4.6 Laser Oscillation Conditions

A. Optical Gain Coefficient, g

- Propagating em-wave in a laser medium along x-direction.

Optical gain coefficient of the medium, g

$g \longrightarrow$ fractional change in the light power (or intensity) per unit distance.

Optical power P is proportional with the concen. of coherent photons, N_{ph} and their energies, $h\nu$.

The photons travel a distance $\boxed{\delta x = \frac{c}{n}\delta t}$ in the medium.

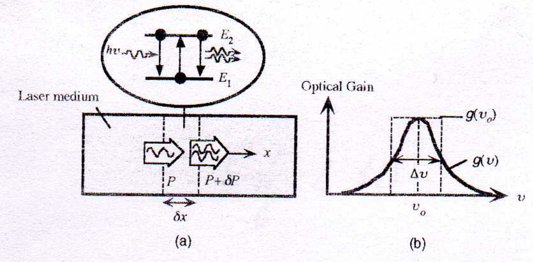
$$\therefore \quad g = \frac{\delta P}{P}\frac{1}{\delta x} = \frac{\delta N_{ph}}{N_{ph}\cdot\delta x} = \frac{n}{cN_{ph}}\frac{\delta N_{ph}}{\delta t}$$

→ The net rate of change in coherent photon concentration:

$$\frac{dN_{ph}}{dt} = \text{Net rate of stimulated photon emission}$$

$$= N_2 B_{21}\rho(h\nu) - N_1 B_{21}\rho(h\nu)$$

$$= (N_2 - N_1) B_{21}\rho(h\nu)$$



(a) A laser medium with an optical gain (b) The optical gain curve of the medium. The dashed line is the approximate derivation in the text.

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5.12.

Emission is within Δν interval (e.g. due to Doppler broadening)

$\therefore \quad g = g(\nu)$

$\therefore \quad \rho(h\nu) \rightarrow$ energy density per unit freq.

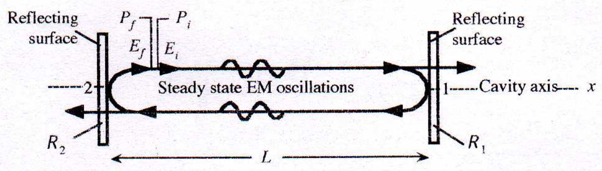
$$\rho(h\nu_0) \approx \frac{N_{ph}\, h\nu_0}{\Delta\nu}$$

Gain Coefficient:

$$\boxed{g(\nu_0) \simeq (N_2 - N_1)\,\frac{B_{21}\, n\, h\nu_0}{c \cdot \Delta\nu}}$$

optical gain at ν_0

B. Threshold Gain g_{th}



Optical cavity resonator

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The net round-trip optical gain G_{op} must be unity (≡ no gain and no loss)

$$G_{op} = \frac{P_f}{P_i} = 1$$

$$\boxed{P_f = P_i R_1 R_2 \exp\left[g(2L)\right] \exp\left[-\gamma(2L)\right]}$$

$\rightarrow$ Attenuation or loss coeffi. of the medium. and represents all losses in the cavity and the walls except transmission losses from mirrors.

In steady state $G_{op} = \frac{P_f}{P_i} = 1$ must be satisfied.

The value of g that makes this eq. is valid is called threshold gain g_{th}.

$$\boxed{g_{th} = \gamma + \frac{1}{2L} \ln\left(\frac{1}{R_1 R_2}\right)}$$

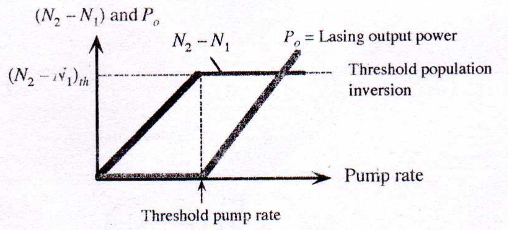
→ The necessary g_{th} has to be obtained by suitably pumping so that $N_2 \gg N_1$

Threshold population inversion → $N_2 - N_1 = (N_2 - N_1)_{th}$:

$$\boxed{(N_2 - N_1)_{th} \approx g_{th} \cdot \frac{c \Delta\nu}{B_{21} \, n h \nu_0}}$$

→ Initially the medium must have a gain coefficient g greater than g_{th}. ($g > g_{th}$)

→ When the steady state is reached, ∴ $g = g_{th}$



Simplified description of a laser oscillator. $(N_2 - N_1)$ and coherent output power (P_o) vs. pump rate under continuous wave steady state operation.

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→ Until the pump rate can bring $(N_2 - N_1)$ to $(N_2 - N_1)_{th}$., no coherent radiation.

→ When the pumping rate exceeds the threshold., $(N_2 - N_1)$ remains clamped at $(N_2 - N_1)_{th}$

→ Additional increase in pumping increases the rate of stimulated transitions and ∴ the optical output power P_o

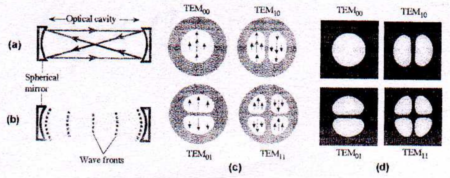
C. Phase Condition and Laser Modes

After one round-trip the phase-change $\Delta\phi_{\text{round-trip}}$

$$\Delta\phi_{\text{round-trip}} = m(2\pi)$$

$$n k_m (2L) = m(2\pi)$$

Mode condition $\longrightarrow$ $\boxed{m\left(\frac{\lambda_m}{2n}\right) = L}$



Laser Modes (a) An off-axis transverse mode is able to self-replicate after one round trip. (b) Wavefronts in a self-replicating wave (c) Four low order transverse cavity modes and their fields. (d) Intensity patterns in the modes of (c).

Transverse modes or Transverse Electromagnetic Modes TEM_{pqm}

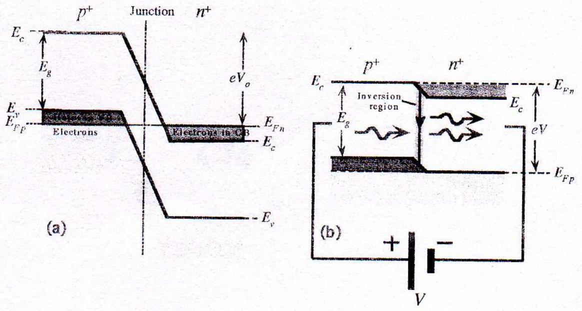
TEM_{pqm}

pq → The number of modes in the field distribution along transverse directions

m → The number of nodes along the cavity axis and generally NOT shown.
($m \sim 10^6$ for gas lasers)

4.7. Principle of the Laser Diode

Degenerately doped direct band gap s/c pn junction



The energy band diagram of a degenerately doped p-n with no bias. (b) Band diagram with a sufficiently large forward bias to cause population inversion and hence stimulated emission.

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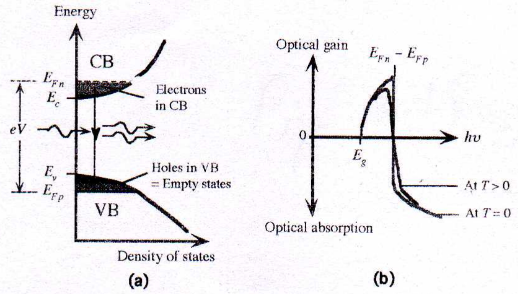
→ Very narrow depletion region.

→ Population inversion (forward biased) is around the junction

Inversion layer or active region.

→ An incoming photon can NOT EXCITE an electron, ∵ the edge of E_v is empty.

However, it can stimulate an electron to fall down.



(a) The density of states and energy distribution of electrons and holes in the conduction and valence bands respectively at T U0 in the SCL under forward bias such that $E_{Fn} - E_{Fp} > E_g$. Holes in the VB are empty states. (b) Gain vs. photon energy.

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→ Photons have energies greater than E_g but less than $(E_{Fn} - E_{Fp})$

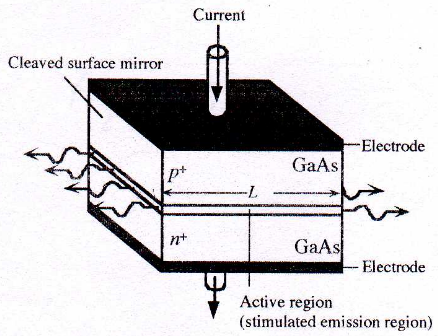
→ As T increases (from $T \approx 0$), F.D. spreads the elec. energies.

∴ a reduction in optical gain

→ The pumping mechanism is : the forward diode current and pumping energy is supplied by the external battery.

This type of injection is INJECTION PUMPING

Besides population inversion, we need to have an OPTICAL CAVITY



A schematic illustration of a GaAs homojunction laser diode. The cleaved surfaces act as reflecting mirrors.

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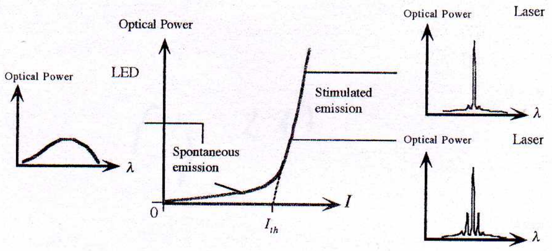
$$\boxed{m\frac{\lambda}{2n} = L}$$

Modes in an optical cavity



Typical output optical power vs. diode current (I) characteristics and the corresponding output spectrum of a laser diode.

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- When $I < I_{th}$ Spontaneous emission
- When $I > I_{th}$ Stimulated emission.

→ Transparency current is the diode current providing just enough current to inject electrons to balance the absorption.

No net photon absorption ⇒ the medium is transparent

→ The main problem with the homojunction laser diode is

J_{th} is so large that it's in the order of ~500 A/mm² for GaAs

→ HOWEVER, J_{th} can be reduced by orders of magnitude by using heterojunction s/c diodes.

4.8. Heterostructure Laser Diodes.

To reduce J_{th} ⟶ improve the rate of stimulated emission.

and improve the efficiency of optical cavity.

→ Confine the electrons and holes to a narrow region → narrowing active region.

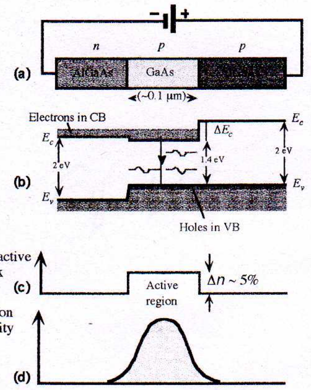
∴ less current is needed for population inversion.

→ Build a dielectric waveguide around active region.

∴ increase the photon concentration and hence prob. of stim em..

∴ We need both

- photon confinement and
- carrier confinement.



(a) A double heterostructure diode has two junctions which are between two different bandgap semiconductors (GaAs and AlGaAs).

(b) Simplified energy band diagram under a large forward bias. Lasing recombination takes place in the *p*-GaAs layer, the *active layer*

(c) Higher bandgap materials have a lower refractive index

(d) AlGaAs layers provide lateral optical confinement.

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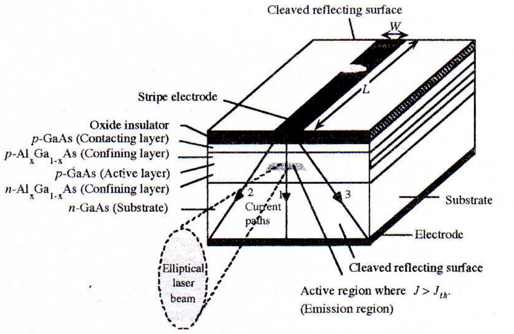

Schematic illustration of the the structure of a double heterojunction stripe contact laser diode

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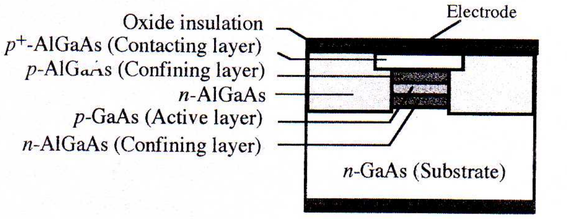

Schematic illustration of the cross sectional structure of a buried heterostructure laser diode.

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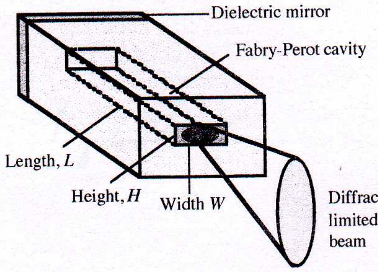
4.9. Elementary Laser Diode Characteristics

Fabry-Perot Cavity

Length ↔ Longitudinal modes

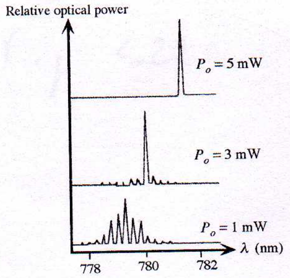
Width and Height → Transverse or Lateral modes.

→ Sufficiently small W and H → TEM_{00} (but it has long. modes.

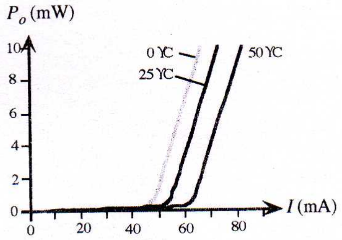


The laser cavity definitions and the output laser beam characteristics.

- At low power outputs LD → single mode
- At high power outputs LD → Multi mode



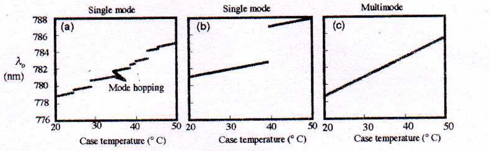
Output spectra of lasing emission from an index guided LD. At sufficiently high diode currents corresponding to high optical power, the operation becomes single mode. (Note: Relative power scale applies to each spectrum individually and not between spectra)



Output optical power vs. diode current as three different temperatures. The threshold current shifts to higher temperatures.

→ As the temp. increases, threshold current increases steeply.

4.19.



Peak wavelength vs. case temperature characteristics. (a) Mode hops in the output spectrum of a single mode LD. (b) Restricted mode hops and none over the temperature range of interest (20 - 40 °C). (c) Output spectrum from a multimode LD.

→ **Mode Hop** → As the temp. rises, another laser mode fulfills the oscillation condition → Due do the slight increase in n and cavity length.

→ Undesirable Mode Hops → Sufficiently separated modes by changing structure

→ A gain guided laser → many modes

Highly stabilized Lasers with TE coolers, controlling the device temp.

Slope Efficiency ; Optical power output in terms of diode current

$$\boxed{\eta_{slope} = \frac{P_o}{I - I_{th}}}$$

4.11 Light Emitters for Optical Fiber Communications

→ For short haul applications (local networks etc.) ⟶ LEDs.
with multimode and graded index fibers. ∵ $\Delta\lambda$ is not a major concern.

LEDs;

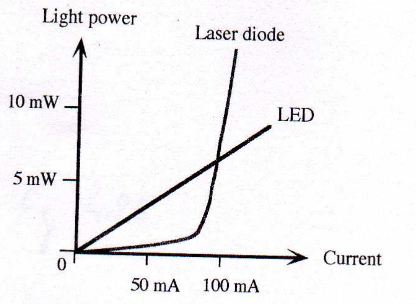
- are simpler to drive
- more economic
- have longer lifetime
- provide necessary output power

LEDs

- are typically used with multimode and graded index fibers.
  ∵ the dispersion is not major concern for these fibers.

→ For long haul and wide bandwidth communications ⟶ Laser Diodes.
∵ of their narrower bandwidth and higher output power.

Allowing only one mode ⟶ Single mode laser with a bandwidth of 0.01–0.1 nm.



Typical optical power output vs. forward current for a LED and a laser diode.

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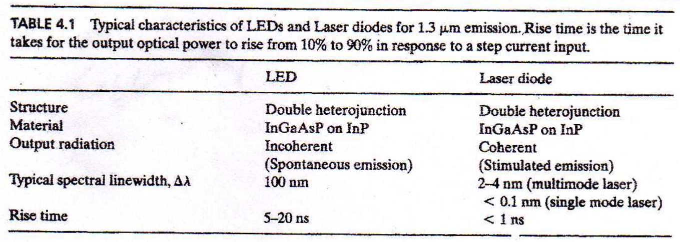


TABLE 4.1 Typical characteristics of LEDs and Laser diodes for 1.3 μm emission. Rise time is the time it takes for the output optical power to rise from 10% to 90% in response to a step current input.

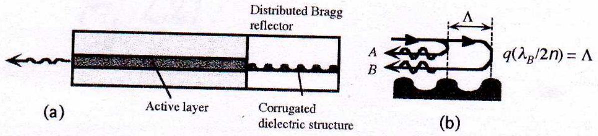
	LED	Laser diode
Structure	Double heterojunction	Double heterojunction
Material	InGaAsP on InP	InGaAsP on InP
Output radiation	Incoherent (Spontaneous emission)	Coherent (Stimulated emission)
Typical spectral linewidth, $\Delta\lambda$	100 nm	2–4 nm (multimode laser) < 0.1 nm (single mode laser)
Rise time	5–20 ns	< 1 ns

Rise time ⟶ The time it takes the output power to rise from 10% to 90%, when a current driving is applied suddenly as a step.

LDs have shorter rise times than LEDs.

4.12. Single frequency Solid State Lasers

One method of ensuring a single mode radiation ⟷ DBR



(a) Distributed Bragg reflection (DBR) laser principle. (b) Partially reflected waves at the corrugations can only constitute a reflected wave when the wavelength satisfies the Bragg condition. Reflected waves *A* and *B* interfere constructive when $q(\lambda_B/2n) = \Lambda$.

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Optical path difference → 2Λ

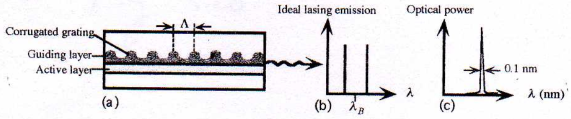
Λ is corrugation period

Diffraction order

$$\boxed{q\frac{\lambda_B}{n} = 2\Lambda}$$

Ref. index of corrugated mat.

DFB (Distributed feedback Lasers)



(a) Distributed feedback (DFB) laser structure. (b) Ideal lasing emission output. (c) Typical output spectrum from a DFB laser.

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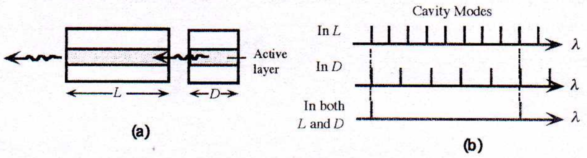
Cleaved Coupled Cavity (C^3) Laser

Two different laser optical cavities L and D are coupled.



Cleaved-coupled-cavity (C^3) laser

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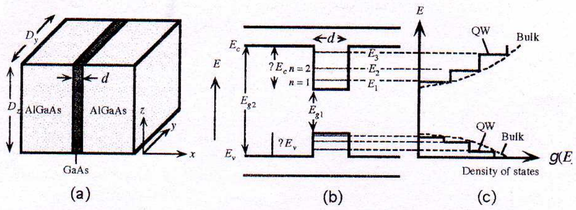
→ Two lasers are pumped by different currents

→ Two different sets of modes coincide only at far spaced intervals!

4.13. Quantum Well Devices

A typical Q.W. device has an ultra-thin (less than 50nm) narrow band-gap semiconductor, such as GaAs, sandwiched between two wider bandgap s/c.

- lattice match is required.
- Because of pot-en. barrier, ΔE_c, conduction electrons in GaAs are confined in x-direction.
- The Confinement can be treated as in 1-D (2D electron gas), due to narrow GaAs layer.



A quantum well (QW) device. (a) Schematic illustration of a quantum well (QW) structure in which a thin layer of GaAs is sandwiched between two wider bandgap semiconductors (AlGaAs). (b) The conduction electrons in the GaAs layer are confined (by ? E_c) in the *x*-direction to a small length *d* so that their energy is quantized. (c) The density of states of a two-dimensional QW. The density of states is constant at each quantized energy level.

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The energy in Q.W

$$E = E_c + \frac{h^2 n^2}{8 m_e^* d^2} + \frac{h^2 n_y^2}{8 m_e^* D_y^2} + \frac{h^2 n_z^2}{8 m_e^* D_z^2}$$

n, n_y and n_z are Q numbers

$D_y, D_z \gg d$, E_1 is associated with "d"

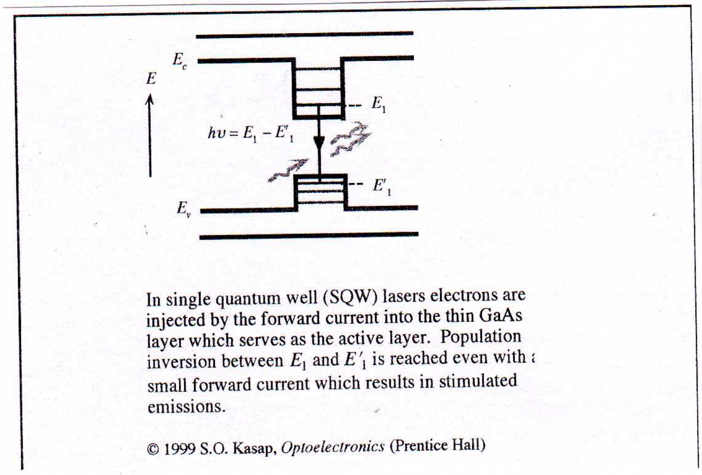
→ A large concentration of electrons occurs at E_1, while almost no electronic (DO) states at E_c for a BULK s/c.

∴ population inversion with a small current is reached between E_1 and E_1'

4.25.

→ Two distinct advantages of single quantum well (SQW) laser

- The treshold current for population inversion is markedly reduced.
- The majority of electrons are at E_1 and holes are at E_1', the range of emitted photon energies are very close to $(E_1 - E_1')$



In single quantum well (SQW) lasers electrons are injected by the forward current into the thin GaAs layer which serves as the active layer. Population inversion between E_1 and E'_1 is reached even with small forward current which results in stimulated emissions.

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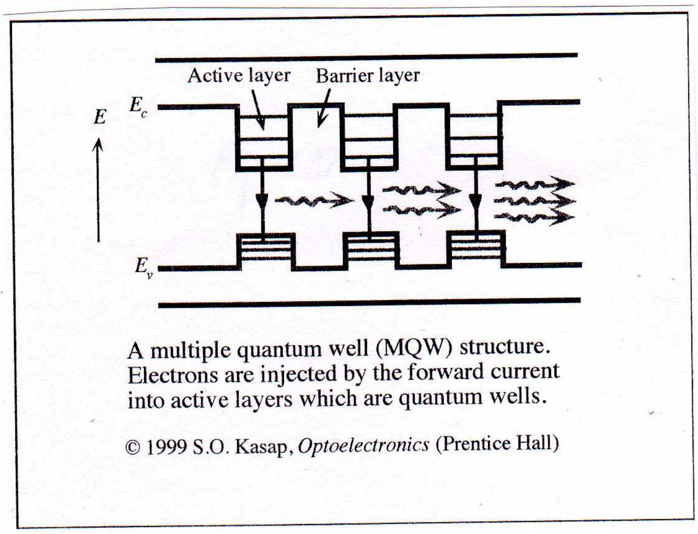
Multiple Q.W. lasers



A multiple quantum well (MQW) structure. Electrons are injected by the forward current into active layers which are quantum wells.

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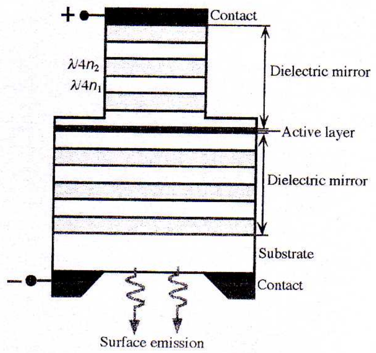
4.11. Vertical Cavity Surface Emitting Lasers (VCSELs)

→ The actual width is very much larger than the thickness of the activelayer, then the radiation emerges from the surface.



A simplified schematic illustration of a vertical cavity surface emitting laser (VCSEL).

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constructive reflections at wavelengths of λ (free space)

$$n_1 d_1 + n_2 d_2 = \frac{1}{2}\lambda$$

which leads constructive interference

Pheripheral contacts are used instead to avoid undesirable voltage drops in the DBRs.

→ High reflectance end mirrors are needed. Because the cavity length $\rightarrow L$ is so short that the optical gain is so small, inasmuch as the optical gain is prop. to. $\exp(gL)$.

- With 20-30 layers in the DBR to obtain required reflectance 99%
- Active layers are very thin (<0.1μm)

for example

→ Active layer : InGaAs as emitting at 980nm
→ Substrate : GaAs transparent at 980nm
→ DBR : Varying compositions of AlGaAs

The vertical cavity,

→ generally circular shaped cross section, so is beam shape.

→ The height of the VC may be as small as microns.

∴ The longitudinal separations are sufficiently large

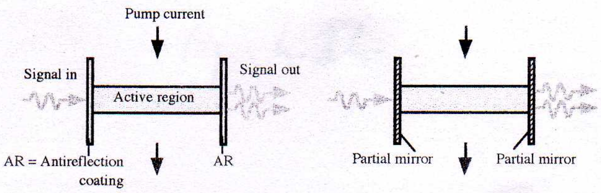
→ There may be ~~[illegible]~~ one or more lateral modes, depending on the lateral size of the cavity.

- Typically less than ~8μm (in diameter) → single mode.

→ They can be arrayed to construct a Matrix emitter.
The matrix emitters have a broad area as a surface emit. source.
They have important applications in

OPTICAL INTERCONNECT and OPTICAL COMPUTING tech.

4.15. Optical Laser Amplifiers



(a) Traveling wave amplifier (a) Fabry-Perot amplifier

Simplified schematic illustrations of two types of laser amplifiers

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A-FP Amp
has higher gain
than T-W Amp
but less stability.

A s/c laser structure can be used as an optical amplifier.

Travelling Wave S/C. laser amp.

with AR coated at two ends of the cavity.

- However light incident on the input is amplified by STIMULATED emissions
- The device is pumped to achieve optical gain. (population inversion)

The Fabry-Perot laser Amp.

similar to conventional laser osc., but operated below the threshold current.

∴ is not self-lasing.

- Input light is amplified by stimulated emissions and multiple reflections.