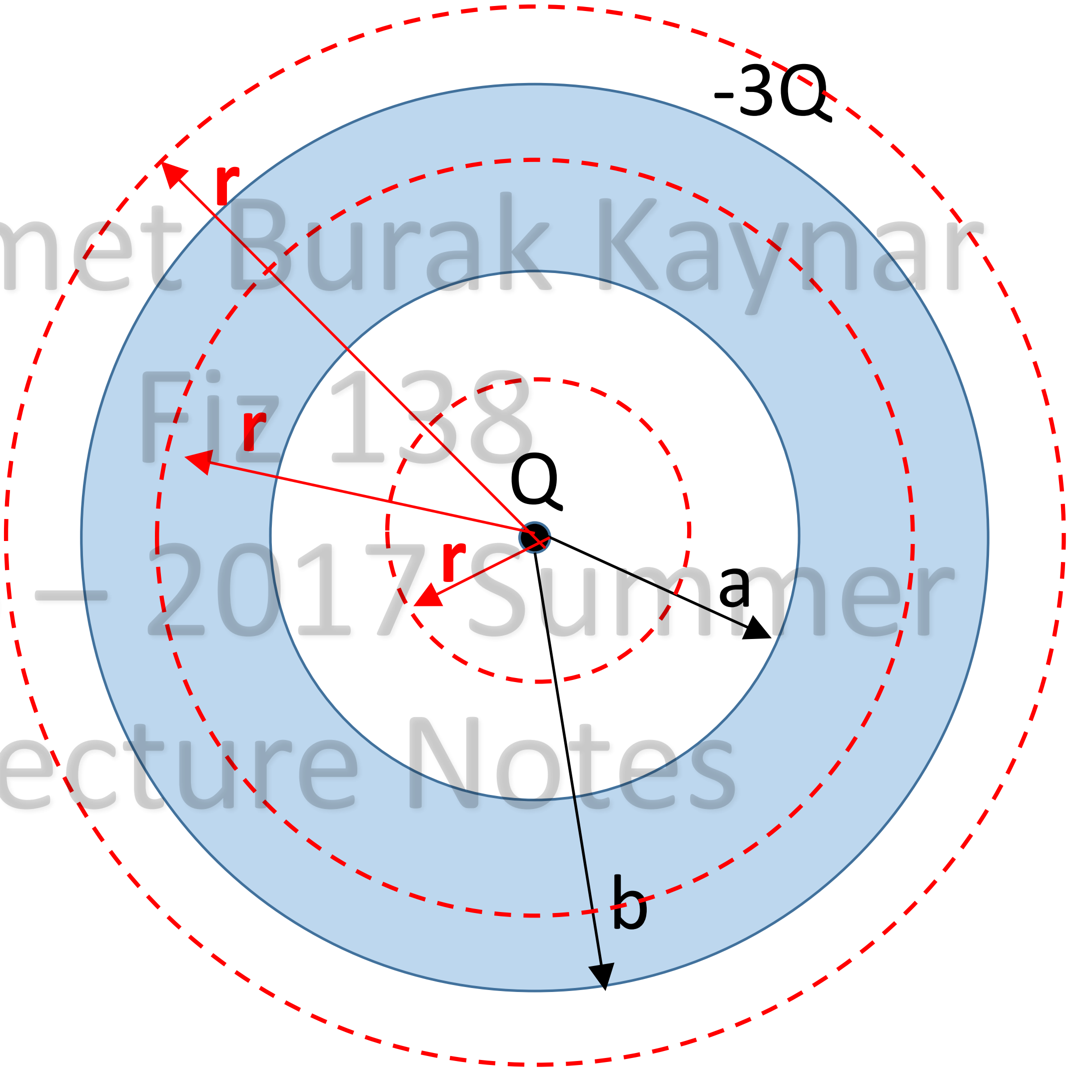
Mehmet Burak Kaynar
Fiz 138
2016 – 2017 Summer
Lecture Notes

For Lecture Notes Fiz138 Physics II Please Check
http://yunus.hacettepe.edu.tr/~bkaynar

Gauss' Law

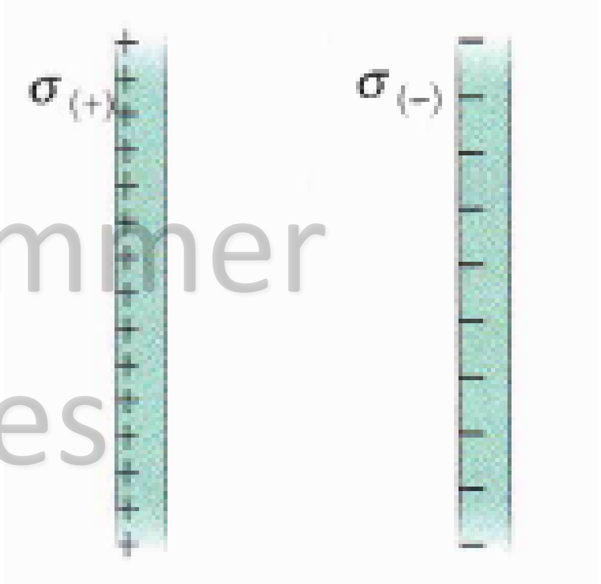
$$\varepsilon_0 \oint \vec{E}.d\vec{A} = q_{enc}$$



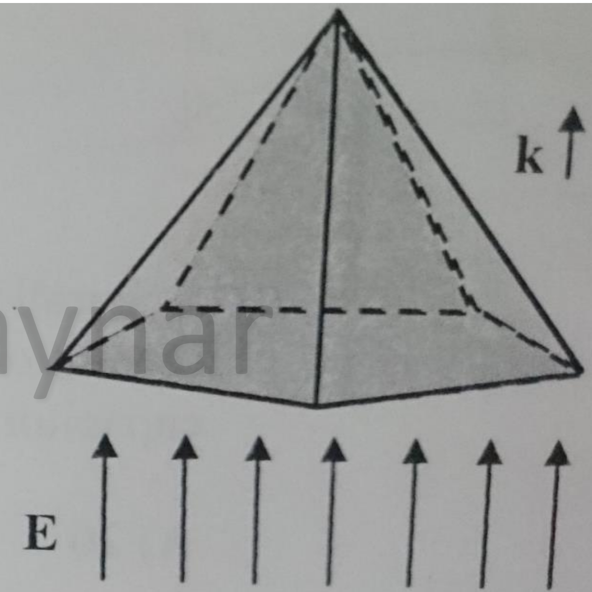
Sample Problem 23-7

Figure 23-17*a* shows portions of two large, parallel, nonconducting sheets, each with a fixed uniform charge on one side. The magnitudes of the surface charge densities are $\sigma_{(+)} = 6.8\ \mu C/m^2$ for the positively charged sheet and $\sigma_{(-)} = 4.3\ \mu C/m^2$ for the negatively charged sheet.

Find the electric field $\vec{E}$ (a) to the left of the sheets, (b) between the sheets, and (c) to the right of the sheets.



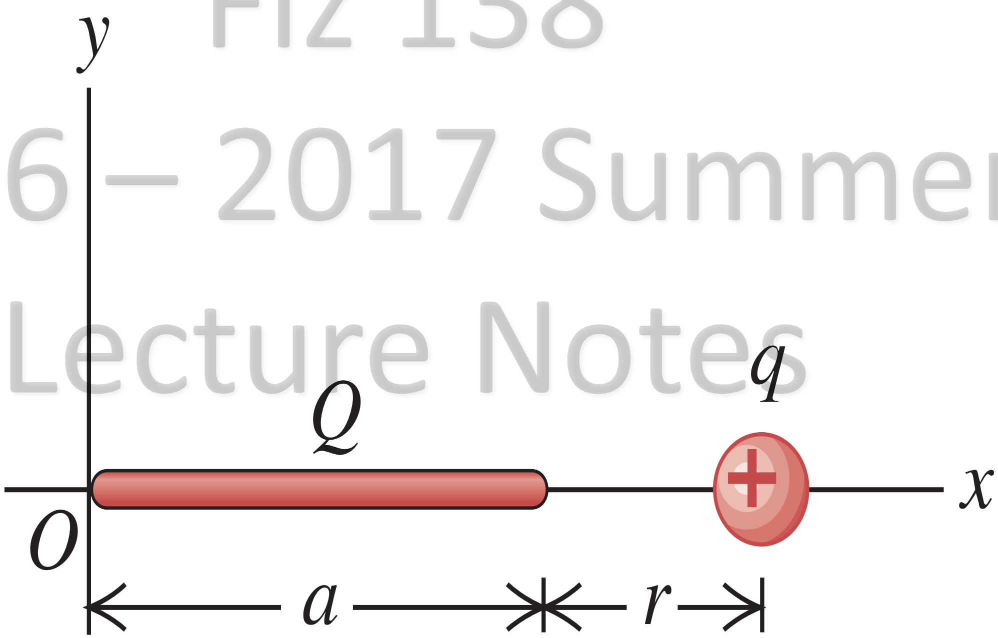
4. The base area of an equilateral pentagonal pyramid is 35 m^2 and placed in a uniform electric field **E** = 36 **k** N/C which is perpendicular to the base of the pyramid as in the figure. Find the electric flux (in $N.m^2/C$) through one of the five triangle slanted surfaces.



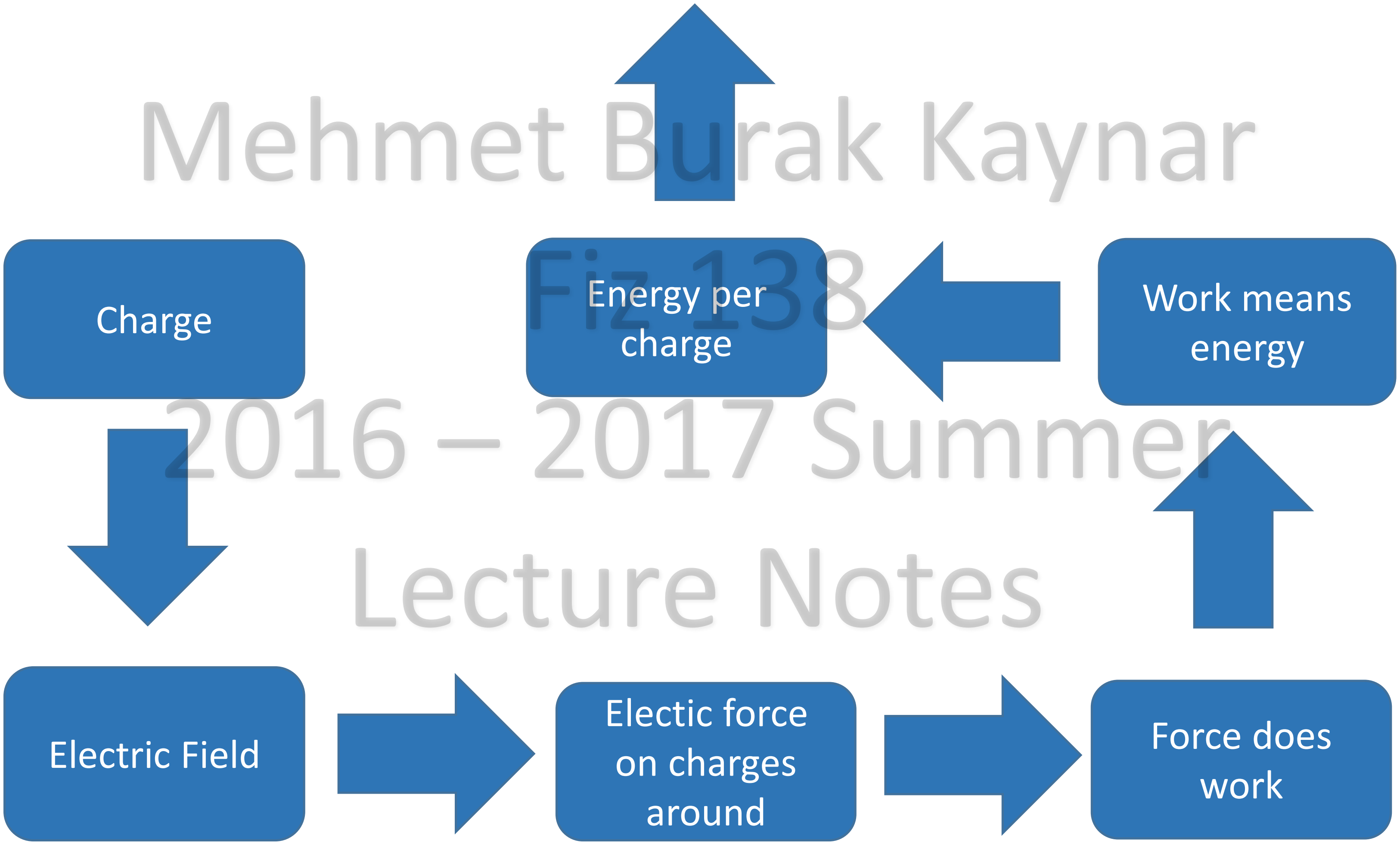
A) 210
B) 252
C) 315
D) 420
E) 630

Example

Positive charge Q is distributed uniformly along the x-axis from x=0 to x=a. A positive point charge q is located on the positive x-axis as shown in the figure. Calculate the x and y components of the electric force exerted on q due the charge distribution.



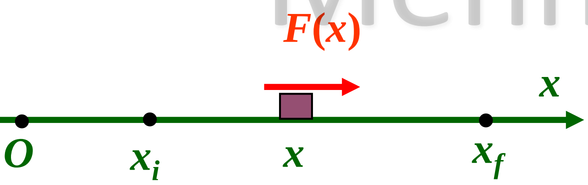
Chapter 24
Electric Potential



Electric (Potential Energy→Potential)

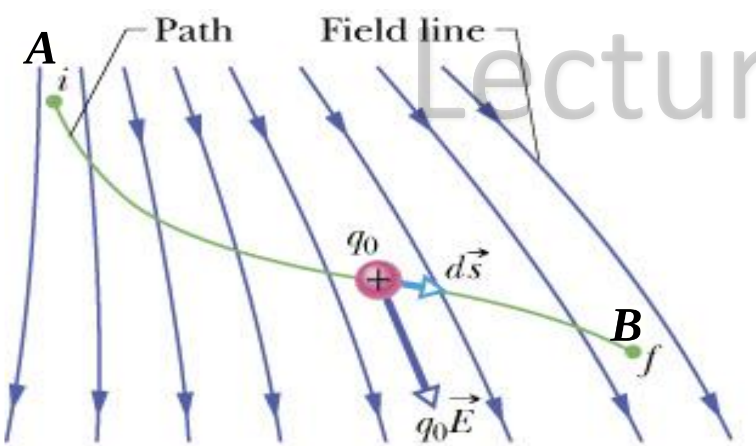
Remember!!
In Mechanics

$$\Delta U = -\int_{x_i}^{x_f} F(x)dx$$



Energy required to move a charge in an ***E*** Field

ΔU



$$DU = -\int \vec{F}.d\vec{s}$$

$$DU = U_f - U_i = -q_0 \int \vec{E}.d\vec{s}$$

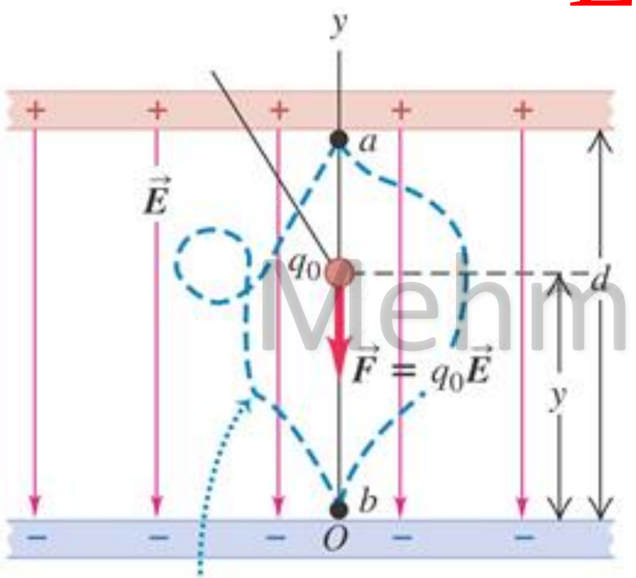
$$DV = \frac{DU}{q_0} = V_f - V_i$$

Initial point→(Infinity=ZERO CHARGE)

$$V = \frac{U}{q_0}$$

Energy per unit charge

Electric Potential

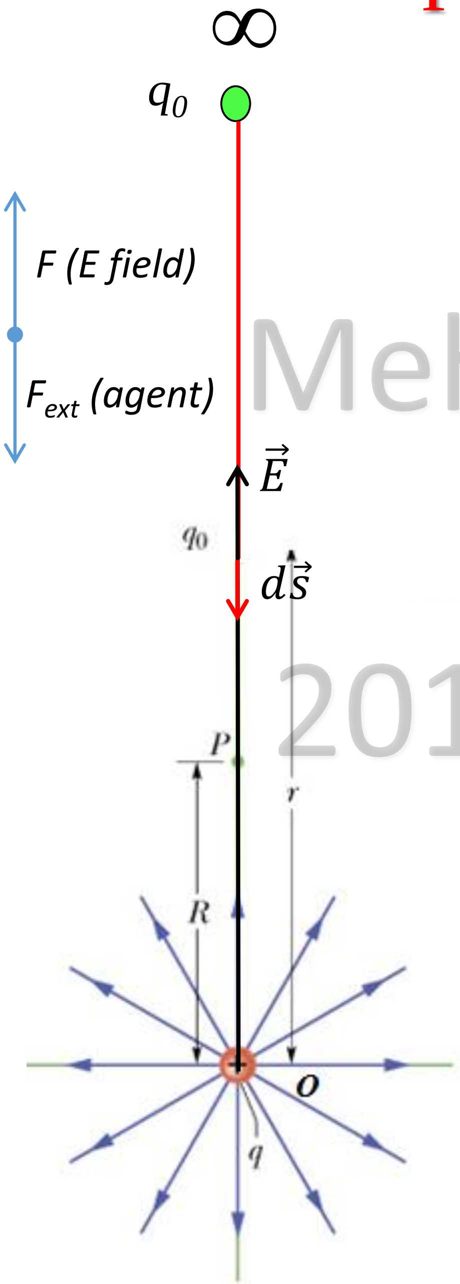


$$Potential = \frac{Potential\ Energy}{Charge}$$

We can think of the potential difference between points *a* and *b* in either of two ways. The potential of *a* with respect to *b* ($V_{ab} = V_a - V_b$) equals:

- the work done by the electric force when a *unit* charge moves from *a* to *b*.
- the work that must be done to move a *unit* charge slowly from *b* to *a* against the electric force.

Potential Due to A Point Charge



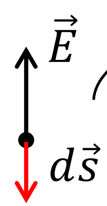
Bringing q from infinity to P is done by an applied force by an external agent. That corresponds to W_{app}. On the other hand, E field does work against the external agent. So $W=-W_{app}$

For the external force, V_i=infinity V_f=point P

Since the work is done by an external force, no negative sign

$$V_f - V_i = \int_i^f \vec{E}.d\vec{s}$$

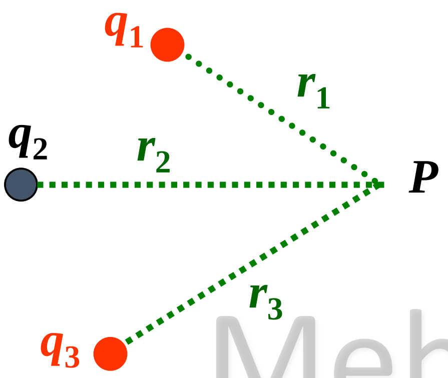
zero (for V_i); $k\dfrac{q}{r^2}$ (for $\vec{E}$); dr (for $d\vec{s}$)



Scalar product

$$V_P = -kq\int_{\infty}^{R}\frac{dr}{r^2}$$

$$V_P = -kq\left(-\frac{1}{r}\right)_{\infty}^{R} \Rightarrow V_P = k\frac{q}{R}$$



$$V = V_1 + V_2 + V_3$$

Potential Due to a Group of Point Charges

Consider the group of three point charges shown in the figure. The potential V generated by this group at any point P is calculated using the principle of superposition.

1. We determine the potentials $V_1, V_2,$ and V_3 generated by each charge at point P:

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1}, \quad V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2}, \quad V_3 = \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_3}$$

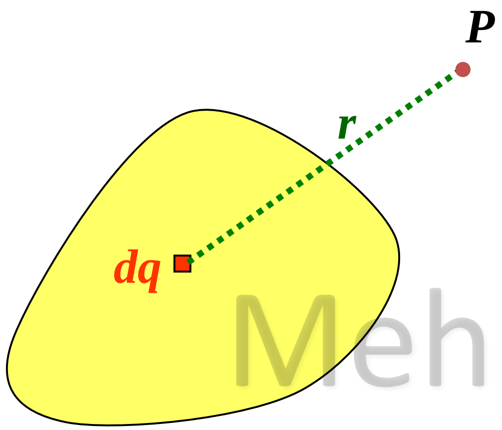
2. We add the three terms:

$$V = V_1 + V_2 + V_3$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2} + \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_3}$$

The previous equation can be generalized for n charges as follows:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2} + \ldots + \frac{1}{4\pi\varepsilon_0}\frac{q_n}{r_n} = \frac{1}{4\pi\varepsilon_0}\sum_{1}^{n}\frac{q_i}{r_i}$$



$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r}$$

Potential Due to a Continuous Charge Distribution

Consider the charge distribution shown in the figure. In order to determine the electric potential V created by the distribution at point P we use the principle of superposition as follows:

1. We divide the distribution into elements of charge dq.

For a volume charge distribution, $dq = \rho dV$.

For a surface charge distribution, $dq = \sigma dA$.

For a linear charge distribution, $dq = \lambda d\ell$.

2. We determine the potential dV created by dq at P: $dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r}$.

3. We sum all the contributions in the form of the integral: $V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r}$.

Note 1 : The integral is taken over the whole charge distribution.

Note 2 : The integral involves only scalar quantities.

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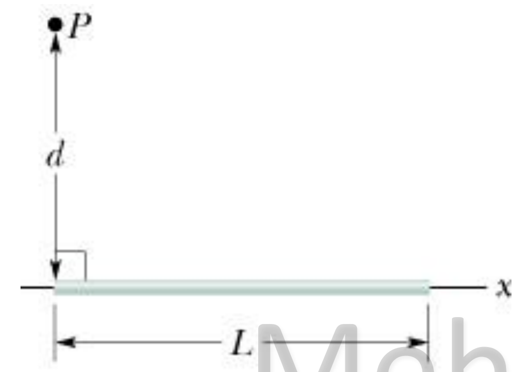
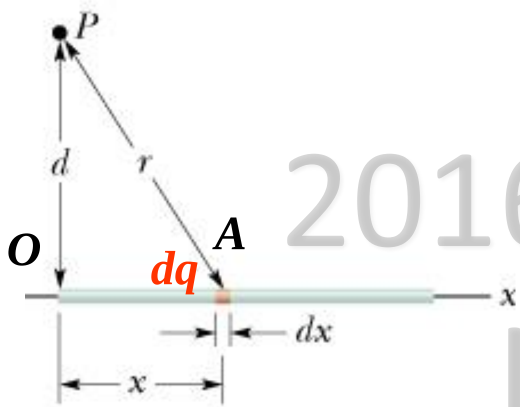


(a)



(b)

Example : Potential created by a line of charge of length L and uniform linear charge density λ at point P. Consider the charge element $dq = \lambda dx$ at point A, a distance x from O. From triangle OAP we have:

$r = \sqrt{d^2 + x^2}$. Here d is the distance OP.

The potential dV created by dq at P is:

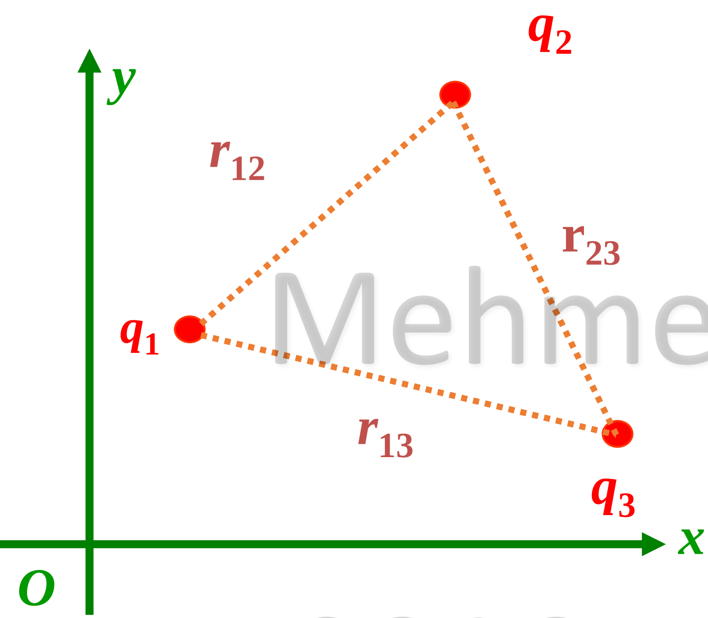
$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{\sqrt{d^2 + x^2}}$$

$$V = \frac{\lambda}{4\pi\varepsilon_0}\int_0^L \frac{dx}{\sqrt{d^2 + x^2}}$$

$$\int \frac{dx}{\sqrt{d^2 + x^2}} = \ln\left(x + \sqrt{d^2 + x^2}\right)$$

$$V = \frac{\lambda}{4\pi\varepsilon_0}\left[\ln\left(x + \sqrt{d^2 + x^2}\right)\right]_0^L$$

$$V = \frac{\lambda}{4\pi\varepsilon_0}\left[\ln\left(L + \sqrt{L^2 + x^2}\right) - \ln d\right]$$



Potential Energy U of a System of Point Charges

We define U as the work required to assemble the system of charges one by one, bringing each charge from infinity to its final position.

Using the above definition we will prove that for a system of three point charges U is given by:

$$U = \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}} + \frac{q_2 q_3}{4\pi\varepsilon_0 r_{23}} + \frac{q_1 q_3}{4\pi\varepsilon_0 r_{13}}$$

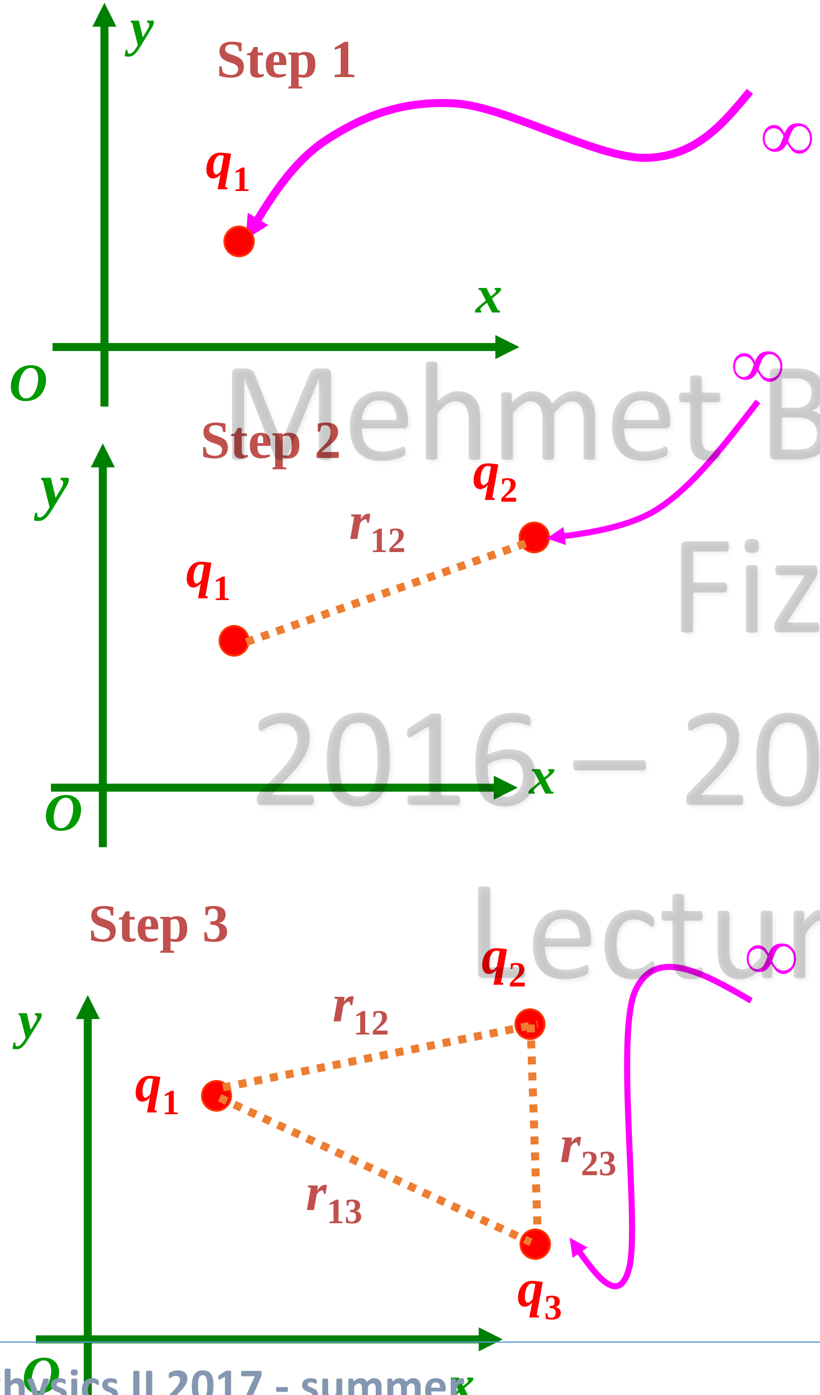
Note : Each pair of charges is counted only once.

For a system of n point charges $\{q_i\}$ the potential energy U is given by:

$$U = \frac{1}{4\pi\varepsilon_0} \sum_{\substack{i,j=1 \\ i<j}}^{n} \frac{q_i q_j}{r_{ij}} .$$

Here r_{ij} is the separation between q_i and q_j.

The summation condition $i < j$ is imposed so that, as in the case of three point charges, each pair of charges is counted only once.



Step 1 : Bring in q_1:

$W_1 = 0$

(no other charges around)

Step 2 : Bring in q_2:

$W_2 = q_2 V(2)$

$$V(2) = \frac{q_1}{4\pi\varepsilon_0 r_{12}} \rightarrow W_2 = \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}$$

Step 3 : Bring in q_3:

$W_3 = q_3 V(3)$

$$V(3) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right) \rightarrow$$

$$W_3 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

$$W = W_1 + W_2 + W_3$$

$$W = \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}} + \frac{q_2 q_3}{4\pi\varepsilon_0 r_{23}} + \frac{q_1 q_3}{4\pi\varepsilon_0 r_{13}}$$

Calculating E Field from Electric Potential

$$V = -\int \vec{E}.d\vec{s}$$ If we know E field, we get potential by integration.

$$E = -\frac{\partial V}{\partial s}$$ If we know the potential, we get E field by derivation.

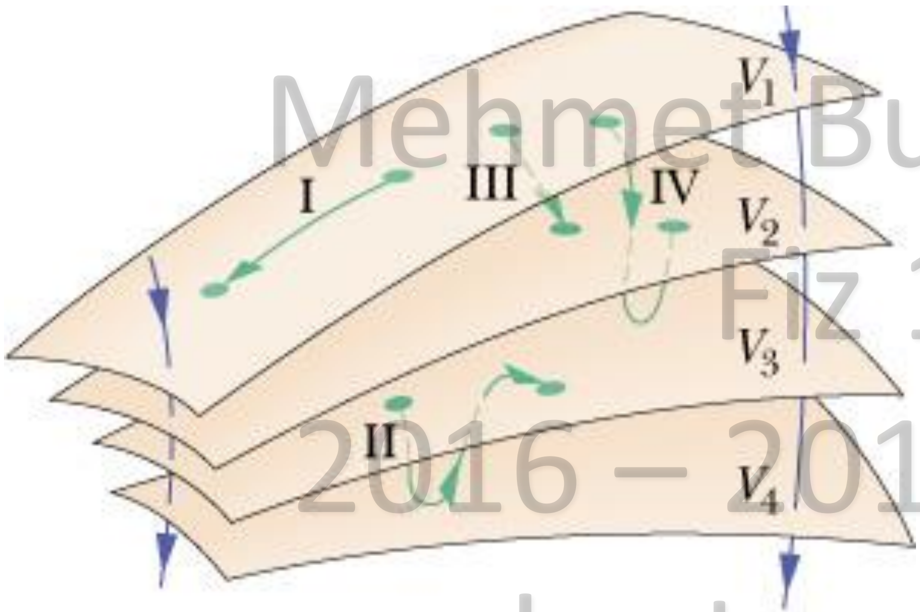
<u>Partial derivative</u>, because E field is a vector and by taking the derivative of electric potential function with respect to a certain direction we get component of E field at that direction. If we need x component of E then we take x derivative of V function.

$$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y} \qquad E_z = -\frac{\partial V}{\partial z}$$

$$E = E_x i + E_y j + E_z k$$

Equipotential Surfaces

A collection of points that have <u>the same potential</u> is known as an equipotential surface.



Equipotential surfaces with different constant potential.

If the potential stays constant then moving a charge on an equipotential surface requires NO WORK.

For path I : $W_I = 0$ because $\Delta V = 0$.

For path II: $W_{II} = 0$ because $\Delta V = 0$.

For path III: $W_{III} = q\Delta V = q(V_2 - V_1)$.

For path IV: $W_{IV} = q\Delta V = q(V_2 - V_1)$.

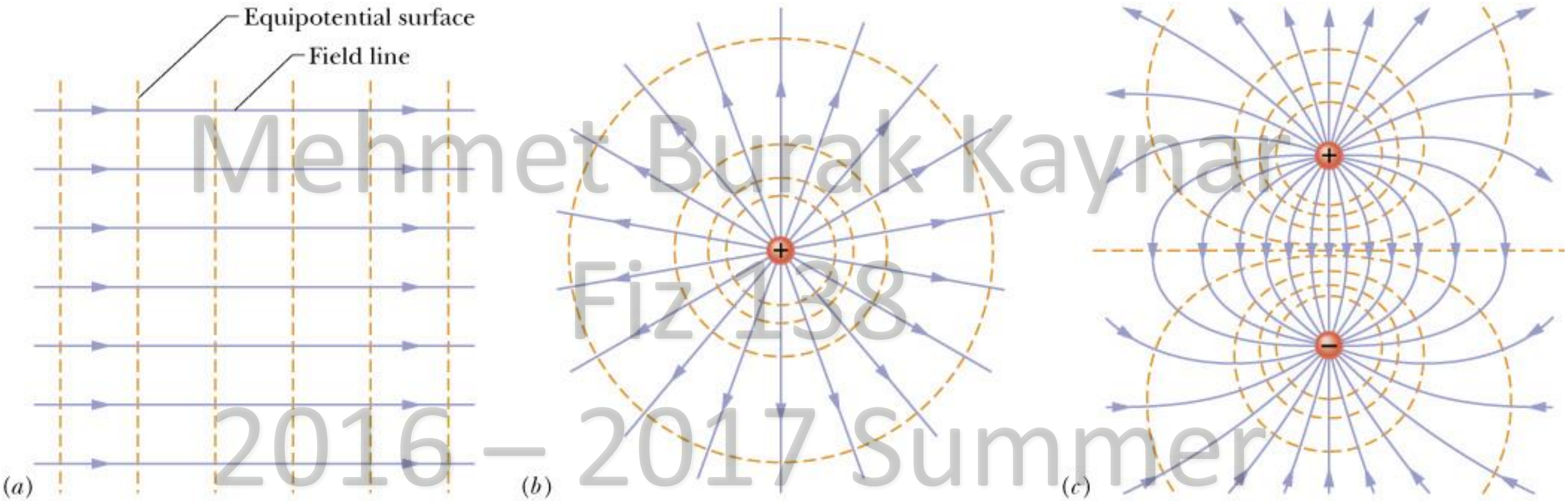
Equipotential surfaces do not cross each other. (Remember E field lines.)

Examples of Equipotential Surfaces and the Corresponding Electric Field Lines

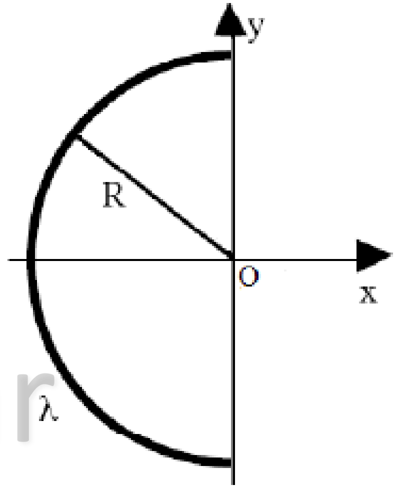
Uniform electric field

Isolated point charge

Electric dipole



. What is the electrical potential at the origin due to a semicircle of radius R with a linear charge density λ?

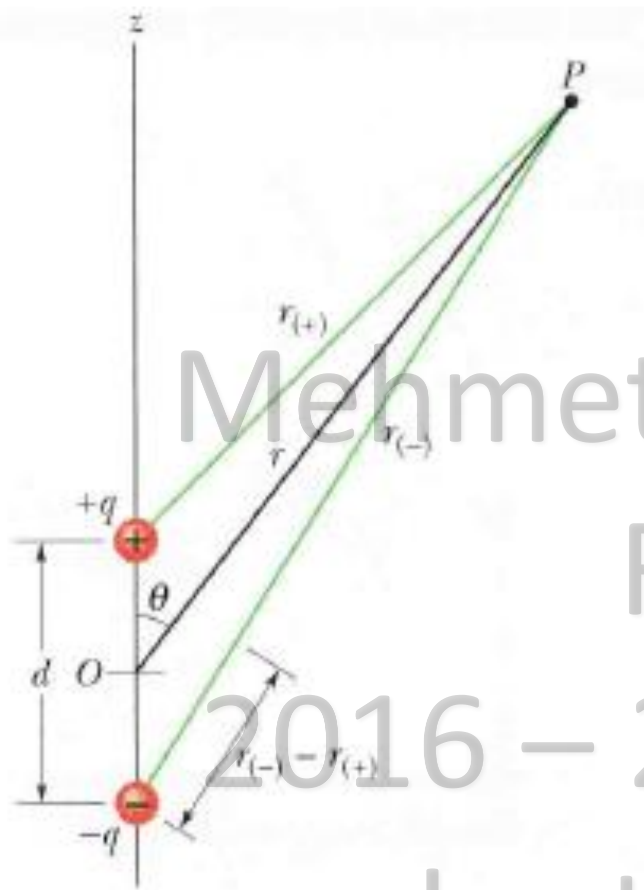


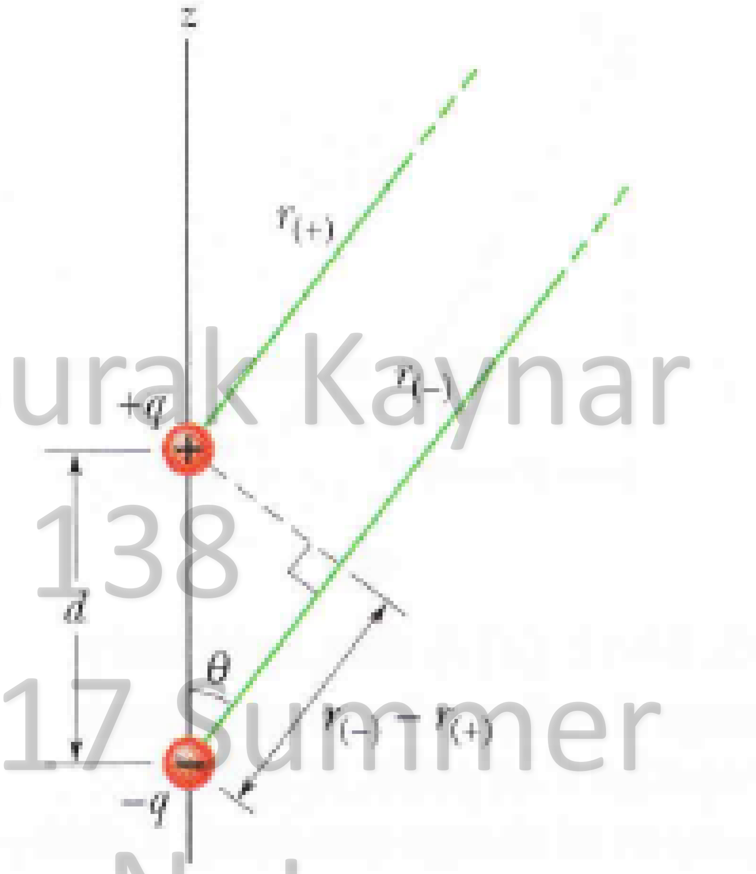
A) $\lambda/2\varepsilon_0$ B) $\lambda/4\varepsilon_0$ C λ/ε_0 D) $\lambda/8\varepsilon_0$ E) $2\lambda/\varepsilon_0$

Sample Problem 24-3

What is the electric potential at point P, located at the center of the square of point charges shown in Fig. 24-8a? The distance d is 1.3 m, and the charges are

$$q_1 = +12\text{ nC}, \quad q_3 = +31\text{ nC},$$
$$q_2 = -24\text{ nC}, \quad q_4 = +17\text{ nC}.$$


z
P
$r_{(+)}$
$r_{(-)}$
r
+q
θ
d O
$r_{(-)} - r_{(+)}$
−q


z
$r_{(+)}$
$r_{(-)}$
+q
d
θ
$r_{(-)} - r_{(+)}$
−q

The electric potential at any point on the central axis of a uniformly charged disk is given by Eq. 24-37,

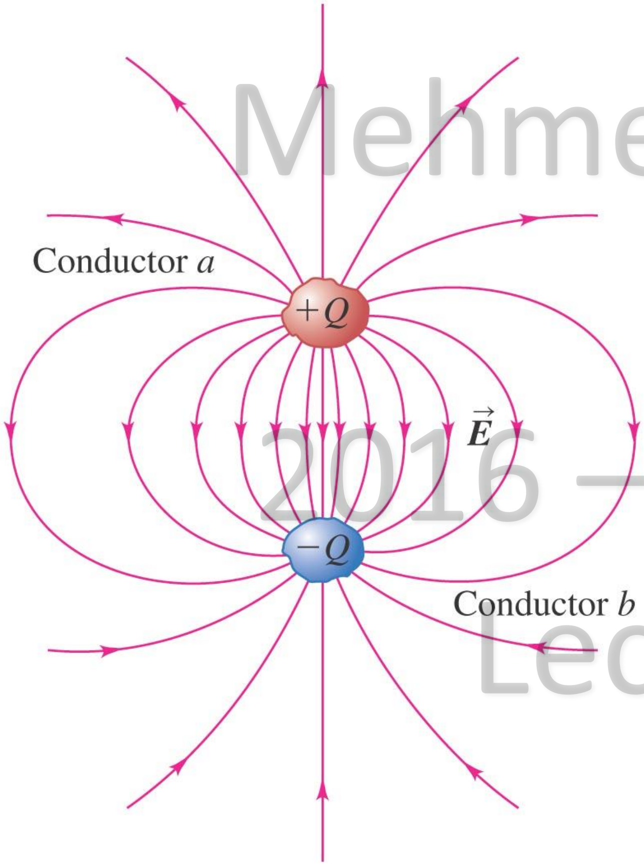
$$V = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2 + R^2} - z\right).$$

Starting with this expression, derive an expression for the electric field at any point on the axis of the disk.

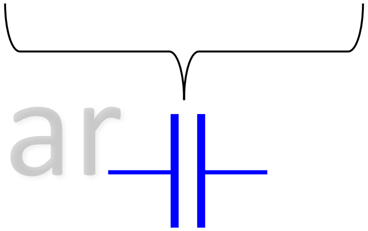
Chapter 25
Capacitance

CAPACITOR

System of Equal and Oppositly Charged *Conductors*

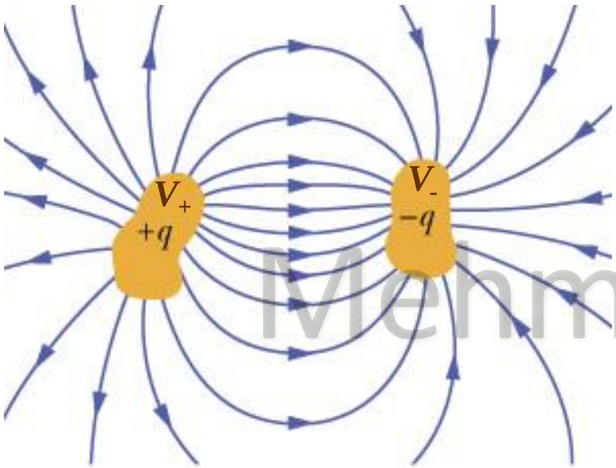


Symbol for a capacitor in electric circuits



$$\frac{CHARGE}{POTENTIAL\ DIFFERENCE} \rightarrow C = \frac{Q}{V_{ab}} \text{ (Farad)}$$

Calculating the Capacitance



The capacitance depends on the geometry of the plates (shape, size, and relative position of one with respect to the other). Below we give a procedure for calculating C.

Recipe :

1. Assume that the plates have charges $+q$ and $-q$.

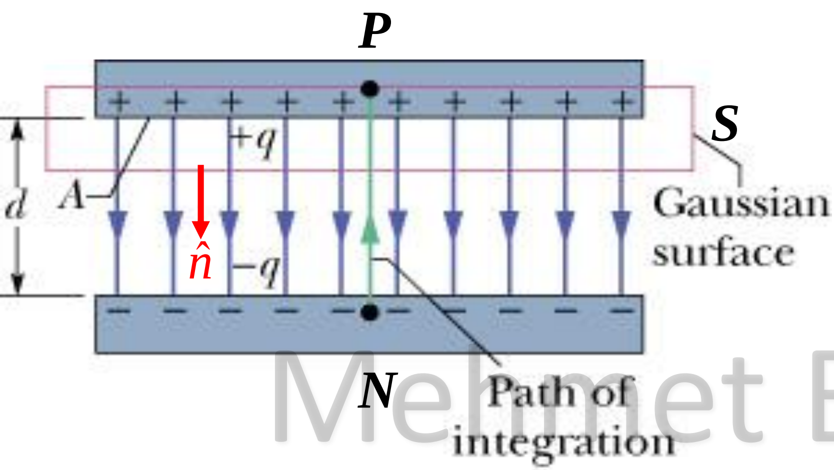
2. Use Gauss' law to determine the electric field $\vec{E}$ between the plates $\left(\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc}\right)$.

3. Determine the potential difference V between the plates using the equation

$V = \int_{-}^{+} \vec{E} \cdot d\vec{s}$ along any path that connects the negative with the positive plate.

4. The capacitance C is given by the equation

$$C = \frac{q}{V}.$$



Capacitance of a Parallel Plate Capacitor

The plates in the figure have area A and are separated by a distance d. The upper plate has a charge $+q$ and the lower plate a charge $-q$.

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s},$$

We apply Gauss' law using the Gaussian surface S shown in the figure.

The electric flux $\Phi = EA\cos 0 = EA.$

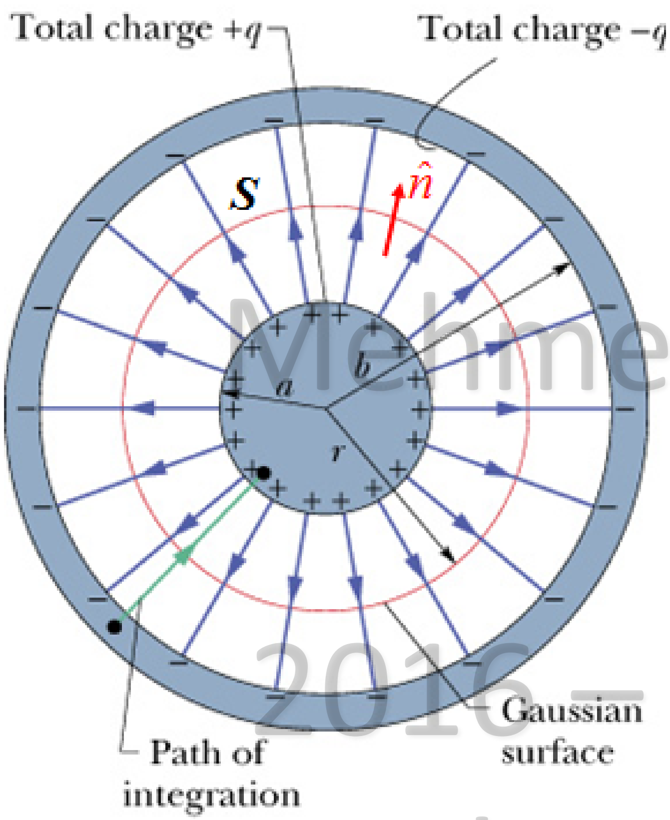
From Gauss' law we have: $\Phi = \frac{q}{\varepsilon_0} \rightarrow EA = \frac{q}{\varepsilon_0} \rightarrow E = \frac{q}{A\varepsilon_0}.$

The potential difference V between the positive and the negative plate is given by: $V = \int_-^+ Eds\cos 0 = E\int_-^+ ds = Ed = \frac{qd}{A\varepsilon_0}.$

The capacitance $C = \frac{q}{V} = \frac{q}{qd / A\varepsilon_0} = \frac{A\varepsilon_0}{d}.$

$$C = e_0 \frac{A}{d}$$

Geometry of the capacitor determines the CAPACITANCE



Cylindrical Capacitor

It consists of two cylinders of radii a and b with a common axis. The two cylinders have a height L. We choose a Gaussian surface S that is also a cylinder with radius r and height L.

The flux of the electric field through S is

$\Phi = 2\pi rLE\cos 0 = 2\pi rLE.$

Using Gauss' law we have: $\Phi = \frac{q}{\varepsilon_0}.$

If we combine these equations we have:

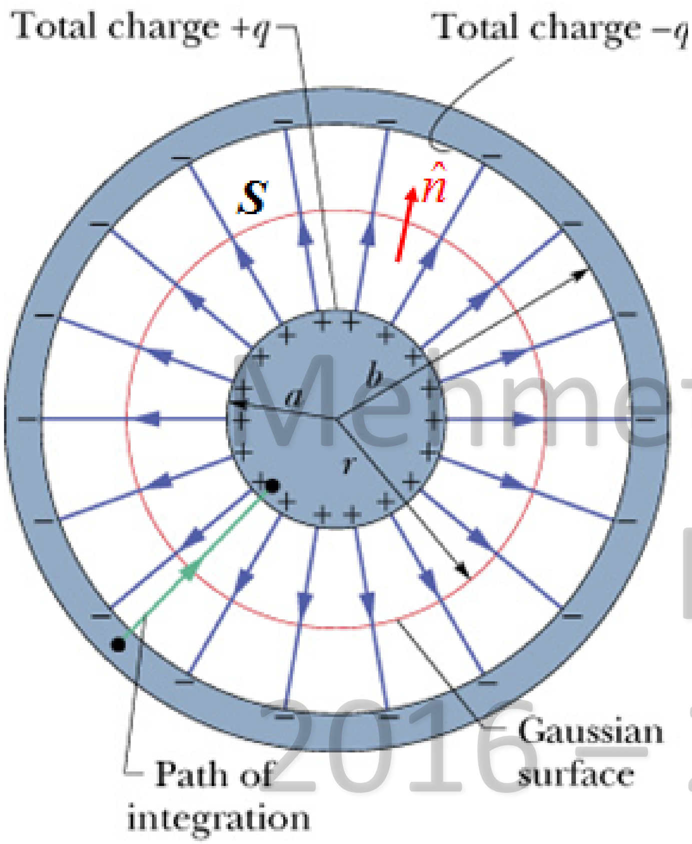
$$E = \frac{q}{\varepsilon_0 2\pi rL}.$$

Comes from the SCALAR product

The potential difference V between the positive and the negative plate is given by:

$$V = \int_{-}^{+} E\,dr = -\frac{q}{\varepsilon_0 2\pi L}\int_b^a \frac{dr}{r} = -\frac{q}{\varepsilon_0 2\pi L}\left[\ln r\right]_b^a = \frac{q}{\varepsilon_0 2\pi L}\ln\left(\frac{b}{a}\right).$$

The capacitance $C = \frac{q}{V} = \frac{q}{\left(q/2\pi L\varepsilon_0\right)\ln\left(b/a\right)} = \frac{2\pi L\varepsilon_0}{\ln\left(b/a\right)}.$



Spherical Capacitor

It consists of two concentric spheres of radii a and b.

We choose a Gaussian surface S that is also a sphere with radius r.

The flux of the electric field through S is:

$\Phi = 4\pi r^2 E\cos 0 = 4\pi r^2 E.$

Using Gauss' law we have: $\Phi = \dfrac{q}{\varepsilon_0}.$

If we combine these equations we have:

$$E = \frac{q}{4\pi\varepsilon_0 r^2}.$$

The potential difference V between the positive and the negative plate is given by:

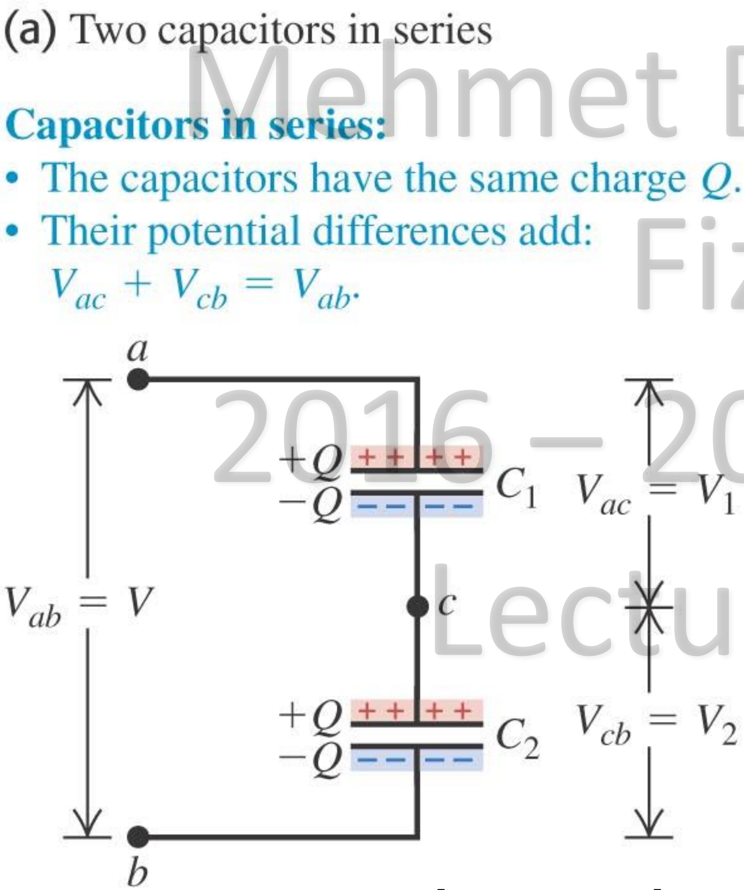
$$V = \int_{-}^{+} E\,dr = -\frac{q}{4\pi\varepsilon_0}\int_b^a \frac{dr}{r^2} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_b^a = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right).$$

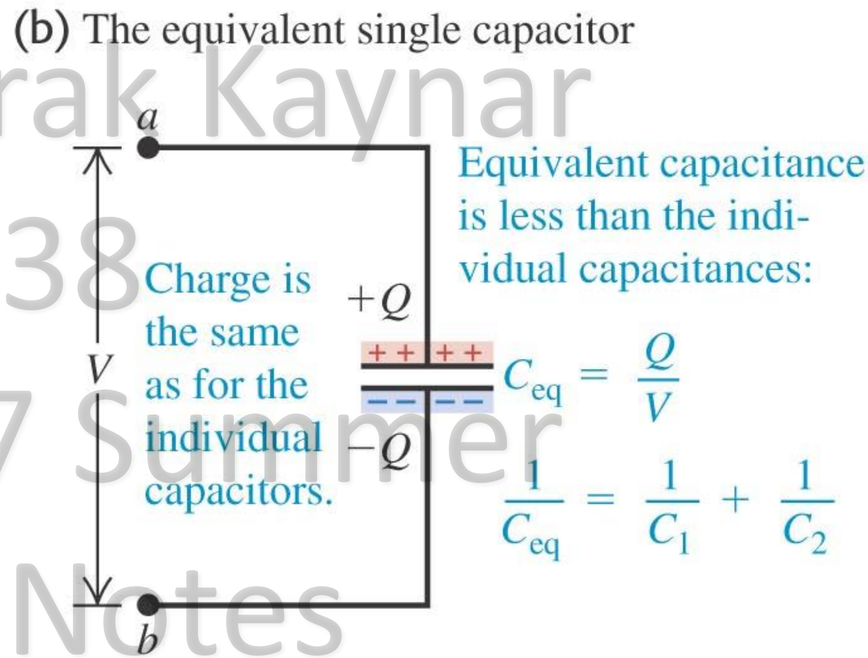
The capacitance

$$C = \frac{q}{V} = \frac{q}{\dfrac{q}{4\pi\varepsilon_0}\left(\dfrac{1}{a}-\dfrac{1}{b}\right)} = \frac{4\pi\varepsilon_0}{\left(\dfrac{1}{a}-\dfrac{1}{b}\right)} = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right).$$

Capacitors are manufactured with certain standards. The capacitance required might not be a standard one therefore we need to make that using the standard one by implementing parallel and series connections

Capacitors in Series

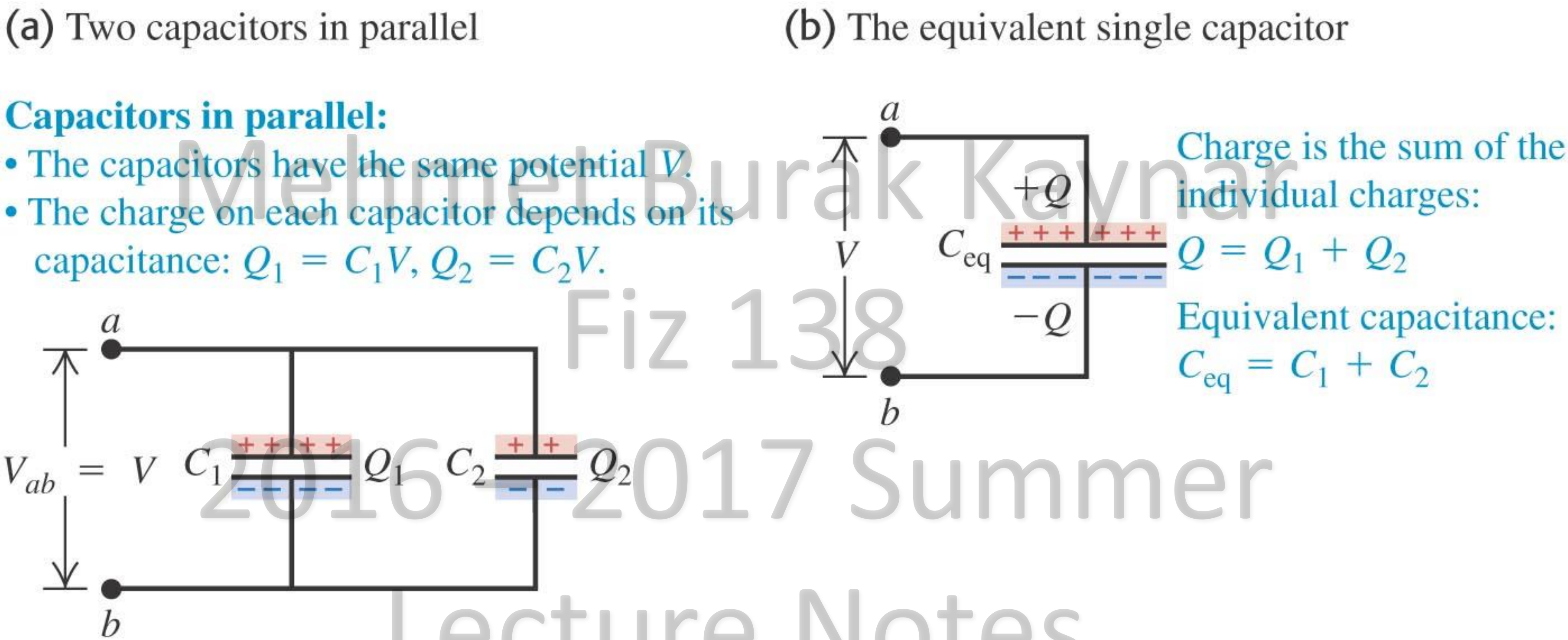




$$V_{ac} = V_1 = \frac{Q}{C_1} \qquad V_{cb} = V_2 = \frac{Q}{C_2}$$

$$V_{ab} = V_1 + V_2 = Q\left[\frac{1}{C_1} + \frac{1}{C_2}\right] \qquad \frac{V_{ab}}{Q} = \frac{1}{C_{eq}} \qquad \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Capacitors in Parallel



$$Q = Q_1 + Q_2 = C_1V + C_2V = (C_1 + C_2)V$$

$$Ceq = C_1 + C_2$$

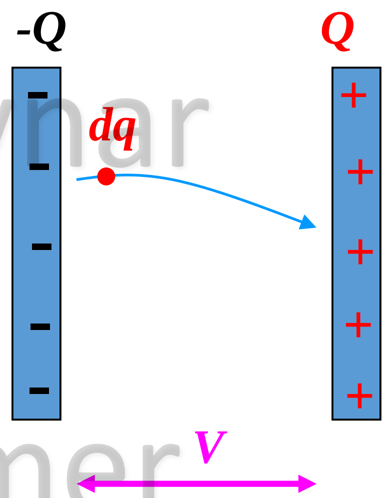
Energy Stored in *E* Field of a Capacitor

It can be found by calculating the WORK required to move a charge from the negative conductor to the positive conductor.

$$dW = dq * V = \frac{q}{C} dq$$

$$W = \frac{1}{C} \int_0^Q q\, dq$$

$$W = \frac{1}{2C} Q^2 = \frac{1}{2} QV = \frac{1}{2} CV^2$$



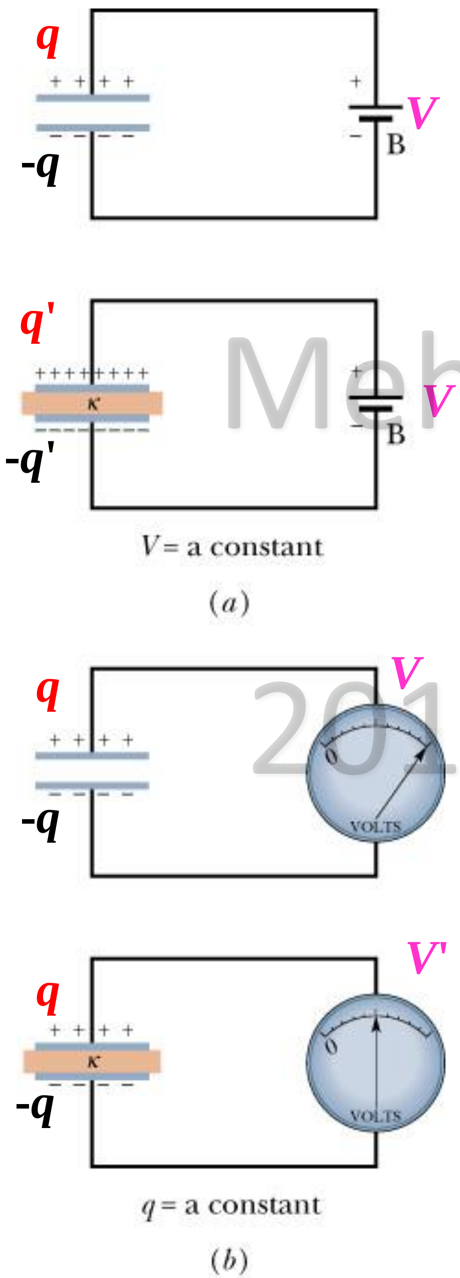
For a parallel plate capacitor

$$C = \varepsilon_0 \frac{A}{d}$$

$$U = \frac{1}{2}\left[\varepsilon_0 \frac{A}{d}\right] V^2 = \frac{1}{2}\varepsilon_0 A d \left(\frac{V}{d}\right)^2$$

$$U = \frac{1}{2}\varepsilon_0 (Volume)\left(\frac{V}{d}\right)^2$$ $\Rightarrow$ Total Energy

$$\frac{U}{Volume} = u = \frac{1}{2}\varepsilon_0 E^2$$ $\Rightarrow$ E field energy density in vacuum



$$C = \kappa C_{air}$$

Capacitor with a Dielectric

In 1837 Michael Faraday investigated what happens to the capacitance C of a capacitor when the gap between the plates is completely filled with an insulator (a.k.a. dielectric).

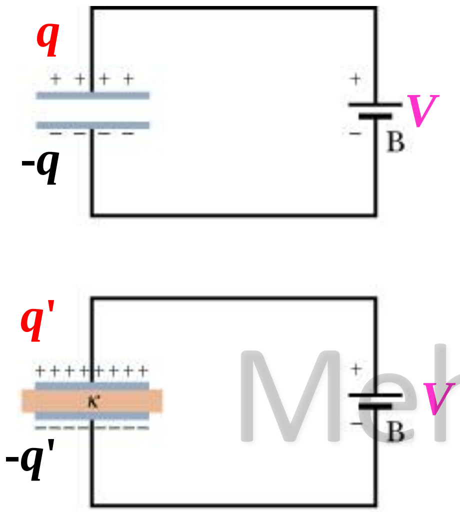
Faraday discovered that the new capacitance is given by $C = \kappa C_{air}$. Here C_{air} is the capacitance before the insertion of the dielectric between the plates. The factor κ is known as the dielectric constant of the material.

Faraday's experiment can be carried out in two ways:

1. With the voltage V across the plates remaining constant. In this case a battery remains connected to the plates. This is shown in fig. *a*.

2. With the charge q of the plates remaining constant. In this case the plates are isolated from the battery. This is shown in fig. *b*.

$C = \kappa C_{\text{air}}$



V = a constant

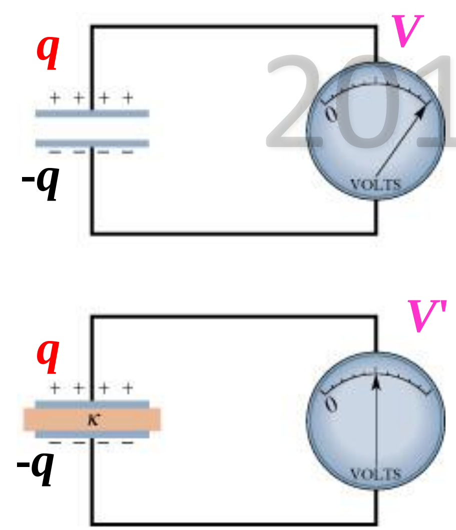
(a)

Fig. a **: Capacitor voltage** V **remains constant**

This is because the battery remains connected to the plates.

After the dielectric is inserted between the capacitor plates the plate charge changes from q to $q' = \kappa q$.

The new capacitance $C = \dfrac{q'}{V} = \dfrac{\kappa q}{V} = \kappa \dfrac{q}{V} = \kappa C_{\text{air}}$.



q = a constant

(b)

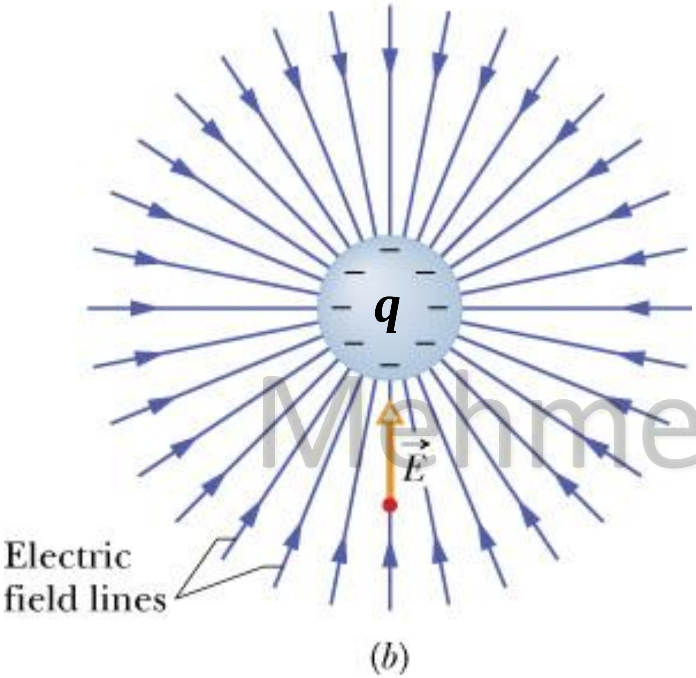
Fig. b **: Capacitor charge** q **remains constant**

This is because the plates are isolated.

After the dielectric is inserted between the capacitor plates

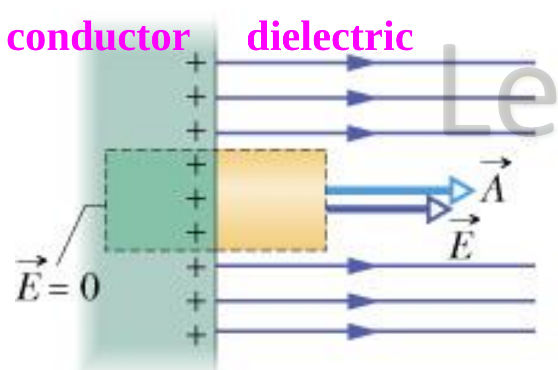
the plate voltage changes from V to $V' = \dfrac{V}{\kappa}$.

The new capacitance $C = \dfrac{q}{V'} = \dfrac{q}{V/\kappa} = \kappa \dfrac{q}{V} = \kappa C_{\text{air}}$.



In a region completely filled with an insulator of dielectric constant κ, all electrostatic equations containing the constant ε_0 are to be modified by replacing ε_0 with $\kappa\varepsilon_0$.

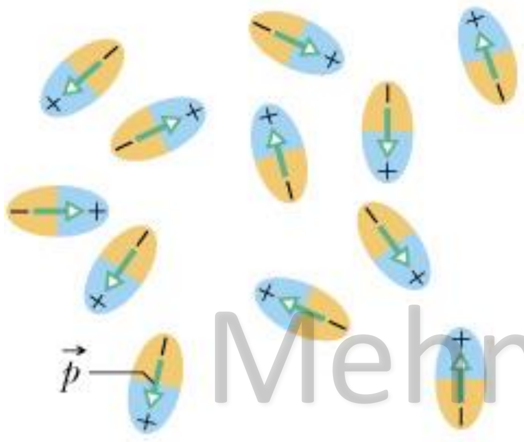
Example 1 : Electric field of a point charge inside a dielectric is: $E = \dfrac{1}{4\pi\kappa\varepsilon_0}\dfrac{q}{r^2}.$



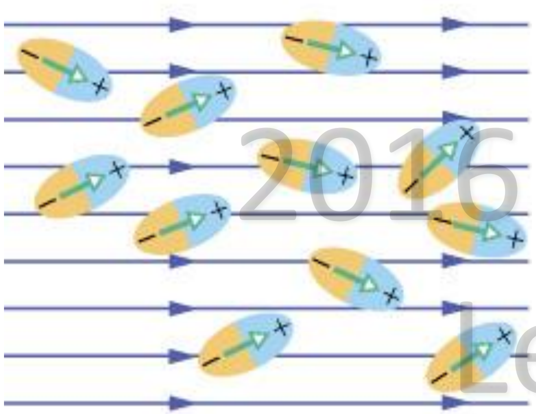
Example 2 :

The electric field outside an isolated conductor immersed in a dielectric becomes:

$$E = \frac{\sigma}{\kappa\varepsilon_0}.$$



(a)



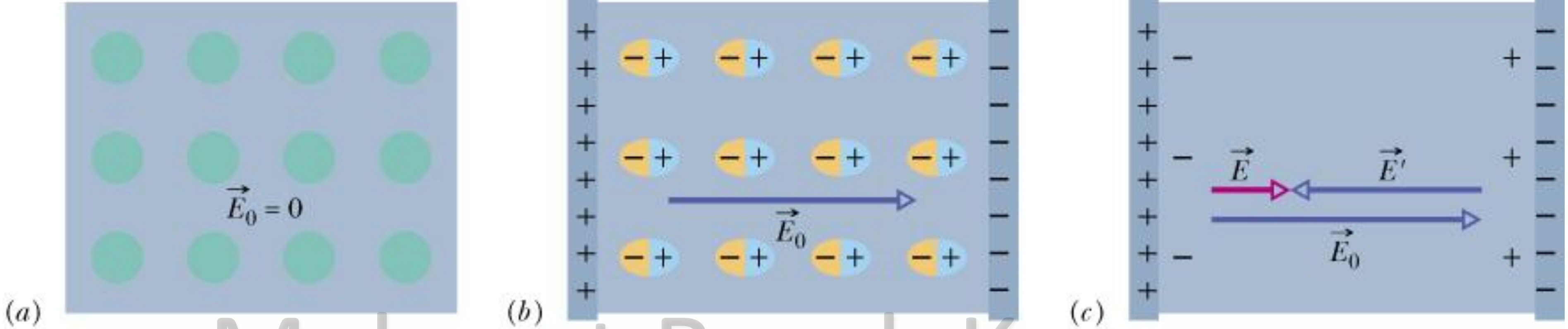
(b)

$$U = -pE\cos\theta$$

Dielectrics : An Atomic View

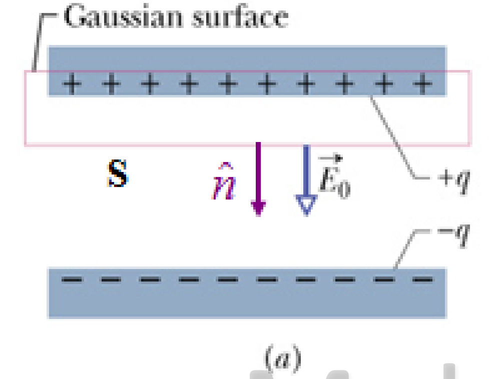
Dielectrics are classified as "polar" and "nonpolar."

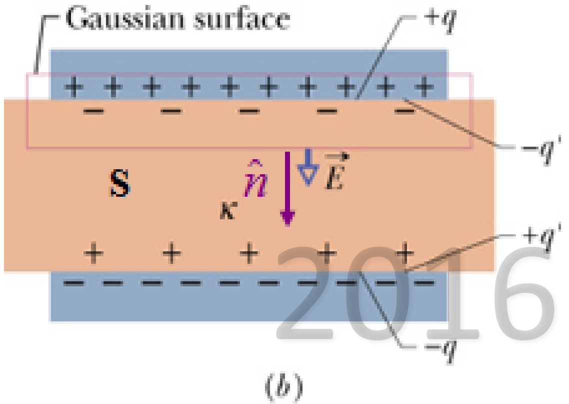
Polar dielectrics consist of molecules that have a nonzero electric dipole moment even at zero electric field due to the asymmetric distribution of charge within the molecule (e.g., H_2O). At zero electric field (see fig. *a*) the electric dipole moments are randomly oriented. When an external electric field $\vec{E}_0$ is applied (see fig. *b*) the electric dipole moments tend to align preferentially along the direction of $\vec{E}_0$ because this configuration corresponds to a minimum of the potential energy and thus is a position of stable equilibrium. Thermal random motion opposes the alignment and thus ordering is incomplete. Even so, the partial alignment produced by the external electric field generates an internal electric field that opposes $\vec{E}_0$. Thus the net electric field $\vec{E}$ is **weaker** than $\vec{E}_0$.



A nonpolar dielectric, on the other hand, consists of molecules that in the absence of an electric field have zero electric dipole moment (see fig. *a*) . If we place the dielectric between the plates of a capacitor the external electric field $\vec{E}_0$ induces an electric dipole moment $\vec{p}$ that becomes aligned with $\vec{E}_0$ (see fig. *b*). The aligned molecules do not create any net charge inside the dielectric. A net charge appears at the left and right surfaces of the dielectric opposite the capacitor plates. These charges come from negative and positive ends of the electric dipoles. These **induced** surface charges have sign **opposite** that of the opposing plate charges. Thus the induced charges create an electric field $\vec{E}'$ that opposes the applied field $\vec{E}_0$ (see fig. *c*). As a result, the net electric field $\vec{E}$ between the capacitor plates is weaker.

$$E = \frac{E_0}{k}$$





Gauss' Law and Dielectrics

In Chapter 22 we formulated Gauss' law assuming that the charges existed in vacuum: $\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q$ or $\varepsilon_0 \Phi = q$.

In this section we will write Gauss' law in a form that is suitable for cases in which dielectrics are present.

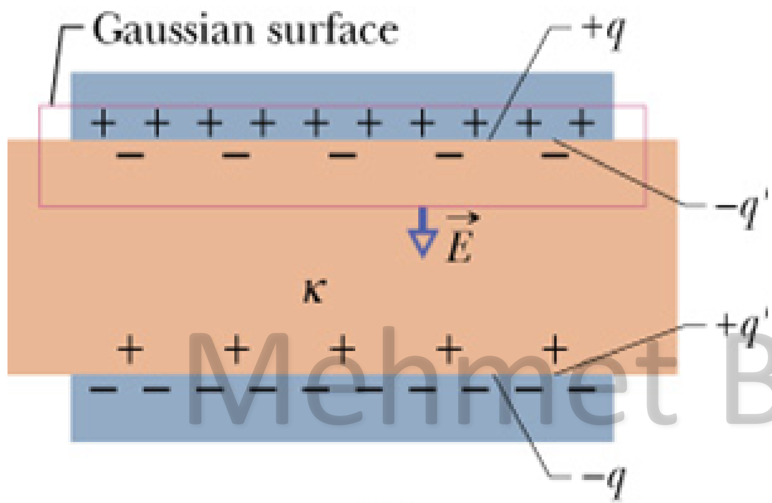
Consider first the parallel plate capacitor shown in fig. a.

We will use the Gaussian surface S. The flux $\Phi = E_0 A = \dfrac{q}{\varepsilon_0}$ $\rightarrow E_0 = \dfrac{q}{\varepsilon_0 A}$. Now we fill the space between the plates with an insulator of dielectric constant κ (see fig. b).

We will apply Gauss' law for the same surface S. Inside S in addition to the plate charge q we also have the induced charge q' on the surface of the dielectric: $\Phi = EA = \dfrac{q-q'}{\varepsilon_0} \rightarrow$

$E = \dfrac{q-q'}{A\varepsilon_0}$ (eq. 1). From Faraday's experiments we have: $E = \dfrac{E_0}{\kappa} = \dfrac{q}{\kappa A \varepsilon_0}$ (eq. 2).

If we compare eq. 1 with eq. 2 we have: $q - q' = \dfrac{q}{\kappa} \rightarrow \varepsilon_0 \oint \kappa \vec{E} \cdot d\vec{A} = q.$



(b)

Gauss' Law in the Presence of Dielectrics

$$\epsilon_0 \oint \kappa \vec{E} \cdot d\vec{A} = q$$

Even though the equation above was derived for the parallel plate capacitor, it is true in general.

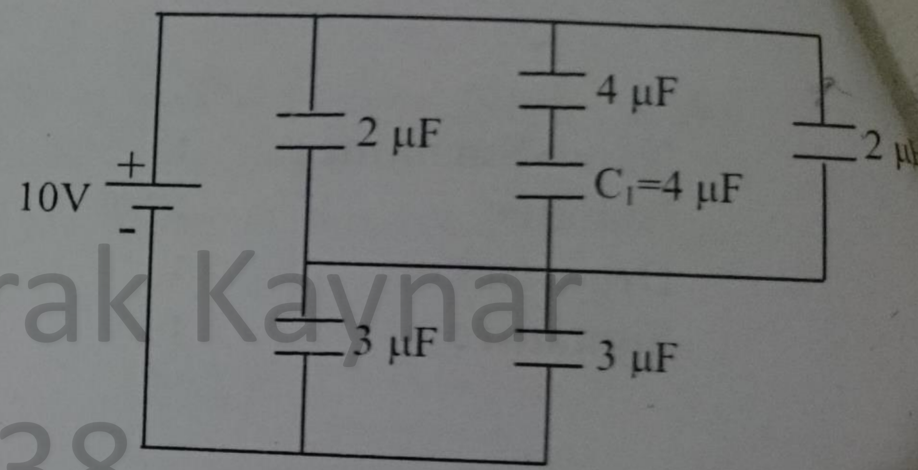
Note 1: The flux integral now involves $\kappa \vec{E}$.

Note 2: The charge q that is used is the plate charge, also known as "free charge." Using the equation above we can ignore the induced charge q'.

Note 3: The dielectric constant κ is kept inside the integral to describe the most general case in which κ is not constant over the Gaussian surface.

29. A charge $Q = 2.5x10^{-12}$ C is distributed uniformly on a circular loop of radius $a = 3$ cm lying on xy-plane. If the potential at any point on the z-axis through from its center is given by $V(z) = kQ(z^2+a^2)^{-1/2}$, what is the magnitude of electric field (in N/C) at a point $z = 4$ cm on the z-axis?

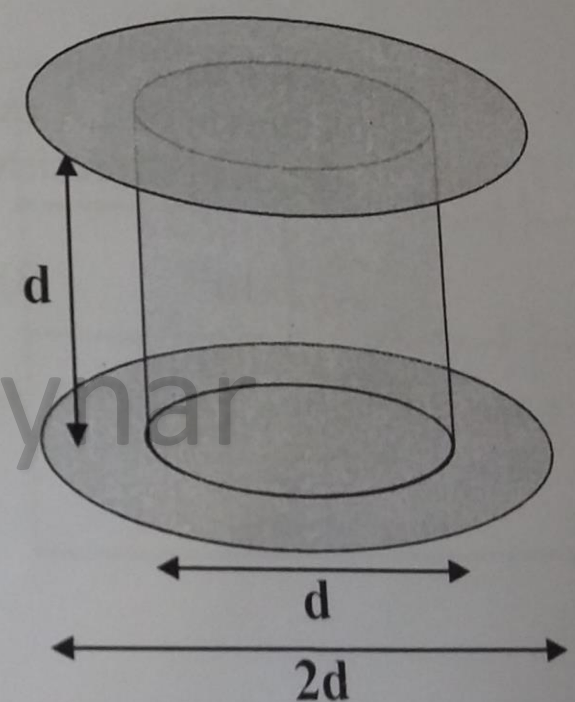
In the figure, the battery has a potential difference 10 V. What is the charge (in μC) on capacitor C_1?



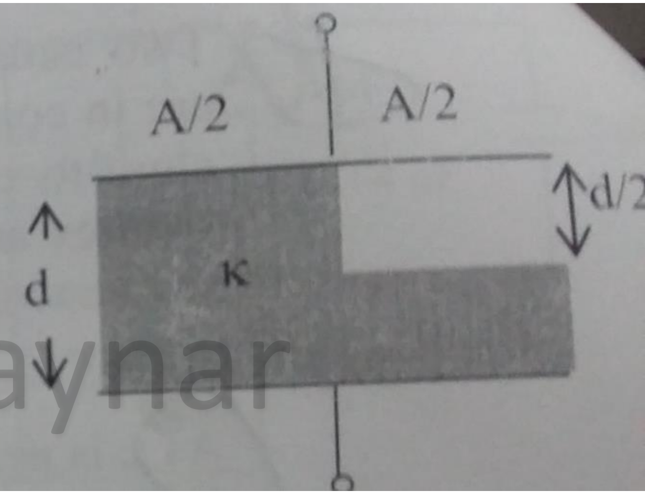
A) 4
B) 6
C) 8
D) 10
E) 12

8. A cylindrical dielectric ($\kappa = 5$) of radius d/2 and height d is placed between two circular metallic plates with radius d. What will be the final capacitance (in pF) of the system, if the plates are d = 10 cm apart?

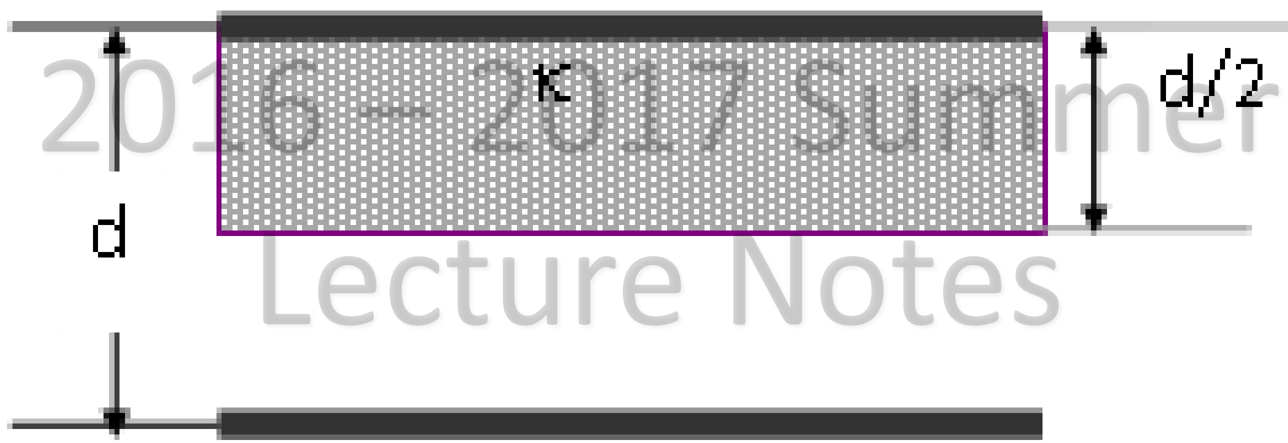
A) 2.7 B) 1.8 C) 0.9
D) 4.5 E) 5.4



A parallel plate capacitor with plate area A is filled with a dielectric material with dielectric constant κ as shown in figure. What is the capacitance of this parallel plate capacitor?



A parallel -plate capacitor has a capacitance C_0 in the absance of a dielectric . A slab of dielectric material of dielectric constant κ and thickness *d/2* is inserted between the plates. What is the new capacitance when the dielectric is present ?



The area of the plates in a plane capacitor is $100cm^2$ and the distance between them is 5mm. A potential difference of 300V is applied to the plates. After capacitor is disconnected from the source of power, the space between the plates is filled with ebonite. What is the surface charge density (in C/m^2) on the plates after filling? ($\kappa_{ebonite}=2.6$)

At a distance of 0.6 m, the magnitude of potential of a solid sphere of radius 0.3 m is 1620V. What is the surface charge density (C/m^2) of the solid sphere?

An electric field is given by $E_x=5x^2$ (kN/C). What is the potential difference $(V_1 - V_2)$ (in kV) between the points on the x axis at $x_1=3$m and $x_2=5$m?

A spherical shell of radius R = 10 cm has a uniform surface charge density σ =4 nC/m^2. What is the electric field (in N/C) at r = 5 cm?