{//} = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2}$$

Ex. $n_1 = 1 \rightarrow n_2 = 1.5$ (glass)

$\sim$%4 of incident radiation on air-glass surface is reflected back.

Transmittance, T, relates the intensities of transmitted and incident waves.

For normal incidence,

$$T_\perp = \frac{n_2 |E_{t0,\perp}|^2}{n_1 |E_{i0,\perp}|^2} = \left(\frac{n_2}{n_1}\right) |t_\perp|^2 \quad \text{and} \quad T_\parallel = \frac{n_2 |E_{t0,\parallel}|^2}{n_1 |E_{i0,\parallel}|^2} = \left(\frac{n_2}{n_1}\right) |t_\parallel|^2$$

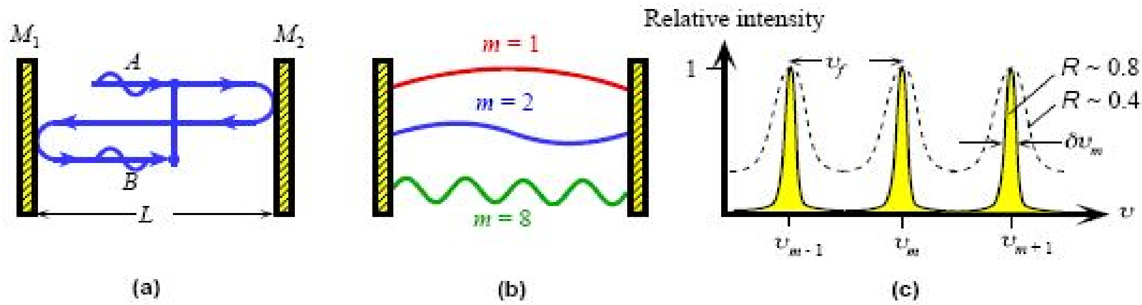
$$T = T_\perp = T_\parallel = \frac{4 n_1 n_2}{(n_1 + n_2)^2}$$

1.7 Multiple Interference and Optical Resonators

- An electrical resonator

→ Parallel LC circuit
- oscillates at the resonant frequency f_0
- stores energy at f_0
- acts as a filter at f_0

- An optical resonator
  - stores energy and ~~acts~~ filters light at f_0



Schematic illustration of the Fabry-Perot optical cavity and its properties. (a) Reflected waves interfere. (b) Only standing EM waves, *modes*, of certain wavelengths are allowed in the cavity. (c) Intensity vs. frequency for various modes. *R* is mirror reflectance and lower *R* means higher loss from the cavity.

Figure 1.16

- Two perfectly reflecting mirrors M_1 and M_2
- A series of allowed standing waves.

∵ $\vec{E}$ is zero at metallic mirrors.

$$\boxed{m\left(\frac{\lambda_m}{2}\right) = L} \quad ; m = 1, 2, 3 \ldots$$

Cavity mode

Resonant frequencies :

$$\boxed{\nu_m = m\left(\frac{c}{2L}\right) = m\nu_f} \qquad \boxed{\nu_f = \frac{c}{2L}}$$

FREE SPECTRAL RANGE

$$\boxed{\Delta\nu_m = \nu_{m+1} - \nu_m = \nu_f}$$

This optical cavity with its mirrors, etalon, serves to STORE en at certain frequencies.

Fabry-Perot Optical Resonator

- A travelling toward right

  After one round trip

- B travelling toward right with a phase difference and with a loss in magnitude $(R<1)$

$M_1 \equiv M_2$ (identical) with ref. coef. "r"

A and B interfere:

two reflections → phase diff. is "$2kL$"

$$A+B = A + Ar^2 \exp(-j2kL)$$

After infinite round-trip reflections:

Resultant field:

$$E_{cavity} = A + B + \cdots$$
$$= A + Ar^2\exp(-j2kL) + Ar^4\exp(-j4kL) + Ar^6\exp(-j6kL) + \cdots$$

$$\boxed{E_{cavity} = \frac{A}{1-r^2\exp(-j2kL)}}$$

$I_{cavity} = |E_{cavity}|^2$ and $R = r^2$

Cavity Intensity

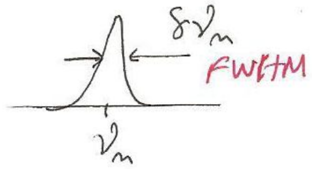
$$I_{cavity} = \frac{I_0}{(1-R)^2 + 4R\sin^2(kL)} \qquad I_0 = A^2$$

Maximum Cavity Intensity

$$\boxed{I_{max} = \frac{I_0}{(1-R)^2}} \; ; \; \boxed{k_m L = m\pi}$$

A Smaller R ⟶ More Radiation Loss from the cavity

Spectral width, $\delta\nu_m$ of the F-P etalon



$$\delta\nu_m = \frac{\nu_f}{F} \qquad ; F = \frac{\pi R^{1/2}}{1-R}$$

Finesse of resonator

$F \uparrow$ as Losses $\downarrow$ $(R\uparrow)$

Application: F-P cavities used in lasers

→ partially reflecting and transmitting plates

→ The ~~output~~ part of incident light enters $\quad I_{incident}(1-R)$

→ only allowed modes exist in cavity

→ A fraction ~~sum~~ of intensity in cavity comes out! $\quad I_{cavity}(1-R)$

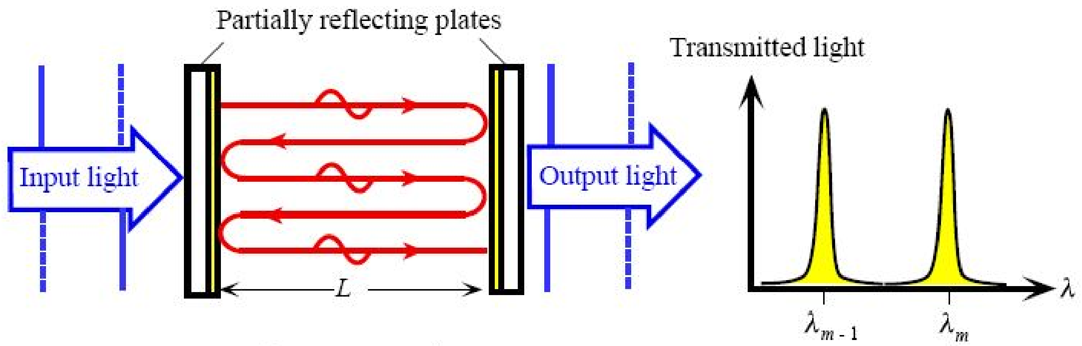
"Tunning Capability" ⟶ cavity length

$$kL = m\pi$$

$$I_{transmitted} = I_{incident}\frac{(1-R)^2}{(1-R)^2 + 4R\sin^2(kL)}$$

→ If medium exists: $k \rightarrow nk$

angle of incidence: $k \rightarrow k\cos\theta$



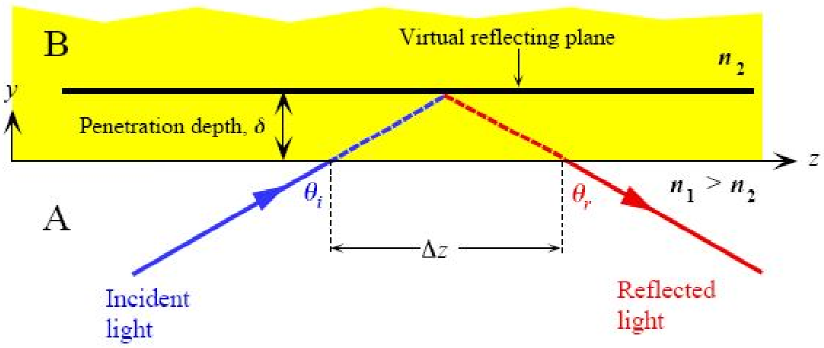
Fabry-Perot etalon

Transmitted light through a Fabry-Perot optical cavity.

Figure 1.17

1-8 Goos Hänchen Shift and Optical Tunnelling

$n_1 \to n_2$ (where $n_1 > n_2$) and $\theta_i > \theta_c$ | TIR



The reflected light beam in total internal reflection appears to have been laterally shifted by an amount Δz at the interface.

Figure 1.18

- Reflection from virtual plane inside the optically less dense medium.
- This lateral shift,
  - → Reflected beam experiences a phase change ϕ
  - → $\vec{E}$ penetrates into the second medium by a depth $\delta = 1/\alpha_2$

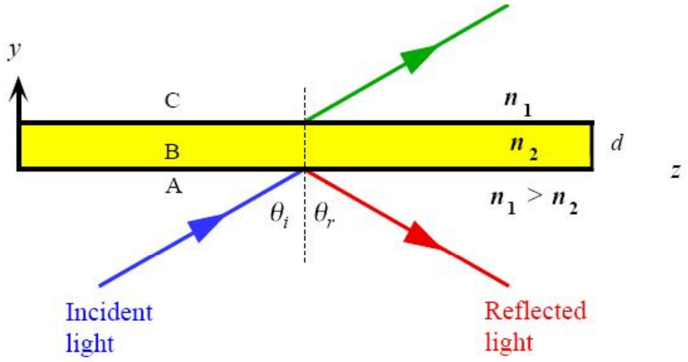
along evanescent wave ← $\boxed{\Delta z = 2\delta \tan\theta_i}$

exp.: $\lambda = 1\mu m$ at $\theta_i = 85^\circ$ at a glass-glass $\left(\begin{smallmatrix} n_1 = 1.450 \\ n_2 = 1.430 \end{smallmatrix}\right)$ interface

from ex. 1.6.2 ← $\delta = 0.78\mu m \longrightarrow \Delta z \approx 18\mu m$

Optical Tunnelling

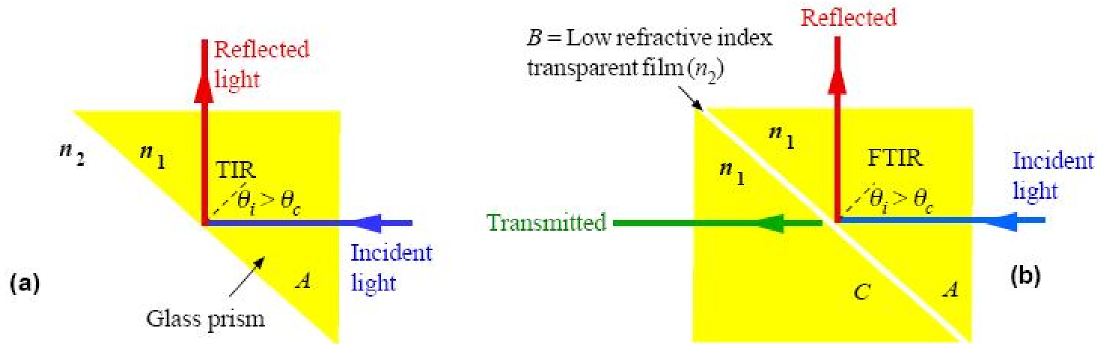
FTIR
"Frustrated Total Internal Ref."



When medium B is thin (thickness d is small), the field penetrates to the BC interface and gives rise to an attenuated wave in medium C. The effect is the tunnelling of the incident beam in A through B to C.

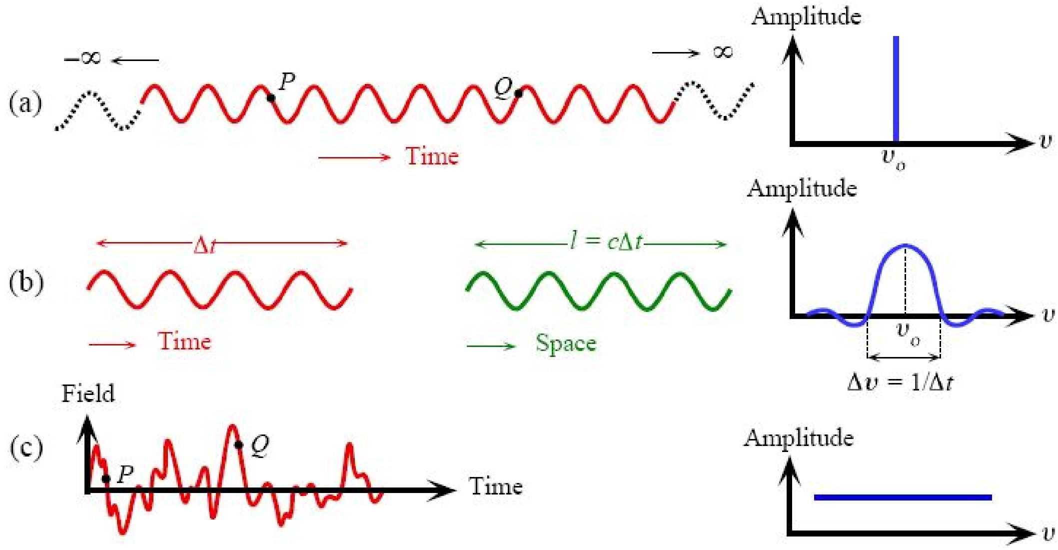
Figure 1.19

Application: Beam splitters



(a) A light incident at the long face of a glass prism suffers TIR; the prism deflects the light.
(b) Two prisms separated by a thin low refractive index film forming a beam-splitter cube. The incident beam is split into two beams by FTIR.

Figure 1.20



(a) A sine wave is perfectly coherent and contains a well-defined frequency υ_o. (b) A finite wave train lasts for a duration Δt and has a length l. Its frequency spectrum extends over $\Delta\upsilon = 1/\Delta t$. It has a coherence time Δt and a coherence length l. (c) White light exhibits practically no coherence.

Figure 1.21

1.9. Temporal and Spatial Coherence

$$E_x = E_o \sin(\omega_o t - k_o z)$$

Fig 1.21

Fig 1.21 (b) → wave-train of length $l = c\Delta t$
(existing only over a finite time interval Δt)

Radiation emission process
Modulation output from a laser
etc.
→ Δt (amplitude not be constant over Δt)

Δt → coherence time
l → ($l = c.\Delta t$) coherence length

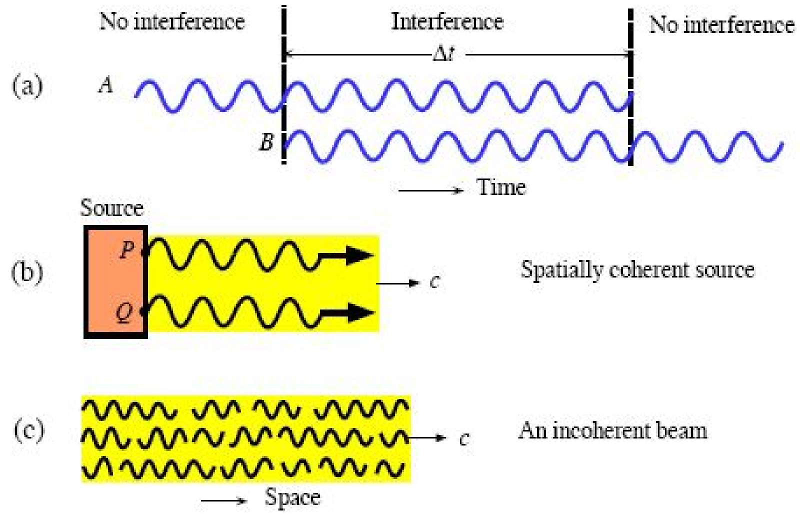
$\Delta \nu$ ⇒ spectral width

$$\Delta \nu = \frac{1}{\Delta t}$$

exp: for a sodium lamp, the spec. width $\Delta \nu \approx 5 \times 10^{11}$ Hz
$\therefore \Delta t \approx 2 \times 10^{-12}$ s $= 2$ps
$\therefore l = 6 \times 10^{-4}$ m $= 0.60$mm

exp: He-Ne laser operating in multimode $\Delta \nu \approx 1\text{-}5 \times 10^9$ Hz
$\therefore l = 200$mm

exp: He-Ne laser in continuous mode. $\Delta \nu = ?$
$\therefore l$ → several hundereds of meters



(a) Two waves can only interfere over the time interval Δt. (b) Spatial coherence involves comparing the coherence of waves emitted from different locations on the source. (c) An incoherent beam.

Waves A and B of ν_0 coincide only within Δt

∴ They ∴ have "Mutual Temporal Coherence" over Δt.

· when they arrive at the destination they can interfere only over a space portion $c \cdot \Delta t$.

exp: Young's double slit exp. can be used to measure mutual temp-coh.

Spatial Coherence;

If the waves emitted from P and Q (Fig 1.22.b) are in phase, then P and Q are spatially coherent

• A light beam emerging from a spatially coherent light source will have exhibit spatial coherence across the beam cross section.

→ that is waves are in phase over coherence length $c \cdot \Delta t$.