Interests

- Simple Interest
- Compound Interest
- Effective Rate
- Nominal Rate
- Future Values
- Present Value

- Time value of Money
- Relation between present and future values.
- Simple and compound interests and the corresponding future and present values of an amount of money invested today

TYPES OF INTEREST

- **Simple Interest (SI);** which is interest earned on only the original amount, called Principal, lent over a period of time at a certain rate.
- **Compound Interest (CI);** which is interest earned on any previous interests earned as well as on the Principal lent.

TYPES OF INTEREST

- What is the difference between the two?

  In the case of simple interest, the amount of interest paid is based ONLY on the amount borrowed, whereas in a compound interest scenario the amount of interest paid is based on the amount invested PLUS the interest accumulated in the account.

SIMPLE INTEREST

The Simple interest is given by

$$SI = P_0 \cdot i \cdot n$$

where

P_0 : Present value today (deposited t = 0),

i : interest rate per period of time,

n : number of time periods

- Question:

  Assume that you deposit $1000 in an account paying 7% annual simple interest for 2 years. What is the accumulated interest at the end of the second year?

- Solution:

$$SI = P_0 \cdot i \cdot n = 1000 \cdot 0.07 \cdot 2 = \$140.$$

Simple Interest and Future Value

- Future Value (FV) is the value at some future time of a present amount of money evaluated at a given interest rate.
- What is the future value of the deposit?

$$FV_n = P_0 + SI = P_0(1 + n \cdot i).$$

- For our example above,

$$FV_n = 1000 + 140 = \$1140.$$

- Note that there are 4 variables in the formula above. Therefore, having any three of them, could be used to find the fourth one.

Simple Interest and Present Value

- Present Value (PV) is the current value of a future amount of money evaluated at a given interest rate.

- What is the present value of the previous problem?

  The present value is simply the $1000 you originally deposited. That is the value today.

Compound Interest

The Compound interest is given by

$$FV_n = P_0 \cdot (1+i)^n$$

or

$$FV_n = P_0 \cdot (1+\frac{r}{m})^{m \cdot t}$$

where

P_0	: Present value today (deposited t = 0),
i	: interest rate per period,
n	: number of time periods
r	: annual interest rate
m	: number of interest periods per year

Compound Interest

Compounding Period	Number of Interest Periods per Year, *m*	Length of Each compounding Period
Annually	1	1 year
Semiannually	2	6 months
Quarterly	4	3 months
Monthly	12	1 month
Daily	365	1 day

Note:

Each compound interest problem involves two rates:

a) the annual rate r;

b) the rate per compounding period, $i = \frac{r}{m}$.

You have to understand the distinction between them. If interest is compounded annually, then

i=r.

COMPOUND INTEREST

Example:

A person wants to know how large his deposit of $10000 today will become at a compound annual interest rate of 10% for 5 years.

Solution: Using the formula:

$$FV_5 = 10000(1 + 0.1)^5 = \$16105.10$$

IMPACT OF FREQUENCY

- A person has $1000 to invest for 2 years at an annual compound interest rate of 12%:
- annual: $FV_2 = 1000(1 + \frac{0.12}{1})^{(1)(2)} = \$1254.40$
- semi-annual: $FV_2 = 1000(1 + \frac{0.12}{2})^{(2)(2)} = \$1262.48$
- quarterly: $FV_2 = 1000(1 + \frac{0.12}{4})^{(4)(2)} = \$1266.77$
- monthly: $FV_2 = 1000(1 + \frac{0.12}{12})^{(12)(2)} = \$1269.73$
- daily: $FV_2 = 1000(1 + \frac{0.12}{365})^{(365)(2)} = \$1271.20$

SIMPLE AND COMPOUND INTERESTS

You invest $100 in two accounts that each pay an interest rate of 10% per year. However, one account pays simple interest and one account pays compound interest. Make a table that shows the growth of each account over a 5 year period. Use the compound interest formula to verify the result in the table for the compound interest case.

	SIMPLE INTEREST ACCOUNT		COMPOUND INTEREST ACCOUNT	
End of year	Interest Paid	New Balance	Interest Paid	New Balance
1	$10\% \times 100 = 10$	$100 + 10 = 110$	$10\% \times 100 = 10$	$100 + 10 = 110$
2	$10\% \times 100 = 10$	$110 + 10 = 120$	$10\% \times 110 = 11$	$110 + 11 = 121$
3	$10\% \times 100 = 10$	$120 + 10 = 130$	$10\% \times 121 = 12.1$	$121 + 12.1 = 133.1$
4	$10\% \times 100 = 10$	$130 + 10 = 140$	$10\% \times 133.1 = 13.31$	$133.1 + 13.1 = 146.41$
5	$10\% \times 100 = 10$	$140 + 10 = 150$	$10\% \times 1464.1 = 14.64$	$146.41 + 14.64 = 161.05$

Continuous Compounding of Interest

- Continuous Compound Interest Formula

$$A = Pe^{rt}$$

where

P = Principal

r = Annual interest rate compounded continuously

t = Time in years

A = Accumulated amount at the end of t years

Continuous Compounding of Interest

- Find the accumulated amount after 3 years if \$1000 is invested at 8% per year compounded (a) daily, and (b) continuously.

<u>Solution</u>

a. Using the compound interest formula with $P = 1000$, $r = 0.08$, $m = 365$, and $t = 3$, we find

$$A = P\left(1+\frac{r}{m}\right)^{mt} = 1000\left(1+\frac{0.08}{365}\right)^{(365)(3)} \approx 1271.22$$

b. Using the continuous compound interest formula with $P = 1000$, $r = 0.08$, and $t = 3$, we find

$$A = Pe^{rt} = 1000e^{(0.08)(3)} \approx 1271.25$$

Note that the two solutions are very close to each other.

Effective Rate of Interest

- The interest actually earned on an investment depends on the frequency with which the interest is compounded.
- For clarity when comparing interest rates, we can use what is called the *effective rate* (also called the *annual percentage yield*):
  - This is the simple interest rate that would produce the same accumulated amount in 1 year as the nominal rate compounded m times a year.
- We want to derive a relation between the nominal compounded rate and the effective rate.

Nominal and Effective Interest rate statements

- The primary difference between simple and compound interest is that compound interest includes interest on the interest earned in the previous record, while simple does not.
- The nominal and effective interest rates have also the same basic relationship.
- The difference here is that the concepts of nominal and effective must be used when interest is compounded more than once each year.
- **Nominal interest rate, r,** is an interest rate that does not include any consideration of compounding.

  r = interest rate per period x number of periods
- A nominal rate r may be stated for any time period, 1 year, 6 months, quarter, month, week, day, etc.

NOMINAL AND EFFECTIVE INTEREST RATE STATEMENTS

- **EX:** The nominal rate of r = 1.5% per month is the same as each of the following rates;

1st case	= 1,5% per month x 24 months	= 36% per 2-year period
2nd case	= 1,5% per month x 12 months	= 18% per year
3rd case	= 1,5% per month x 6 months	= 9 % per semiannual period
4th case	= 1,5% per month x 3 months	= 4,5% per quarter

Nominal and Effective Interest rate statements

- **Effective interest rate** is the actual rate that applies for a stated period of time. The compounding of interest during the time period of the corresponding nominal rate is accounted for by the effective interest rate.
- It is commonly expressed on an annual basis as the effective annual rate i_a, but any time basis can be used.
- An effective rate has the compounding frequency attached to the nominal rate statement.
- **EX:** 12% per year, compounded monthly

  12% per year, compounded quarterly

  3% per quarter, compounded monthly

  6% per 6 months, compounded weekly

  3% per quarter, compounded quarterly (compounding same as time period)

- All examples are nominal rate statements; however, they will not have the same effective interest rate value over all time periods, due to the different compounding frequencies.
- In the last example, the nominal rate of 3% per quarter is the same as the effective rate of 3% per quarter, compounded quarterly.

NOMINAL AND EFFECTIVE INTEREST RATE STATEMENTS

- Time units associated with an interest rate statement
  - Time period: The basic time unit of the interest rate
  - Compounding period (CP): The time unit used to determine the effect of interest.
  - The compounding frequency, m: The number of times that compounding occurs within the time period t.
- **EX:**
  - 8% per year, compounded monthly, has a compounding frequency of m=12 times per year.
  - A rate of 8% per year, compounded weekly, has a frequency of m=52.

$$\text{Effective rate per CP} = \frac{r\% \text{ per time period } t}{m \text{ compounding periods per } t} = \frac{r}{m}$$

Nominal and Effective Interest rate statements

EX: Determine the effective rate on the basis of the compounding period for each interest rate.

a) 9% per year, compounded yearly
b) 6% per year, compounded quarterly
c) 8% per year, compounded monthly
d) 5% per 6- months, compounded weekly

Nominal r% per t	Compounding Period	m	Effective Rate per CP
9% per year	Year	1	9/1 = 9%
6% per year	Quarter	4	6/4 = 1,5%
8% per year	Month	12	8/12 = 0,667%
5% per 6- months	Week	26	5/26 = 0,192%

Nominal and Effective Interest rate statements

- Sometimes it is not obvious whether a stated rate is nominal or an effective rate. Basically there are three ways to express interest rates.

Format of Rate Statement	**Examples of Statement**	**What about the Effective Rate?**
Nominal rate stated, compounding period stated	8% per year, compounded quarterly	Find effective rate
Effective rate stated	Effective 2,5% per year, compounded quarterly	Use effective rate directly
Interest rate stated, no compounding period stated	5% per year or 3% per quarter or 4,5 & per month	Rate is effective only for time period stated, find effective rate for all other time periods

Effective Annual Interest Rates

- The effective interest rate is the one rate that truly represents the interest earned in a year.
- Like compound interest, the effective interest rate at any point during the year includes (compounds) the interest rate at any point during the year.
- The following formula should be used for the future worth calculation;

$$F = P(1+i)^m$$

- Where;
  - F is the future worth
  - P is the present worth
  - i is the effective interest rate per compounding period (CP) = r/m
  - m is the number of compounding periods per year

Effective Annual Interest Rates

EX: Suppose you deposit 10.000 TL in a savings account that pays you at an interest rate of 9% compounded quarterly. Here, 9% represents the nominal interest rate, and the interest rate per quarter is 2,25% (9%/4). Below table shows the example of how the interest is compounded when it is paid quarterly.

End of period	Present Amount	Interest Earned	New (Future) Amount
First Quarter	10.000,00 TL	2,25% x 10.000,00 TL =225,00 TL	10.225,00 TL
Second Quarter	10.225,00 TL	2,25% x 10.255,00 TL =230,06 TL	10.455,06 TL
Third Quarter	10.455,06 TL	2,25% x 10.455,06 TL =235,24 TL	10.690,30 TL
Fourth Quarter	10.690,30 TL	2,25% x 10.690,30 TL =240,53 TL	10.930,83 TL

- The total amount annual interest payment for a principal amount of 10.000 TL can also be calculated with the formula.

  $F = P(1+i)^m$

  P = 10.000 TL, i = 2,25%, m= 4

  $F = 10.000 \text{ TL } (1+0,0225)^4$

  F= 10.930,83 TL

Effective Annual Interest Rates

- You are earning more than 9% on your original deposit. In fact, you are earning 9,3083% (930.83 TL / 10.000 TL).
- Effective annual interest rate per year, i_a, = 930.83 / 10.000 = 0,093083 = 9,3083%
- Earning 2,25% interest per quarter for four quarters is equivalent to earning 9,3083% interest just one time each year.

Effective annual interest rate, $i_a = (1+i)^m - 1$

- Again for the same example, but this time by the formula;

$$i_a = (1+i)^m - 1 = (1+0{,}0225)^4 - 1 = 0{,}093083 = 9{,}3083\ \%$$

Effective Annual Interest Rates

EX: For a 1.000 TL balance at the beginning of the year, if the stated rate is 18% per year, compounded monthly, find the effective annual rate and the total amount owned after 1 year.

There are 12 compounded periods per year, thus m = 12.

i = 18%/ 12 = 1,5% per month

$i_a = (1+i)^m - 1 = (1+0{,}015)^{12} - 1 = 1{,}19562 - 1 = 0{,}19562$

$F = P(1+i)^m = 1.000\ \text{TL}\ (1{,}19562) = 1195{,}62\ \text{TL}$

Hence, 19,562% or 195,62 TL owned!!

EFFECTIVE INTEREST RATES FOR ANY TIME PAYMENT PERIOD

- The payment period is the frequency of the payment or receipts in other words, is cash flow transaction period.
- It is important to distinguish the compounding period and the payment period!

EX: If company deposits money each month into an account that pays a nominal interest rate of 14% per year, compounded semiannually, the payment period is 1 month while the compounding period is 6 months.

- The effective annual interest rate formula is easily generalized to any nominal rate by substituting r/m for the period interest rate

$$\textbf{Effective } i = (1+r/M)^{c} - 1 \qquad \textbf{Effective } i = (1+r/CK)^{c} - 1$$

Where;

M is the number of interest per year
C is the number of interest periods per payment period
K is the number of payment periods per year

EX: Suppose that you make quarterly deposits in a savings account that earns 9% interest compounded monthly. Compute the effective interest rate per quarter.

R = 9%, C = three interest periods per quarter, K = four quarterly payments per year, M = 12 interest periods per year.

Using Effective $i = (1+r/CK)^c - 1$

$i = (1+ 0{,}09/12)^3 - 1 = 2{,}27\ \%$

EX: A dot-com company plans to place money in a new venture capital that fund that currently returns 18% per year, compounded daily. What effective rate is this a) yearly and b) semiannually?

a) r = 18%, C = 365 interest periods per year, K = 1 payment per year, M = 365 interest periods per year

Using Effective $i = (1+r/CK)^c - 1$

$i = (1+ 0{,}18/365)^{365} - 1 = 19{,}716\ \%$

b) r = 18%, C = 182 interest periods per semiannual, K = 2 payment per year, M = 365 interest periods per year

Using Effective $i = (1+r/CK)^c - 1$

$i = (1+ 0{,}18/365)^{182} - 1 = 9{,}415\ \%$

Effective Rate of Interest

- The accumulated amount after 1 year at a simple interest rate *R* per year is

$$A = P(1+R)$$

- The accumulated amount after 1 year at a nominal interest rate *r* per year compounded *m* times a year is

$$A = P\left(1+\frac{r}{m}\right)^m$$

- Equating the two expressions gives

$$P(1+R) = P\left(1+\frac{r}{m}\right)^m$$

$$1+R = \left(1+\frac{r}{m}\right)^m$$

Effective Rate of Interest

- Solving the last equation for *R* we obtain the formula for computing the effective rate of interest:

$$r_{\text{eff}} = \left(1 + \frac{r}{m}\right)^m - 1$$

where

r_{eff}	= Effective rate of interest
r	= Nominal interest rate per year
m	= Number of conversion periods per year

Effective Rate of Interest

- Find the *effective* rate of interest corresponding to a nominal rate of 8% per year compounded
  a. Annually
  b. Semiannually
  c. Quarterly
  d. Monthly
  e. Daily

Effective Rate of Interest

a. Annually.

Let r = 0.08 and m = 1. Then

$$r_{\text{eff}} = \left(1 + \frac{0.08}{1}\right)^1 - 1$$
$$= 1.08 - 1$$
$$= 0.08$$

or 8%.

Effective Rate of Interest

b. Semiannually.

Let $r = 0.08$ and $m = 2$. Then

$$
\begin{aligned}
r_{\text{eff}} &= \left(1 + \frac{0.08}{2}\right)^2 - 1 \\
&= 1.0816 - 1 \\
&= 0.0816
\end{aligned}
$$

or 8.16%.

Effective Rate of Interest

c. Quarterly.

Let r = 0.08 and m = 4. Then

$$
\begin{aligned}
r_{\text{eff}} &= \left(1+\frac{0.08}{4}\right)^4 - 1 \\
&\approx 1.08243 - 1 \\
&= 0.08243
\end{aligned}
$$

or 8.243%.

Present Value

- Consider the compound interest formula:

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$

- The principal P is often referred to as the present value, and the accumulated value A is called the future value, since it is realized at a future date.
- On occasion, investors may wish to determine how much money they should invest now, at a fixed rate of interest, so that they will realize a certain sum at some future date.
- This problem may be solved by expressing P in terms of A.
- Present value formula for compound interest

$$P = A(1+i)^{-n}$$

Where $i = \frac{r}{m}$ and $n = mt$

- How much money should be deposited in a bank paying a yearly interest rate of 6% compounded monthly so that after 3 years the accumulated amount will be $20,000?

Solution

- Here, A = 20,000, r = 0.06, m = 12, and t = 3.
- Using the present value formula we get

$$P = A\left(1+\frac{r}{m}\right)^{-mt}$$

$$= 20,000\left(1+\frac{0.06}{12}\right)^{-(12)(3)}$$

$$\approx 16,713$$

EQUIVALENCE RELATIONS: COMPARING PAYMENT PERIOD AND COMPOUNDING PERIOD LENGHTS

- All the examples up to here assumed annual payments and annual compounding. However, a number of situations involve cash flows that occur at intervals that are not the same as the compounding intervals often used in practice.
- Whenever payments and compounding periods differ from each other, one or the other must be transformed so that both conform to the same unit of time.

- **Equivalence Relations: Single Amounts with PP ≥ CP**
- When only single-amount cash flows are involved, there are two equally correct ways to determine i and n for P/F and F/P factors.
- Method 1 is easier to apply, because the interest tables can usually provide the factor value.
- Method 2 likely requires a factor formula calculation, because the resulting effective interest rate is not an integer.

Equivalence Relations: Single Amounts with PP ≥ CP

- **Method 1**

Determine the effective interest rate over the compounding period CP, and set n equal to the number of compounding periods between P and F. The relations to calculate P and F are

  - P = F (P/F, effective i% per CP, total number of periods n)
  - F = P (F/P, effective i% per CP, total number of periods n)

- **!!!** The CP is the best because only over the CP can the effective rate have the same numerical value as the nominal rate over the same time period as the CP.

Equivalence Relations: Single Amounts with PP ≥ CP

- **Method 2**
- Determine the effective interest rate for the time period t of the nominal rate, and set n equal to the total number of periods using this same time period
- The P and F relations are the same as in above equations with the term effective i% per t substituted for the interest rate.

Equivalence Relations: Series with PP ≥ CP

- When uniform or gradient series are included in the cash flow sequence, the procedure is basically the same as method 2 above, except that PP is now defined by the frequency of the cash flows.
- This also establishes the time unit of the effective interest rate.
- For example, if cash flows occur on a quarterly basis, PP is a quarter and the effective quarterly rate is necessary.
- Then n value is the total number of quarters. If PP is a quarter, 5 years translates to an n value of 20 quarters.
- This is a direct application of the following general guideline:
  - When cash flows involve a series (i.e. A, G, g) and the payment period equals or exceeds the compounding period in length,
  - Find the effective I per payment period.
  - Determine n as the total number of payment periods.

- Following table shows the correct formulation for several cash flow series and interest rates. Note that n is always equal to the total number of payment periods and I is an effective rate expressed over the same time period as n.

Examples of n and i values where PP = CP or PP > CP			
Cash Flow Series	**Interest Rate**	**What to Find?**	**Standard Notation**
500 TL semiannually for 5 years	16% per year, compounded semiannually	Find P, given A	P = 500(P/A, 8%, 10)
75 TL monthly for 3 years	24% per year, compounded monthly	Find F, given A	F = 75(F/A, 2%, 36)
180 TL quarterly for 15 years	5% per quarter	Find F, given A	F = 180(F/A, 5%, 60)
25 TL per month increase for 4 years	1% per month	Find P, given G	P = 25(P/G, 1%, 48)
5.000 TL per quarter for 6 years	1% per month	Find A, given P	A = 5.000(A/P, 3,03%, 24)

Equivalence Relations: Single Amounts and Series with PP < CP

The computational procedure for establishing economic equivalence is as follows.

- **Step 1:** Identify the number of compounding periods per year (M), the number payment periods per year (K); and the number of interest periods per payment period (C):
- **Step 2:** Compute the effective interest rate per payment period.
  - For discrete compounding, compute

    $i = (1+r/M)c - 1$

  - For continuous compounding, compute

    $i = er/K - 1$

- **Step 3:** Find the total number of payment periods:

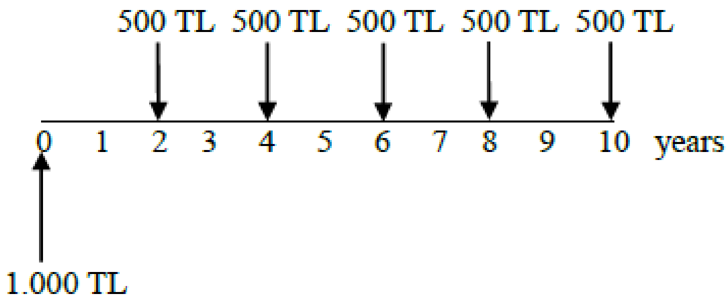
    $N = K \times (\text{number of years})$

- **Step 4:** Use i and N in the appropriate formulas from the table.

- **EX:** Suppose you make equal quarterly deposits of 1.500 TL into a fund that pays interest at a rate of 6% compounded monthly. Find the balance at the end of year 2.
- **Given:** A = 1.500 TL per quarter, r = 6% per year, M = 12 compounding periods per year and N = 8 quarters
- **Find:** F!

- **Step 1:** Identify the parameter values for M, K and C, where
  - M = 12 compounding periods per year
  - K = 4 payment periods per year
  - C = 3 interest periods per payment period (quarter)
- **Step 2:** Compute the effective interest:
  - $i = (1+r/M)^{c} - 1$
  - $= (1+ 0{,}06/12)^{3} - 1$
  - = 1,5075% per quarter
- **Step 3:** Find the total number of payment periods, N:
  - N = K(number of years) = 4(2) = 8 quarters
- **Step 4:** Use I and N in the appropriate equivalence formulas:
  - F = 1.500 TL (F/A, 1,5075%, 8) = 12,652.61 TL

EX: Calculate the Present Value of the cash flow given below where the nominal interest rate is 3% per quarter compounding monthly.



- In this example it is seen that nominal rate is given per quarter with a monthly compounding rate whereas the payments are made with 2-year time period. Therefore, actual interest rate, the effective interest rate that applies for 2-year time period should be calculated.

- **Step 1:** Effective interest rate for Compounding Period (CP)

  $i_{CP}=r/m$ where m is compounding frequency for the given period for nominal interest. Therefore, effective monthly interest rate is:

  $i_{CP}=3/3=1\%$.

- **Step 2:** Effective interest rate for Payment Period (PP)

  $(1+i_{PP})=(1+i_{CP})^n$ where n is compounding frequency in payment periods.

  Shortly, $i_{PP}=(1+i_{CP})^n-1$.

- In this example, payment periods for the given cash flow is 2 years, therefore, effective interest rate for 2-year period is;

  $i_{2-year}=(1+0,01)^{24}-1=0,2697=26,97\%$.

- **Step 3**: Cash Flow Calculation

  PW = – 1000 + 500 (P/A, 26,97%, 5)

  (P/A, 26,97%, 5) = $[(1+0,2697)^5 - 1] / [0,2697\,(1+0,2697)^5] = 2,5842$

  PW = – 1000 + 500 (2,5842)

  PW = – 1000 + 1292,1

  PW = 292,1 TL

Comparison of Alternatives

1. Present worth analysis (Güncel değer yöntemi)
2. Annual equivalent method (Senelik eşdeğer maliyet yöntemi)
3. Rate of return analyses (İç verimlilik)

- **The minimum attractive rate of return,** simply called as **MARR**, is the minimum rate of return that the company is willing to accept on the money it invests. It is also called as minimum acceptable rate of return or hurdle rate.

MARR

- MARR is generally determined by the board of the company according to financial reports prepared by the financial analyst(s) of the company.
- It is not a constant rate, it can vary from project to project, or time to time.
- Many parameters can change the value of MARR in a project.

MARR

- The most common parameters in the determination of the MARR in a project are as follow:
  - **Project Risks**

  Project specific risk covers risks in the market such as unavailable raw materials, ineffective labor force, price deviation in raw materials etc.; social, economic, political and environmental risk in the project location. Therefore, the company is expected to increase the value of MARR as the risk gets greater. For instance, MARR for the highway project in Libya in 2014 is expected to be higher than the one in 2006 at the same location; since the political instability may increase the completion time of the project and may cause delay in the progress payments. Therefore, risk level in the project changes the MARR value.
  - **Investment Opportunities**

  The company may have alternative investment opportunities. For instance, investing the money into bank is a do nothing option. If the company gets higher return than the one in the investment project, it does not need to invest in the project. The company may want to decrease its profit rate to expand in a market. Then, the MARR value is decreased for the projects in that market.
  - **Limits on Available Capital**

  If the capital of the company is limited, it needs to get credit from a bank and pay the bank higher amount of money. Therefore, the company must increase its total profit in the project to get the same profit amount after paying the credit to bank. Therefore, the MARR value is increased.
  - **Rate of Return of Other Companies**

  In a competitive market, the company must consider MARR value of its rivals to compete with them. For instance, in an airport project tender, if the company does not take into consideration the MARR values of its rivals, it may bid much higher price for the project when it is compared to other companies and one of the other companies is expected to get the tender.

EXAMPLE

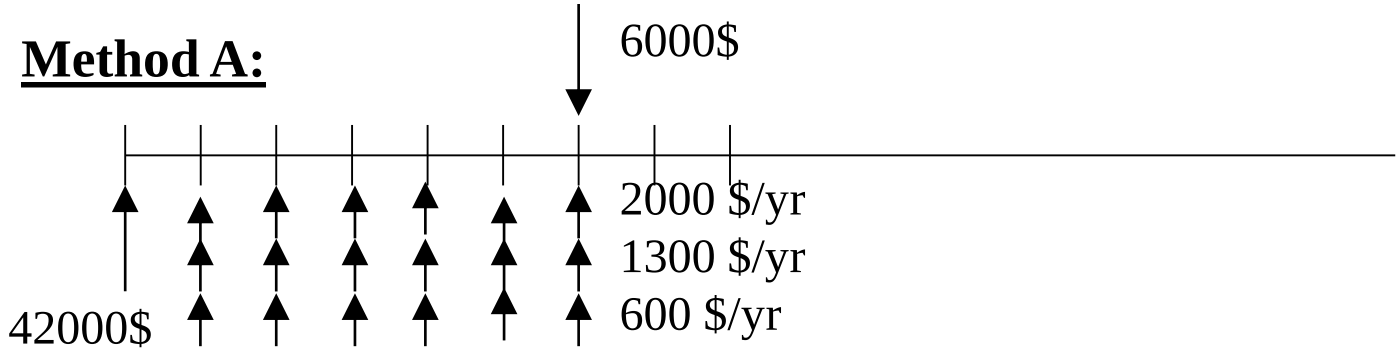
- Two different methods are being considered for elevating rock into a crusher. It is expected that the rock crusher will be used for 6 years. Cost estimates for the two methods are as follows:

	Method A	Method B
First Cost	42000 $	28000 $
Salvage Value	60000 $	10000 $
Annual Fuel Cost	2000 $/yr	4500 $/yr
Annual Maintenance Cost	1300 $/yr	3000 $/yr
Extra Annual Cost	600 $	-

- Compare the present worth and annual equivalent of these two alternatives by using a MARR of 12%.

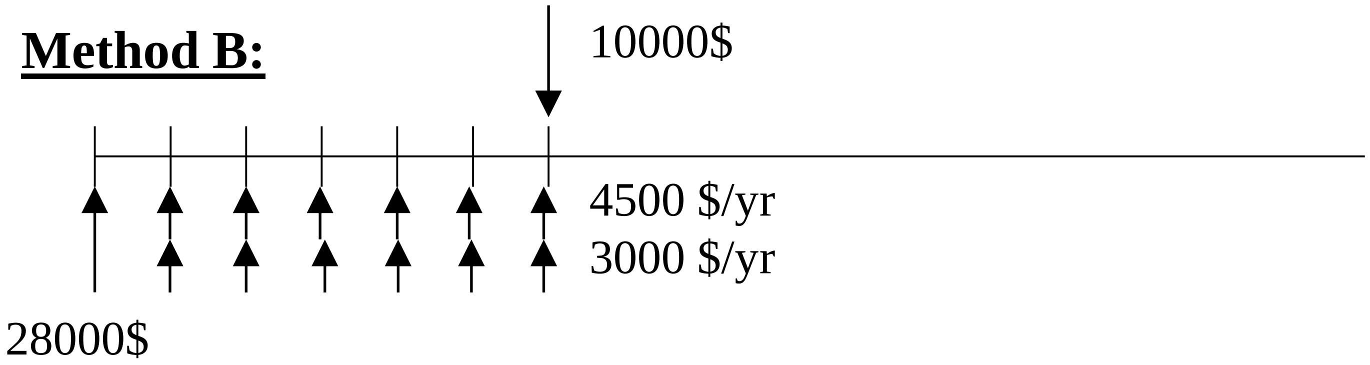
Solution (Annual Equivalent)

Method A:



AE_A= -42000(A/P,12%,6)+6000(A/F,12%,6)-2000-1300-600=-13,376 $/yr

Method B:

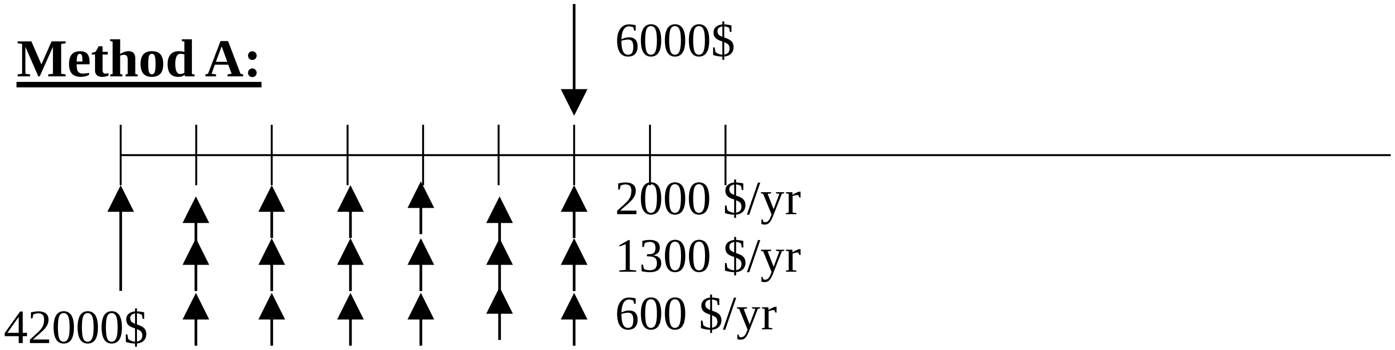


AE_B= -28000(A/P,12%,6)+10000(A/F,12%,6)-4500-3000=-13,078 $/yr

Select Alternative B!

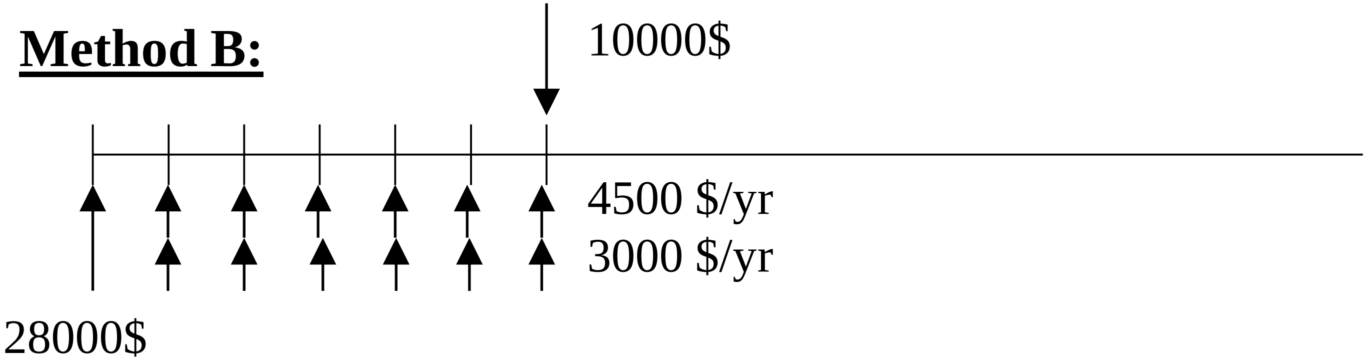
SOLUTION (PRESENT WORTH)

Method A:



PW_A= -42000+(2000+1300+600)(P/A,12%,6)+6000(P/F,12%,6)=-54.993 $

Method B:



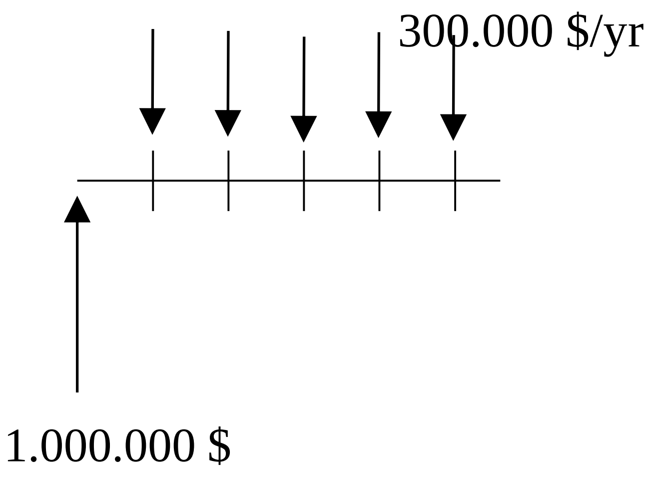
PW_B= -28000-(4500+3000)(P/A,12%,6)+10000(P/F,12%,6)=-53.766 $

Select Alternative B!

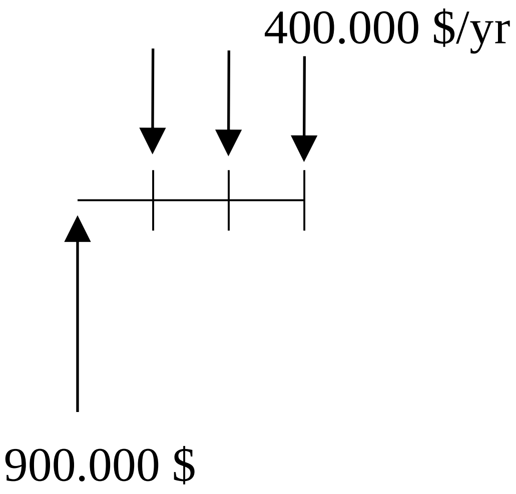
ALTERNATIVES WITH UNEQUAL SERVICE LIVES

- PW: Common multiplier of lives
- AE: No need such a thing as we will look at the annual costs.

EX: Find the present worth of the two alternatives whose cash flows are given below if MARR is 10%.



Method A



Method B

SOLUTION

AE_A= -100000(A/P,10%,5)+300000=36200 $/yr

AE_A= -90000(A/P,10%,5)+400000=38101 $/yr SELECT B!!!!

PW_A= 36200 (P/A,10%,15)=275337 $

PW_B= 38101 (P/A,10%,15)=289767 $ SELECT B!!!

Example

	Plan R	Plan S
First Cost	25 000 $	45 000 $
Useful Life	20 years	30 years
Salvage Value	5000 $	-
Annual Disbursements	5500 $/yr	2800 $/yr
MARR	8%	

SOLUTION:

AE_R= -25000 (A/P,8%,20)-5500+5000 (A/F,8,20) = -7988 $/yr

AE_S= -45000 (A/P,8%,30)-2800 = -6796 $/yr

PW_R= -7988 (P/A,8%,60) = -98245 $

PW_S= -6796 (P/A,8%,60) = -844111 $

SELECT S!!!

Replacement

- Basic reasons for replacement:
  - Physical impairment
  - Obsolescense
- Methods:
  - The outsider viewpoint method
  - Comparative use value method
  - Receipts and disbursement method