Physics II
Fiz138-22
Summer 2017

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Office hours: Wednesday 10:00 – 11:00

Evaluation

Attendance: Mandatory For **Everyone**

- **Minimum %50 for who pass the attendance rule before**
- **Minimum %70 for who the first timer or did not pass the attendence rule before**

Midterm: 03.08.2017 09:30 – 11:30 (%50)
Final: 22.08.2017 13:00 – 15:00 (%50)
Make up for the midterm: 18.08.2017 13:00 – 15:00

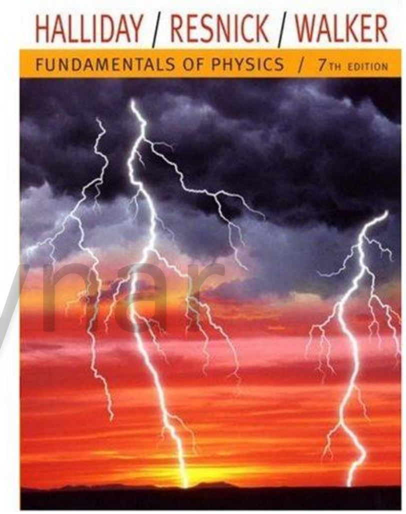
Rules

- Be silent during the lecture.
- Listen or leave the classroom.
- Raise your hand for permission to talk.
- No food no drink during the lectures. (Except water)
- No cell phone use.
- Taking pictures or videos of the lecture is prohibited.
- If you are late to the lecture then wait for the break.

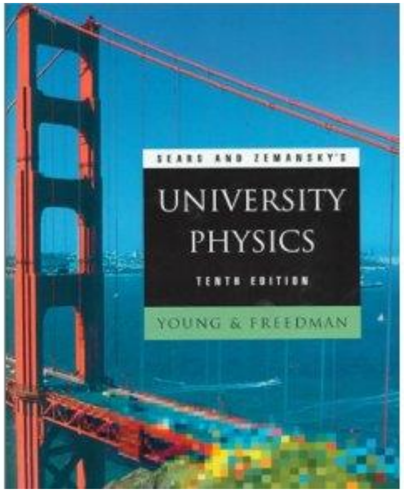
OUTLINE

1. Electric Charge
2. Electric Field
3. Gauss' Law
4. Electric Potential
5. Capacitance
6. Current and Resistance
7. DC Circuits
8. Magnetic Fields
9. Magnetic Fields due to Currents
10. Induction and Inductance
11. AC Circuits

Text Book



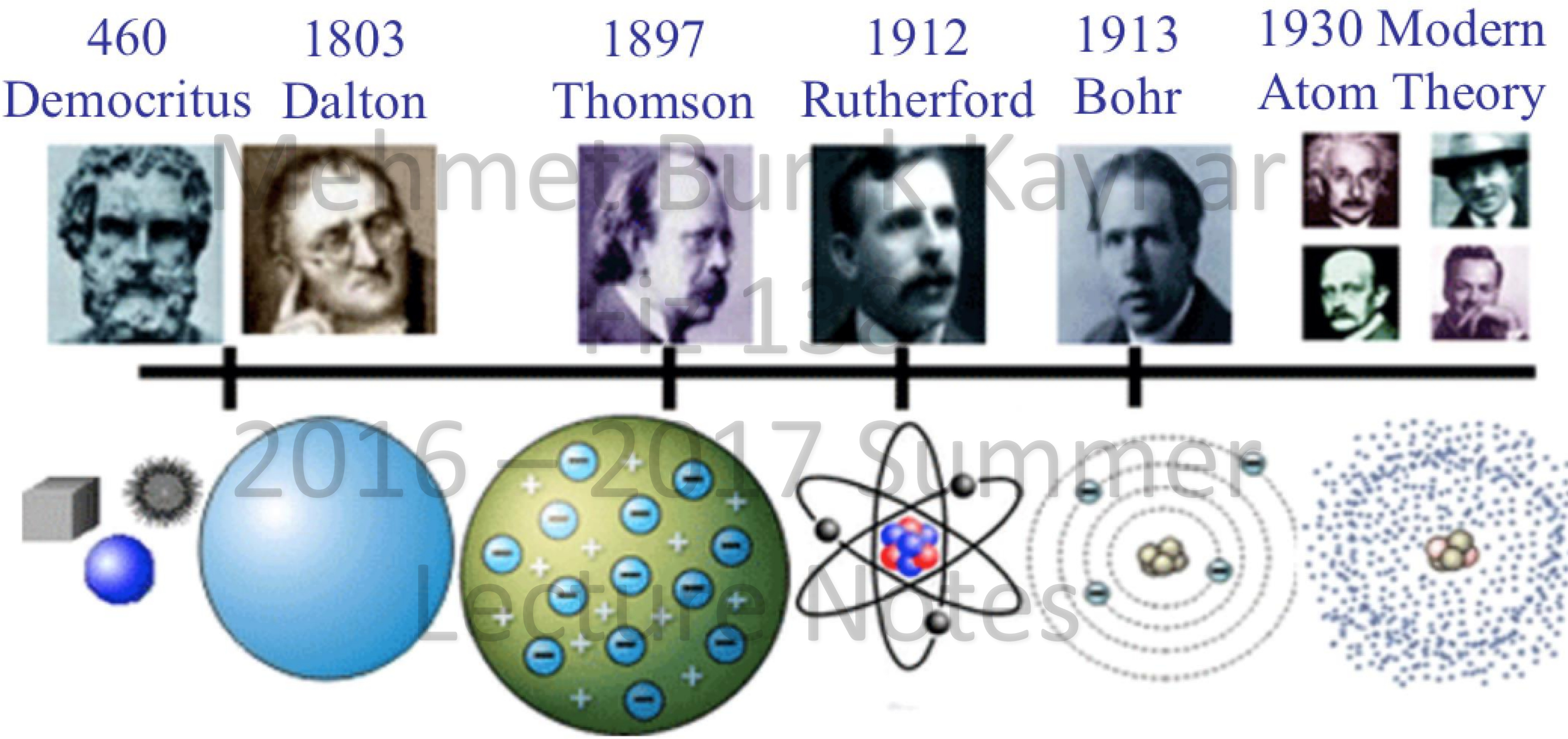
Reference Book
Sears and Zemansky

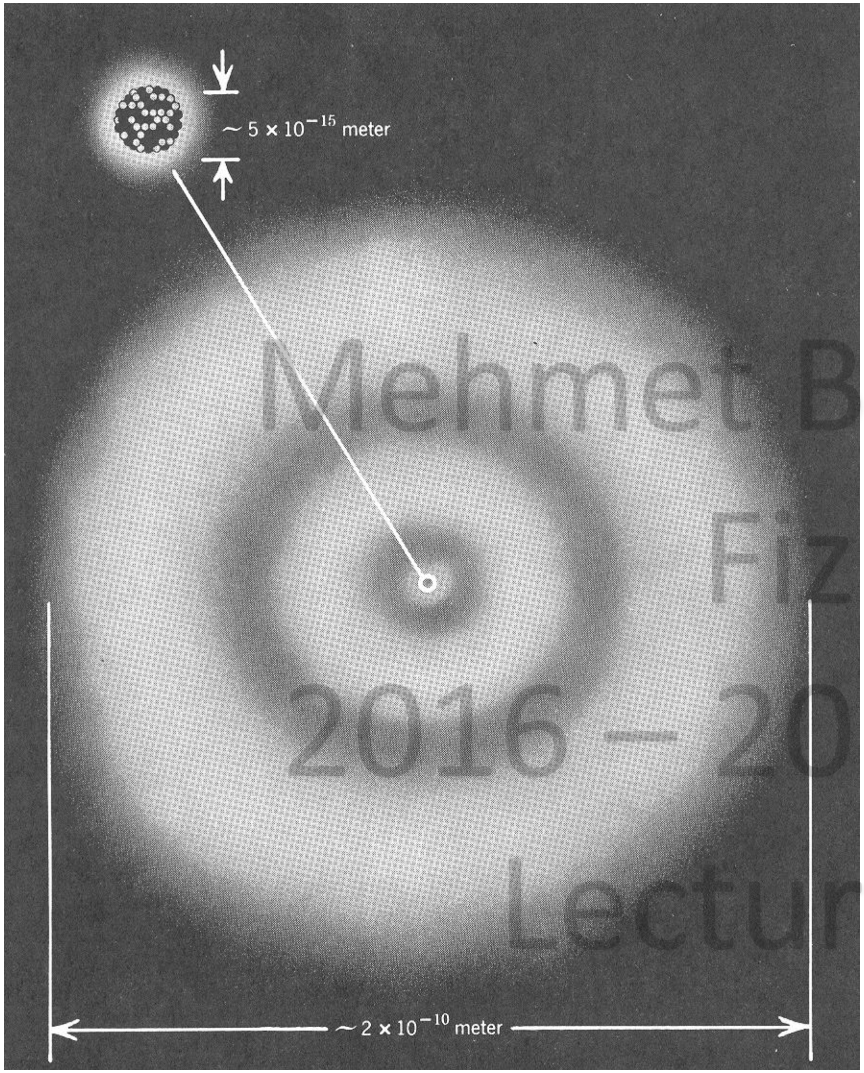


Chapter 21&22
Electric Charge and Electrical Field

- Atom and Charges
- Types of electric charges
- Types of Materials
- Coulomb’s law
- Electric Field (discrete and continuous charges)
- Electric Dipole (Torque and Potential Energy)

Models of the Atom





Atoms consist of **electrons** and the **nucleus**.

Atoms have sizes ≈ 5 x10^{-10} m.

Nuclei have sizes ≈ 5 x10^{-15} m.

The nucleus itself consists of two types of particles: protons and neutrons.

*The electrons are **negatively** charged. The protons are **positively** charged. The neutrons are **neutral** (zero charge).*

Electric charge is a fundamental property of the elementary particles (electrons, protons, neutrons) out of which atoms are made.

Mass and Charge of Atomic Constituents

Neutron (n) : Mass $m = 1.675\text{x}10^{-27}$ kg; Charge $q = 0$

Proton (p) : Mass $m = 1.673\text{x}10^{-27}$ kg; Charge $q = +1.602\text{x}10^{-19}$ C

Electron (e) : Mass $m = 9.11\text{x}10^{-31}$ kg; Charge $q = -1.602\text{x}10^{-19}$ C

Note 1: We use the symbols “-e” and “+e” for the electron and proton charge, respectively. This is known as the **elementary charge.**

Note 2: Atoms are electrically neutral. The number of electrons is equal to the number of protons. This number is known as the “ **atomic number** ” (symbol: *Z*). The chemical properties of atoms are determined **exclusively** by *Z*.

Total Charge q or Q (positive or negative)

- *Quantizied: Takes integer multiple of e.*
- *Always conserved: Net charge of an isolated system stays constant Even in nuclear reactions charge is conserved*

Types of Materials (wrt moving ability of charges)

Conducting Materials

There are freely moving electrons (conduction *e*).

Ag (silver) is the best conductor amongst the metals

Insulating (nonconductor) Materials

No conduction electrons.
There are electrons but they are bound to atoms

Rubber, plastic, glass etc.

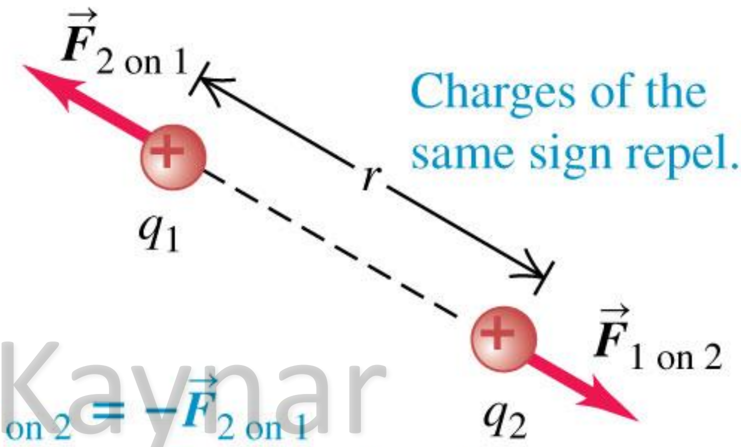
Semiconducting Materials

There ***might be freely*** moving electrons
Electrons are semi-bound to atoms.

(Si, Ge, etc.)

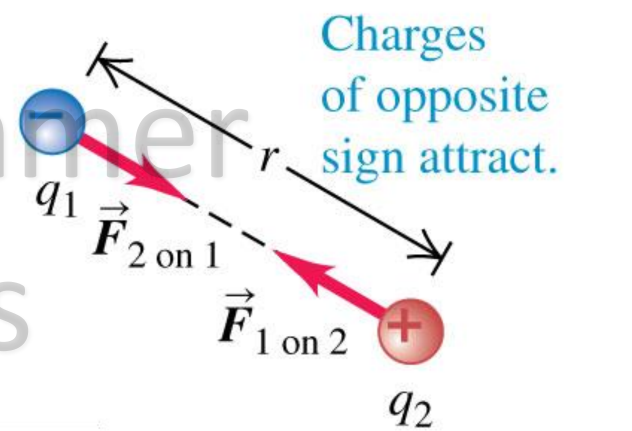
Coulomb’s law

Coulomb’s Law: The magnitude of the electric force between two point charges is directly proportional to the product of their charges and inversely proportional to the square of the
the
distance between them.



$\vec{F}_{1\ on\ 2} = -\vec{F}_{2\ on\ 1}$

$F_{1\ on\ 2} = F_{2\ on\ 1} = k\frac{|q_1 q_2|}{r^2}$

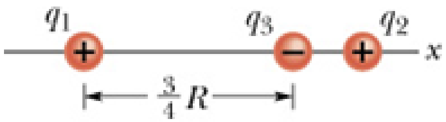


$$k = \frac{1}{4\pi\epsilon_0} = 8.99x10^{-9}(\frac{N.m^2}{C^2})$$

the permitivity of free space

$$\epsilon_0 = 8.85x10^{-12}(\frac{C^2}{N.m^2})$$

- **This force has same spatial dependence as gravitational force, BUT there is NO mention of mass here!!**
- **The strength of the FORCE between two objects is determined by the charge of the two objects.**



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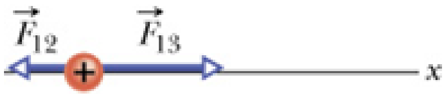


(d)

Coulomb's Law and the Principle of Superposition

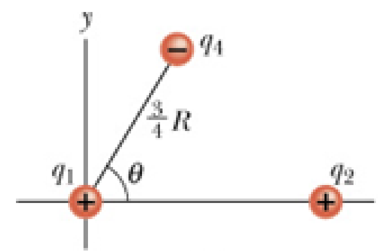
The net electric force exerted by a group of charges is equal to the vector sum of the contribution from each charge.

For example, the net force $\vec{F}_1$ exerted on q_1 by q_2 and q_3 is equal to $\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13}$.
Here $\vec{F}_{12}$ and $\vec{F}_{13}$ are the forces exerted on q_1 by q_2 and q_3, respectively.
In general, the force exerted on q_1 by n charges is given by the equation

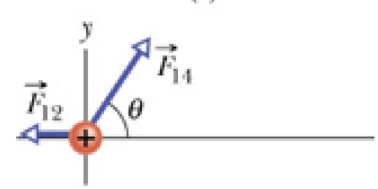
$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \ldots + \vec{F}_{1n} = \sum_{i=2}^{n} \vec{F}_{1i}$$

One must remember that $\vec{F}_{12}$, $\vec{F}_{13}$, ... are vectors and thus we must use vector addition. In the example of fig. f we have:

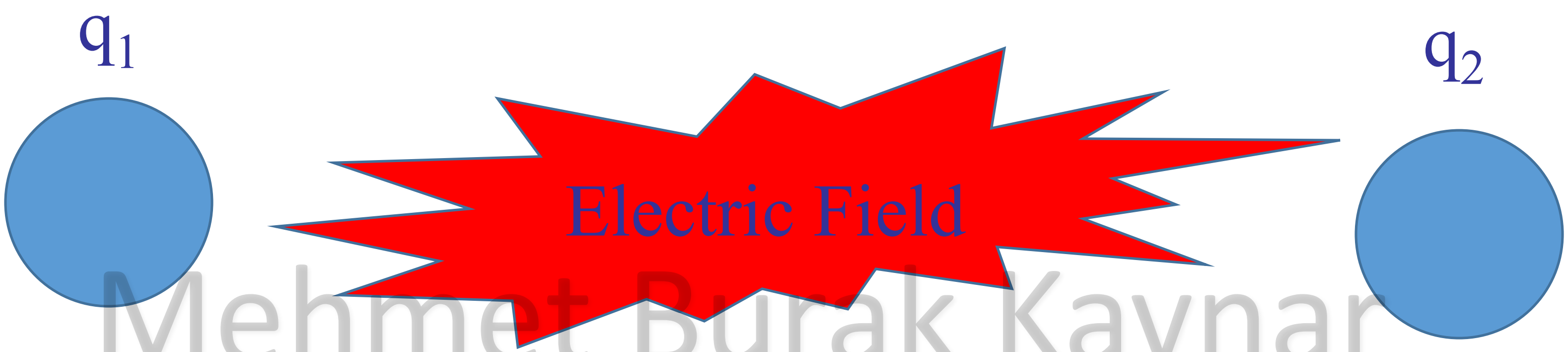
$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{14}$$



(e)



(f)

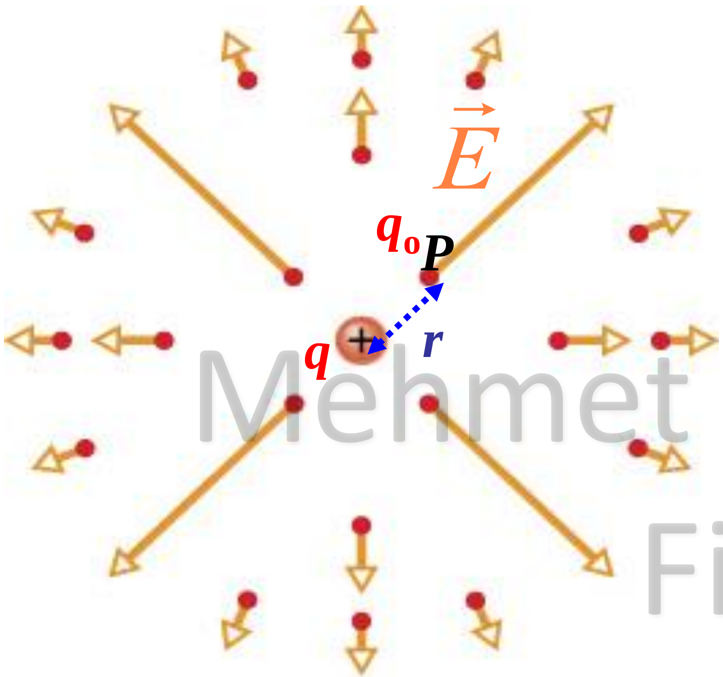


- Charge creates its own E field and

  charges intract with each other via this field.
- E field is parallel to the force (Coulomb Force)
- (Newton/Coulomb) (N/C)

If a charged object expriences a force given by E.q in anywhere in space then that position is said to have an Electric Field.

Electric Field Generated by a Point Charge

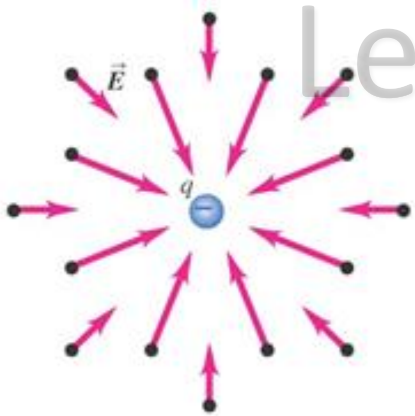


Consider the positive charge q shown in the figure. At point P a distance r from q we place the test charge q_0. The force exerted on q_0 by q is equal to:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{|q||q_0|}{r^2}$$

$$E = \frac{F}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{|q||q_0|}{q_0 r^2} = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2}$$



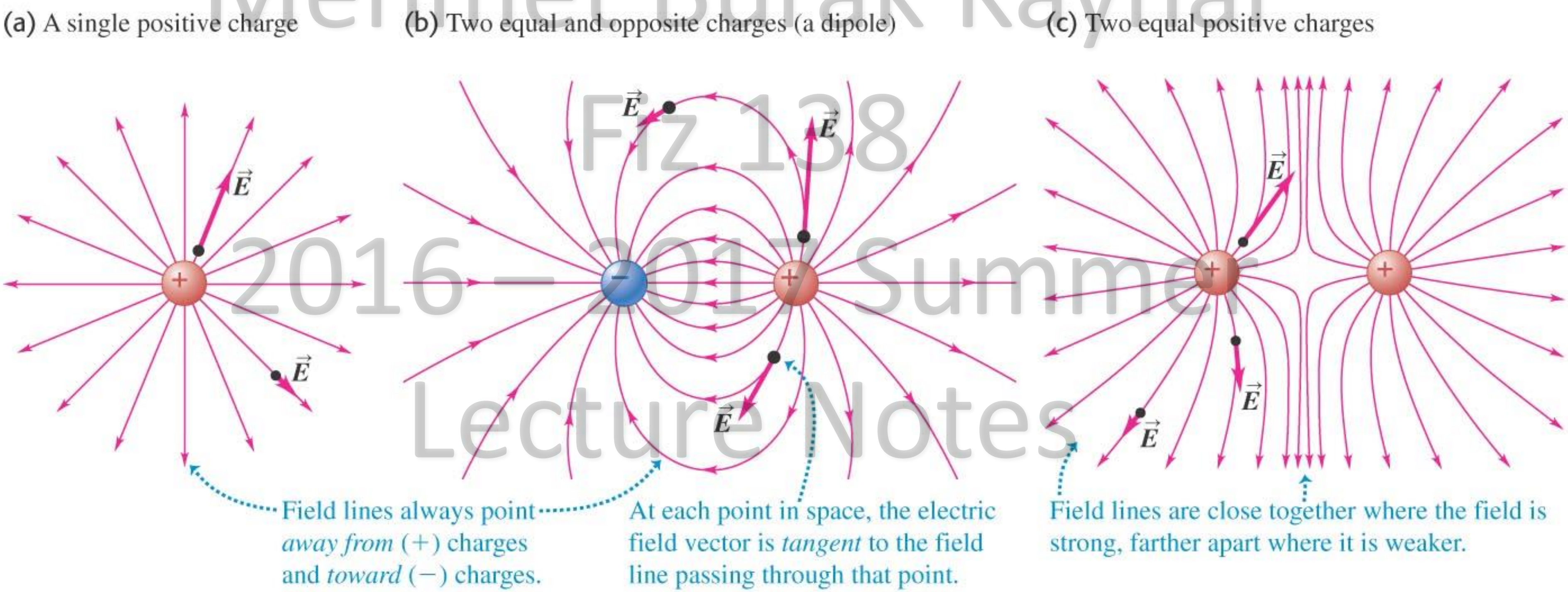
The magnitude of $\vec{E}$ is a positive number.

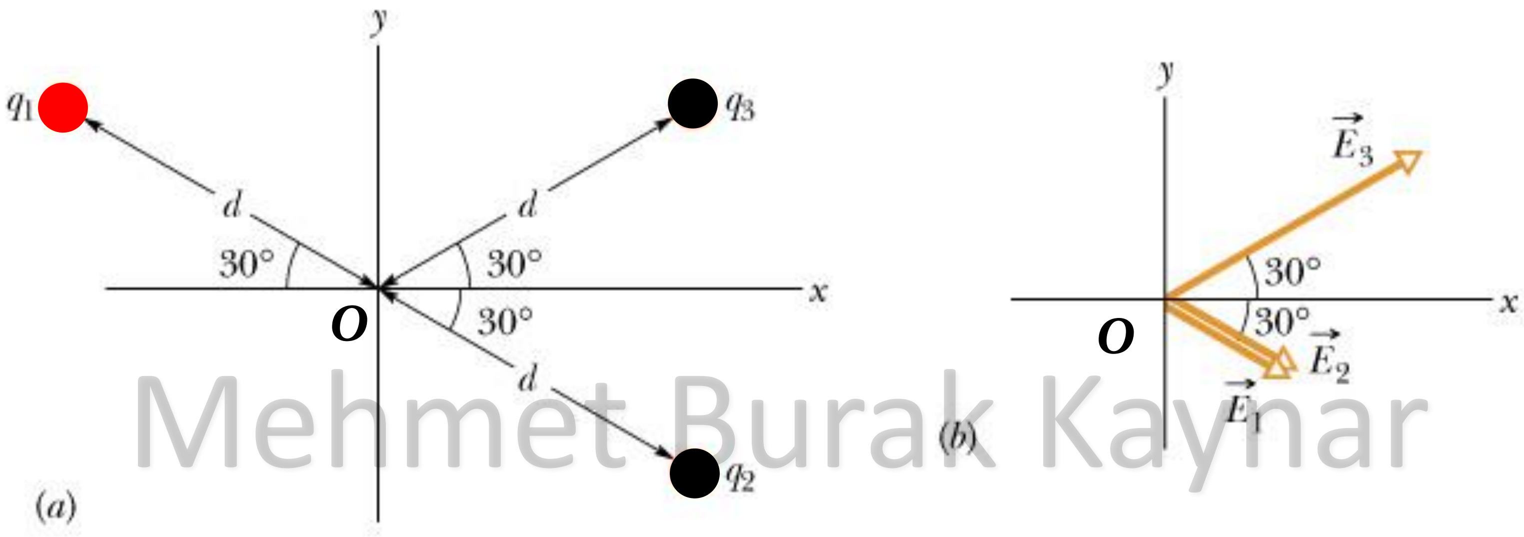
In terms of direction, $\vec{E}$ points radially **outward** as shown in the figure.

If q were a negative charge the magnitude of $\vec{E}$ would remain the same. The direction of $\vec{E}$ would point radially **inward** instead.

Electric field lines of point charges

The figure below shows the electric field lines of a single point charge and for two charges of opposite sign and of equal sign.





Electric Field Generated by a Group of Point Charges. Superposition

The net electric electric field $\vec{E}$ generated by a group of point charges is equal to the vector sum of the electric field vectors generated by each charge.

In the example shown in the figure, $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$.

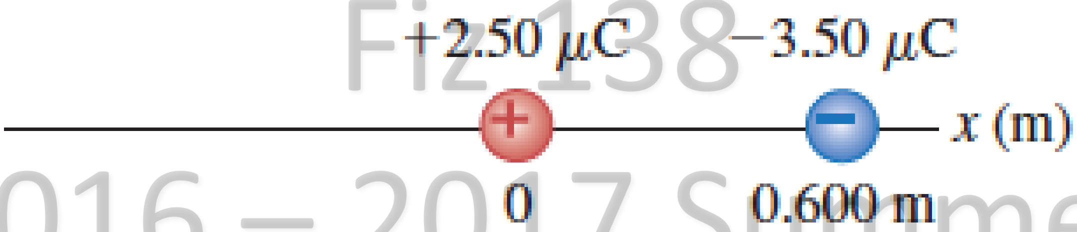
Here $\vec{E}_1$, $\vec{E}_2$, and $\vec{E}_3$ are the electric field vectors generated by q_1, q_1, and q_3, respectively.

Note: $\vec{E}_1$, $\vec{E}_2$, and $\vec{E}_3$ must be added as vectors:

$$E_x = E_{1x} + E_{2x} + E_{3x}, \quad E_y = E_{1y} + E_{2y} + E_{3y}, \quad E_z = E_{1z} + E_{2z} + E_{3z}$$

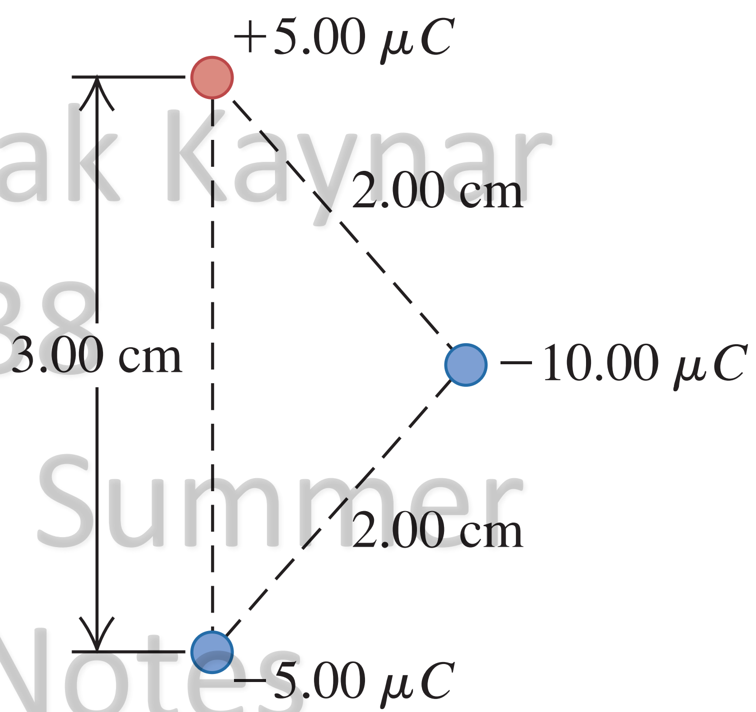
21.64 ••• Two charges, one of 2.50 μC and the other of -3.50 μC, are placed on the x-axis, one at the origin and the other at $x = 0.600$ m, as shown in Fig. P21.64. Find the position on the x-axis where the net force on a small charge $+q$ would be zero.

Figure **P21.64**

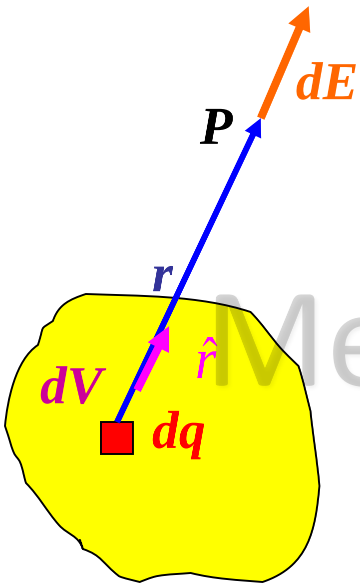


21.61 • Three charges are at the corners of an isosceles triangle as shown in Fig. E21.61. The ± 5.00-μC charges form a dipole. (a) Find the force (magnitude and direction) the -10.00-μC charge exerts on the dipole. (b) For an axis perpendicular to the line connecting the ± 5.00-μC charges at the midpoint of this line, find the torque (magnitude and direction) exerted on the dipole by the -10.00-μC charge.

Figure **E21.61**



Electric Field Generated by a Continuous Charge Distribution



Consider the continuous charge distribution shown in the figure. We assume that we know the volume density ρ of the electric charge. This is defined as $\rho = \dfrac{dq}{dV}$ (Units: C/m^3).

Our goal is to determine the electric field $d\vec{E}$ generated by the distribution at a given point P. This type of problem can be solved using the principle of superposition as described below.

1. Divide the charge distribution into "elements" of volume dV. Each element has charge $dq = \rho dV$. We assume that point P is at a distance r from dq.

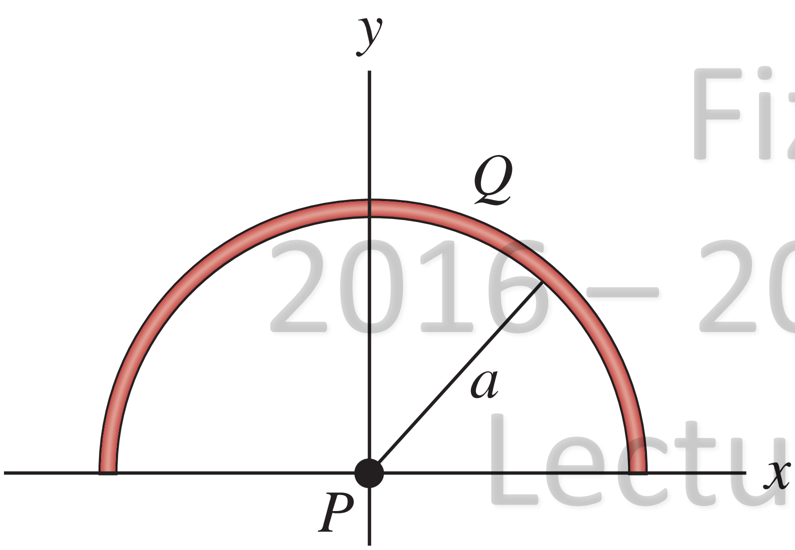
2. Determine the electric field $d\vec{E}$ generated by dq at point P.

The magnitude dE of $d\vec{E}$ is given by the equation $dE = \dfrac{dq}{4\pi\varepsilon_0 r^2}$.

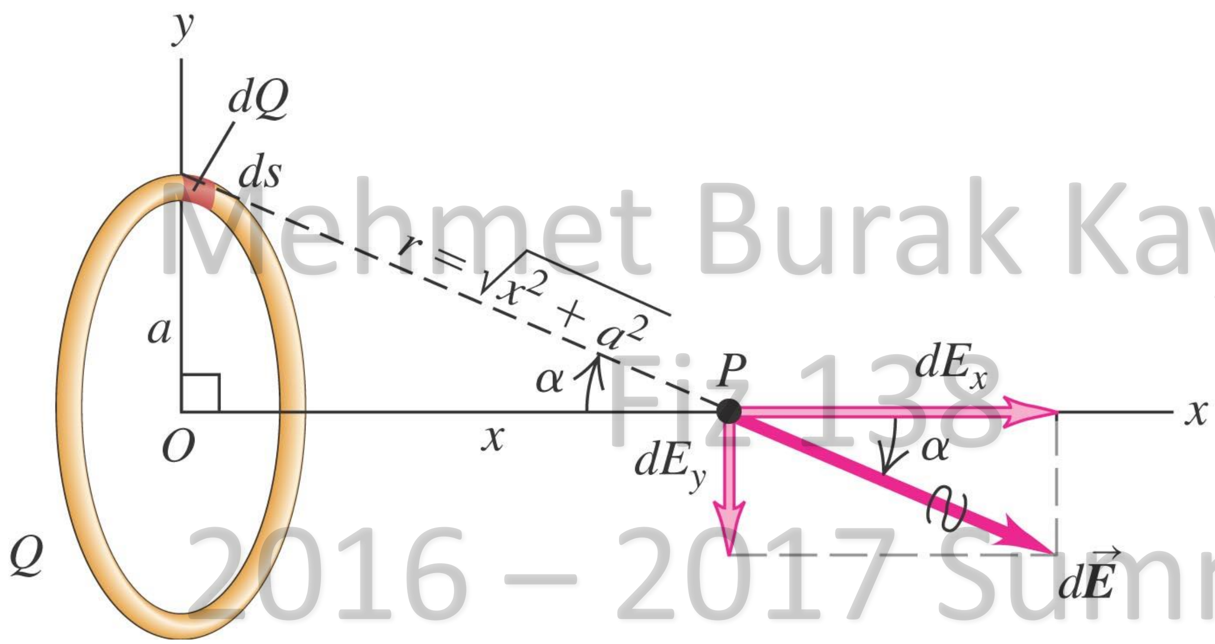
3. Sum all the contributions: $\vec{E} = \dfrac{1}{4\pi\varepsilon_0}\displaystyle\int \frac{\rho dV \hat{r}}{r^2}$.

Example 1: Field of a half a ring of charge

Positive charge Q is uniformly distributed around a semicircle of radius a as shown in figure below. Find the magnitude and direction of the resulting electric field at point P, the center of curvature of the semicircle.



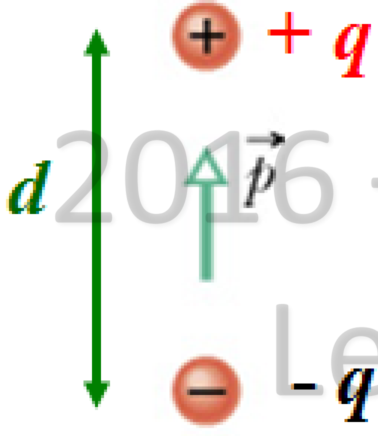
Example 2: Field of a ring of charge

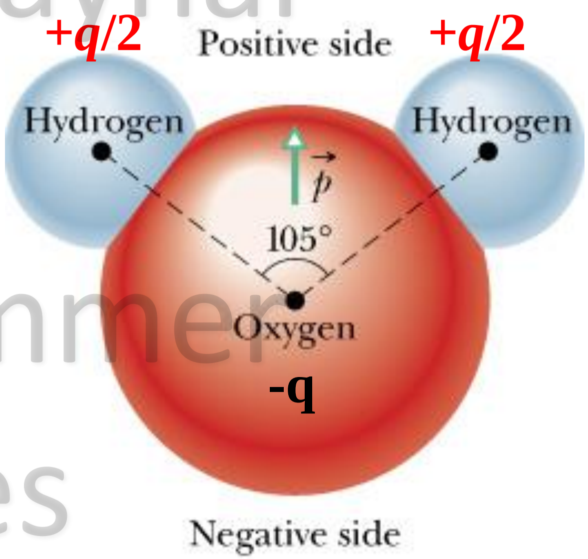


Electric Dipole

Dipoles are important because many physical systems are described as electric dipoles.

System of two equal and opposite charges seperated by a distance *d*.

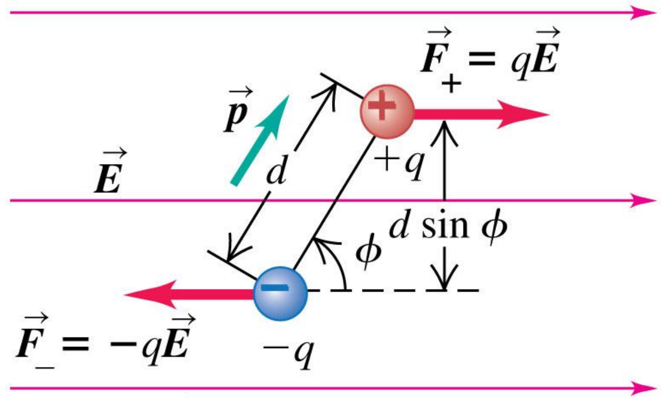




For every electric dipole we define a dipole moment vector

$$\vec{p} = q\vec{d}$$

Force and Torque on an Electric Dipole



Torque's direction is into the page

Net force on the electric dipole is ZERO however forces do not act along the same line, so their torques don't add to zero. If we calculate the torques wrt the center of the dipole then we get;

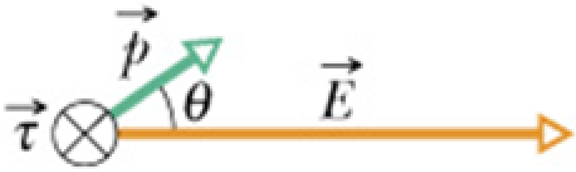
$$t = t_{+} + t_{-}$$

$$t = qE(\frac{d}{2}\sin f) + qE(\frac{d}{2}\sin f)$$

$$t = qd(E\sin f) = pE\sin f$$

$$\vec{t} = \vec{p} x \vec{E} \text{ (in vector form)}$$

Potential Energy (negative of the work) of an Electric Dipole



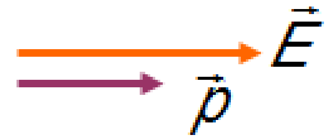
$$U = -\int_{90^\circ}^{\theta} \tau d\theta' = -\int_{90^\circ}^{\theta} pE\sin\theta d\theta'$$

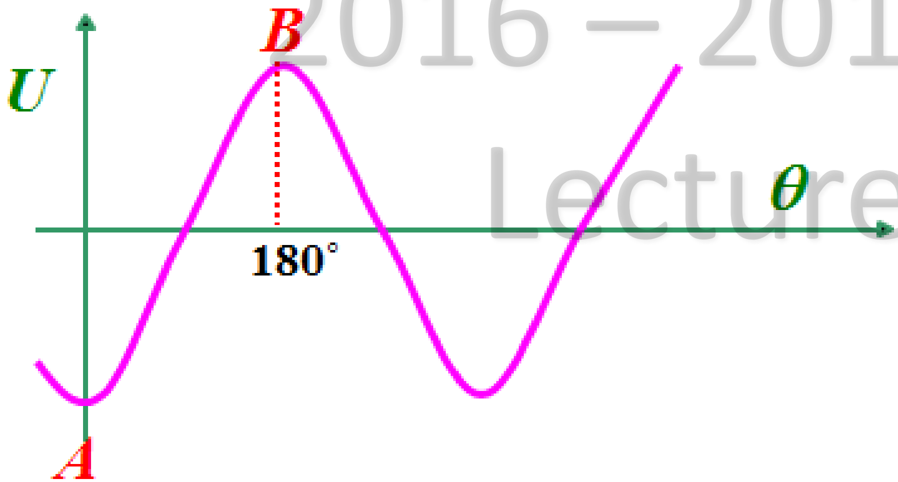
Note that θ decreases.

$$U = -pE\int_{90^\circ}^{\theta} \sin\theta d\theta' = -pE\cos\theta = -\vec{p}\cdot\vec{E}$$

$$U = -pE\cos\theta$$

$$U = -\vec{p}\cdot\vec{E}$$



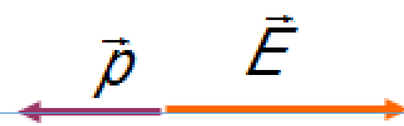


At point A $(\theta = 0)$, U has a minimum value $U_{min} = -pE$.

It is a position of stable equilibrium.

At point B $(\theta = 180^\circ)$, U has a maximum value $U_{max} = +pE$.

It is a position of unstable equilibrium.



Work Done by an External Agent to Rotate an Electric Dipole in a Uniform Electric Field

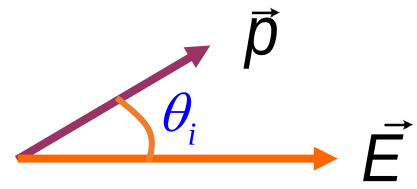


Fig. *a*

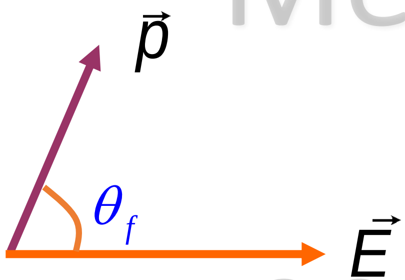


Fig. *b*

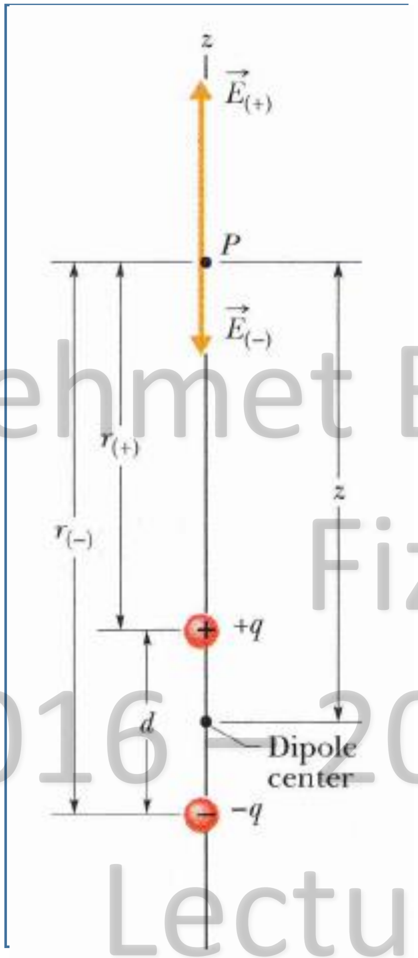
Consider the electric dipole in fig. *a*. It has an electric dipole moment $\vec{p}$ and is positioned so that $\vec{p}$ is at an angle θ_i with respect to a uniform electric field $\vec{E}$.

An external agent rotates the electric dipole and brings it to its final position shown in fig. *b*. In this position $\vec{p}$ is at an angle θ_f with respect to $\vec{E}$.

The work W done by the external agent on the dipole is equal to the difference between the initial and final potential energy of the dipole:

$$W = U_f - U_i = -pE\cos\theta_f - \left(-pE\cos\theta_i\right)$$

$$W = pE\left(\cos\theta_i - \cos\theta_f\right)$$



$$\begin{aligned} E &= E_{(+)} - E_{(-)} \\ &= \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{(+)}^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{(-)}^2} \\ &= \frac{q}{4\pi\varepsilon_0(z-\frac{1}{2}d)^2} - \frac{q}{4\pi\varepsilon_0(z+\frac{1}{2}d)^2}. \end{aligned}$$

$$E = \frac{q}{4\pi\varepsilon_0 z^2}\left(\frac{1}{\left(1-\frac{d}{2z}\right)^2} - \frac{1}{\left(1+\frac{d}{2z}\right)^2}\right).$$

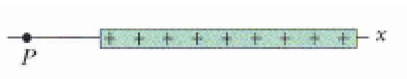
$$E = \frac{q}{4\pi\varepsilon_0 z^2}\frac{2d/z}{\left(1-\left(\frac{d}{2z}\right)^2\right)^2} = \frac{q}{2\pi\varepsilon_0 z^3}\frac{d}{\left(1-\left(\frac{d}{2z}\right)^2\right)^2}.$$

$d/2z \ll 1$ ⟹

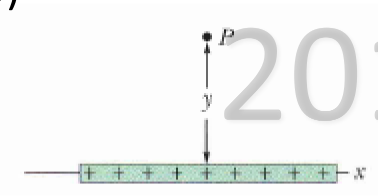
$$E = \frac{1}{2\pi\varepsilon_0}\frac{qd}{z^3}.$$

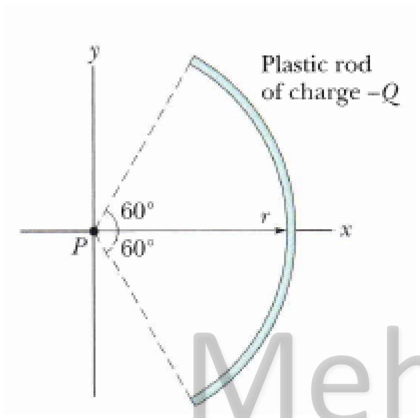
$$E = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3}$$

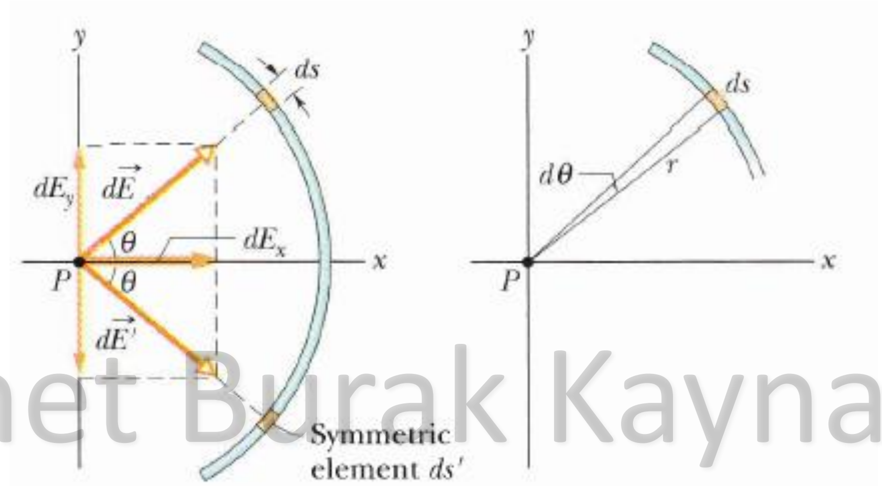
a)



b)







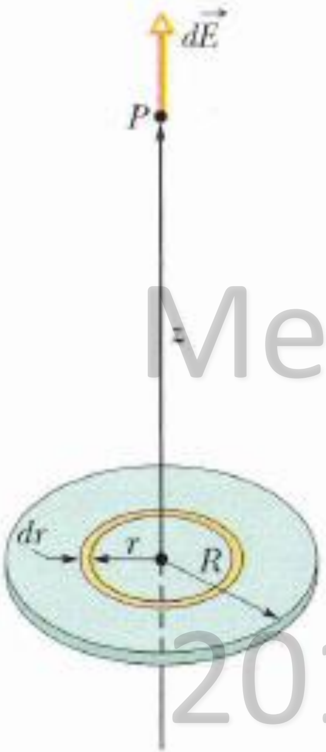
$$dq = \lambda\, ds.$$

$$ds = r\, d\theta,$$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{r^2}.$$

$$dE_x = dE\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{r^2}\cos\theta\, ds.$$

$$\begin{aligned} E &= \int dE_x = \int_{-60^\circ}^{60^\circ} \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{r^2}\cos\theta\, r\, d\theta \\ &= \frac{\lambda}{4\pi\varepsilon_0 r}\int_{-60^\circ}^{60^\circ}\cos\theta\, d\theta = \frac{\lambda}{4\pi\varepsilon_0 r}\Big[\sin\theta\Big]_{-60^\circ}^{60^\circ} \\ &= \frac{\lambda}{4\pi\varepsilon_0 r}[\sin 60^\circ - \sin(-60^\circ)] \\ &= \frac{1.73\lambda}{4\pi\varepsilon_0 r}. \end{aligned}$$



$$dq = \sigma\, dA = \sigma(2\pi r\, dr),$$

$$dE = \frac{z\sigma 2\pi r\, dr}{4\pi\varepsilon_0 (z^2 + r^2)^{3/2}}$$

$$dE = \frac{\sigma z}{4\varepsilon_0} \frac{2r\, dr}{(z^2 + r^2)^{3/2}}$$

$$E = \int dE = \frac{\sigma z}{4\varepsilon_0} \int_0^R (z^2 + r^2)^{-3/2}(2r)\, dr.$$

$$E = \frac{\sigma z}{4\varepsilon_0}\left[\frac{(z^2 + r^2)^{-1/2}}{-\frac{1}{2}}\right]_0^R$$

$$X = (z^2 + r^2),$$
$$dX = (2r)\, dr.$$

$$E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)$$

Chapter 23
Gauss' Law

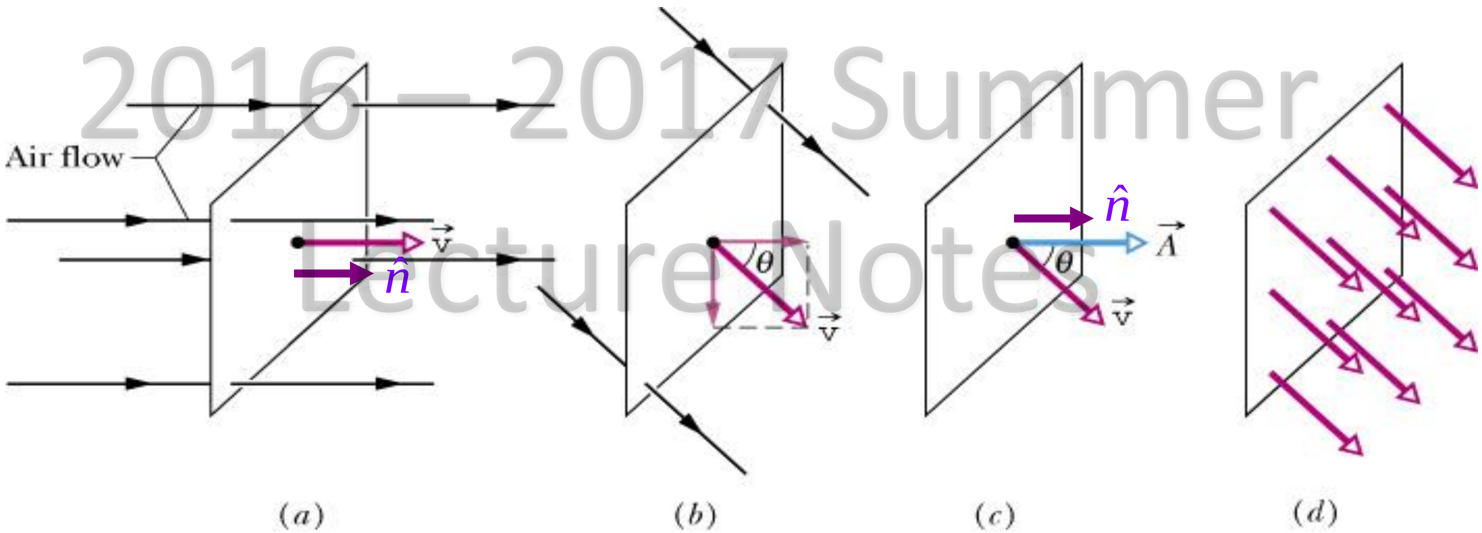
Method for Calculating E Field of Symmetric Charge Distributions

- Symmetry properties play an important role in physics.
- Gauss's law will allow us to do electric-field calculations using symmetry principles.

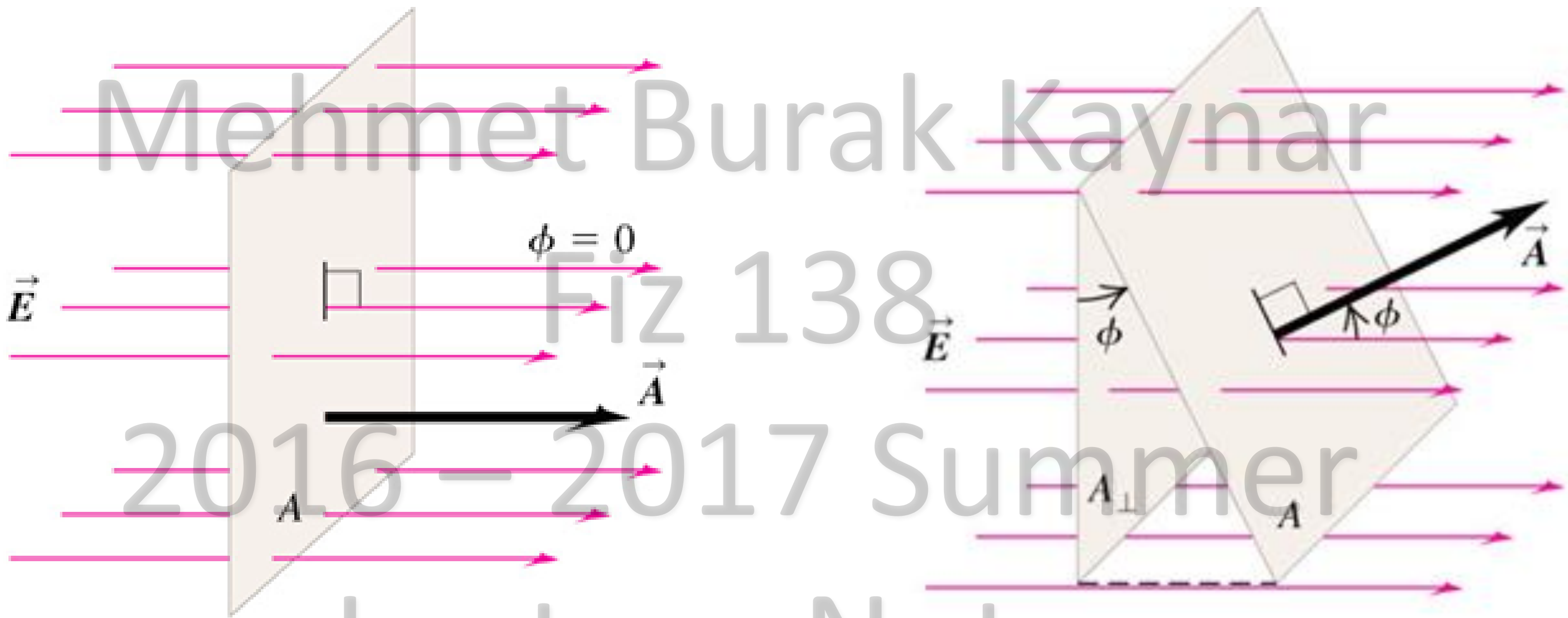
Flux of A Vector

- A measure of the strength of a physical quantity that is represented by a vector.
- Higher the flux, higher the strength of the physical quantity.

FLUX=Total number of normal vectors on a surface

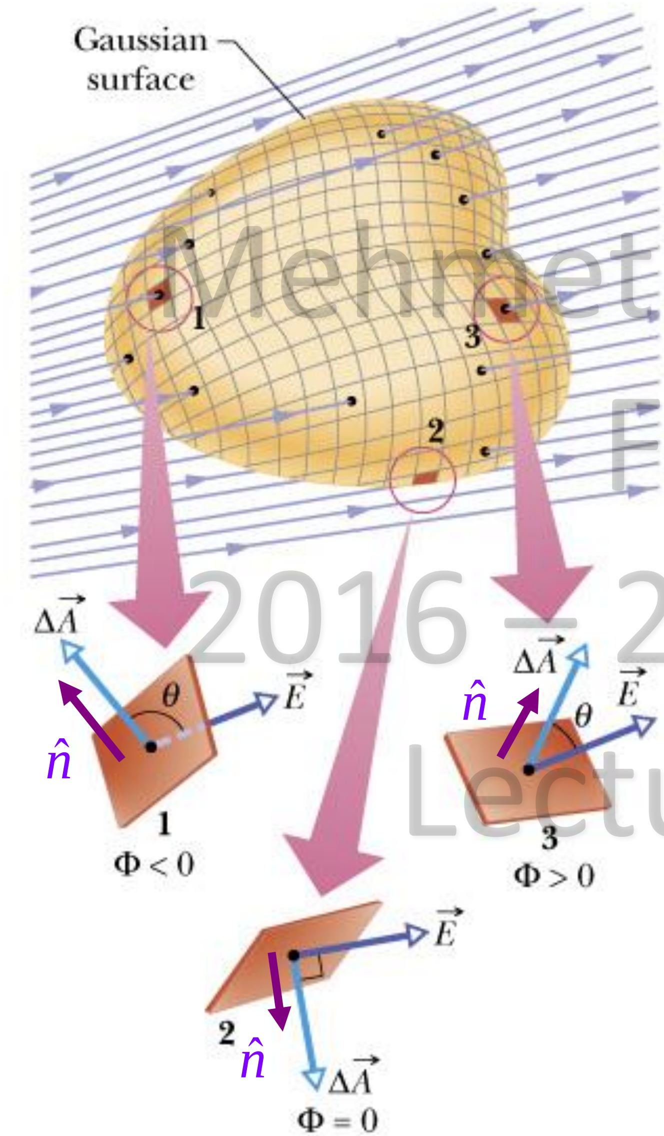


Calculating Electric Flux



$$F_E = \vec{E}.\vec{A} = EA\cos f$$

Gaussian Surface



- *Usefull for E field calculation.*
- *Might be any imaginary surface.*
- *For simplicity, equal E field points are used to produce Gaussian Surface.*
- *Equal E field points are generated in symmetric (charge distribution) problems.*

We want to know the strength of E field

↓

FLUX

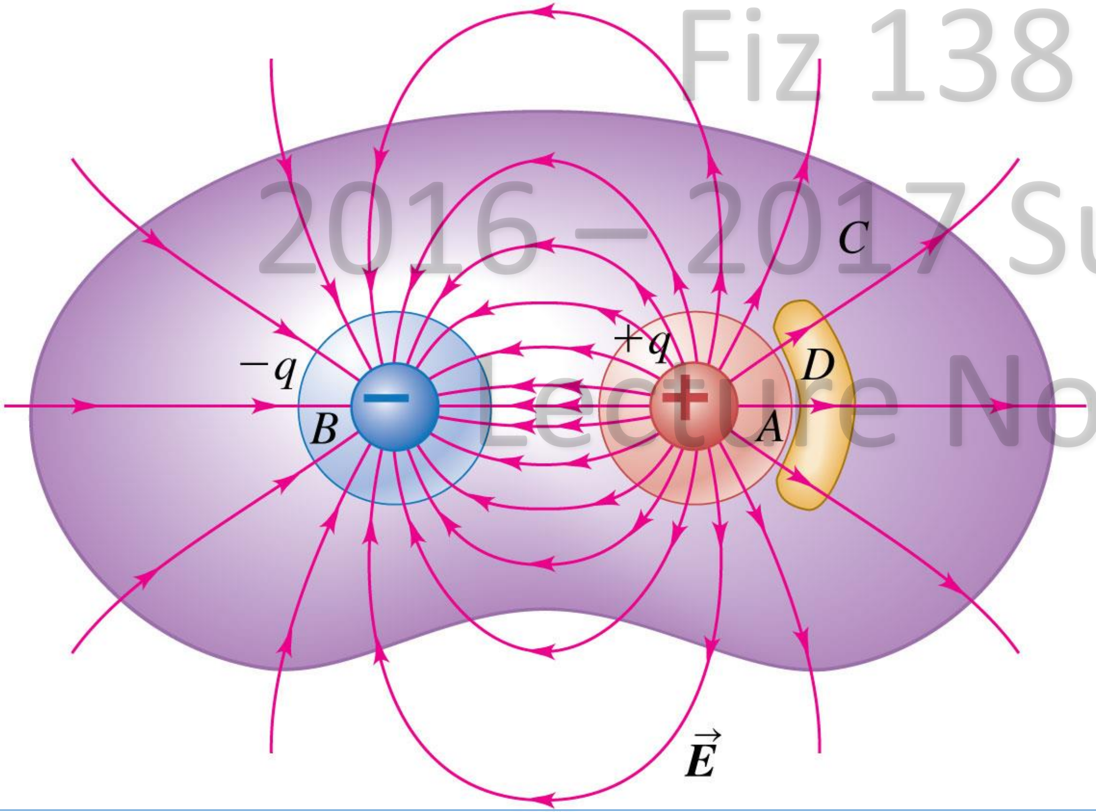
↓

$$\sum \vec{E}.\Delta\vec{A} \longrightarrow \oint \vec{E}.d\vec{A}$$

Gauss' Law

- Gauss's law is an alternative to Coulomb's law and is completely equivalent to it.
- Valid for any closed surface

ε_0 X (TOTAL *E* FLUX)=NET CHARGE INSIDE ANY CLOSED SURFACE



$$\varepsilon_0 \oint \vec{E}.d\vec{A} = q_{enc}$$

$$\Phi_A = \frac{+q}{\varepsilon_0}$$

$$\Phi_B = \frac{-q}{\varepsilon_0}$$

$$\Phi_C = \frac{(+q)+(-q)}{\varepsilon_0} = 0$$

$$\Phi_D = 0, \text{ no net charge at all.}$$

Recipe for Applying Gauss's Law

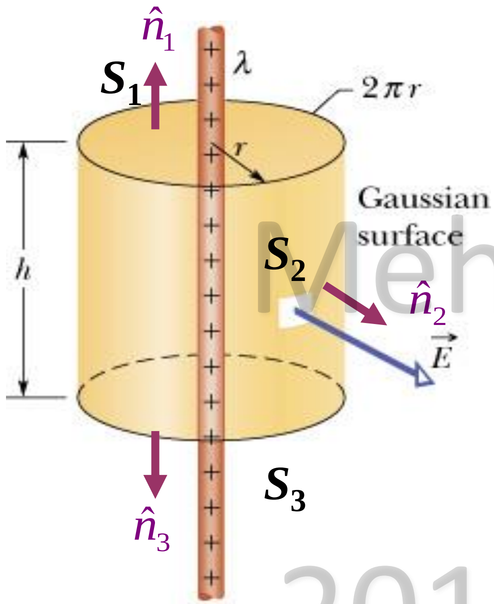
1. Make a sketch of the charge distribution.

2. Identify the symmetry of the distribution and its effect on the electric field.

3. Gauss's law is true for **any** closed surface. Choose one that makes the calculation of the flux as easy as possible.

4. Use Gauss's law to determine the electric field vector:

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$



Electric Field Generated by a Long, Uniformly Charged Rod

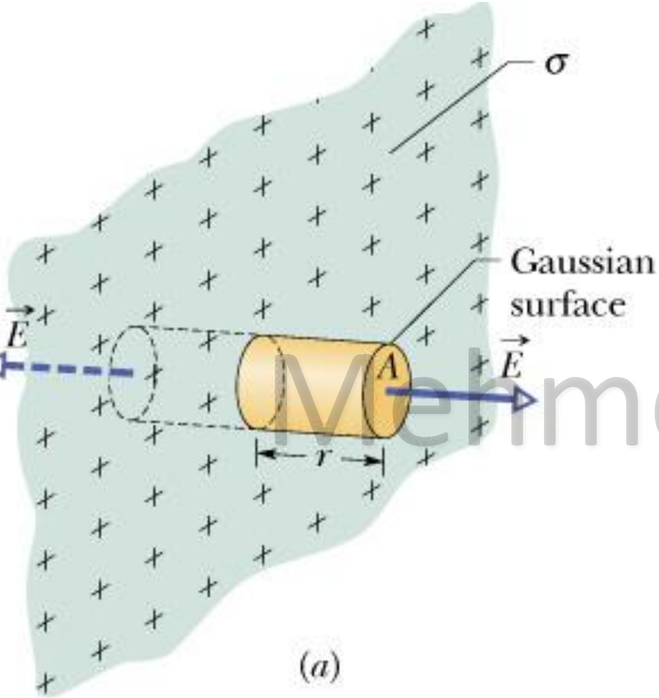
Consider the long rod shown in the figure. It is uniformly charged with linear charge density λ. Using symmetry arguments we can show that the electric field vector points radially outward and has the same magnitude for points at the same distance r from the rod. We use a Gaussian surface S that has the same symmetry. It is a cylinder of radius r and height h whose axis coincides with the charged rod.

We divide S into three sections: Top flat section S_1, middle curved section S_2, and bottom flat section S_3. The net flux through S is $\Phi = \Phi_1 + \Phi_2 + \Phi_3$. Fluxes Φ_1 and Φ_2 vanish because the electric field is at right angles with the normal to the surface:

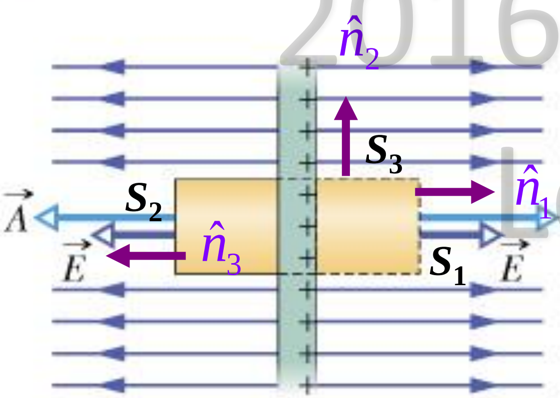
$\Phi_3 = 2\pi r h E \cos 0 = 2\pi r h E \rightarrow \Phi = 2\pi r h E$. From Gauss's law we have: $\Phi = \dfrac{q_{enc}}{\varepsilon_0} = \dfrac{\lambda h}{\varepsilon_0}$.

If we compare these two equations we get: $2\pi r h E = \dfrac{\lambda h}{\varepsilon_0} \rightarrow E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$.

E Field of a Thin, Infinitely Large, Nonconducting Uniformly Charged Sheet



(a)



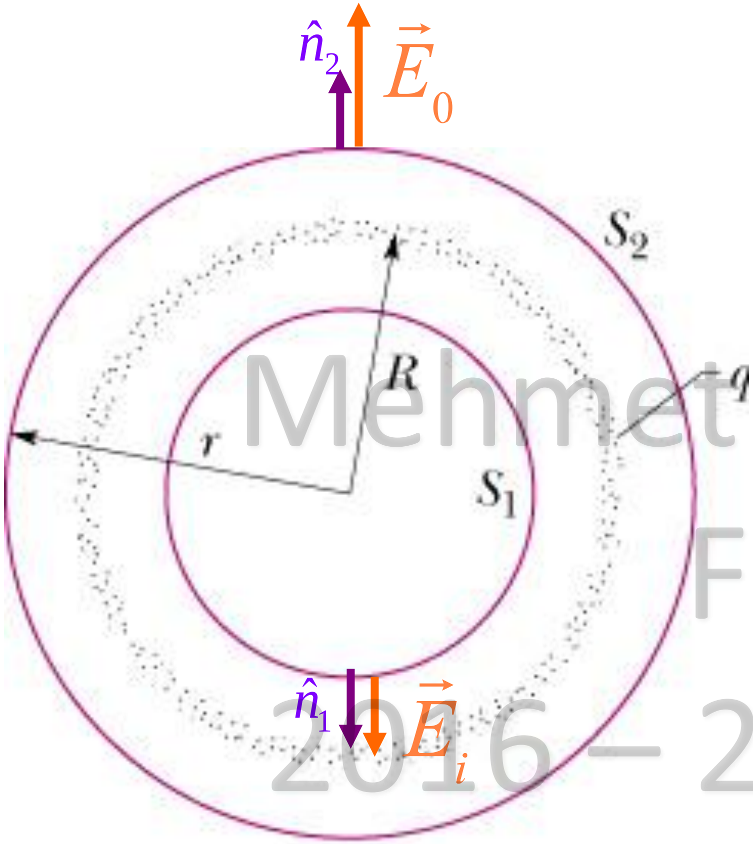
(b)

The net flux through S is $\Phi = \Phi_1 + \Phi_2 + \Phi_3$.

$$\Phi_1 = \Phi_2 = EA\cos 0 = EA. \quad \Phi_3 = 0 \quad (\theta = 90°)$$

$\Phi = 2EA.$ From Gauss's law we have: $\Phi = \dfrac{q_{enc}}{\varepsilon_0} = \dfrac{\sigma A}{\varepsilon_0}$

$$2EA = \frac{\sigma A}{\varepsilon_0} \rightarrow \quad E = \frac{\sigma}{2\varepsilon_0}.$$



$$E_i = 0$$

$$E_0 = \frac{q}{4\pi\varepsilon_0 r^2}$$

The Electric Field Generated by a Spherical Shell of Charge *q* and Radius *R*

Inside the shell : Consider a Gaussian surface S_1 that is a sphere with radius $r < R$ and whose center coincides with that of the charged shell.

The electric field flux $\Phi = 4\pi r^2 E_i = \frac{q_{\text{enc}}}{\varepsilon_0} = 0.$

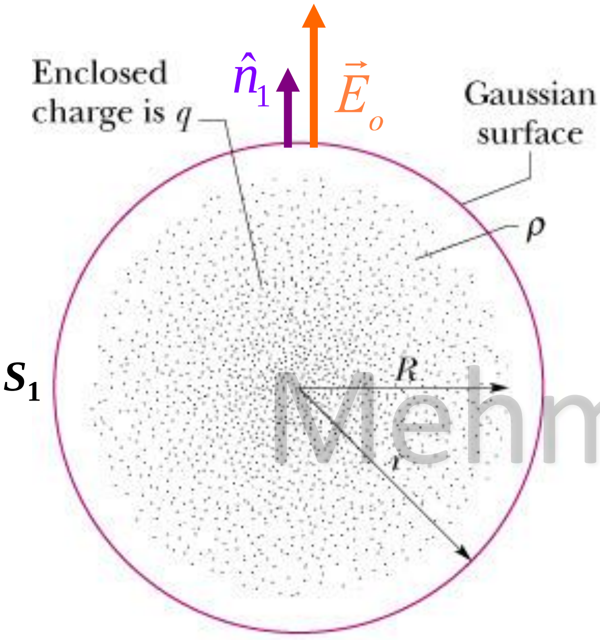
Thus $E_i = 0.$

Outside the shell : Consider a Gaussian surface S_2 that is a sphere with radius $r > R$ and whose center coincides with that of the charged shell.

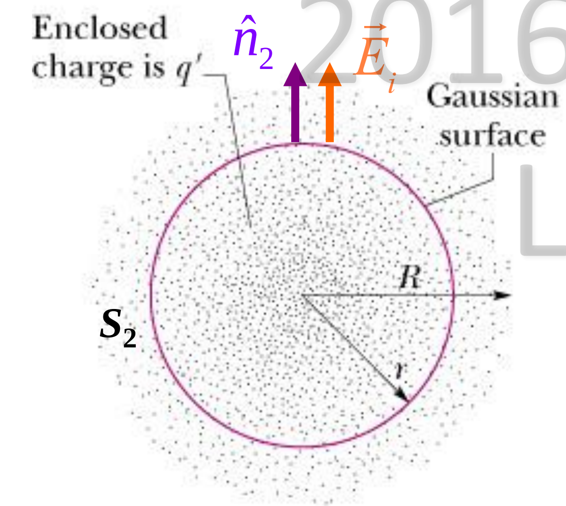
The electric field flux $\Phi = 4\pi r^2 E_0 = \frac{q_{\text{enc}}}{\varepsilon_0} = \frac{q}{\varepsilon_0}.$

Thus $E_0 = \frac{q}{4\pi\varepsilon_0 r^2}.$

Note : Outside the shell the electric field is the same as if all the charge of the shell were concentrated at the shell center.



(a)



(b)

Electric Field Generated by a Uniformly Charged Sphere of Radius *R* and Charge *q*

Outside the sphere : Consider a Gaussian surface S_1 that is a sphere with radius $r > R$ and whose center coincides with that of the charged shell.

The electric field flux $\Phi = 4\pi r^2 E_0 = q_{\text{enc}} / \varepsilon_0 = q / \varepsilon_0$

Thus $E_0 = \dfrac{q}{4\pi\varepsilon_0 r^2}$.

Inside the sphere : Consider a Gaussian surface S_2 that is a sphere with radius $r < R$ and whose center coincides with that of the charged shell.

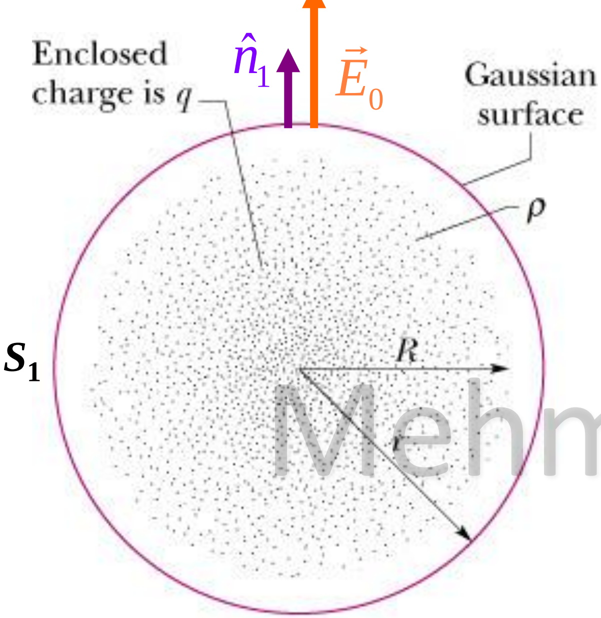
The electric field flux $\Phi = 4\pi r^2 E_i = \dfrac{q_{\text{enc}}}{\varepsilon_0}$.

$$q_{enc} = \frac{\left(\frac{4}{3}\right)\pi r^3}{\left(\frac{4}{3}\right)\pi R^3} q = \frac{r^3}{R^3} q \rightarrow 4\pi r^2 E_i = \frac{1}{\varepsilon_0}\frac{r^3}{R^3} q$$

Thus $E_i = \left(\dfrac{q}{4\pi\varepsilon_0 R^3}\right) r$.

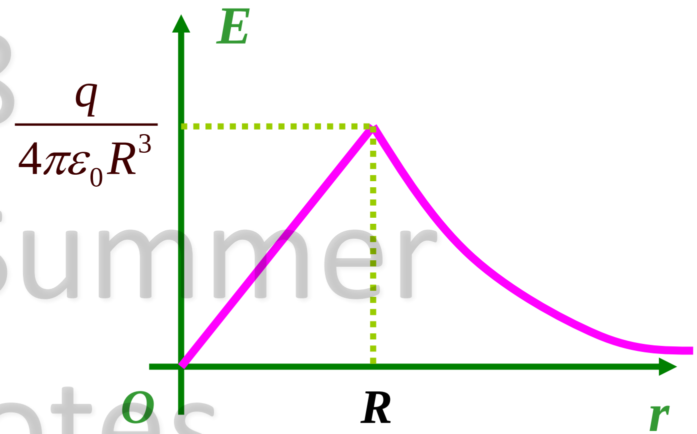
Electric Field Generated by a Uniformly Charged Sphere of Radius *R* and Charge *q*

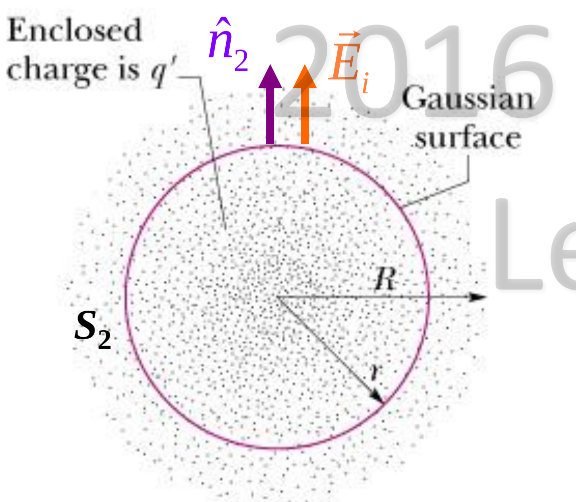
Summary :



(*a*)

$$E_i = \left(\frac{q}{4\pi\varepsilon_0 R^3}\right) r$$





(*b*)

$$E_0 = \frac{q}{4\pi\varepsilon_0 r^2}$$

Example

A conducting spherical shell with inner radius a and outer radius b has positive point charge Q located at its center. The total charge on the shell is $-3Q$ and it is insulated from its surroundings. A) Derive expressions for the electric field magnitude in terms of the distance r from the center for the regions $r<a$, $a<r<b$, and $r>b$. B) What is the surface charge density on the inner surface of the conducting shell? C) Graph the electric field magnitude as a function of r.