PILE FOUNDATIONS

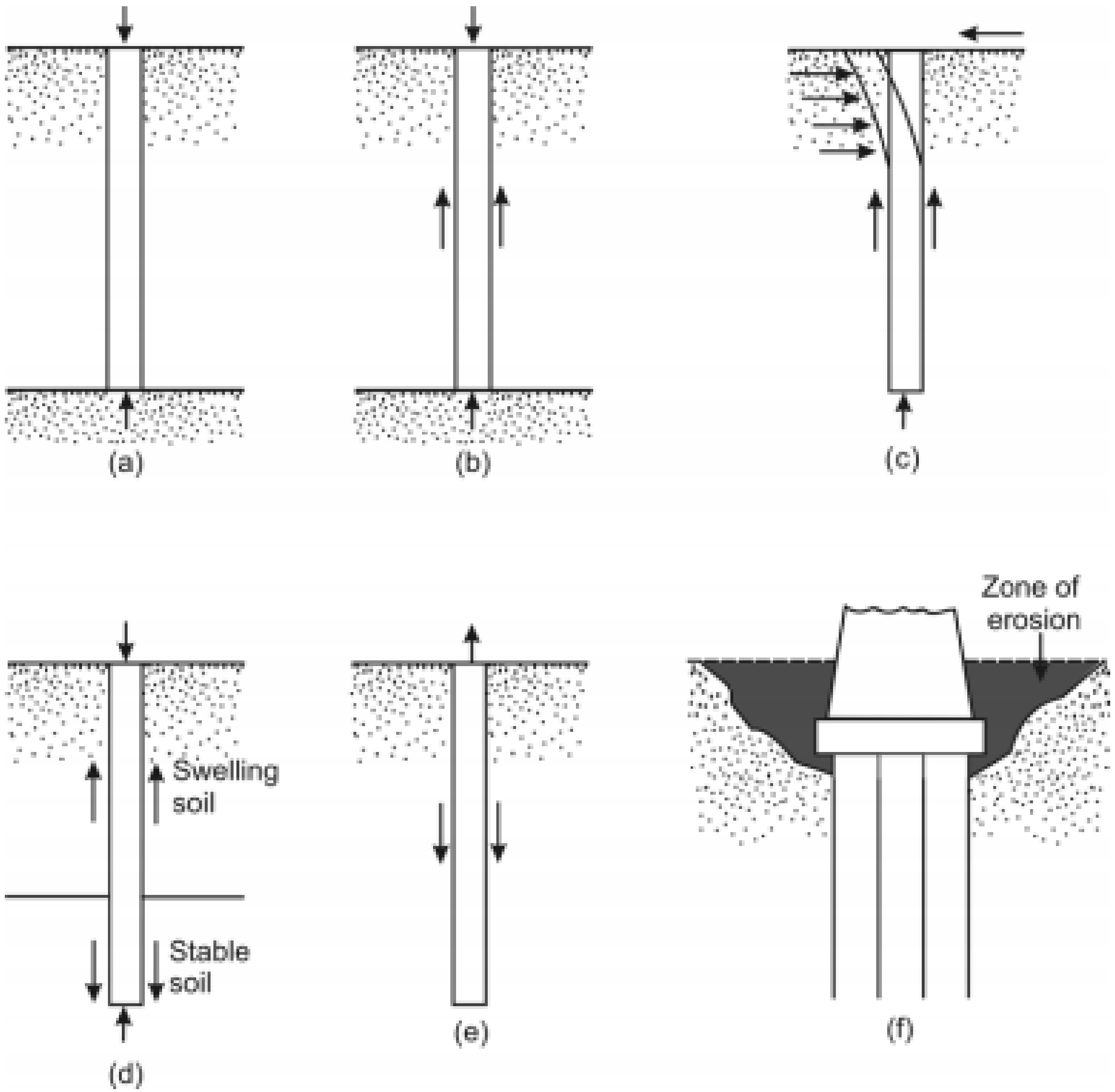
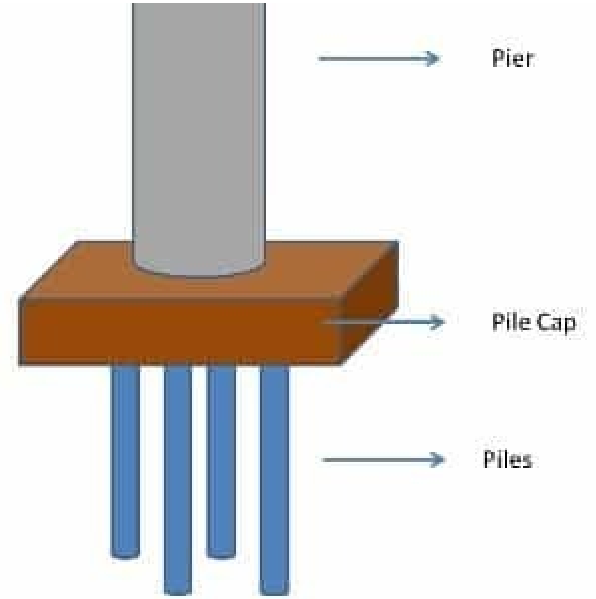
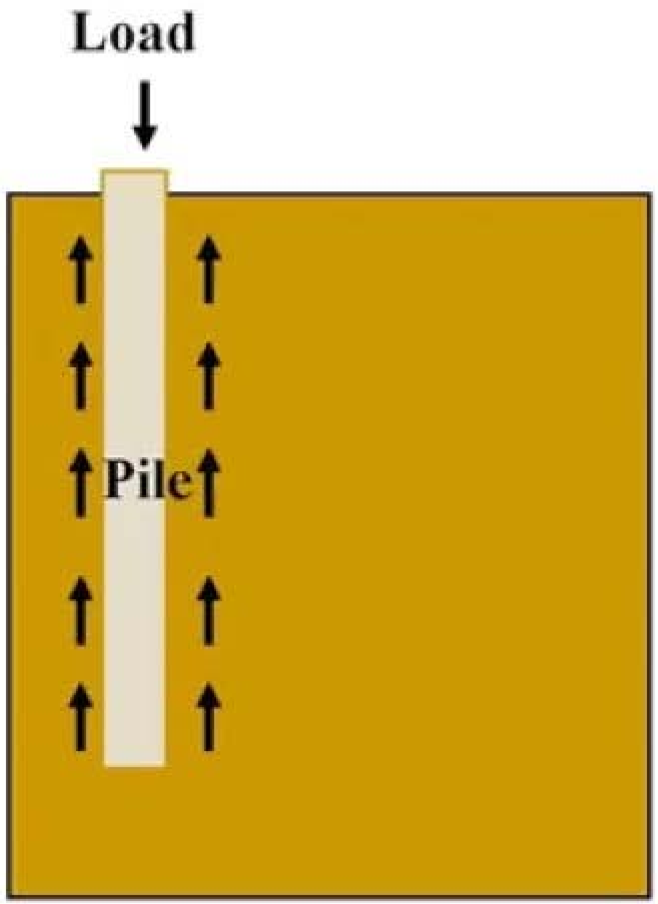
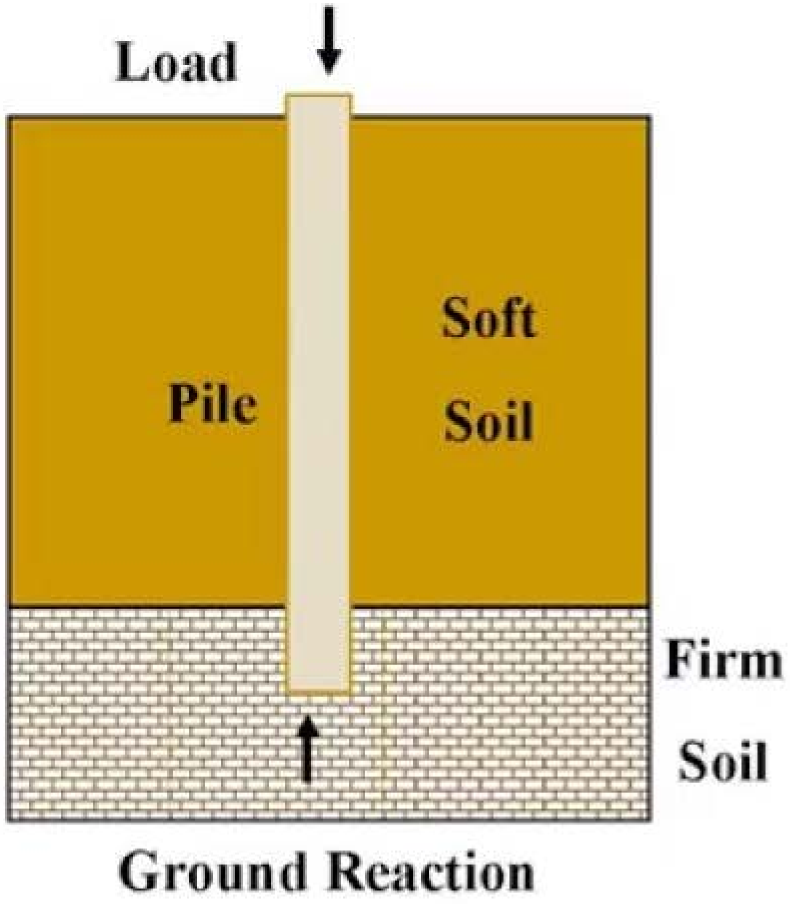


Figure 8.1 Conditions for use of pile foundations


Pier
Pile Cap
Piles



Friction Piles



End-bearing Piles

Types of piles

Different types of piles are used in construction work, depending on:

- the type of load to be carried,
- the subsoil conditions,
- the location of the water table.

Piles can be divided into the following categories:

- steel piles
- concrete piles,
- wooden (timber) piles,
- composite piles.

Piles can be divided into three major categories, depending on their lengths and the mechanisms of load transfer to the soil: ,

- point bearing piles,
- friction piles,
- compaction piles

Types of piles

Based on the nature of their placement, piles may be divided into two categories:

- displacement piles
- nondisplacement piles

Driven piles are displacement piles because they move some soil laterally; hence there is a tendency for densification of soil surrounding them. Concrete piles and closed-ended pile piles are high-displacement piles. However, steel H-piles displace less soil laterally during driving, and so they are low displacement piles. In contrast, bored piles are nondisplacement piles because their placement causes very little change in the state of stress in the soil.

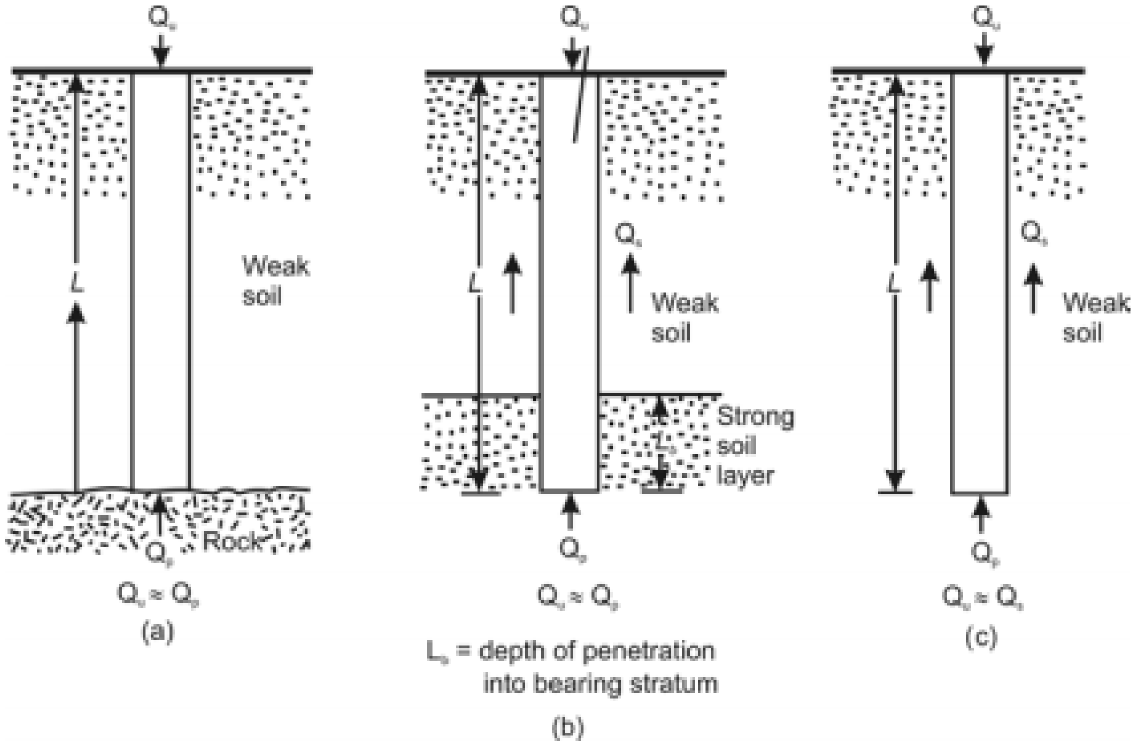


Figure 8.6 (a) and (b) Point bearing piles; (c) friction piles

$$Q_u + Q_p + Q_s \qquad [8.4]$$

Where

Q_p = load carried at the pile point

Q_s = load carried by skin friction developed at the side of the pile (caused by shearing resistance between the soil and the pile)

Friction Piles

When no layer of rock or rocklike material is present at a reasonable depth at a site, point bearing piles become very long and uneconomical. For this type o subsoil condition, piles are driven through the softer material to specified depths (figure 8. 6c).

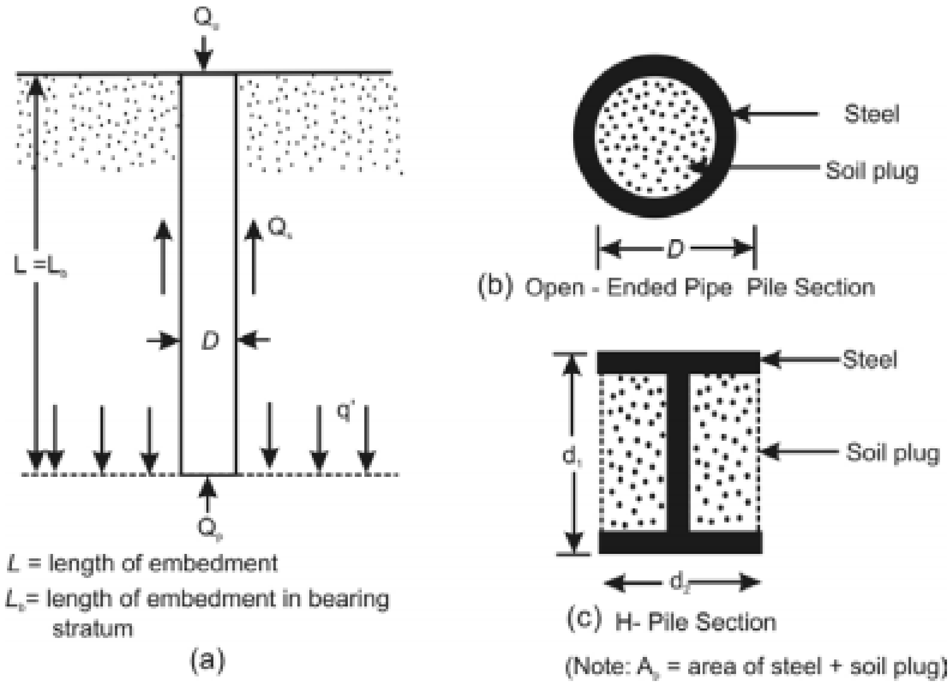
The ultimate load of these piles may be expressed by equation below. However, if the value of Q p is relatively small,

$$Q_u \approx Q_s$$

These piles are called friction piles because most of the resistance is derived from skin friction. However, the term friction pile, although used often in the literature, is a misnomer: in clayey soils, the resistance to applied load is also caused by adhesion. The length of friction of piles depends on the shear strength of the soil, the applied load and the pile size. To determine the necessary lengths of these piles, an engineer needs a good understanding of soil-pile interaction, good judgment, and experience.

ESTIMATING PILE CAPACITY

$$Q_u = Q_p + Q_s \qquad [8.8]$$



Q_u = ultimate pile capacity
Q_p = load – carrying capacity of the pile point
Q_s = frictional resistance

Point Bearing Capacity, $\boldsymbol{Q}$p

$$Q_p = A_p q_p = A_p(cN_c^* + q'N_q^*) \qquad [8.11]$$

Where

A_p = area of pile tip
c = cohesion of the soil supporting the pile tip
q_p = unit point resistance
q' = effective vertical stress at the level of the pile tip
$N_c^* + N_q^*$ = the bearing capacity factors

Frictional Resistance, $\boldsymbol{Q}$s

$$Q_s = \Sigma\, p\, \Delta L f \qquad [8.12]$$

Where

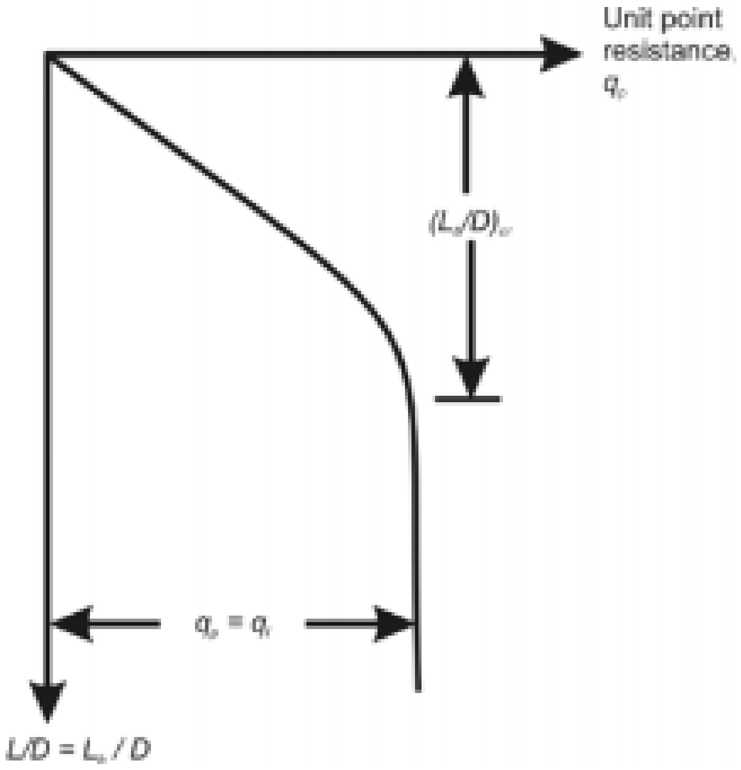
p = perimeter of the pile section
ΔL = incremental pile length over which p and f are taken constant
f = unit friction resistance at any depth z

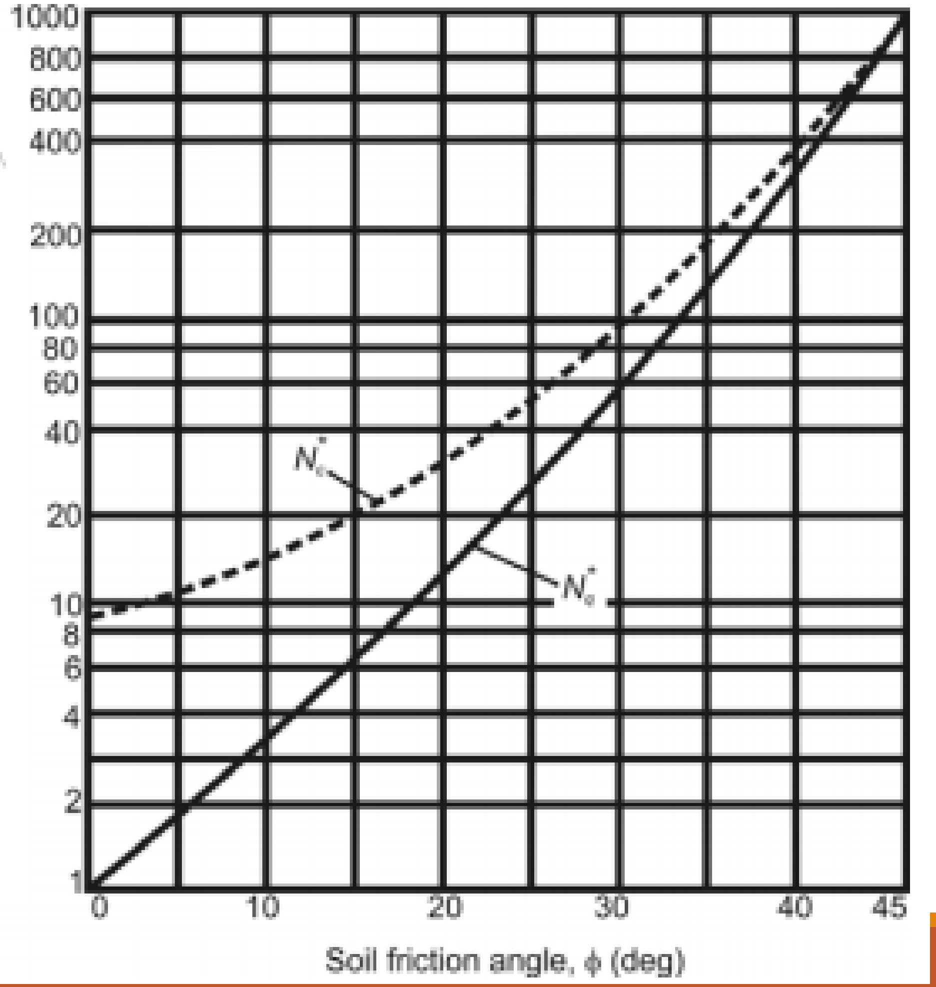
There are several methods for estimating Qp and Qs.

It needs to be reemphasized that, in the field, for full mobilization of the point resistance (Qp), the pile tip must go through a displacement of 10 to 25% of the pile width (or diameter).

MEYERHOF’S METHODS – Qp -- SAND

Sand:





$$Q_p = A_p q_p = A_p q' N_q^*$$

$$Q_p = A_p q' N_q^* \le A_p q_1$$

The limiting point resistance is

$$q_1(\text{kN/m}^2) = 50 N_q^* \tan\phi$$

Where

ϕ = soil friction angle in the bearing stratum

MEYERHOF’S METHODS – Qp – SAND

Based on field observations, Meyerhof (1976) also suggested that the ultimate point resistance, q_p, in a homogeneous granular soil ($L = L_b$) may be obtained from standard penetration numbers as

$$q_p(\mathrm{kN/m^2}) = 40\, N_{\mathrm{cor}}\, L/D \leq 400 N_{\mathrm{cor}} \quad [8.17]$$

Where

N_{cor} = average corrected standard penetration number near the pile point (about $10D$ above and $4D$ below the pile point)

In English units,

$$q_b(\mathrm{lb/ft^2}) = 800\, N_{\mathrm{cor}}\, L/D \leq 8000 N_{\mathrm{cor}} \quad [8.18]$$

MEYERHOF’S METHODS – Qp – CLAY

Clay (ϕ = 0 condition)

For piles in *saturated clays* in undrained conditions ($\phi = 0$).

$$Q_p = N_c^* c_u A_p = 9 c_u A_p \qquad [8.19]$$

Where

c_u = undrained cohesion of the soil below the pile tip

VESIC'S METHOD- $\boldsymbol{Q}$p

$$Q_p = A_p q_p = A_p(cN_c^* + \sigma'_0 N_\sigma^*) \qquad [8.20]$$

Where

σ'_0 = mean normal ground stress (effective)at the level of the pile point.

$$= \left(\frac{1+2K_0}{3}\right) q' \qquad [8.21]$$

K_0 = earth pressure coefficient at rest $= 1 - \sin\phi$ [8.22]

N_c^*, N_σ^* = bearing capacity factors

Note that equation (20) is a mobilization of equation (11) with

$$N_\sigma^* = \frac{3N_q^*}{(1+2K_0)} \qquad [8.23]$$

VESIC'S METHOD- $\boldsymbol{Q}$p

$N_c^* = (N_q^* - 1)\cot\phi$

According to Vesic's theory,

$N_\sigma^* = f(I_{rr})$

Where
I_{rr} = reduced rigidity index for the soil

However,

$I_{rr} = \frac{I_r}{1+I_r\Delta}$
Where

I_r = rigidity index $= \frac{E_s}{2(1+\mu_s)(c+q'\tan\phi)} = \frac{G_s}{c+q'\tan\phi}$

E_s = modulus of elasticity of soil

μ_s = Poisson's ratio of soil

G_s = shear modulus of soil

Δ = average volumatric strain in the plastic zone below t

For conditions of no volume change (dense sand or saturate

$I_r = I_{rr}$

Table D.6 (Appendix D) gives the values of N_c^* and N_σ^* f
friction angle (ϕ) and I_{rr}. For $\phi = 0$ (undrained condition),

$N_c^* = \frac{4}{3}(\ln I_{rr} + 1) + \frac{\pi}{2} + 1$

VESIC'S METHOD- $\boldsymbol{Q}$p

The values of I_r can be estimated from laboratory consolidation and triaxial tests corresponding to the proper stress levels. However, for preliminary use the following values are recommended:

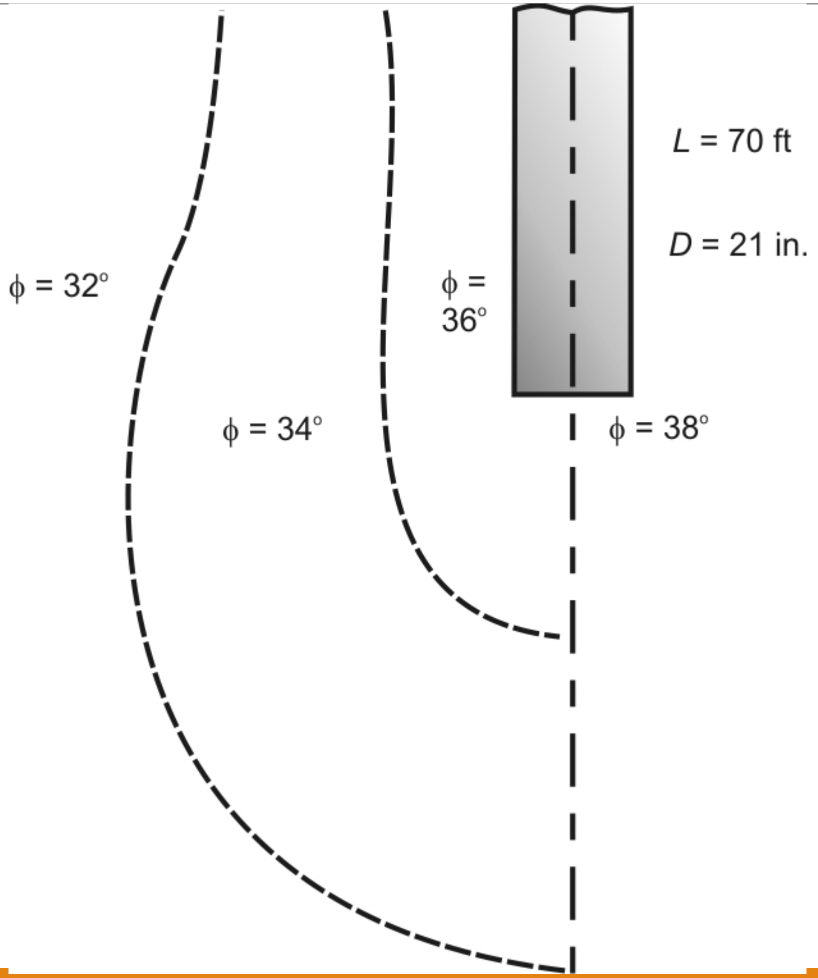
Soil type	I_r
Sand	70-150
Silts and clays (drained condition)	50-100
Clays (udrained condition)	100-200

FRICTIONAL RESISTANCE ($\boldsymbol{Qs}$) IN SAND

$$Q_s = \Sigma\, p\, \Delta\, L f$$

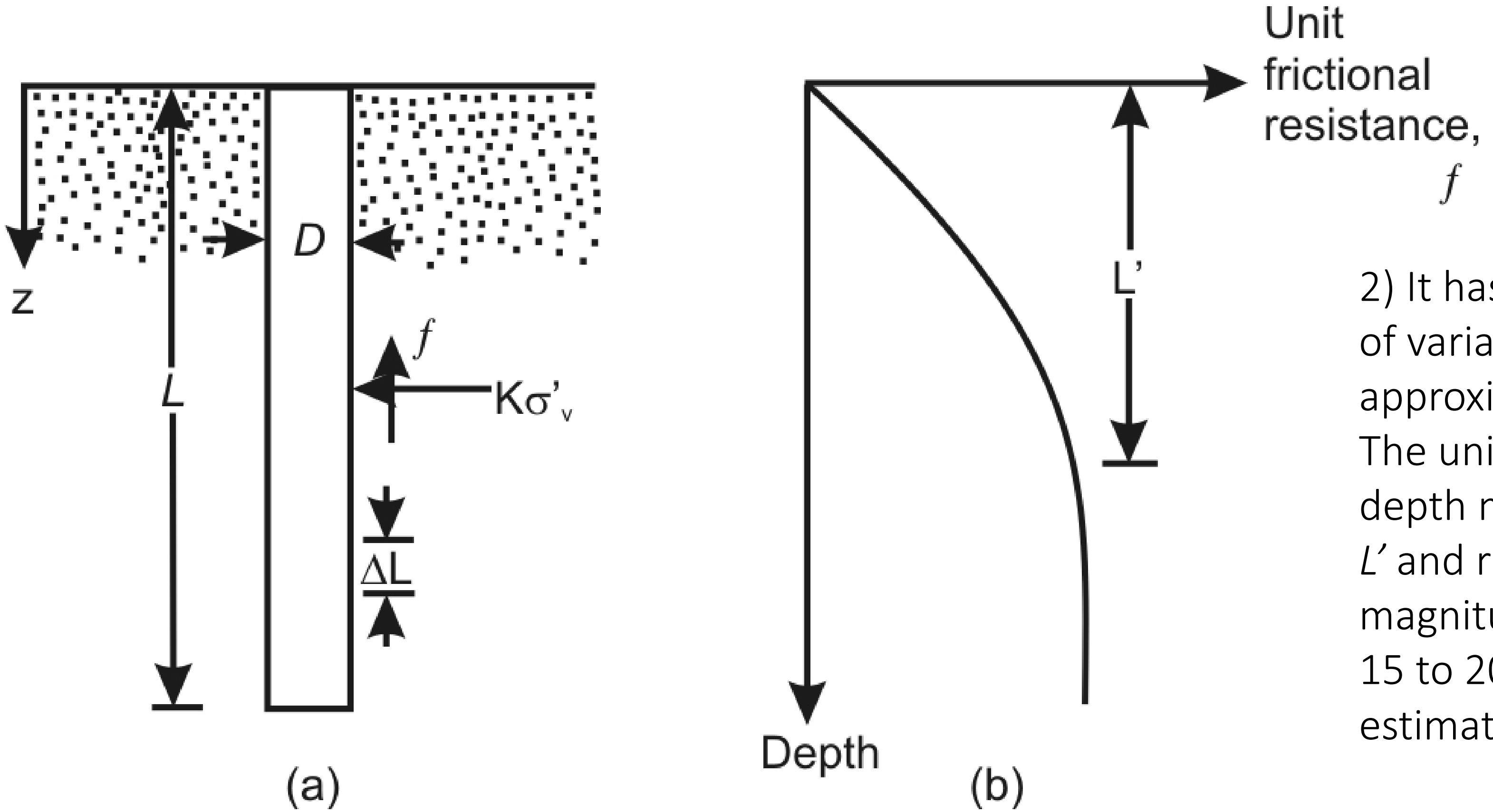
The unit frictional resistance, *f*, is hard to estimate. In making an estimation of *f*, several important factors must be kept in mind. They are as follows:

1) The nature of pile installation. For driven piles in sand, the vibration caused during pile driving helps densify the soil around the pile.



Compaction of sand near driven piles (after Meyerhof, 1961)

FRICTIONAL RESISTANCE ($\boldsymbol{Qs}$) IN SAND



Unit frictional resistances for piles in sand

2) It has been observed that the nature of variation of *f* in the field is approximately as shown in figure.
The unit skin friction increases with depth more or less linearly to a depth of *L′* and remains constant thereafter. The magnitude of the critical depth *L′* may be 15 to 20 pile diameters. A conservative estimate would be $L' \cong 15D$

FRICTIONAL RESISTANCE (Qs) IN SAND

3. At similar depths, the unit skin friction in loose sand is higher for a high displacement pile as compared to a low-displacement pile.

4. At similar depth, bored, or jetted, piles will have a lower unit skin friction as compared to driven piles.

Considering the above factors, an approximate relationship for$f$ can be given as follows

For $z = 0$ to L'

$$f = K\sigma'_v \tan\delta$$

And for $z =$ " to L

$$f = f_{z=L'}$$

Where

K = effective earth coefficient
σ'_v = effective vertical stress at the depth under consideration
δ = soil − pile friction angle

FRICTIONAL RESISTANCE ($\boldsymbol{Qs}$) IN SAND

Pile type	K
Bored or jetted	$\approx K_0 = 1 - \sin\phi$
Low-displacement driven	$\approx K_0 = 1 - \sin\phi$ to $1.4K_o = 1.4(1 - \sin\phi)$
High-displacement driven	$\approx K_0 = 1 - \sin\phi$ to $1.8K_o = 1.8(1 - \sin\phi)$

The values of δ from various investigations appear to be in the range of 0.5ϕ to 0.8ϕ.

FRICTIONAL RESISTANCE (Qs) IN SAND

For high displacement driven piles,

Bhusan (1982) recommended

$$K \tan \delta = 0.18 + 0.0065 D_r$$

And

$$K = 0.5 + 0.008 D_r$$

Where

D_r = relative density (%)

Meyerhof (1976) also indicated that the average unit frictional resistance, f_{av},

for high-displacement $\quad f_{av} = (\mathrm{kN/m^2}) = 2\overline{\mathrm{N}}_{cor}$

for low-displacement $\quad f_{av}(\mathrm{kN/m^2}) = \overline{\mathrm{N}}_{cor}$

Where

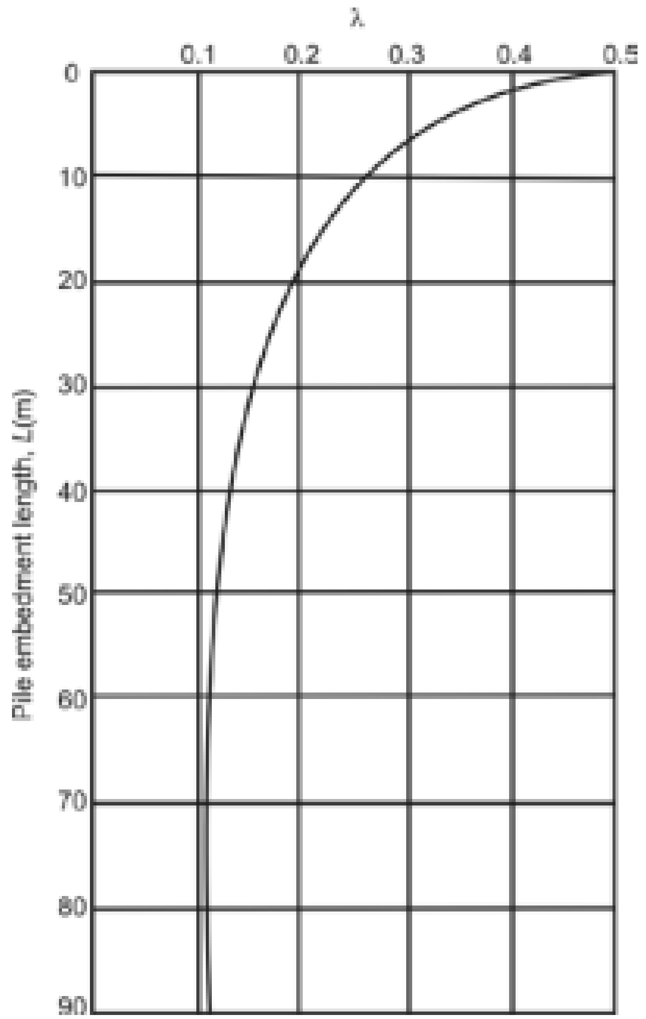
N_{cor}=average corrected value of standard penetration resistance

FRICTIONAL RESISTANCE IN CLAY -- λ Method

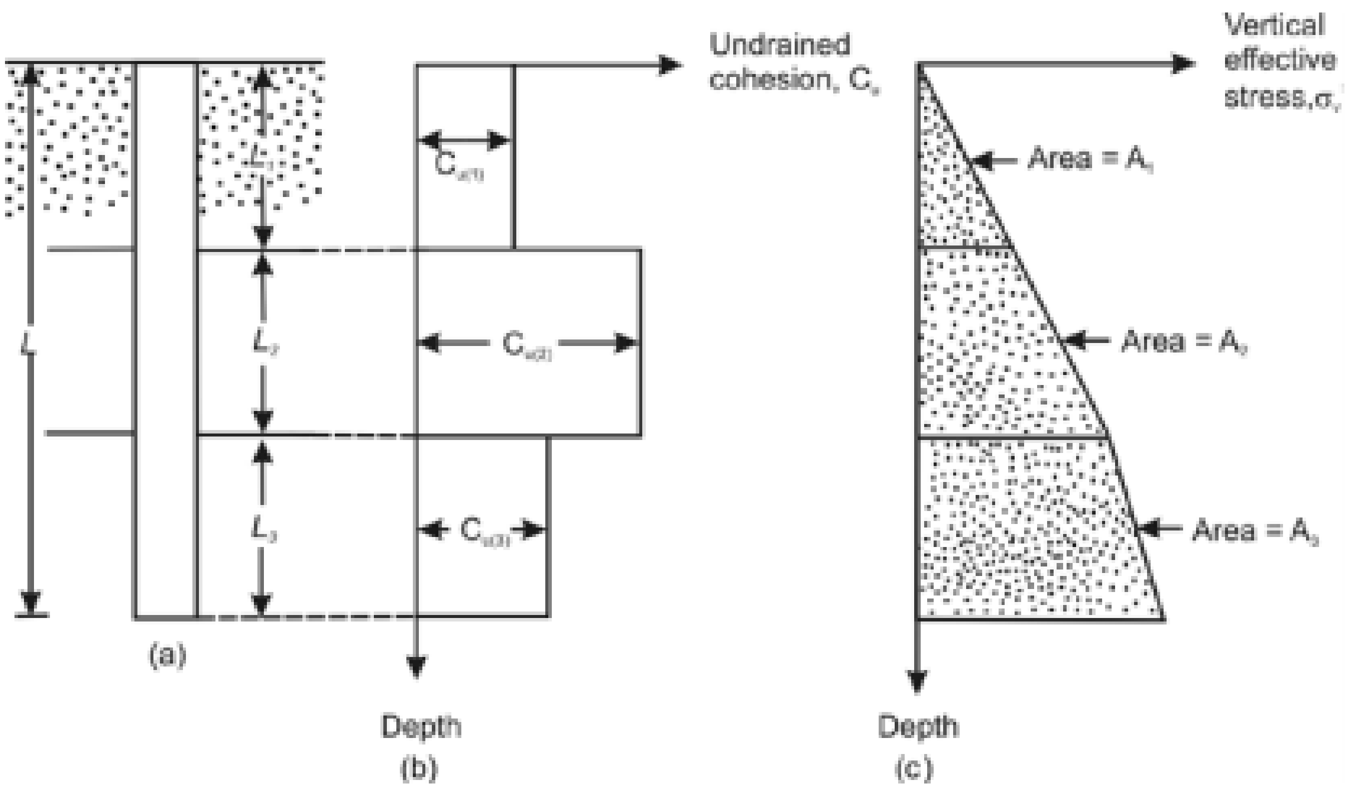
$$f_{av} = \lambda(\bar{\sigma}'_v + 2c_u)$$

$\bar{\sigma}'_v$ = mean effective vertical stress for the entire embedment length
c_u = mean undrained shear strength ($\phi = 0$ concept)

$$Q_s = pLf_{av}$$

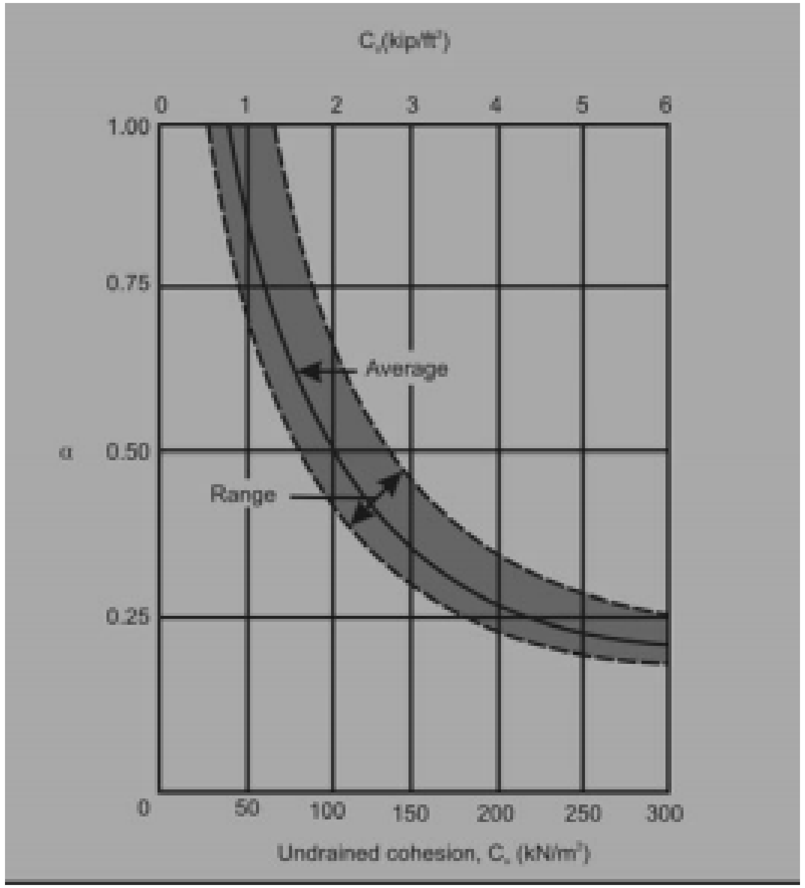


FRICTIONAL RESISTANCE IN CLAY -- λ Method



$$\bar{\sigma}'_v = \frac{A_1 + A_2 + A_3 + \cdots}{L}$$

FRICTIONAL RESISTANCE IN CLAY -- α Method



$f = \alpha c_u$

Where

α = empirical adhesion factor

$$Q_s = \Sigma\, f p\, \Delta L = \Sigma\, \alpha c_u\, p \Delta L$$

FRICTIONAL RESISTANCE IN CLAY -- β Method

$$f = \beta \sigma'_v$$

$$Q_s = \Sigma f p \, \Delta L$$

σ'_v = vertical effective stress
$\beta = K \tan \phi_R$
ϕ_R = drianed friction angle of remolded clay

K = earth pressure coefficient

$K = (1 - \sin \phi_R)$ (for normally consolidated clays

$K = (1 - \sin \phi_R)\sqrt{OCR}$ (for overconsolidated clays

OCR= overconsolidation ratio

POINT BEARING CAPACITY OF PILES RESTING ON ROCK

$$q_p = q_u(N_\phi + 1)$$

$N_\phi = tan^2(45 + \phi/2)$

$q_u = unconfined\ compression\ strength\ of\ rock$

$\phi = drained\ angle\ of\ friction$

The unconfined compression strength of rock can be determined by laboratory tests on rock specimens collected during field investigation. However, extreme caution should be used in obtaining the proper vale of qu because laboratory specimens usually are small in diameter.

As the diameter of the specimen increases, the unconfined compression strength decreases, this is referred to as the scale effect. For specimens larger than about 3 ft (1 m) in diameter, the value of qu remains approximately constant. There appears to be a fourfold to fivefold reduction of the magnitude of qu in this process.

The scale effect in rock is primarily caused by randomly distributed large and small fractures and also by progressive ruptures along the slip lines. Hence, it is recommended that

$$Q_{u(design)} = \frac{q_{u(lab)}}{5}$$

POINT BEARING CAPACITY OF PILES RESTING ON ROCK

	q_u	
Rock type	lb/in^2	MN/m^2
Sand stone	10,000-20,000	70-140
Limestone	15,000-30,000	105-210
Shale	5,000-10,000	35-70
Granite	20,000-30,000	140-210
Marble	8,500-10,000	60-70

Typical Unconfined Compressive Strength of Rocks

POINT BEARING CAPACITY OF PILES RESTING ON ROCK

Rock type	Angle of friction, ϕ (deg)
Sandstone	27-45
Limestone	30-40
Shale	10-20
Granite	40-50
Marble	25-30

Typical Values of Angle of Friction, $\phi\phi$, of Rocks

POINT BEARING CAPACITY OF PILES RESTING ON ROCK

A factor of safety of at least 3 should be used to determine the allowable point bearing capacity of piles. Thus :

$$Q_{p(all)} = \frac{[q_{u(design\ )}(N_{\phi}+1)A_p}{FS}$$

SETTLEMENT OF PILES

$$s = s_1 + s_2 + s_3$$

$s = total\ pile\ settlement$

$s_1 = elastic\ settlement\ of\ pile$

$s_2 = settlement\ of\ pile\ caused\ by\ the\ load\ at\ the\ pile\ tip$

$s_3 = settlement\ of\ pile\ casued\ by\ the\ load\ transmitted\ along\ the\ pile\ shaft$

SETTLEMENT OF PILES – S1

If the pile material is assumed to be elastic, the deformation of the pile shaft can be evaluated using the fundamental principles of mechanics of materials.

$$S_1 = \frac{(Q_{wp} + \xi Q_{ws})L}{A_p E_p}$$

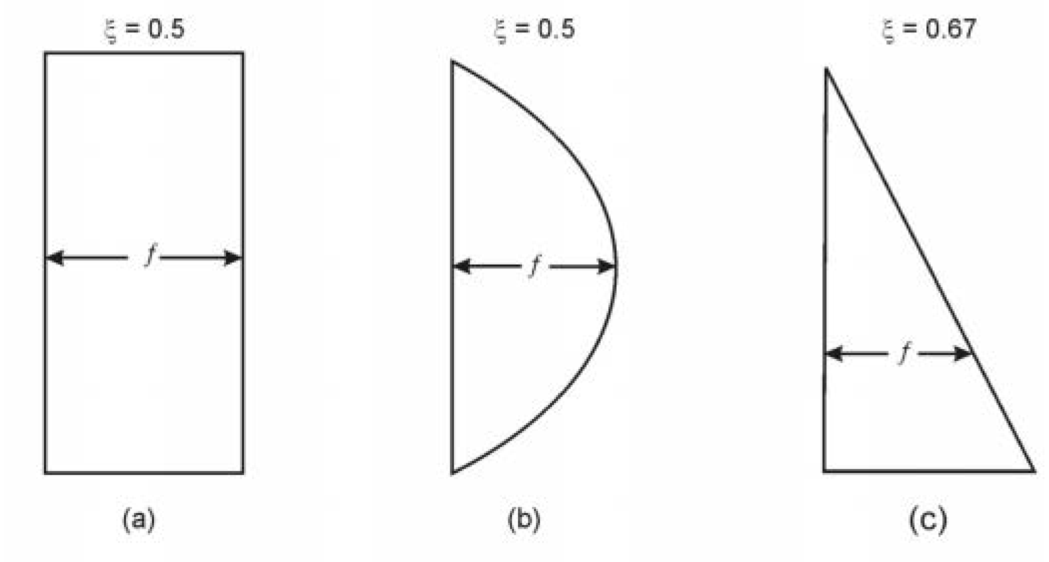
Q_{wp} = *load carried at the pile point under working load condition*

Q_{ws} = *load carried by frictional (skin) resistance under working load condition*

A_p = *area of pile cross section*

L = *length of pile*

E_p = *modulus of elasticity of the pile material*



The magnitude of ξ will depend on the nature of unit friction (skin) resistance distribution along the pile shaft. If the distribution of f is uniform or parabolic, ξ = 0.5. However, for triangular distribution of f, the magnitude of ξ is about 0.67 (Vesic, 1977).

SETTLEMENT OF PILES – S2

The settlement of a pile caused by the load carried at the pile point may be expressed in a form similar to that given for shallow foundations:

$$s_2 = \frac{q_{wp} D}{E_s}(1 - \mu_s^2) I_{wp}$$

$D = width\ or\ diameter\ of\ pile$

$q_{wp} = point\ load\ per\ unit\ area\ at\ the\ pile\ point = Q_{wp}/A_p$

$E_s = modulus\ of\ elasticity\ of\ soil\ at\ or\ below\ the\ pile\ point$

$\mu_s = Poisson's\ ratio\ of\ soil$

$I_{wp} = influence\ factor \approx 0.85$

Vesic (1977) also proposed a semi-empirical method to obtain the magnitude of the settlement, $s2$:

$$s_2 = \frac{Q_{wp} C_p}{D q_p}$$

$q_p = ultimate\ point\ resistance\ of\ the\ pile$

$C_p = an\ empirical\ coefficient$

Soil type	Driven pile	Bored pile
Sand (dense to loose)	0.02-0.04	0.09-0.18
Clay (stiff to soft)	0.02-0.03	0.03-0.06
Silt (dense to loose)	0.03-0.05	0.09-0.12
From "Design of Pile Foundations," by A. S. Vesic in NCHRP *Synthesis of Highway Practice 42*, Transportation Research Board, 1977. Reprinted by permission		

SETTLEMENT OF PILES – S3

The settlement of a pile caused by the load carried by the pile shaft is given by a relation:

$$S_3 = \left(\frac{Q_{ws}}{pL}\right)\frac{D}{E_s}(1 - \mu_s^2)I_{ws}$$

$p = perimeter\ of\ the\ pile$

$L = embedded\ length\ of\ pile$

$I_{ws} = influence\ factor$

$$I_{ws} = 2 + 0.35\sqrt{\frac{L}{D}}$$

Note that the term Qws/pL is the average value of along the pile shaft.

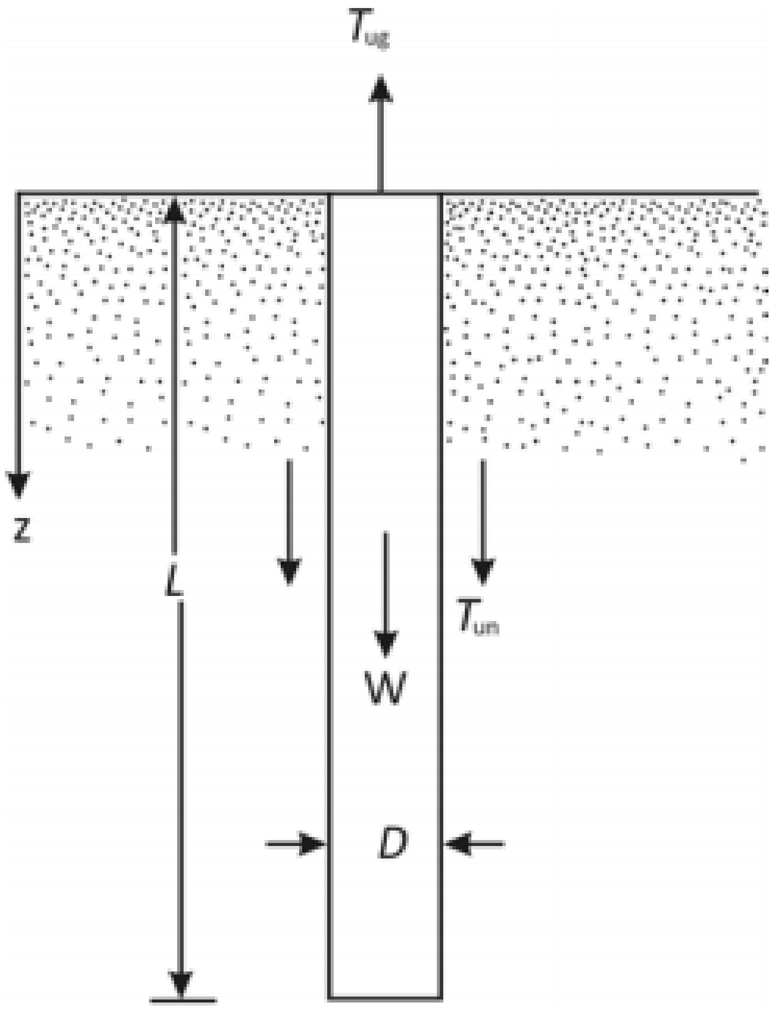
PULLOUT RESISTANCE OF PILES

$$T_{ug} = T_{un} + W$$

T_{ug} = *gross uplift capacity*

T_{un} = *net uplift capacity*

W = *effective weight of the pile*



D = diameter or width of pile

PULLOUT RESISTANCE OF PILES – CLAYS

The net ultimate uplift capacity of piles embedded in saturated clays was studied by Das and Seeley (1982). According to that study,

$$T_{un} = Lp\alpha' c_u$$

$L = length\ of\ the\ pile$

$p = perimeter\ of\ pile\ section$

$\alpha' = adhesion\ coefficient\ at\ soil - pile\ interface$

$c_u = undrained\ cohesion\ of\ clay$

For cast-in-situ concrete piles $\alpha' = 0.9 - 0.00625c_u \quad (for\ c_u \leq 80\ kN/m^2)$

$\alpha' = 0.4\ (for\ c_u > 80\ kN/m^2)$

For pile piles, $\alpha' = 0.715 - 0.0191c_u\ (for\ c_u \leq 27\ kN/m^2)$

$\alpha' = 0.2\ (for\ c_u > 27\ kN/m^2)$

PULLOUT RESISTANCE OF PILES – SANDS

When piles are embedded in granular soils ($c = 0$), the net ultimate uplift capacity (Das and Seeley, 1975) is:

$$T_{un} = \int_0^L (f_u p)\,dz$$

$f_u = unit\ skin\ friction\ during\ uplift$

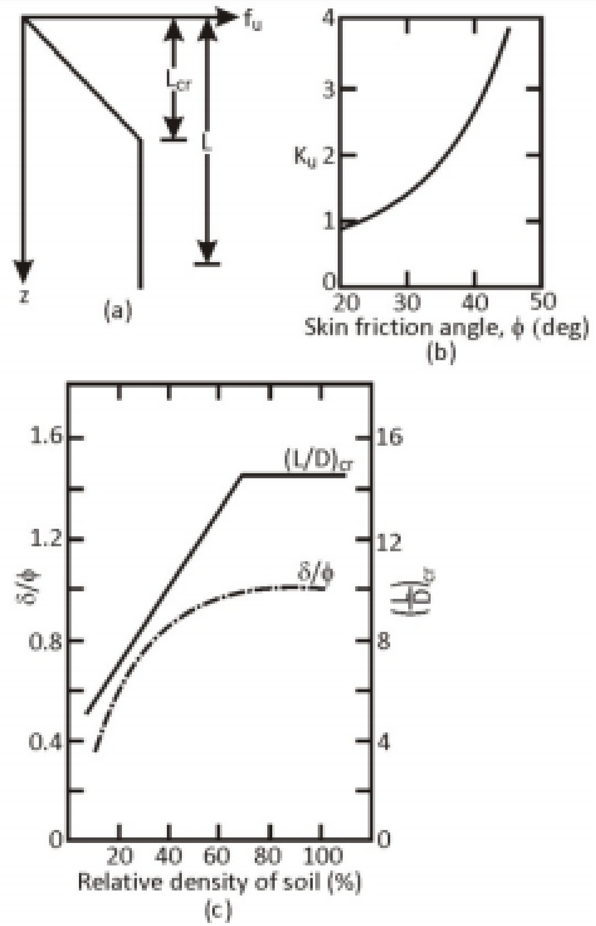
$p = perimeter\ of\ pile\ cross\ section$

$$f_u = K_u \sigma'_v \tan\delta$$

$K_u = uplift\ coefficient$

$\sigma'_v = effective\ vertical\ stress\ at\ a\ depth\ of\ z$

$\delta = soil - pile\ friction\ angle$



(a) Nature of variation of f_u; (b) uplift coefficient K_u; (c) variation of δ/ϕ *and* $(L/D)_{cr}$ with relative density of sand

For calculating the net ultimate uplift capacity of piles, the following procedure is suggested

1) Determine the relative density of the soil and, using figure c, obtain the value of L_{cr}

2) If the length of the pile, L, is less than or equal to L_{cr}.

$$T_{un} = p\int_0^L f_u\,dz = p\int_0^L (\sigma'_v K_u \tan\delta)dz$$

3) For L>Lcr

$$T_{un} = p\int_0^L f_u\,dz = p\left[\int_0^{L_{cr}} f_u\,dz + \int_{L_{cr}}^L f_u\,dz\right]$$

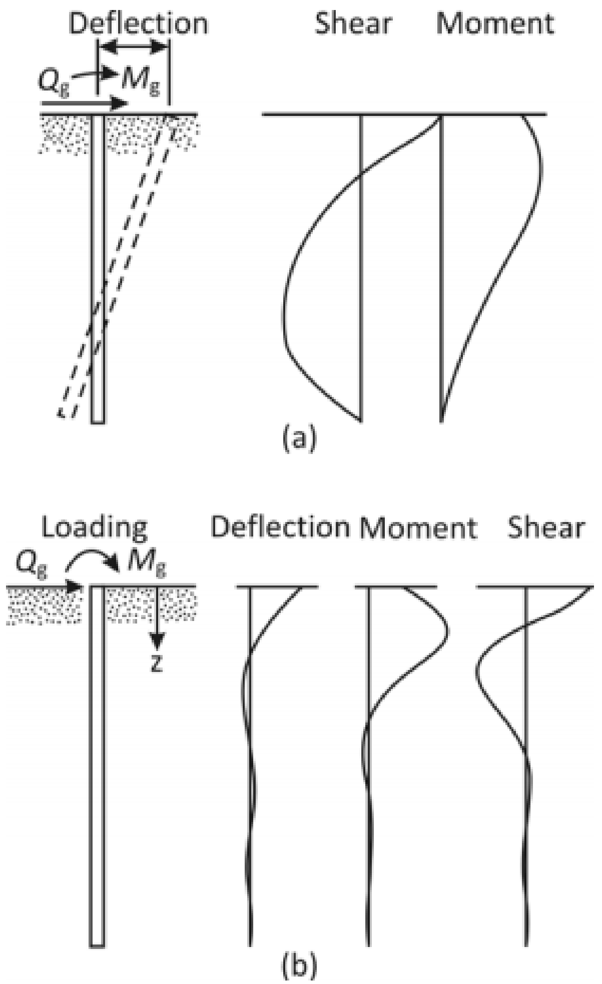
$$= p\left\{\int_0^{L_{cr}} [\sigma'_v K_u \tan\delta]dz + \int_{L_{cr}}^L [\sigma'_{v(at\ z=L_{cr})} K_u \tan\delta]dz\right\}$$

For dry soils:

$$T_{un} = \tfrac{1}{2}p\gamma L_{cr}^2 K_u \tan\delta + p\gamma L_{cr} K_u \tan\delta(L - L_{cr})$$

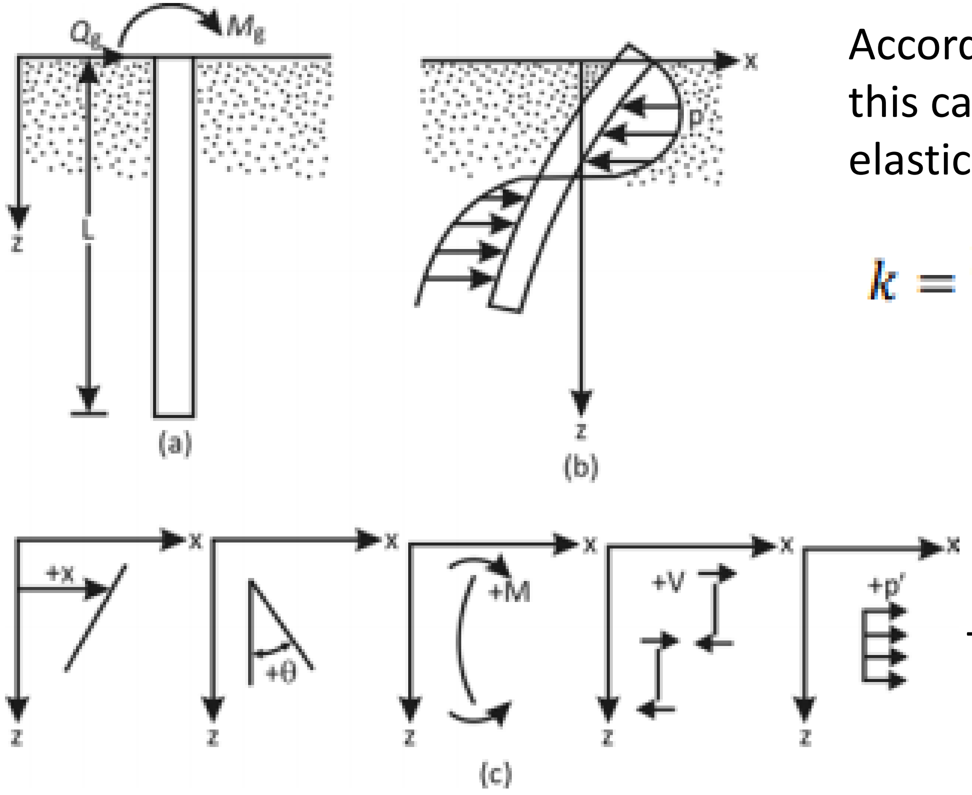
A factor of safety of 2-3 is recommended → $T_{u(all)} = \dfrac{T_{ug}}{FS}$

LATERALLY LOADED PILES



Nature of variation of pile deflection, moment, and shear force for (a) rigid pile, (b) elastic pile

Laterally Loaded Piles – Elastic Solution



According to a simpler Winkler's model, an elastic medium (soil in this case) can be replaced by a series of infinitely close independent elastic springs. Based on this assumption

$$k = \frac{p'\,(kN/m \text{ or } lb/ft)}{x(m \text{ or } ft)}$$

$k = modulus\ of\ subgrade\ reaction$

$p' = pressure\ on\ soil$

$x = deflection$

The subgrade modulus for granular soils at a depth z is defined as

$$k_z = n_h z$$

$n_h = constant\ of\ modulus\ of\ horizontal\ subgrade\ reaction$

Laterally Loaded Piles – Elastic Solution – SAND

Pile Deflection at Any Depth [$x_z(z)$]

$$x_z(z) = A_x \frac{Q_g T^3}{E_p I_p} + B_x \frac{M_g T^2}{E_p I_p}$$

Slope of Pile at Any Depth [$\theta_z(z)$]

$$\theta_z(z) = A_\theta \frac{Q_g T^2}{E_p I_p} + B_\theta \frac{M_g T}{E_p I_p}$$

Moment of Pile at Any Depth [$M_z(z)$]

$$M_z(z) = A_m Q_g T + B_m M_g$$

Shear Force on Pile at Any Depth [$V_z(z)$]

$$V_z(z) = A_v Q_g + B_v \frac{M_g}{T}$$

Soil Reaction at Any Depth [$p'_z(z)$]

$$p'_z(z) = A_{p'} \frac{Q_g}{T} + B_{p'} \frac{M_g}{T^2}$$

Where $A_x, B_x, A_\theta, B_\theta, A_m, B_m, A_v, B_v, A_{p'}$, $and\ B_{p'}$ are coefficients

$$T = characteristics\ length\ of\ the\ soil - pile\ system$$

$$= \sqrt[5]{\frac{E_p I_p}{n_h}}$$

$$k_z = n_h z$$

When $L \geq 5T$, the piles is considered to be a long pile. For $L \leq 2T$, the pile is considered to be a rigid pile.

Z, is the nondimensional depth, or

$$Z = \frac{z}{T}$$

When L/T is greater than about 5, the coefficients do no change, which is true of long piles only.

Z	A_x	A_θ	A_m	A_v	A'_p	B_x	B_θ	B_m	B_v	B'_p
0.0	2.435	-1.623	0.000	1.000	0.000	1.623	-1.750	1.000	0.000	0.000
0.1	2.273	-1.618	0.100	0.989	-0.227	1.453	-1.650	1.000	-0.007	-0.145
0.2	2.112	-1.603	0.198	0.956	-0.422	1.293	-1.550	0.999	-0.028	-0.259
0.3	1.952	-1.578	0.291	0.906	-0.586	1.143	-1.450	0.994	-0.058	-0.343
0.4	1.796	-1.545	0.379	0.840	-0.718	1.003	-1.351	0.987	-0.095	-0.401
0.5	1.644	-1.503	0.459	0.764	-0.822	0.873	-1.253	0.976	-0.137	-0.436
0.6	1.496	-1.454	0.532	0.677	-0.897	0.752	-1.156	0.960	-0.181	-0.451
0.7	1.353	-1.397	0.595	0.585	-0.947	0.642	-1.061	0.939	-0.226	-0.449
0.8	1.216	-1.335	0.649	0.489	-0.973	0.540	-0.968	0.914	-0.270	-0.432
0.9	1.086	-1.268	0.693	0.392	-0.977	0.448	-0.878	0.885	-0.312	-0.403
1.0	0.962	-1.197	0.727	0.295	-0.962	0.364	-0.792	0.852	-0.350	-0.364
1.2	0.738	-1.047	0.767	0.109	-0.885	0.223	-0.629	0.775	-0.414	-0.268
1.4	0.544	-0.893	0.772	-0.056	-0.761	0.112	-0.482	0.688	-0.456	-0.157
1.6	0.381	-0.741	0.746	-0.193	-0.609	0.029	-0.354	0.594	-0.477	-0.047
1.8	0.247	-0.596	0.696	-0.298	-0.445	-0.030	-0.245	0.498	-0.476	0.054
2.0	0.142	-0.464	0.628	-0.371	-0.283	-0.070	-0.155	0.404	-0.456	0.140
3.0	-0.075	-0.040	0.225	-0.349	0.226	-0.089	0.057	0.059	-0.213	0.268
4.0	-0.050	0.052	0.000	-0.106	0.201	-0.028	0.049	-0.042	0.017	0.112
5.0	-0.009	0.025	-0.033	0.015	0.046	0.000	-0.011	-0.026	0.029	-0.002

Coefficients for Long Piles, $kz = n_h z$

Representative Values of n_h

n_h		
Soil	lb/in^3	kN/m^3
Dry or moist and		
Loose	6.5-8.0	1800-2200
Medium	20-25	5500-7000
Dense	55-65	15,000-18,000
Submerged sand		
Loose	3.5-5.0	1000-1400
Medium	12-18	3500-4500
Dense	32-45	9000-12,000

Laterally Loaded Piles – Elastic Solution – CLAY

$$\boldsymbol{x_z(z)} = A'_x \frac{Q_g R^3}{E_p I_p} + B'_x \frac{M_g R^2}{E_p I_p}$$

$$M_z(z) = A'_m Q_g R + B'_m M_g$$

Where

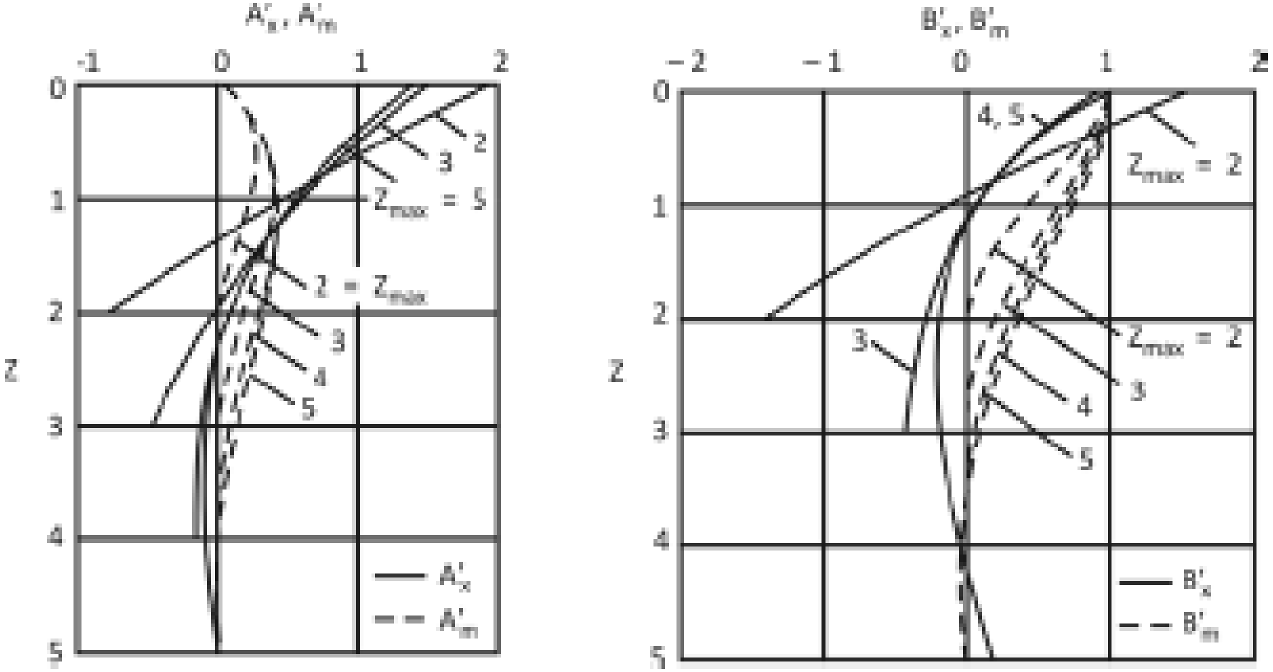
$A'_x, B'_x, A'_m, and\ B'_m$ are coefficients

$$R = \sqrt[4]{\frac{E_p I_p}{k}}$$

$$Z = \frac{z}{R}$$

And

$$Z_{max} = \frac{L}{R}$$



40 Variation of $A'_x, B'_x, A'_m, and\ B'_m$ *with* Z (after Davisson and Gill, 1960)

Laterally Loaded Piles – Elastic Solution – CLAY

For sands, the coefficient of subgrade reaction was showed a linear variation with depth. However, in cohesive soils, the subgrade reaction may be assumed to be approximately constant with depth. Vesic (1961) proposed the following equation to estimate the value of k:

$$k = 0.65\sqrt[12]{\frac{E_s D^4}{E_p I_p}}\frac{E_s}{1-\mu_s^2}$$

Where

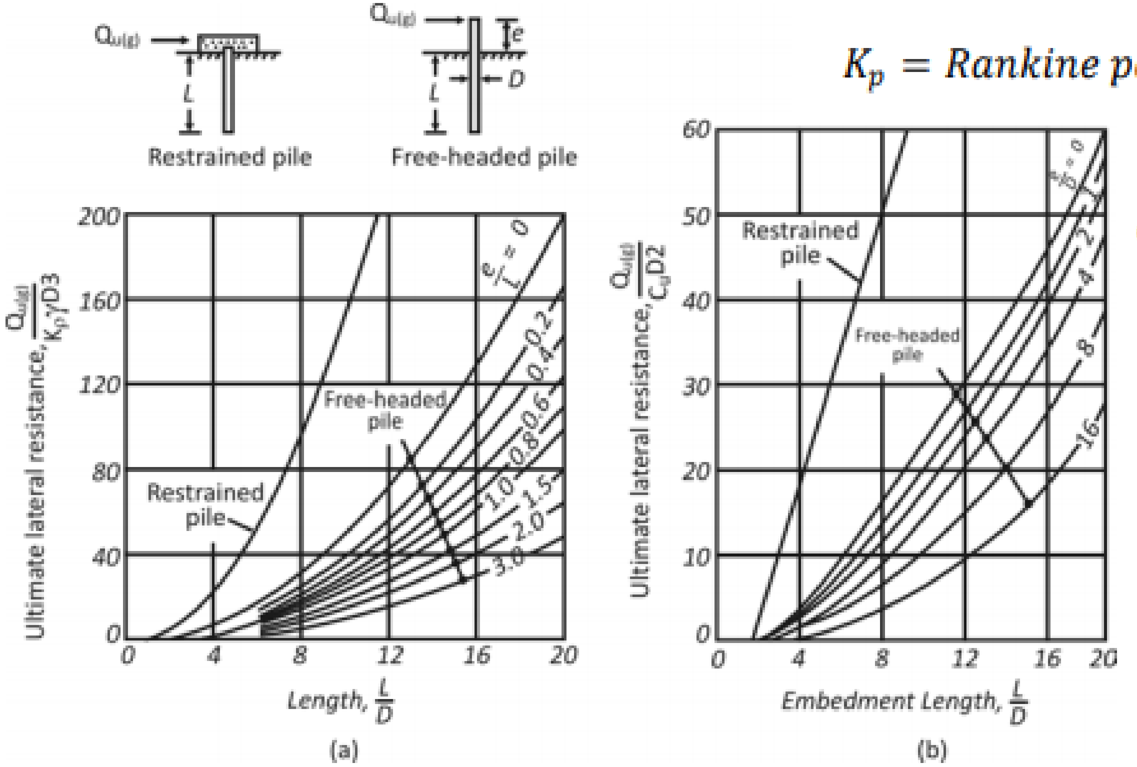
$E_s = modulus\ of\ elasticity\ of\ soil$

$d = pile\ width\ (or\ diameter)$

$\mu_s = Poisson's\ ratio\ of\ the\ soil$

Ultimate Load Analysis-Brom’s Method – Short Piles

Broms (1965) developed a simplified solution for laterally loaded piles based on the assumptions of (a) shear failure in soil, which is the case for short piles, and (b) bending of the pile governed by plastic yield resistance of the pile section, which is applicable for long piles. Brom’s solution for calculating the ultimate load resistance, $Qu(g)$, for short piles is given in figure 8.41a. A similar solution for piles embedded in cohesive soil is shown in figure 8.41b.



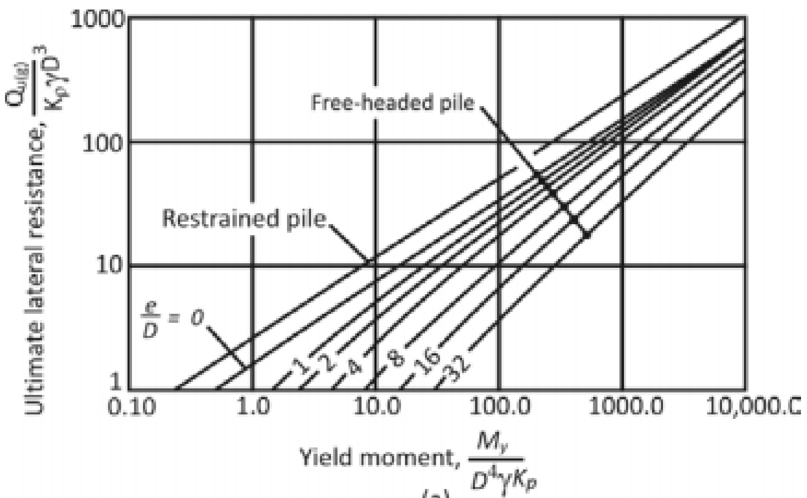
$$K_p = Rankine\ passive\ earth\ pressure\ coefficient = tan^2\left(45 + \frac{\phi}{2}\right)$$

$$c_u = undrained\ cohesion \approx \frac{0.75q_u}{FS} = \frac{0.75q_u}{2} = 0.375q_u$$

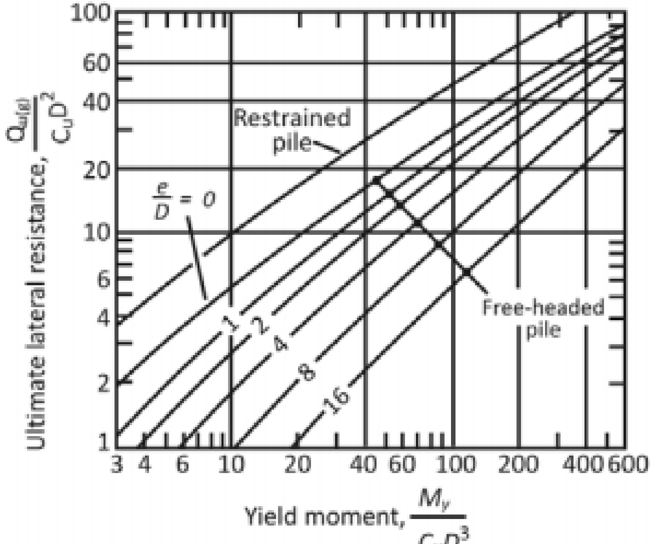
$$q_u = unconfined\ compression\ strength$$

Figure 8.41 Brom’s solution for ultimate lateral resistance of short piles (a) in sand, (b) in clay

Ultimate Load Analysis-Brom's Method – Long Piles



(a)

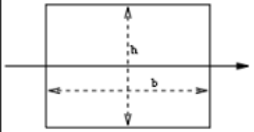
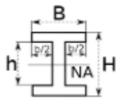
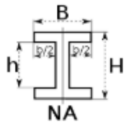
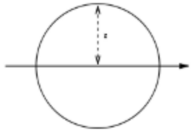


(b)

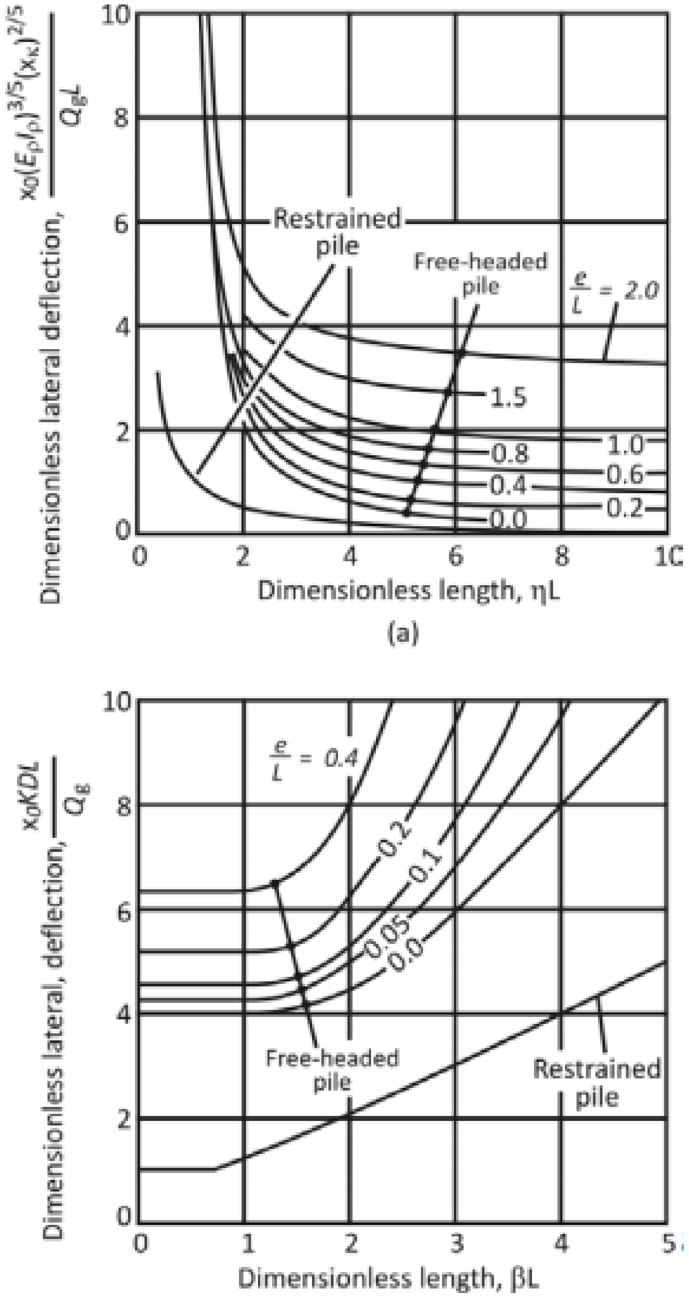
$$M_y = SF_y$$

$S = \textit{section modulus of the pile section}$

$F_y = \textit{yield stress of the pile material}$

	$S = \frac{bh^2}{6}$	Solid arrow represents neutral axis
	$S = \frac{BH^2}{6} - \frac{bh^3}{6H}$	NA indicates neutral axis
	$S = \frac{B^2(H-h)}{6} + \frac{(B-b)^3 h}{6B}$	NA indicates neutral axis
	$S = \frac{\pi r^3}{4} = \frac{\pi d^3}{32}$	Solid arrow represents neutral axis

Brom's solution for ultimate lateral resistance of long piles (a) in sand, (b) in clay



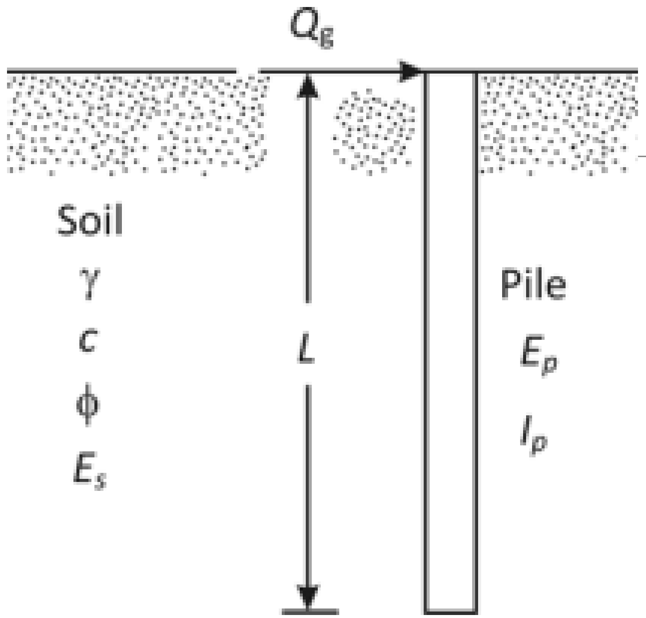
$$\eta = \sqrt[5]{\frac{n_h}{E_p I_p}}$$

Qg is the working load.
$x0$ is the deflection of the pile head,

$$\beta = \sqrt[4]{\frac{KD}{4E_p I_p}}$$

Brom's solution for estimating deflection of pile head (a) in sand, and (b) in clay

Ultimate Load Analysis-Meyerhof's Method



$$K_r = relative\ stiffness\ of\ pile = \frac{E_p I_p}{E_s L^4} < 0.01$$

$E_s = average\ horizontal\ soil\ modulus\ of\ elasticity$

Piles in Sand -Meyerhof's Method

For short (rigid) piles in sand, the ultimate load resistance can be givenas

$$Q_{u(g)} = 0.12\, \gamma D L^2 K_{br} \leq 0.4 p_l D L$$

$\gamma = unit\ weight\ of\ soil$

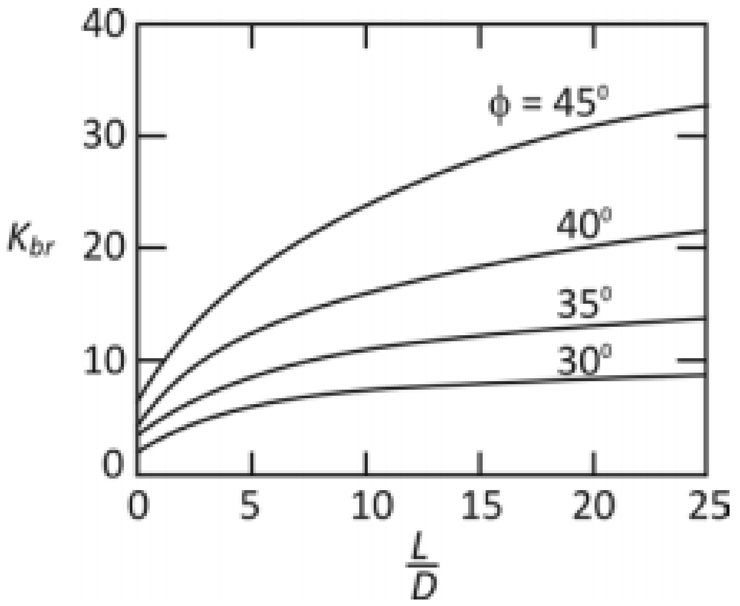
$K_{br} = resultant\ net\ soil\ pressure\ coefficient$

$p_l = limit\ pressure\ obtained\ from\ pressuremeter\ tests$

$p_l = 40 N_q \tan\phi\ (kPa) \quad (for\ Menard\ pressuremeter)$

$p_l =$
$60 N_q \tan\phi\ (kPa) \quad (for\ self-boring\ and\ full\ displacement\ pressuremeters)$

$N_q = bearing\ capacity\ factor$



The maximum moment, Mmax , in the pile due to the lateral load, $Qu(g)$, is

$$M_{max} = 0.35 Q_{u(g)} L \leq M_y$$

Piles in Sand -Meyerhof's Method

For long (flexible) piles in sand, the ultimate lateral load, $Q_{u(g)}$, can be estimated from equation (106) by substituting an effective length (L_e) for L where

$$\frac{L_e}{L} = 1.65K_r^{0.12} \leq 1$$

The maximum moment in a flexible pile due to a working lateral load Q_g applied at the ground surface is

$$M_{max} = 0.3K_r^{0.2}Q_gL \leq 0.3Q_gL$$

Piles in Clay-Meyerhof's Method – Short Pile

The ultimate lateral load, $Qu(g)$, applied at the ground surface for short (rigid) pile embedded in clay can be given as

$$Q_{u(g)} = 0.4c_u K_{cr} DL \leq 0.4p_l DL$$

$p_l = limit\ pressure\ from\ pressuremeter\ test$

$K_{cr} = net\ soil\ pressure\ coefficient\ (\boldsymbol{figure\ 46})$

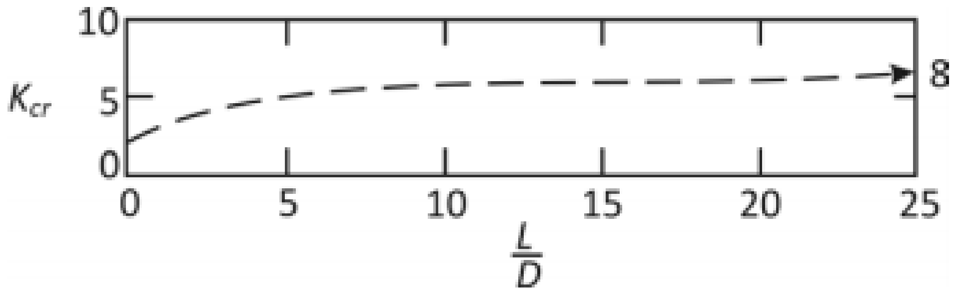


Figure 8.46 Variation of K_{cr}

The limit pressure in clay is

$p_l \approx 6c_u \quad (for\ Menard\ pressuremeter)$

$p_l \approx 8c_u \quad (for\ self-boring\ and\ full\ displacement\ pressuremeter)$

The maximum bending moment in the pile due to $Qu(g)$ is

$$M_{max} = 0.22Q_{u(g)}L \leq M_y$$

Piles in Clay-Meyerhof's Method – Long Pile

For long (flexible) piles, use $Q_{u(g)} = 0.4 c_u K_{cr} D L \leq 0.4 p_l D L$ by substituting the effective length (Le) in place of L.

$$\frac{L_e}{L} = 1.5 K_r^{0.12} \leq 1$$

The maximum moment in a flexible pile due to a working lateral load Qg applied at the ground surface is

$$M_{max} = 0.3 K_r^{0.2} Q_g L \leq 0.15 Q_g L$$

NEGATIVE SKIN FRICTION – NSF

Negative skin friction is a downward drag force exerted on the pile by the soil surrounding it. This action can occur under conditions such as the following:

1. If a fill of clay soil is placed over a granular soil layer into which a pile is driven, the fill will gradually consolidate. This consolidation process will exert a downward drag force on the pile **(figure 8.48a)** during the period of consolidation.
2. If a fill of granular soil is placed over a layer of soft clay, as shown in figure 8. 48b, it will induce the process of consolidation in the clay layer and thus exert a downward drag on the pile.
3. Lowering of the water table will increase the vertical effective stress on the soil at any depth, which will induce consolidation settlement in clay. If a pile is located in the clay layer, it will be subjected to a downward drag force.

In some cases, the downward drag force may be excessive and cause foundation failure.

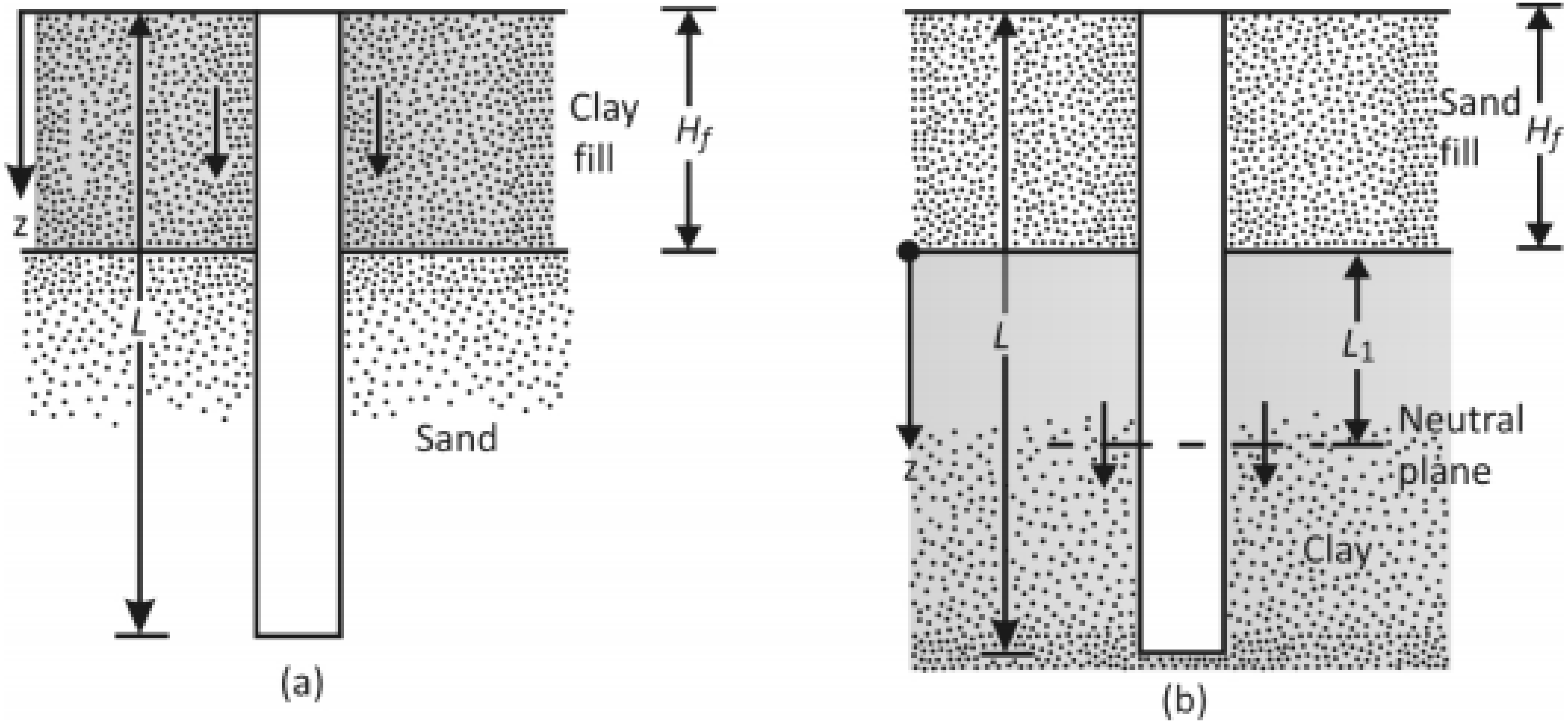


Figure 8.48 Negative skin friction

NSF – Clay Fill over Granular Soil

Similar to the β method presented in section 12, the negative (downward) skin stress on the pile is

$$f_n = K' \sigma'_v \tan\delta \qquad [8.122]$$

Where

$K' = earth\ pressure\ coefficient = K_o = 1 - \sin\phi$

$\sigma'_v = vertical\ effective\ stress\ at\ any\ depth\ z = \gamma'_f z$

$\gamma'_f = effective\ unit\ weight\ of\ fill$

$\delta = soil - pile\ friction\ angle\ \approx 0.5 - 0.7\phi$

Hence the total downward drag force, Q_n, on a pile is

$$Q_n = \int_0^{H_f} (pK'\gamma'_f \tan\delta) z\, dz = \frac{pK'\gamma'_f H_f^2 \tan\delta}{2} \qquad [8.123]$$

Where

$H_f = height\ of\ the\ fill$

If the fill is above the water table, the effective unit weight, γ'_f, should be replaced by the moist unit weight.

NSF – Granular Soil Fill over Clay

In this case, the evidence indicates that the negative skin stress on the pile may exist from $z = 0 \; to \; z = L_1$, which is referred to as the *neutral depth* (see Vesic, 1977, pp. 25-26, for discussion). The neutral depth may be given as (Bowles, 1982):

$$L_1 = \frac{(L-H_f)}{L_1}\left[\frac{L-H_f}{2} + \frac{\gamma'_f H_f}{\gamma'}\right] - \frac{2\gamma'_f H_f}{\gamma'} \qquad [8.124]$$

Where

$\gamma'_f \; and \; \gamma' =$
$effective \; unit \; weights \; of \; the \; fill \; and \; the \; underlying \; clay \; layer, respectively$

For end-bearing piles, the neutral depth may be assumed to be located at the pile tip (that is, $L_1 = L - H_f$).

Once the value of L_1 is determined, the downward drag force is obtained in the following manner. The unit negative skin friction at any depth from $z = 0 \; to \; z = L_1$ is

$$f_n = K' \sigma'_v \tan\delta \qquad [8.125]$$

Where

$K' = K_o = 1 - \sin\phi$

$\sigma'_v = \gamma'_f H_f + \gamma' z$

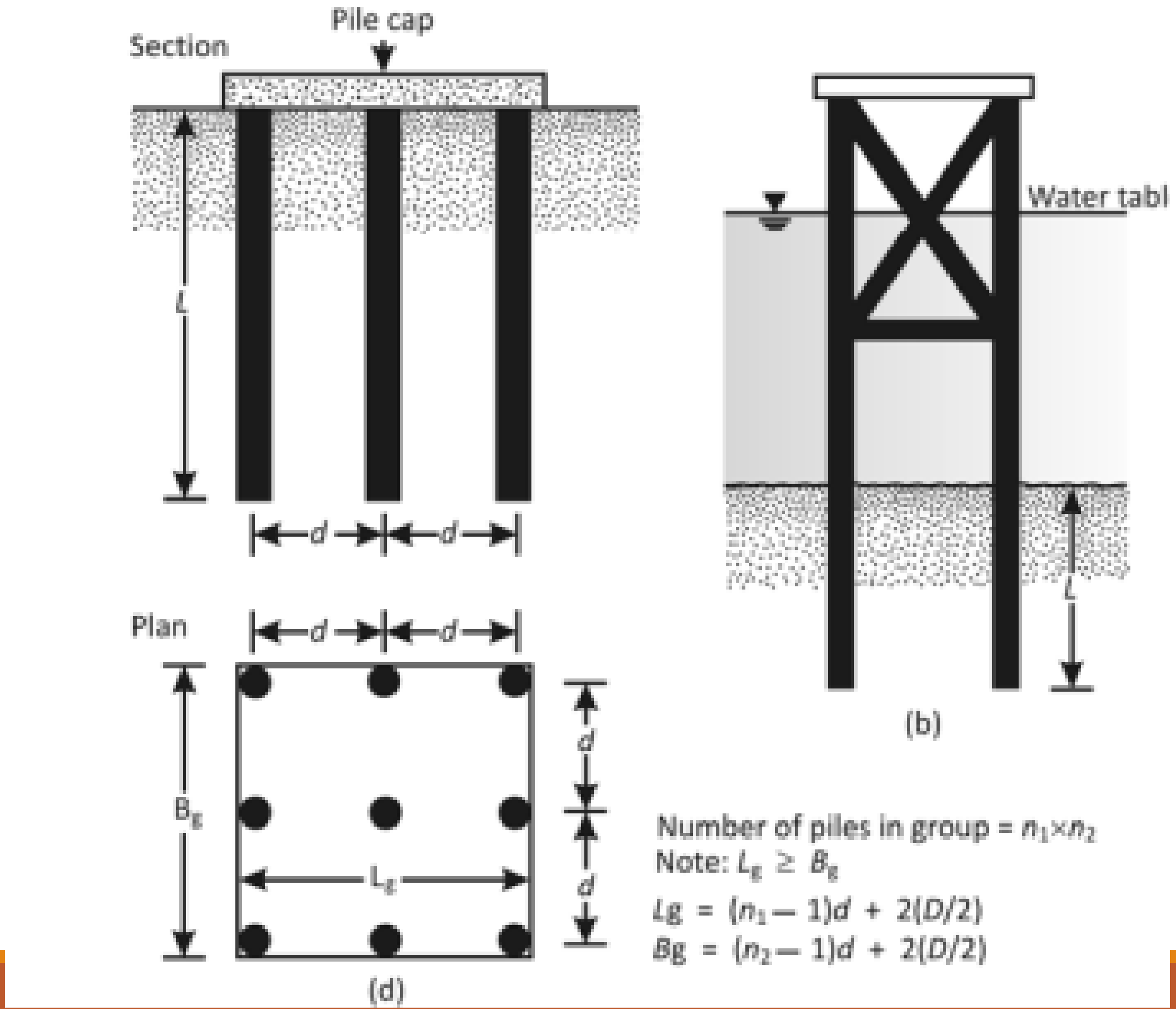
$\delta = 0.5 - 0.7\phi$

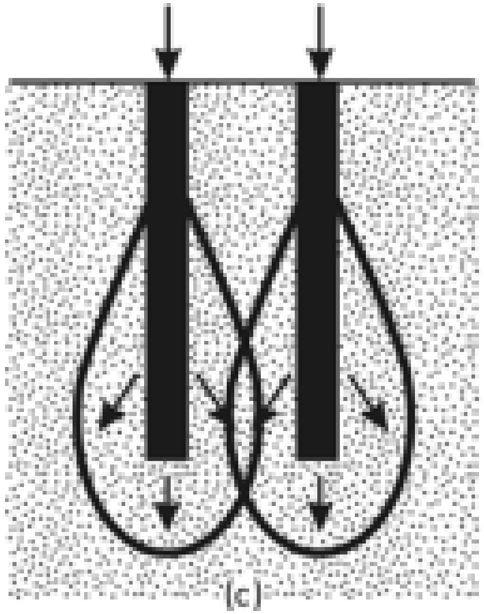
$$Q_n = \int_0^{L_1} p f_n \, dz = \int_0^{L_1} pK' \left(\gamma'_f H_f \gamma' z\right) \tan\delta \, dz$$

$$= (pK' \gamma'_f H_f \tan\delta) L_1 + \frac{L_1^2 pK' \gamma' \tan\delta}{2} \qquad [8.126]$$

If the soil and the fill are above the water table, the effective unit weights should be replaced by moist unit weights.

GROUP PILES – GROUP EFFICIENCY





$$\eta = \frac{Q_{g(u)}}{\Sigma Q_u}$$

GROUP PILES – GROUP EFFICIENCY

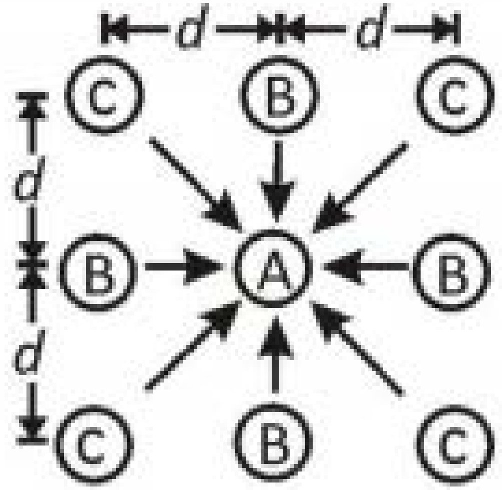
$$\eta = \frac{Q_{g(u)}}{\Sigma\, Q_u} = \frac{f_{av}\,[2(n_1+n_2-2)d+4D]L}{n_1 n_2 p L f_{av}} = \frac{2(n_1+n_2-2)d+4D}{p n_1 n_2}$$

Table 13 Equations for Group Efficiency of Friction Piles

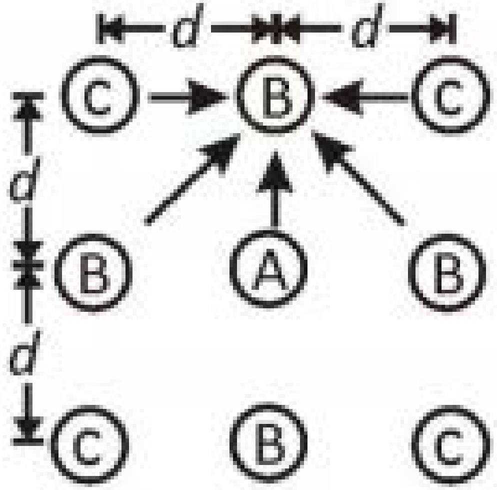
Converse-Labarre equation	$\eta = 1 - \left[\frac{(n_1-1)n_2+(n_2-1)n_1}{90 n_1 n_2}\right]\theta$ $where\ \theta\ (deg) = tan^{-1}(D/d)$
Los Angles Group Action equation	$\eta = 1 - \frac{D}{\pi d n_1 n_2}[n_1(n_2-1)] + n_2(n_1-1) + \sqrt{2}(n_1-1)(n_2-1)]$
Seiler and Keeney equation (Seiler and Keeney, 1944)	$\eta = \left\{1 - \left[\frac{111d}{7(d^2-1)}\right]\left[\frac{n_1+n_2-2}{n_1+n_2-1}\right]\right\} + \frac{0.3}{n_1+n_2}$ $where\ d\ is\ in\ ft$

$$Q_{g(u)} = \eta\, \Sigma\, Q_u$$

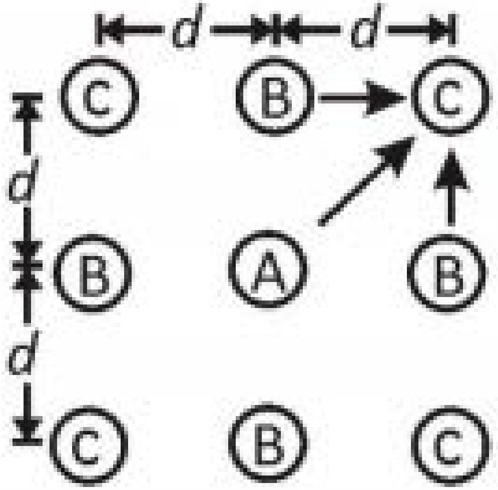
Feld's method for estimation of group capacity of friction piles



(a)



(b)



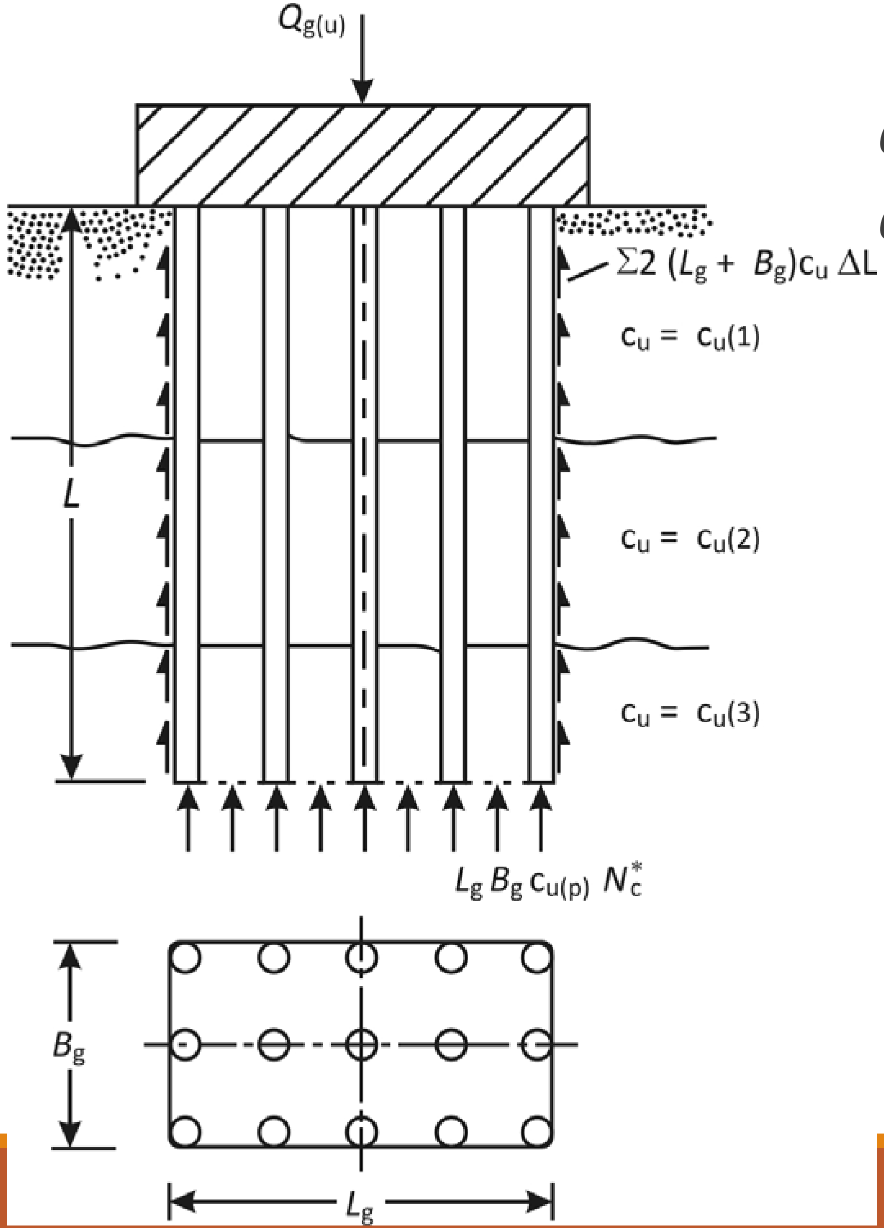
(c)

Pile type	No. of Piles	No. of adjacent piles/pile	Reduction factor for each pile	Ultimate capacity
A	1	8	$1-\frac{8}{16}$	$0.5Q_u$
B	4	5	$1-\frac{5}{16}$	$2.75Q_u$
C	4	3	$1-\frac{3}{16}$	$3.25Q_u$
				$\Sigma\, 6.5\, Q_u = Q_{g(u)}$

(No. of piles)(Q_u) (reduction factor)

$Q_u = ultimate\ capacity\ for\ an\ isolated\ piles$

ULTIMATE CAPACITY OF A GROUP



1) Determine $\Sigma\, Qu = n_1 n_2 (Qp + Qs)$

$QQp = Ap[9c_{u(p)}]$

$QQs = \Sigma\, \alpha p c_u \Delta L$

2) Determine the ultimate capacity by assuming that the pile in the group act as a block with dimensions of $Lg \times Bg \times L$. The skin resistance of the block is:

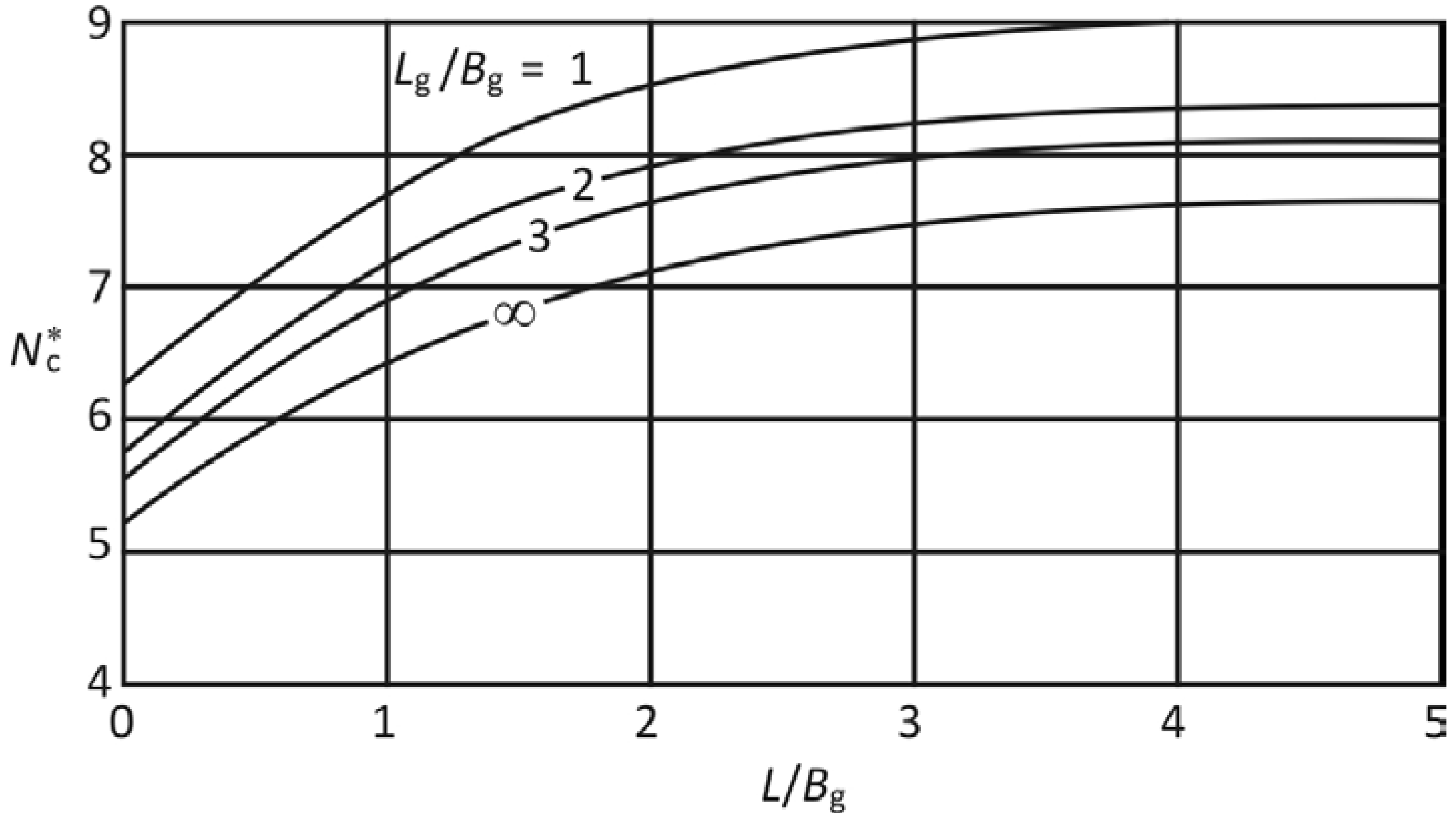
$$\Sigma\, p_g c_u\, \Delta L = \Sigma\, 2(L_g + B_g) c_u \Delta L$$

Calculate the point bearing capacity:

$$A_p q_p = A_p c_{u(p)} N_c^* = (L_g B_g) c_{u(p)} N_c^*$$

$$\Sigma\, Q_u = (L_g B_g) c_{u(p)} N_c^* + \Sigma\, 2(L_g + B_g) c_u \Delta L$$

Compare the values obtained from equations above. The *lower* of the two values is $Qg(u)$.



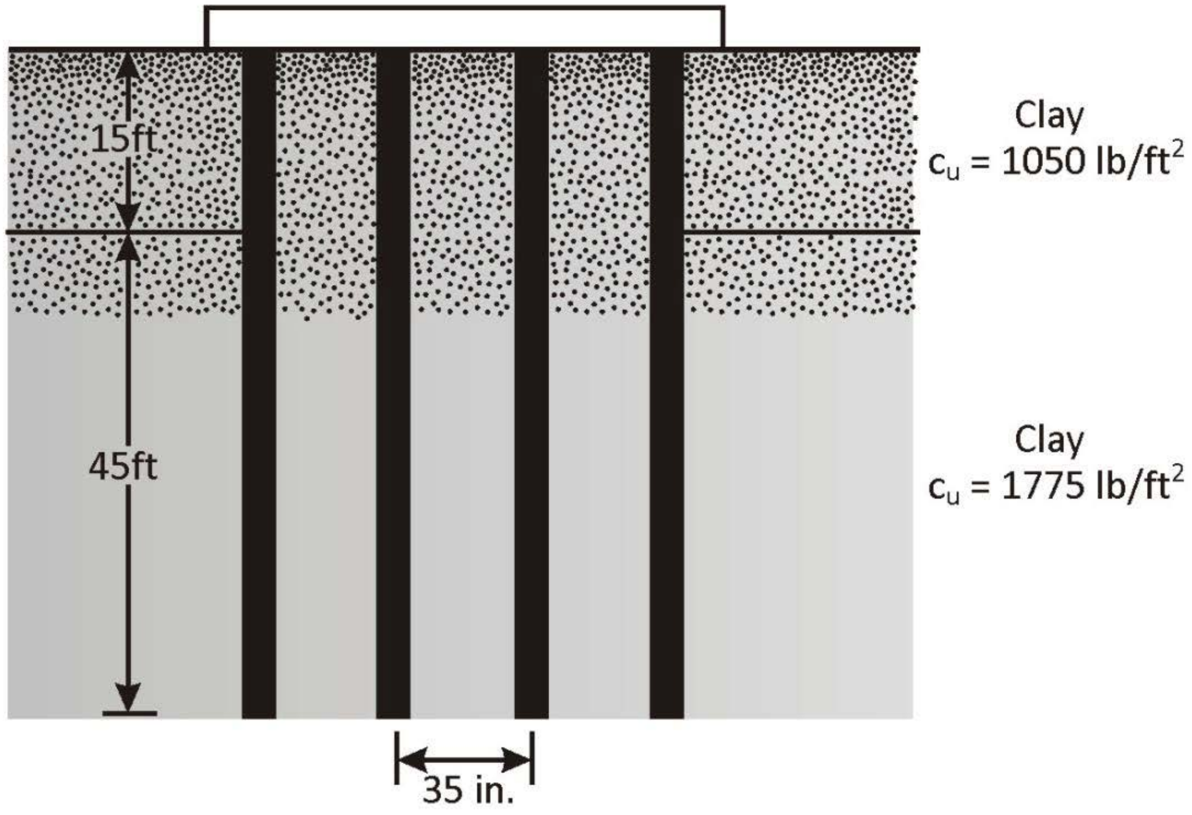
Variation of N_c^* with L_g/B_g and L/B_g

PILES IN ROCK

For point bearing piles resting on rock, most building codes specify that $Qg(u)=\Sigma\ Qu$, provided that the minimum center-to-center spacing of piles is D+300 mm. For H-piles and piles with square cross section, the magnitude of D is equal to the diagonals dimensions if the pile cross section.

Example

The section of a 3×4 group pile in layered saturated clay is shown in figure. The piles are square in cross section (14 $in.$×14 $in.$). The center-to-center spacing, *d,* of the piles n 35 in. Determine the allowable load-bearing capacity of the pile group. Use FS=4.



$$\Sigma\, Q_u = n_1 n_2[9A_p c_{u(p)} + \alpha_1 p c_{u(1)} L_1 + \alpha_2 p c_{u(2)} L_2]$$

From figure 8. 22, $c_{u(1)} = 1050\ lb/ft^2$; $\alpha_1 = 0.86$ and $c_{u(2)} = 1775\ lb/ft^2$; $\alpha_2 = 0.6$.

$$\Sigma\, Q_u = \frac{(3)(4)}{1000}\left[(9)\left(\frac{14}{12}\right)^2(1775) + (0.86)\left(4 \times \frac{14}{12}\right)(1050)(15) + (0.6)\left(4 \times \frac{14}{12}\right)(1775)(45) = 3703\ kip\right]$$

For piles acting as a group,

$$L_g = (3)(35) + 14 = 119\ in. = 9.92\ ft$$

$$B_g = (2)(35) + 14 = 84\ in. = 7\ ft$$

$$\frac{L_g}{B_g} = \frac{9.92}{7} = 1.42$$

$$\frac{L}{B_g} = \frac{60}{7} = 8.57$$

From figure 8. 58, $N_c^* = 8.75$. From equation (131)

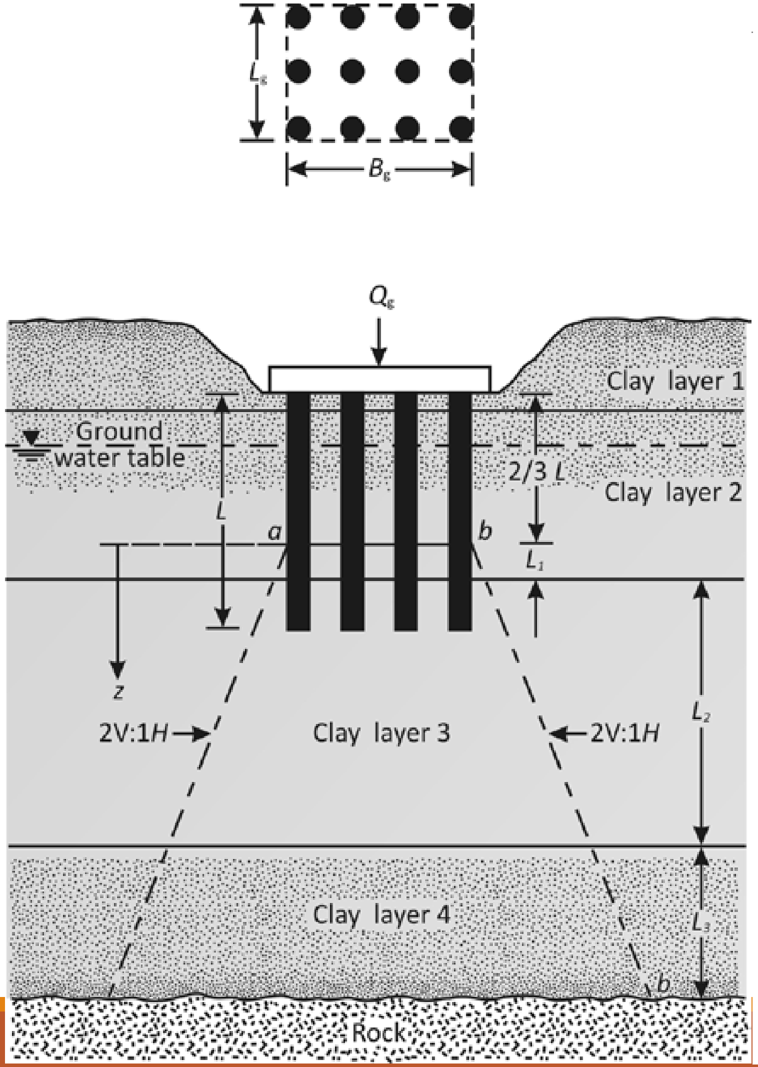
$$\Sigma\, Q_u = L_g B_g c_{u(p)} N_c^* + \Sigma\, 2(L_g + B_g) c_u \Delta L$$

$$= (9.92)(7)(1775)(8.75) + (2)(9.92 + 7)[(1050)(15) + (1775)(45)] = 4313\ kip$$

Hence, $\Sigma\, Q_u = 3703\ kip$.

$$\Sigma\, Q_{all} = \frac{3703}{FS} = \frac{3703}{4} \approx 926\ kip$$

CONSOLIDATION SETTLEMENT OF GROUP PILES



The consolidation settlement of a group pile in clay can be approximately estimated by using the 2:1 stress distribution method. The procedure of calculation involves the following steps:

1. Let the depth of embedment of the piles be L. the group is subjected to a total load of Q_g. If the pile cap is below the original ground surface, Q_g equals the total load of the superstructure on the piles minus the effective weight of soil above the pile group removed by excavation.
2. Assume that the load Q_g is transmitted to the soil beginning at a depth of $2L/3$ from the top of the pile, as shown in figure 8. 60. The load Q_g spreads out along 2 vertical: 1 horizontal line from this depth. Lines aa' and bb' are two 2:1 lines.
3. Calculate the stress increase caused at the middle of each soil layer by the load Q_g:

$$\Delta p_i = \frac{Q_g}{(B_g+z_i)(L_g+z_i)} \qquad [8.132]$$

Where

$\Delta p_i = stress\ increase\ at\ the\ middle\ if\ layer\ i$
$L_g, B_g = length\ and\ width\ of\ the\ plan\ of\ pile\ group, respectively$
$z_i = distance\ from\ z = 0\ to\ the\ middle\ of\ the\ clay\ layer, i$

For example, in figure 8. 60 for layer 2, $z_i = L_1/2$; for layer 3, $z_i = L_1 + L_2/2$; and for layer 4, $z_i = L_1 + L_2 + L_3/2$. Note, however, that there will be no stress increase in clay layer 1 because it is above the horizontal plane ($z = 0$) from which the stress distribution to the soil starts.

4. Calculate the settlement of each layer caused by the increased stress:

$$\Delta s_i = \left[\frac{\Delta e_{(i)}}{1+e_{o(i)}}\right] H_i \qquad [8.133]$$

Where

$\Delta s_i = consolidation\ settlement\ of\ layer\ i$
$\Delta e_{(i)} = change\ of\ void\ ratio\ caused\ by\ the\ stress\ increase\ in\ layer\ i$
$e_o = initial\ void\ ratio\ of\ layer\ i\ (before\ construction)$
$H_i = thickness\ of\ layer\ i\ (Note: In\ figure\ 60, for\ layer\ 2$
$H_i = L_1; for\ layer\ 3, H_i = L_2; and\ for\ layer\ 4, H_i = L_3).$

Relations for $\Delta e_{(i)}$ are given in chapter 1.

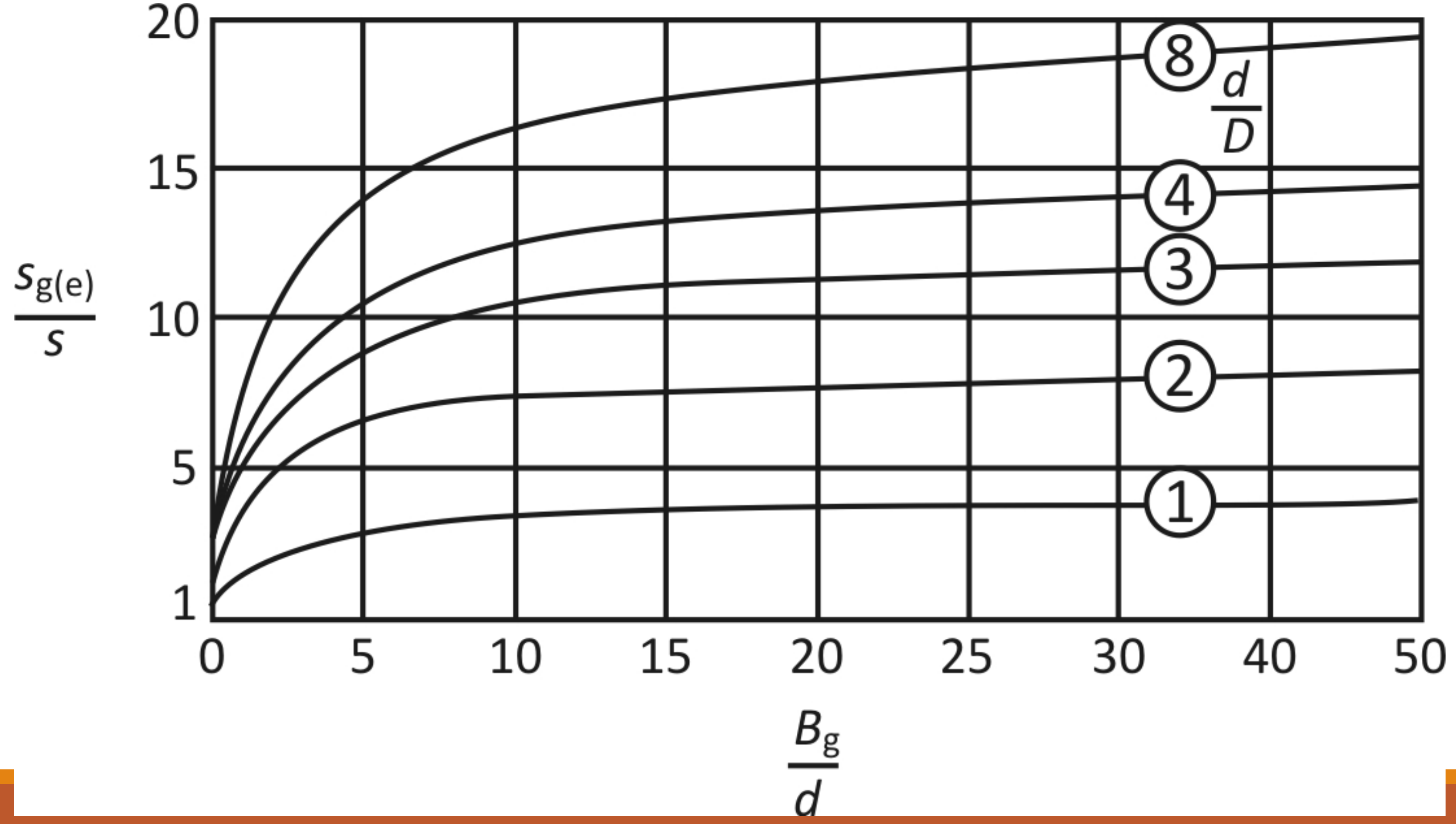
5. Total consolidation settlement of the pile group is then

$$\Delta s_g = \Sigma \Delta s_i \qquad [8.134]$$

Note that consolidation settlement of piles may be initiated by fills placed nearby, adjacent floor loads, and lowering of water tables.

ELASTIC SETTLEMENT OF GROUP PILES

In general, the settlement of a pile group under similar working load per pile increases with the width of the group (Bg) and the center-to-center spacing of piles (d). This fact is demonstrated in figure below obtained from the experimental results of Meyerhof (1961) for pile groups in sand. In this figure , $s_{g(e)}$ is the settlement of the pile group and s is the settlement of isolated piles under similar working load.



ELASTIC SETTLEMENT OF GROUP PILES

Several investigations relating to the settlement of group piles with widely varying results have been reported in the literature. The simplest relation for the settlement of group piles was given by Vesic (1969) as

$$s_{g(e)} = \sqrt{\frac{B_g}{D}}\, s$$

Where

$s_{g(e)} = \textit{elastic settlement of group piles}$

$B_g = \textit{width of pile group section}$

$D = \textit{width or diameter of each pile in the group}$

$s = \textit{elastic settlement of each pile at comparable working load}$

ELASTIC SETTLEMENT OF GROUP PILES

For pile groups in sand and gravel, Meyerhof (1976) suggested the following empirical relation for elastic settlement:

$$s_{g(e)}(in.) = \frac{2q\sqrt{B_g}I}{N_{cor}} \quad [8.136]$$

Where

$$q = Q_g/(L_g/B_g)\ (in\ U.S.Ton/ft^2) \quad [8.137]$$

L_g and B_g = length and width of the pile group section respectively (ft)

N_{cor} = average corrected standard penetration number within seat of settlement ($\approx B_g$ deep below the tip of the piles)

$$I = influence\ factor = 1 - L/8B_g \geq 0.5 \quad [8.138]$$

L = length of embedment of piles

ELASTIC SETTLEMENT OF GROUP PILES

Similarly, the pile group settlement is related to the cone penetration resistance as:

$$S_{g(e)} = \frac{q B_g I}{2q_c} \qquad [8.139]$$

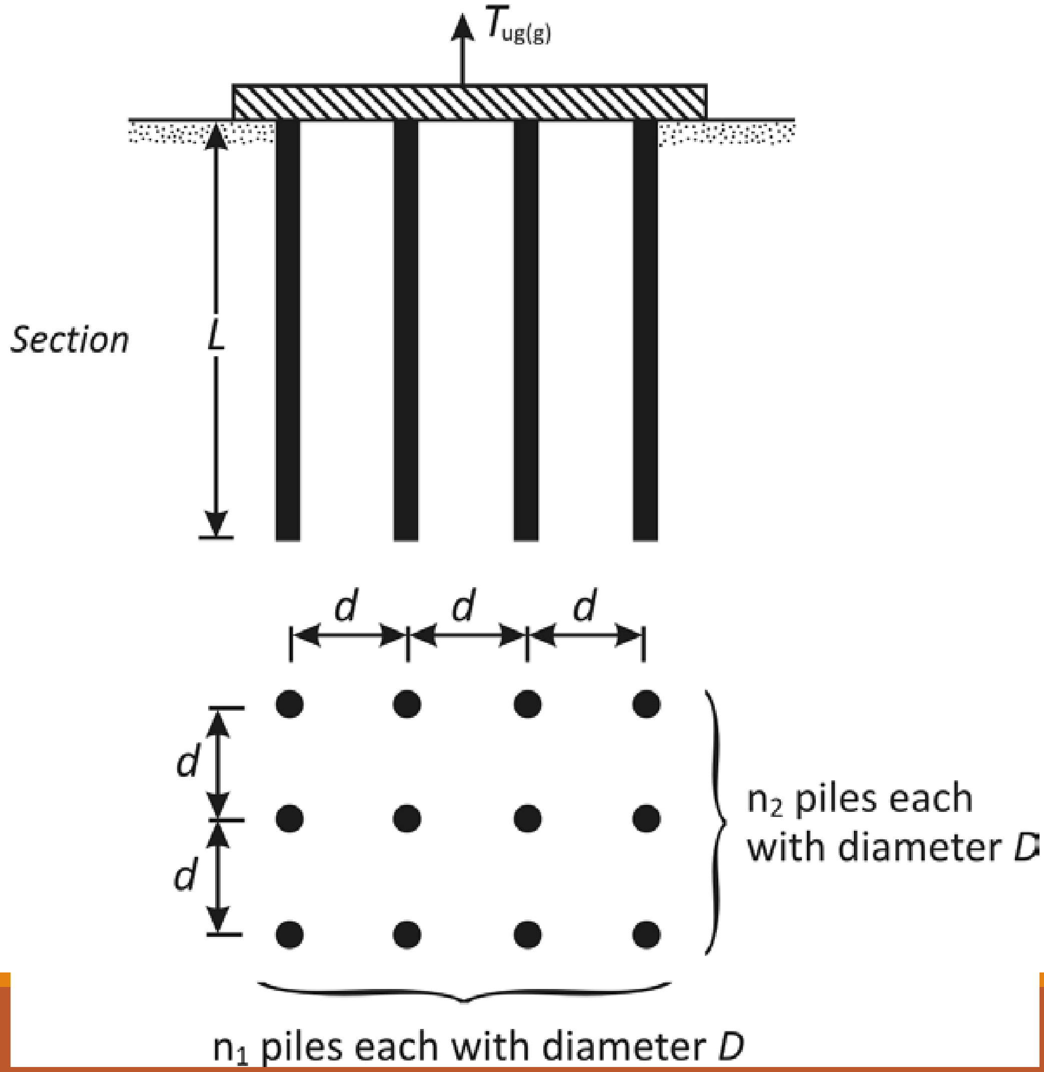
Where

$q_c = average\ cone\ penetration\ resistance\ within\ the\ seat\ of\ settlement$

In equation (139), all symbols are in consistent units.

UPLIFT CAPACITY OF GROUP PILES

Under certain circumstances, group piles may be used for construction of foundations subjected to uplifting load. The group efficiency under uplift may be expressed as



$$\eta_T = \frac{T_{un\,(g)}}{T_{un}}$$

Where

$\eta_T = group\ efficiency\ under\ uplift$

$T_{un\,(g)} = net\ ultimate\ uplift\ capacity\ of\ pile\ group$

$T_{un} = net\ ultimate\ uplift\ capacity\ of\ single\ pile$

Note that

$$T_{un\,(g)} = T_{ug\,(g)} - (n_1 \times n_2)W - W_{cap}$$

Where

$T_{ug\,(g)} = gross\ ultimate\ uplift\ capacity\ of\ group\ piles$

$W = effective\ self - weight\ of\ each\ pile$

$n_1 \times n_2 = number\ of\ piles\ in\ the\ group$

$W_{cap} = effective\ weight\ of\ pile\ cap$