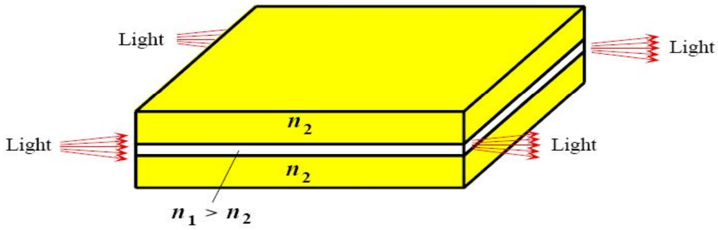
2. DIELECTRIC WAVEGUIDES and OPTICAL FIBRES

2.1 Symmetric Planar Dielectric Slab Waveguide

A. Waveguide condition

The (central) region of higher ref. index (n_1) ⟶ CORE

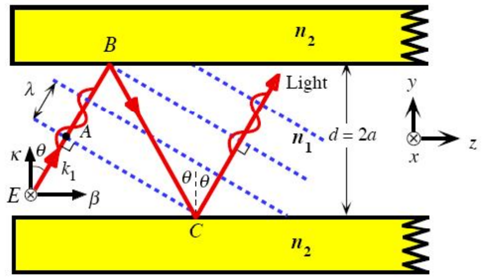
The lower ref. in. region (n_2) sandwiching the core ⟶ CLADDING



A planar dielectric waveguide has a central rectangular region of higher refractive index n_1 than the surrounding region which has a refractive index n_2. It is assumed that the waveguide is infinitely wide and the central region is of thickness $2a$. It is illuminated at one end by a monochromatic light source.

Figure 2.1

→ Beam diameter is less than the slab thickness $2a$



A light ray travelling in the guide must interfere constructively with itself to propagate successfully. Otherwise destructive interference will destroy the wave.

- Two TIRs at B and C each introduces a phase change ϕ.
- In n_1 the wavevector $k_1 = kn_1 = \frac{2\pi}{\lambda} n_1$
- For constructive interference, the phase diff. ($\Delta\phi$) between A and C must be a multiple of 2π

$$\Delta\phi(AC) = k_1 \underbrace{(AB+BC)}_{2d\cos\theta} - 2\phi = \underbrace{m}_{\text{Integer}}(2\pi)$$

$m = 0, 1, 2 \ldots$

$$\therefore \quad \boxed{k_1 \big[\underbrace{2d}_{2a} \cos\theta\big] - 2\phi = m(2\pi)}$$

Constructive Interference

→ Only certain θ and ϕ values satisfy this eq. for a given m.

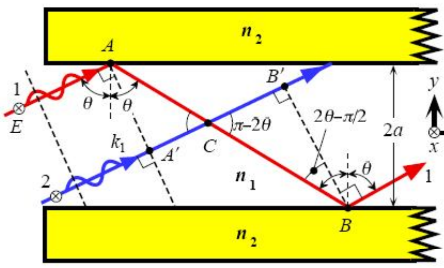
→ ϕ depends on θ

→ ϕ depends on the polarization state of the wave

$$\boxed{\left[\frac{2\pi n_1 (2a)}{\lambda}\right] \cos\theta_m - \phi_m = m\pi}$$

Waveguide Condition

For wider angles :



Two arbitrary waves 1 and 2 that are initially in phase must remain in phase after reflections. Otherwise the two will interfere destructively and cancel each other.

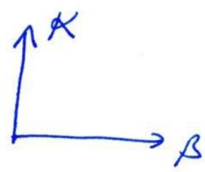
Figure 2.3

Ray 1 $\xrightarrow{\text{at } B}$ $k_1 AB - 2\phi$

Ray 2 $\xrightarrow{\text{at } B'}$ $k_1 (A'B')$

Difference must be $m(2\pi)$

Two propagation constants, β and κ



Prop. constant along the guide

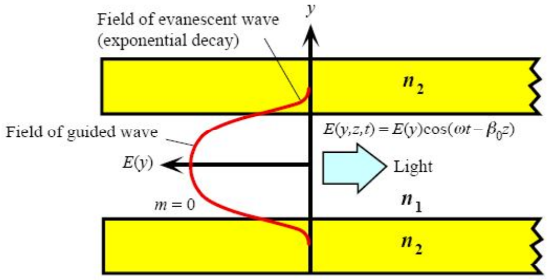
$$\beta_m = k_1 \sin\theta_m = \frac{2\pi n_1}{\lambda} \sin\theta_m$$

$$\kappa_m = k_1 \cos\theta_m = \frac{2\pi n_1}{\lambda} \cos\theta_m$$

$m = 0, 1, 2 \ldots$

Transverse prop. constant

If we consider interference of many rays →
we would find stationary $\vec{E}$ pattern along y-dir;
this pattern travels along z-dir. with a prop. const. β_m



The electric field pattern of the lowest mode traveling wave along the guide. This mode has $m = 0$ and the lowest θ. It is often referred to as the glazing incidence ray. It has the highest phase velocity along the guide.

Figure 2.5

2.3.

The two rays meet at C.

→ The optical path diff. between A'C and AC plus phase ch. ϕ_m (reflection at A)

The phase diff. between 1 and 2 is

$$\Phi_m = (k_1 AC - \phi_m) - k_1 A'C$$

$$\Phi_m = 2k_1(a-y)\cos\theta_m - \phi_m$$

Φ_m : phase diff.

ϕ_m : phase change due to reflec.

by substituting Eq.3. (waveguide condition)

$$\Phi_m = \Phi_m(y) = m\pi - \frac{y}{a}(m\pi + \phi_m)$$

→ phase diff. as a func. of y !

→ Just before C, 1 and 2 have opposite k_y terms due to the travel in opposite y-directions.

∴ Electric fields of 1 and 2.

$$E_1(y,z,t) = E_0 \cos(\omega t - \beta_m z + k_m y + \Phi_m)$$

$$E_2(y,z,t) = E_0 \cos(\omega t - \beta_m z - k_m y)$$

Interference :

$$\boxed{E(y,z,t) = 2E_0 \underbrace{\left(\cos k_m y + \tfrac{1}{2}\Phi_m\right)} \underbrace{\cos\left(\omega t - \beta_m z + \tfrac{1}{2}\Phi_m\right)}}$$

- No time dependence
- Modulates amp. along y-dir.
- Standing wave pattern along y-dir.

Travelling along z-dir.

$$\Phi_m = 2k_1(a-y)\cos\theta_m - \phi_m$$

$$k_1 2a\cos\theta_m - \phi_m = m\pi$$

$$2k_1\cos\theta_m = \frac{1}{a}\left\{m\pi + \phi_m\right\}$$

$$= (a-y)\frac{1}{a}\left\{m\pi + \phi_m\right\} - \phi_m$$

$$= m\pi + \phi_m - \frac{y}{a}\left\{m\pi + \phi_m\right\} - \phi_m$$

$$\Phi_m = m\pi - \frac{y}{a}\left\{m\pi + \phi_m\right\}$$

→ Each (m) value gives a different K_m and Φ_m

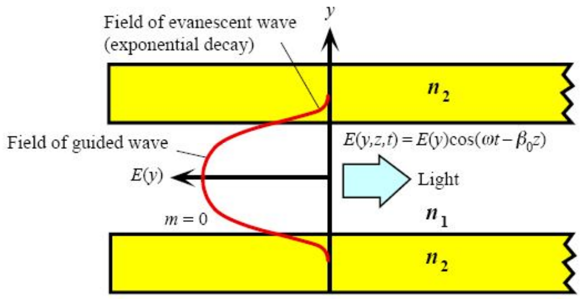
→ for each (m), we get a distinct field pattern along y.

∴ Along the guide the wave ~~...~~ propagates with the form of

$$\boxed{E(y,z,t) = 2E_m(y)\cos(\omega t - \beta_m z)}$$ Possible waves in the guide

Field distribution for a given m.

→ The field distribution across the guide travels down the guide along z.



The electric field pattern of the lowest mode traveling wave along the guide. This mode has $m = 0$ and the lowest θ. It is often referred to as the glazing incidence ray. It has the highest phase velocity along the guide.

Figure 2.5

$m \rightarrow \theta_m$

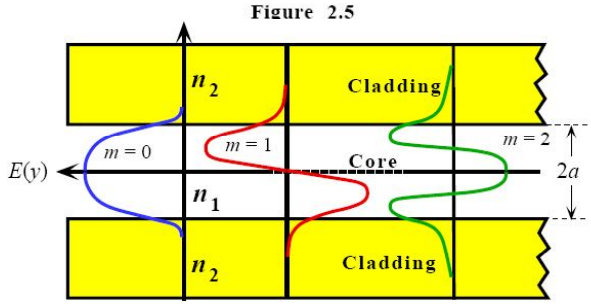
↓ Particular wavevectors

β_m

↓ A distinct field pattern

$E_m(y)$ → Mode of propagation

→ m: mode number



The electric field patterns of the first three modes ($m = 0, 1, 2$) traveling wave along the guide. Notice different extents of field penetration into the cladding.

Figure 2.6

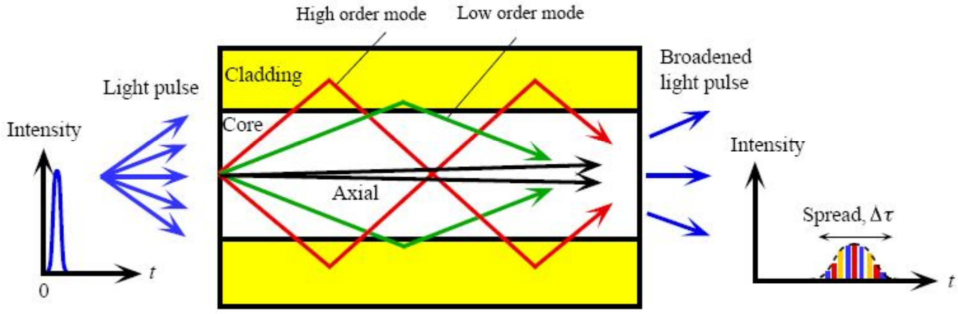
m↑ ⇒ θ_m↓

∴ more reflections

also more penetration into the CLADDING

Each mode travels at a diff. group velocity.

∴ Broadened beam.



chematic illustration of light propagation in a slab dielectric waveguide. Light pulse ntering the waveguide breaks up into various modes which then propagate at different roup velocities down the guide. At the end of the guide, the modes combine to onstitute the output light pulse which is broader than the input light pulse.

Figure 2.7

B. Single and Multimode Waveguides

To satisfy TIR, θ_m must satisfy $\theta_m > \theta_c$

$$\therefore \sin\theta_m > \sin\theta_c$$

using "Waveguide condition"

$$\boxed{m \leq (2V - \phi)/\pi}$$

V-number:

$$\boxed{V = \frac{2\pi a}{\lambda}\left(n_1^2 - n_2^2\right)^{1/2}}$$

V-number (often in fiberoptics) $\equiv$ V-parameter $\equiv$ normalized thickness (often in planar guides) $\equiv$ normalized freq.

$V = ?$ that makes $m = 0$ (lowest mode)

→ glazing incidence: $\theta_m \to 90^\circ \longrightarrow \phi \to \pi$

$$V = \frac{m\pi + \phi}{2} = \frac{\pi}{2}$$

! • When $V < \frac{\pi}{2}$ ⟶ there is only one mode propagating !

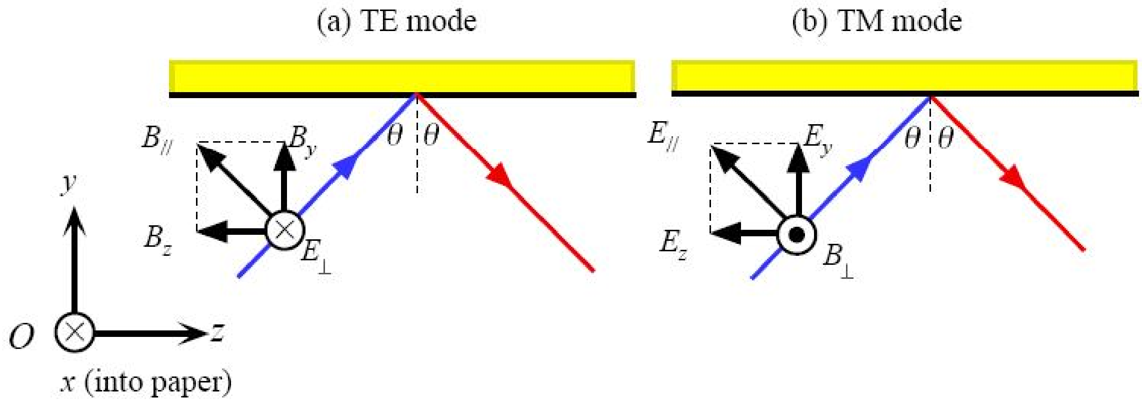
$m = 0$

that is called SINGLE MODE PLANAR WAVEGUIDE.

→ • λ_c (free space) leading to $V = \frac{\pi}{2}$ is CUT-OFF WAVELENGTH.

Above λ_c, only one-mode, fundamental mode, will propagate.

C. TE and TM modes



Possible modes can be classified in terms of (a) transelectric field (TE) and (b) transmagnetic field (TM). Plane of incidence is the paper.

Figure 2.8

→ The modes associ. with $E_\perp$ (or E_x) are termed TRANSVERSE ELECTRICAL FIELD MODES denoted by TE_m ($E_\perp$ is $\perp$ to the direction of propagation, z)

→ The modes assoc. with $E_{//}$ (or $B_\perp$) are termed TRANSV. MAG. FIELD MODES denoted by TM_m.

$E_{//}$ has a component along z, direction of propagation,

Impossible in free space, such a longitudinal comp exists.

Phase Change $\emptyset$, (when TIR) depends on the polarization and is different for $E_\perp$ and $E_{//}$.

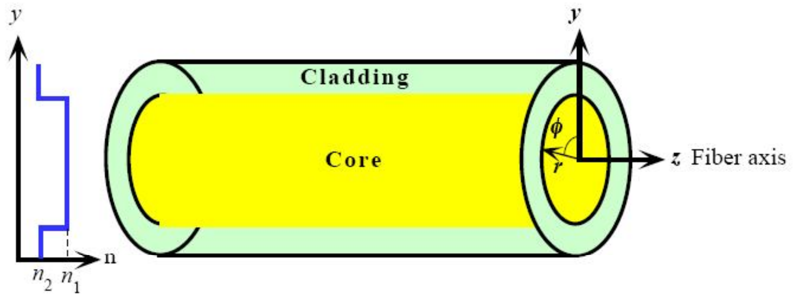
However; when $n_1 - n_2 \ll 1$, the diff. in phase ch. is negligibly small.

∴ We can TAKE

$$\lambda_c \text{ (for TE Modes)} = \lambda_c \text{ (for TM Modes)} \qquad (n_1 - n_2 \ll 1)$$

2.3. STEP INDEX FIBER

planar dielectric waveguide ⟶ step index fiber



The step index optical fiber. The central region, the core, has greater refractive index than the outer region, the cladding. The fiber has cylindrical symmetry. We use the coordinates r, ϕ, z to represent any point in the fiber. Cladding is normally much thicker than shown.

Figure 2.12

Normalized index difference;

$$\boxed{\Delta = \frac{n_1 - n_2}{n_1}}$$

Normalized index difference

for all practical fibers in communications $\Delta \ll 1$ (less than a few percent)

→ 2D case ; we need two integers l and m to label all possible travelling waves in the guide.

- A meridional ray enters the fiber by crossing the axis, and crosses the fiber axis after every reflection.
- A skew ray enters the fiber off the fiber axis, and zigzags down the fiber without crossing the fiber axis.

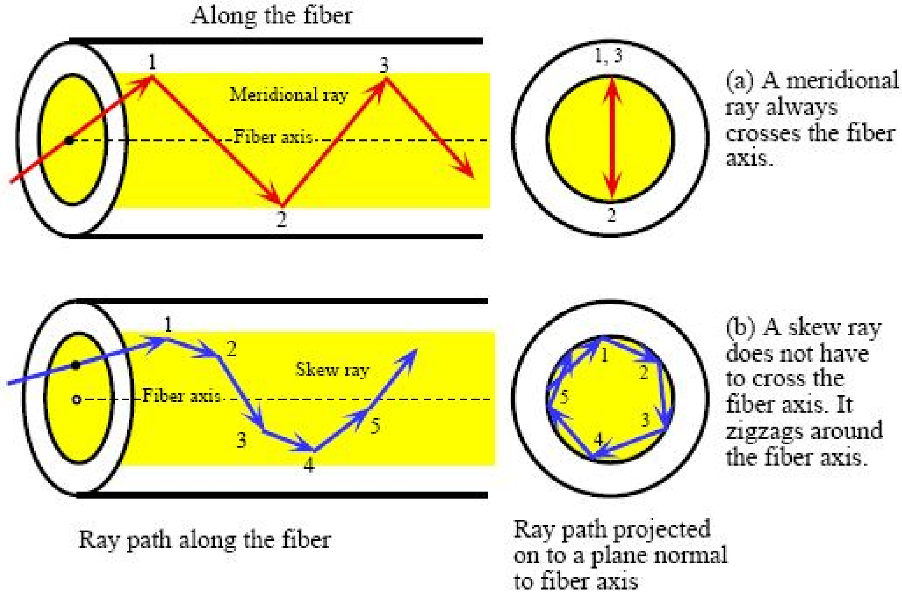


Illustration of the difference between a meridional ray and a skew ray. Numbers represent reflections of the ray.

Figure 2.13

In a STEP-INDEX FIBER

- Both meridional and skew rays give rise to guided modes, each with prop. c. β.
- Guided modes are (from meridional rays) either TE or TM type (as in planar waveguide)
- But, skew rays give rise to Hybrid Modes (HE or EH) because they have both E_z and B_z components, ∴ not TE or TM waves.

In a STEP-INDEX FIBER with $\Delta \ll 1$ (weakly guiding fibers)

- are generally visualized by travelling waves that are almost PLANE POLARIZED.
- They have transverse E and B fields (field magnitudes are not constant in plane)
- These waves are called LINEARLY POLARIZED, (LP) and have transverse $\vec{E}$ and $\vec{B}$ field characteristics.
- A guided LP mode along the fiber can be represented by the propagation of an E field distribution $E(r,\phi)$ along z.

Since this distribution is normal to the fiber axis, ∴ depends on r,ϕ not z.

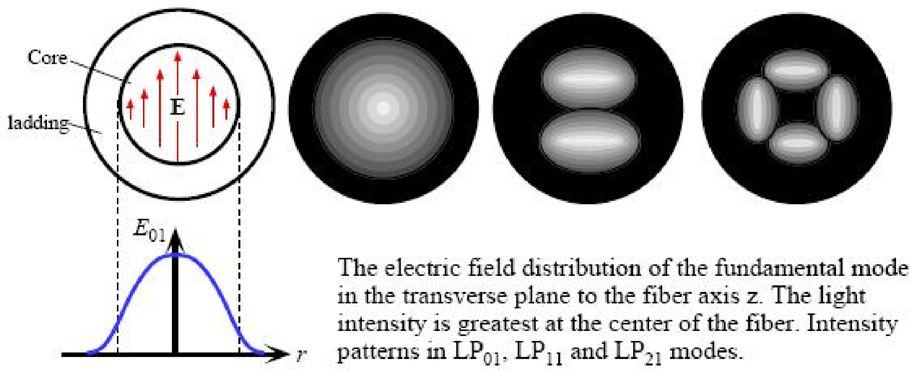
$$E_{LP} = E_{\ell m}(r,\phi)\exp j(\omega t - \beta_{\ell m} z)$$

$\beta_{\ell m}$ → prop. const. along z

m → # of maxima along r

2ℓ → # of maxima along circumference

(a) The electric field of the fundamental mode (b) The intensity in the fundamental mode LP_{01} (c) The intensity in LP_{11} (d) The intensity in LP_{21}



The electric field distribution of the fundamental mode in the transverse plane to the fiber axis z. The light intensity is greatest at the center of the fiber. Intensity patterns in LP_{01}, LP_{11} and LP_{21} modes.

Figure 2.14

For a step-index fiber, V-number or normalized freq. is similar:

$$\boxed{V = \frac{2\pi a}{\lambda}\left(n_1^2 - n_2^2\right)^{1/2} = \frac{2\pi a}{\lambda}\left(2n_1 n \Delta\right)^{1/2}}$$

Here $n = \frac{n_1 + n_2}{2}$

Normalized index diff.

$$\Delta = \frac{n_1 - n_2}{n_1} \approx \frac{n_1^2 - n_2^2}{2n_1^2}$$

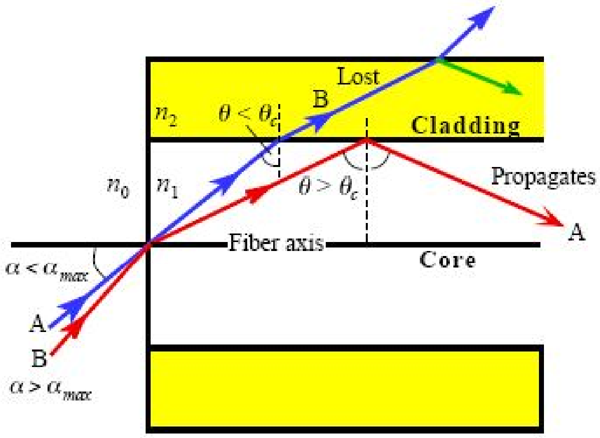
- for $V < 2.405 \longrightarrow$ single mode $\rightarrow$ fundamental mode LP_{01} !

The cut-off wavelength λ_c, above which the fiber becomes single mode

$$\boxed{V_{cut\text{-}off} = \frac{2\pi a}{\lambda_c}\left(n_1^2 - n_2^2\right)^{1/2} = 2.405}$$

2.4 Numerical Aperture

Only the rays within a certain cone can be guided along the optical fiber.



Maximum acceptance angle α_{max} is that which just gives total internal reflection at the core-cladding interface, i.e. when $\alpha = \alpha_{max}$ then $\theta = \theta_c$. Rays with $\alpha > \alpha_{max}$ (e.g. ray B) become refracted and penetrate the cladding and are eventually lost.

Figure 2.16

→ Light comes from n_0 into n_1 (core) at an angle α.

→ This ray makes an angle θ inside the core.

→ α_{max} is when $\theta = \theta_c$

→ Snell's Law ($n_0 \to n_1$) $\quad \sin\alpha_{max} / \sin(90° - \theta_c) = n_1/n_0$

→ Snell's Law (TIR) ($n_1 \to n_2$) $\quad \sin\theta_c = n_2/n_1$

$$\therefore \quad \boxed{\sin\alpha_{max} = \frac{(n_1^2 - n_2^2)^{1/2}}{n_0}}$$

Numerical Aperture → $\boxed{NA = (n_1^2 - n_2^2)^{1/2}}$ Maximum acceptance angle, α_{max}

→ Total Acceptance angle $2\alpha_{max}$;

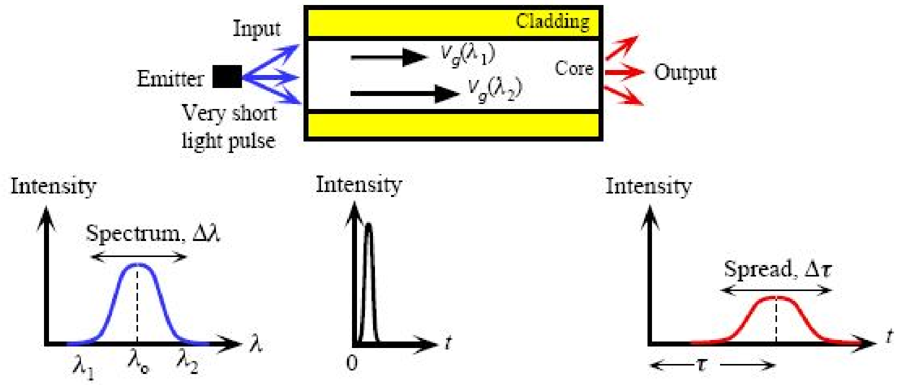
NA vs. V-number:

$$\boxed{V = \frac{2\pi a}{\lambda} NA}$$

2.5 Dispersion in Single Mode Lasers

Modal Dispersion: $\Delta\tau = \frac{L}{v_{g_{min}}} - \frac{L}{v_{g_{max}}}$ $\frac{\Delta\tau}{L} = \frac{n_1 - n_2}{c}$

A: Material Dispersion



$$\frac{\Delta\tau}{L} = |D_m| \Delta\lambda$$

$$D_m \approx -\frac{\lambda}{c}\left(\frac{\partial^2 n}{\partial\lambda^2}\right)$$

Group Delay: $\tau_g = \frac{1}{v_g} = \frac{d\beta_{01}}{d\omega}$

All excitation sources are inherently non-monochromatic and emit within a spectrum, $\Delta\lambda$, of wavelengths. Waves in the guide with different free space wavelengths travel at different group velocities due to the wavelength dependence of n_1. The waves arrive at the end of the fiber at different times and hence result in a broadened output pulse.

Figure 2.17

B: Waveguide Dispersion

$$\frac{\Delta\tau}{L} = |D_w| \Delta\lambda$$

$$D_w \approx \frac{1.984\, N_{g_2}}{(2\pi a)^2\, 2 c n_2^2}$$

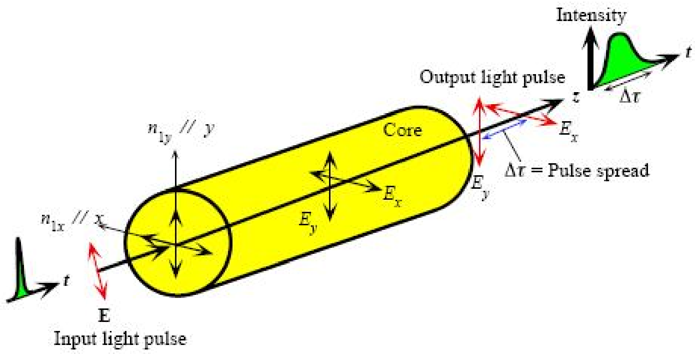
C: Chromatic Disp. or Total Disp

$$\frac{\Delta\tau}{L} = |D_m + D_w| \Delta\lambda$$

D: Profile and Polarization Disp.

$\Delta = \Delta(\lambda)$

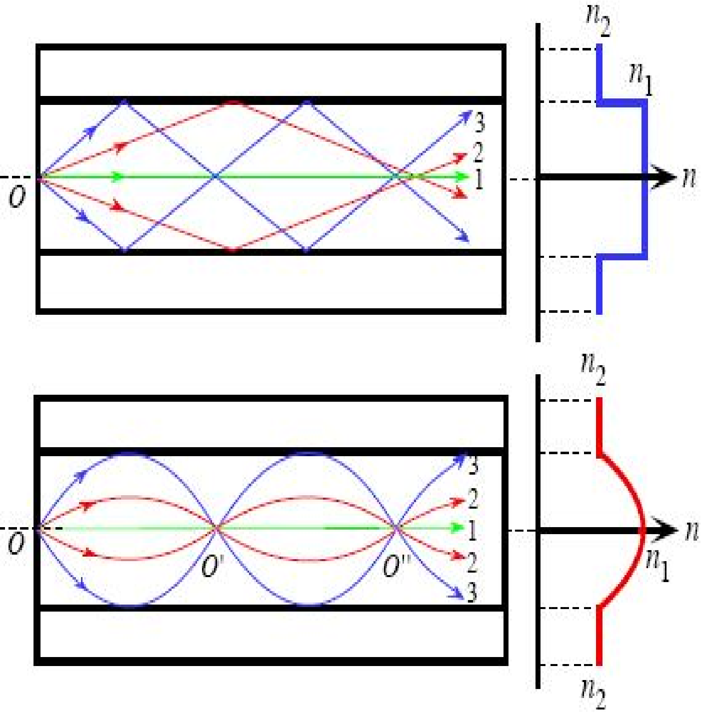
Profile Disp. $\frac{\Delta\tau}{L} = |D_p| \Delta\lambda$



Suppose that the core refractive index has different values along two orthogonal directions corresponding to electric field oscillation direction (polarizations). We can take x and y axes along these directions. An input light will travel along the fiber with E_x and E_y polarizations having different group velocities and hence arrive at the output at different times

Figure 2.19

2.7. The Graded Index (GRIN) OPTICAL FIBER



(a) Multimode step index fiber. Ray paths are different so that rays arrive at different times.

(b) Graded index fiber. Ray paths are different but so are the velocities along the paths so that all the rays arrive at the same time.

Multimode step index fiber.
Rays are different so that rays arrive at different times

Graded index fiber.
Ray paths are different but so are the velocities along the paths so that all the rays arrive at the same time.

2.9. Attenuation in Optical Fibers

$\alpha \rightarrow$ The attenuation coef.

defined as : the fractional decrease in the optical power per unit distance.

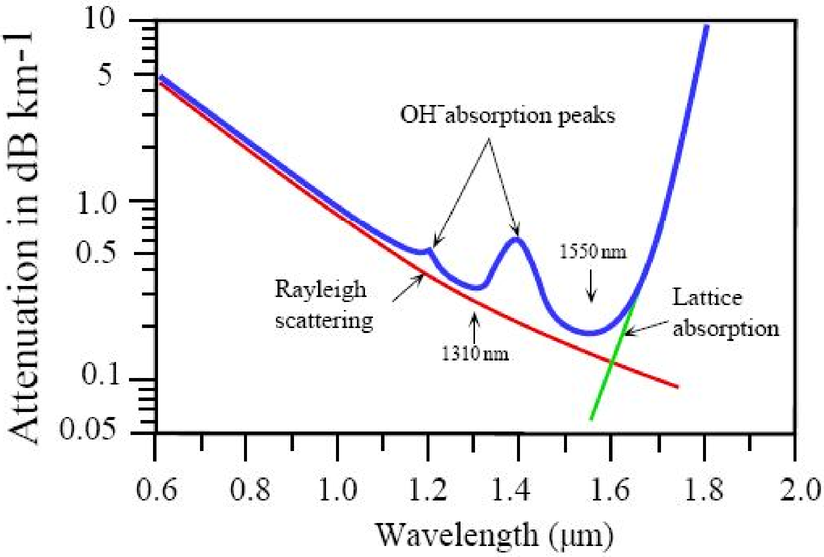
$$\alpha = -\frac{1}{P}\frac{dP}{dx}$$

Integral

$$\alpha = \frac{1}{L}\ln\left(\frac{P_{in}}{P_{out}}\right)$$

$$P_{out} = P_{in}\exp(-\alpha L)$$

$$\alpha_{dB} = \frac{1}{L}10\log\left(\frac{P_{in}}{P_{out}}\right)$$



Two communication channels at 1310nm and 1550nm

Illustration of a typical attenuation vs. wavelength characteristics of a silica based optical fiber. There are two communications channels at 1310 nm and 1550 nm.

Figure 2.31