

The area of the plates in a plane capacitor is 100cm^2 and the distance between them is 5mm . A potential difference of 300V is applied to the plates. After capacitor is disconnected from the source of power, the space between the plates is filled with ebonite. What is the surface charge density (in C/m^2) on the plates after filling? ($\kappa_{\text{ebonite}}=2.6$)

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Lecture Notes

At a distance of 0.6 m, the magnitude of potential of a solid sphere of radius 0.3 m is 1620V. What is the surface charge density (C/m^2) of the solid sphere?

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2016 – 2017 Summer

Lecture Notes

An electric field is given by $E_x = 5x^2$ (kN/C). What is the potential difference ($V_1 - V_2$) (in kV) between the points on the x axis at $x_1 = 3\text{m}$ and $x_2 = 5\text{m}$?

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Lecture Notes

A spherical shell of radius $R = 10$ cm has a uniform surface charge density $\sigma = 4$ nC/m². What is the electric field (in N/C) at $r = 5$ cm?

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Lecture Notes

Chapter 26

Current, Resistance, and Electromotive Force

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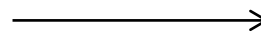
Electric Charges

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Electromotive Force

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Moving Charges



Response to the Charge Motion

Lecture Notes

Electric Current

Resistance



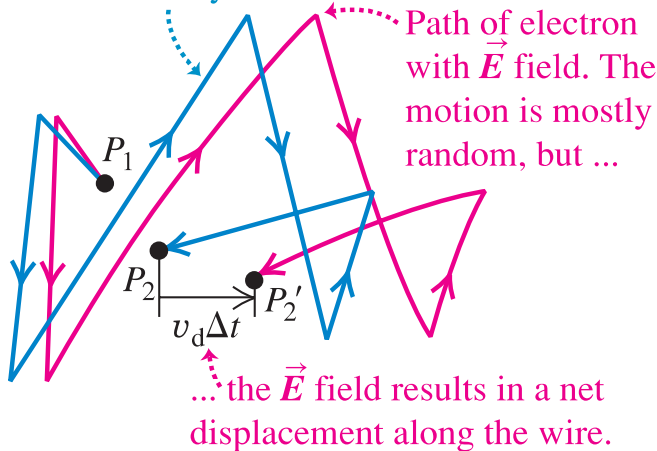
Current

A **current** is any motion of charge from one region to another

Conductor without internal \vec{E} field



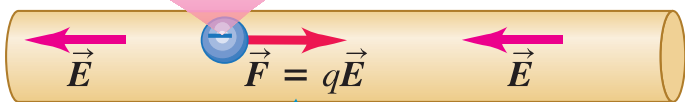
Path of electron without \vec{E} field. Electron moves randomly.



Path of electron with \vec{E} field. The motion is mostly random, but ...

... the \vec{E} field results in a net displacement along the wire.

Conductor with internal \vec{E} field



An electron has a negative charge q , so the force on it due to the \vec{E} field is in the direction opposite to \vec{E} .

Average Current, $I = \frac{DQ}{Dt}$

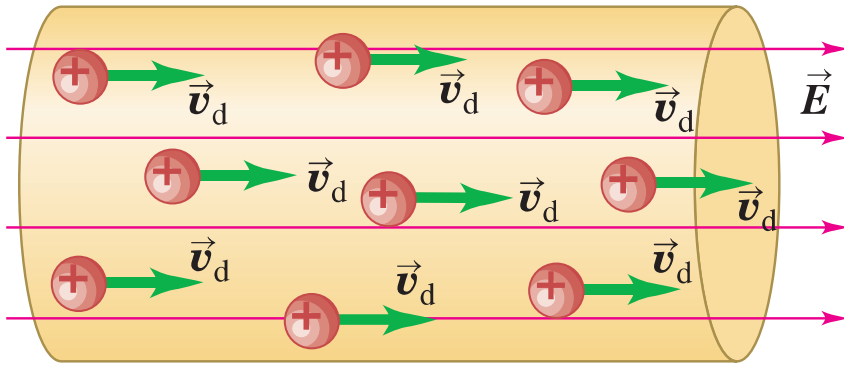
Flow rate of charges is the same.

Instantaneous Current, $I = \frac{dQ}{dt}$

Flow rate of charges is time dependent.

Direction of Electric Current

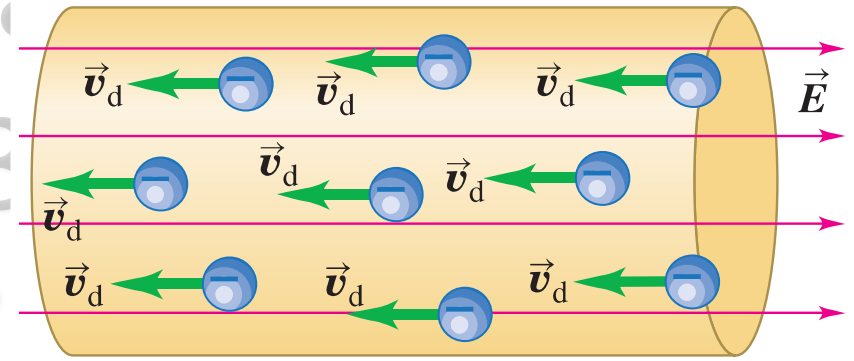
(a)



I

A **conventional current** is treated as a flow of positive charges, regardless of whether the free charges in the conductor are positive, negative, or both.

(b)



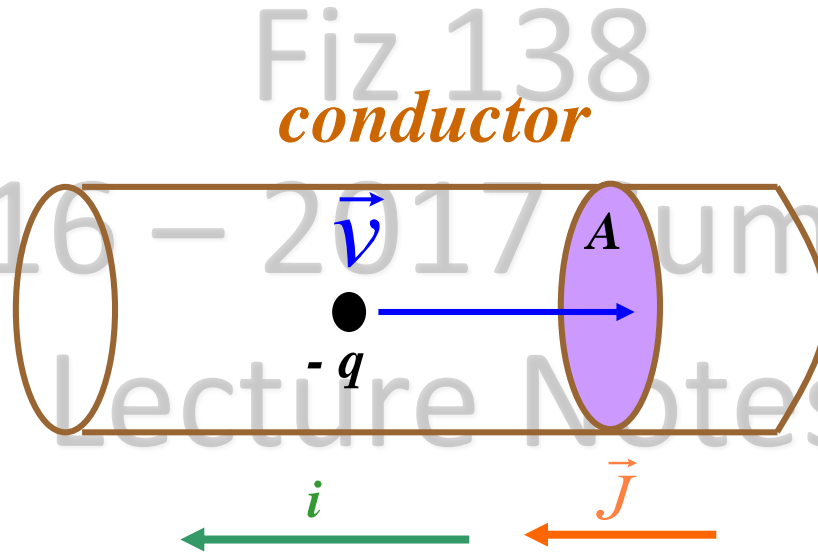
I

In a metallic conductor, the moving charges are electrons — but the *current* still points in the direction positive charges would flow.

(Current Density) → *Vector*
Scalar

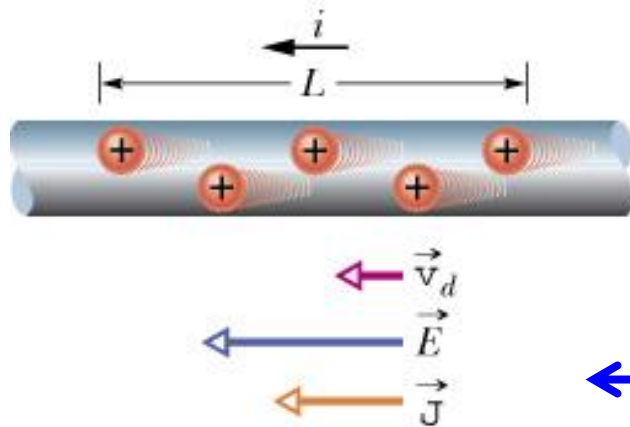
The current per unit cross sectional area is called the current density

$$J = \frac{i}{A} \leftarrow \text{Magnitude}$$



Direction: Flow direction of current is the direction of current density vector.

Drift (Sürüklenme) Speed (V_d)



An external electric field gives rise to current

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How fast is the charges move due to the E field is called as DRIFT SPEED.

Why DRIFT?

Because thermal energy of moving charges causes them to randomly move around while

E field forces them to move in a certain direction. That's why they are drifted.

The relationship between the current density and drift velocity

$$J = nqV_d$$

n is the number charges in unit volume

Conductivity ↔ Resistivity

(A measure of the current that can be generated by applying E Field)

Conductivity

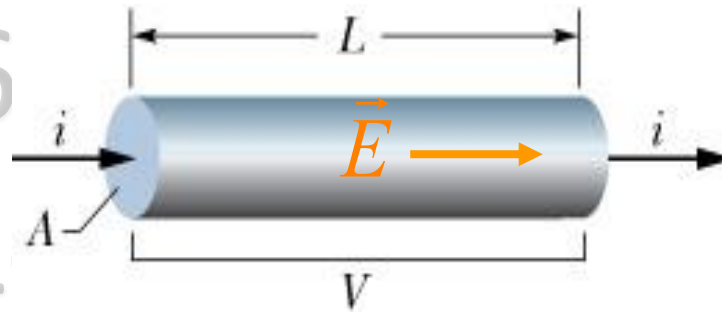
$$S = \frac{J}{E}$$

Inverse Conductivity

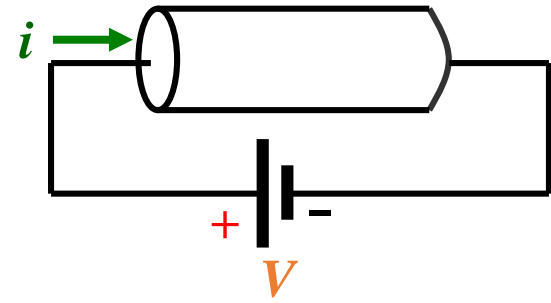
Resistivity

$$r = \frac{1}{S} = \frac{E}{J}$$

For metals, current density is proportional with the E field. Proportionality constant is known as CONDUCTIVITY. Inverse conductivity gives us the RESISTIVITY of the material.



Consider the conductor shown in the figure above. The electric field inside the conductor is $E = \frac{V}{L}$. The current density is $J = \frac{i}{A}$. We substitute E and J into equation $\rho = \frac{E}{J}$ and get: $\rho = \frac{V/L}{i/A} = \frac{V}{i} \frac{A}{L} = R \frac{A}{L} \rightarrow R = \rho \frac{L}{A}$. → RESISTANCE



$$R = \frac{V}{i}$$

Resistance

If we apply a voltage V across a conductor (see figure) a current i will flow through the conductor.

We define the conductor resistance as the ratio $R = \frac{V}{i}$.

SI Unit for R : $\frac{V}{A} =$ the ohm (symbol Ω)

A conductor across which we apply a voltage $V = 1$ volt and results in a current $i = 1$ ampere is defined as having resistance of 1Ω .

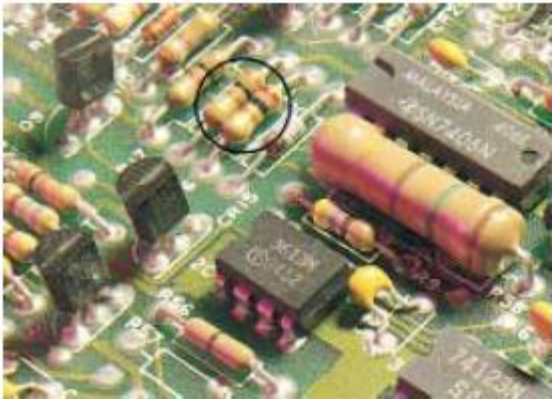
Q: Why not use the symbol "O" instead of " Ω "?

A: Suppose we had a 1000Ω resistor.

We would then write: 1000 O , which can easily be mistakenly read as 10000Ω .

A conductor whose function is to provide a specified resistance is known as a "resistor."

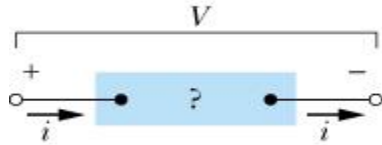
The symbol is given to the left.



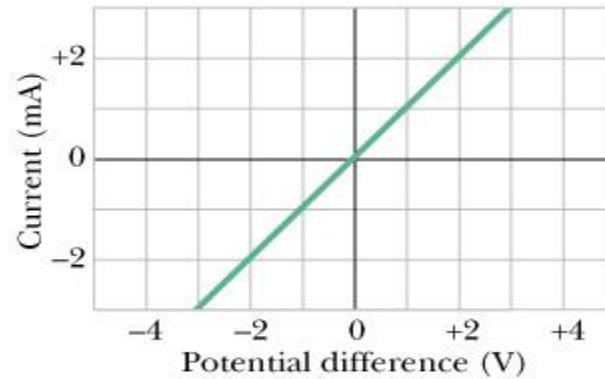
R



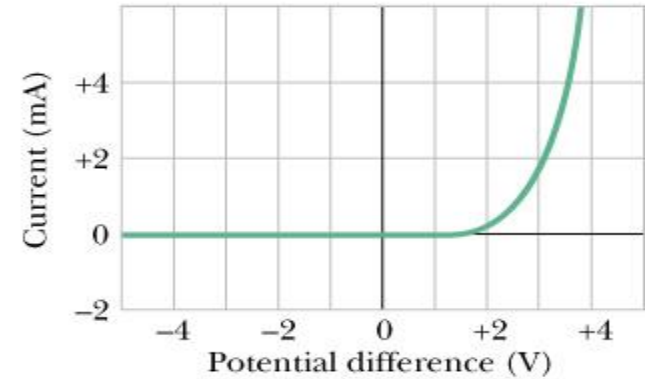
READING



(a)



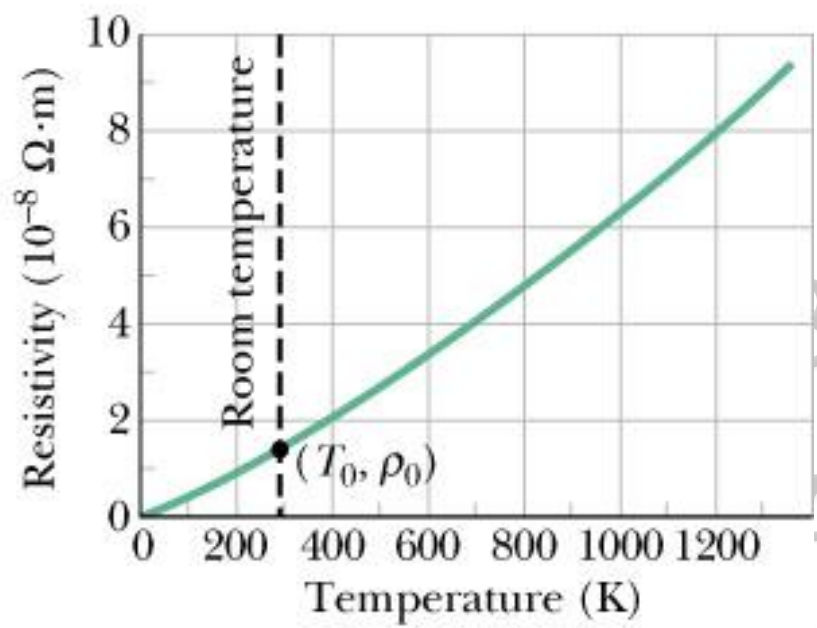
(b)



(c)

Ohm's Law. A resistor was defined as a conductor whose resistance does not change with the voltage V applied across it. In fig. *b* we plot the current i through a resistor as a function of V . The plot (known as the " i - V curve") is a straight line that passes through the origin. Such a conductor is said to be "**Ohmic**" and it obeys Ohm's law, which states: **The current i through a conductor is proportional to the voltage V applied across it.** Not all conductors obey Ohm's law (these are known as "**non - Ohmic**"). An example is given in fig. *c* where we plot i versus V for a semiconductor diode. The ratio V / i (and thus the resistance R) is not constant. As a matter of fact, the diode does not conduct for negative voltage values.

Note: Ohm's "law" is in reality a definition of Ohmic conductors (defined as the conductors that obey Ohm's law).



Variation of Resistivity with Temperature

In the figure we plot the resistivity ρ of copper as a function of temperature T . The dependence of ρ on T is almost linear. Similar dependence is observed in many conductors.

$$\rho - \rho_0 = \nu_0 \alpha (T - T_0)$$

The following empirical equation is used for many practical applications:

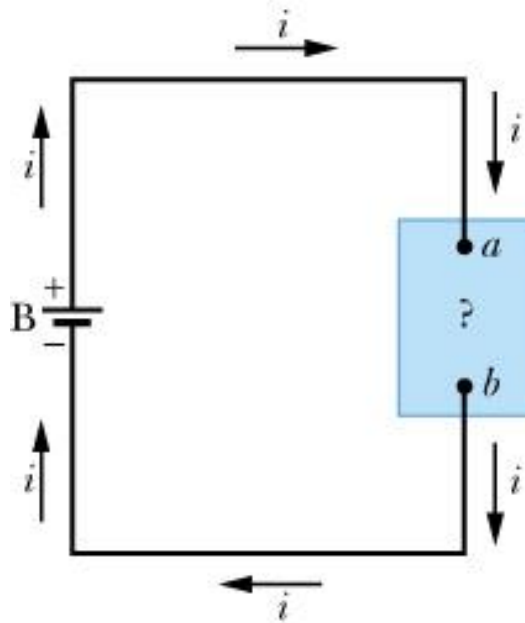
$\rho - \rho_0 = \nu_0 \alpha (T - T_0)$. The constant α is known as the

"temperature coefficient of resistivity." The constant T_0 is a reference temperature usually taken to be room temperature ($T_0 = 293 \text{ K}$), and ρ_0 is the resistivity at T_0 . For copper, $\rho_0 = 1.69 \times 10^{-8} \text{ } \Omega \cdot \text{m}$.

Note: Temperature enters the equation above as a difference ($T - T_0$).

Thus either the Celsius or the Kelvin temperature scale can be used.

$$P = iV$$



Power in Electric Circuits

Consider the circuit shown in the figure. A battery of voltage V is connected across the terminals a and b of a device. This can be a resistor, a motor, etc. The battery maintains a potential difference V between the terminals a and b and thus a current i flows in the circuit as shown in the figure. During the time interval dt a charge $dq = idt$ moves between the terminals. We note that $V_a > V_b$.

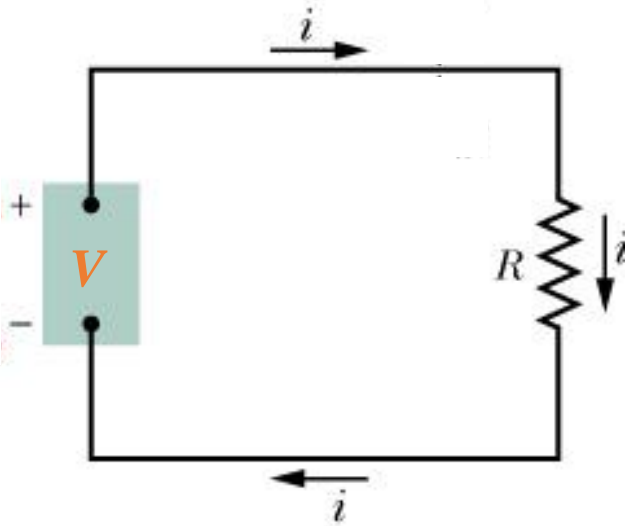
The potential energy of the charge decreases by an amount $dU = Vdq = Vidt$.

Using energy conservation we conclude that the lost energy has been transferred by the battery to the device and has been converted into some other form of energy.

The rate at which energy is transferred to the device is known as "**power**" and it is

$$\text{equal to } P = \frac{dU}{dt} = \frac{Vid t}{dt} = Vi.$$

SI unit for P: $V \times A$ It is known as the "watt" (symbol W).



$$P = i^2 R$$

$$P = \frac{V^2}{R}$$

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iz 138

If the device connected to the battery is a resistor R then the energy transferred by the battery is converted as **heat** that appears on R . If we combine the equation $P = iV$ with Ohm's law $i = \frac{V}{R}$, we get the following two equivalent expressions for the rate at which heat is dissipated on R :

$$P = i^2 R \quad \text{and} \quad P = \frac{V^2}{R}$$

Current ?

Current Density ?

Conductivity ?

Resistivity ?

Power ?

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2016 – 2017 Summer

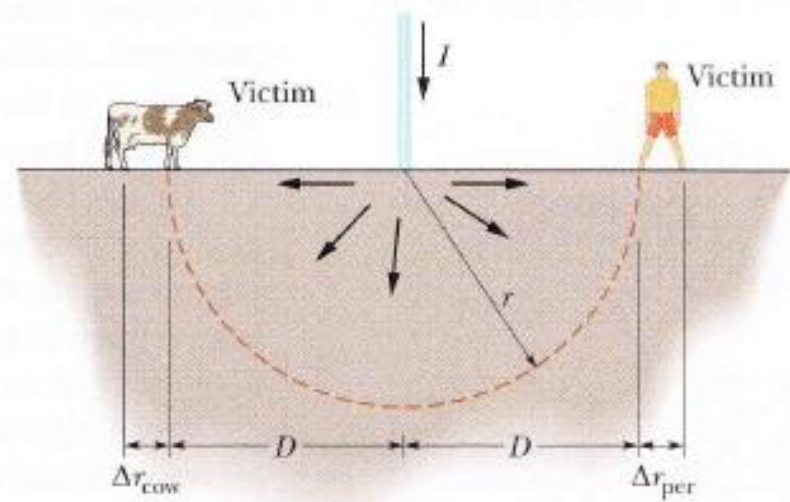
Lecture Notes

A rectangular block of iron has dimensions $1.2 \text{ cm} \times 1.2 \text{ cm} \times 15 \text{ cm}$. A potential difference is to be applied to the block between parallel sides and in such a way that those sides are equipotential surfaces (as in Fig. 26-8*b*). What is the resistance of the block if the two parallel sides are (1) the square ends (with dimensions $1.2 \text{ cm} \times 1.2 \text{ cm}$) and (2) two rectangular sides (with dimensions $1.2 \text{ cm} \times 15 \text{ cm}$)?

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Figure 26-11 shows a person and a cow, each a radial distance $D = 60.0$ m from the point where lightning of current $I = 100$ kA strikes the ground. The current spreads through the ground uniformly over a hemisphere centered on the strike point. The person's feet are separated by radial distance $\Delta r_{\text{per}} = 0.50$ m; the cow's front and rear hooves are separated by radial distance $\Delta r_{\text{cow}} = 1.50$ m. The resistivity of the ground is $\rho_{gr} = 100 \Omega \cdot \text{m}$. The resistance both across the person, between left and right feet, and across the cow, between front and rear hooves, is $R = 4.00$ k Ω .



(a) What is the current i_p through the person?

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Lecture Notes

$$(\vec{J} = i/A),$$

$$(\rho = E/J),$$

You are given a length of uniform heating wire made of a nickel–chromium–iron alloy called Nichrome; it has a resistance R of $72\ \Omega$. At what rate is energy dissipated in each of the following situations? (1) A potential difference of $120\ \text{V}$ is applied across the full length of the wire. (2) The wire is cut in half, and a potential difference of $120\ \text{V}$ is applied across the length of each half.

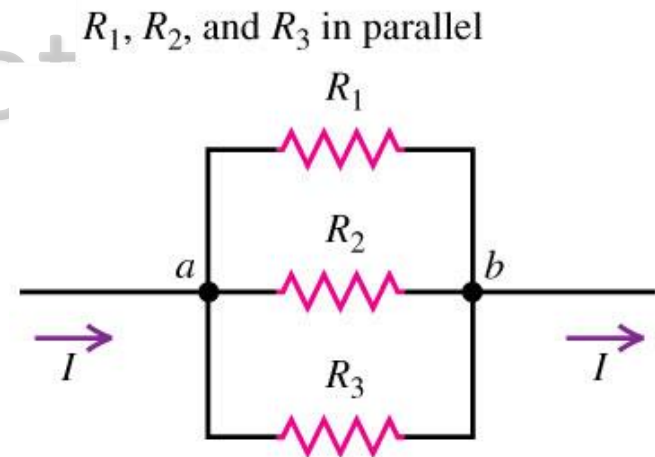
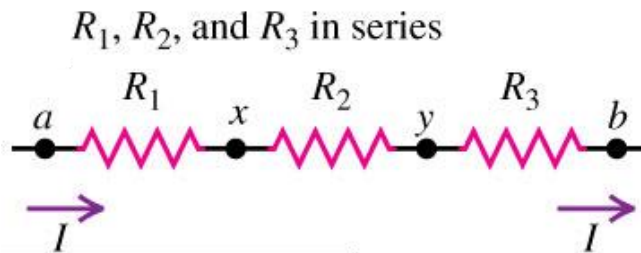
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Resistors in series and parallel

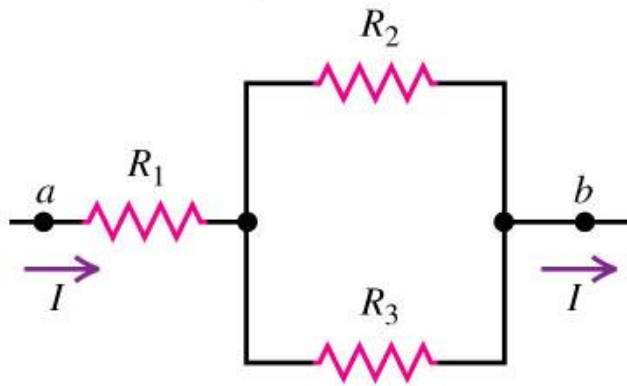
- The *equivalent resistance* of a series combination is the *sum* of the individual resistances: $R_{\text{eq}} = R_1 + R_2 + R_3 + \dots$
- The *equivalent resistance* of a parallel combination is given by $1/R_{\text{eq}} = 1/R_1 + 1/R_2 + 1/R_3 + \dots$



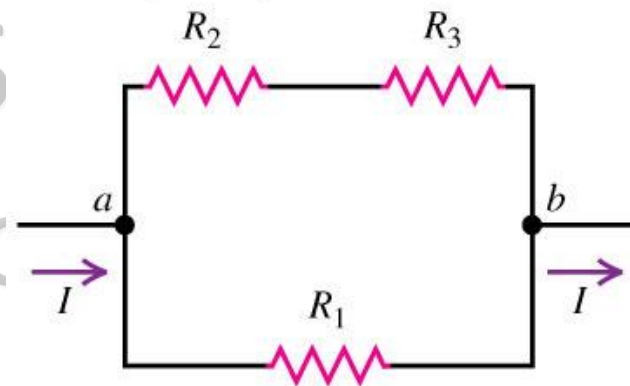
Series and parallel combinations

Resistors can also be connected in combinations of series and parallel.

(c) R_1 in series with parallel combination of R_2 and R_3



(d) R_1 in parallel with series combination of R_2 and R_3

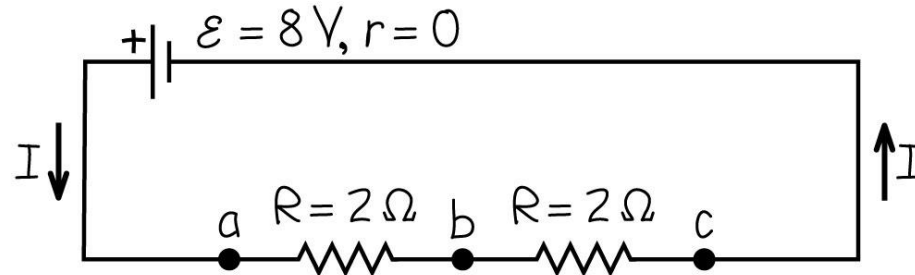


Series vs. Parallel Connections

Example : Current, potential difference and power across each bulb for connecting in series and parallel.

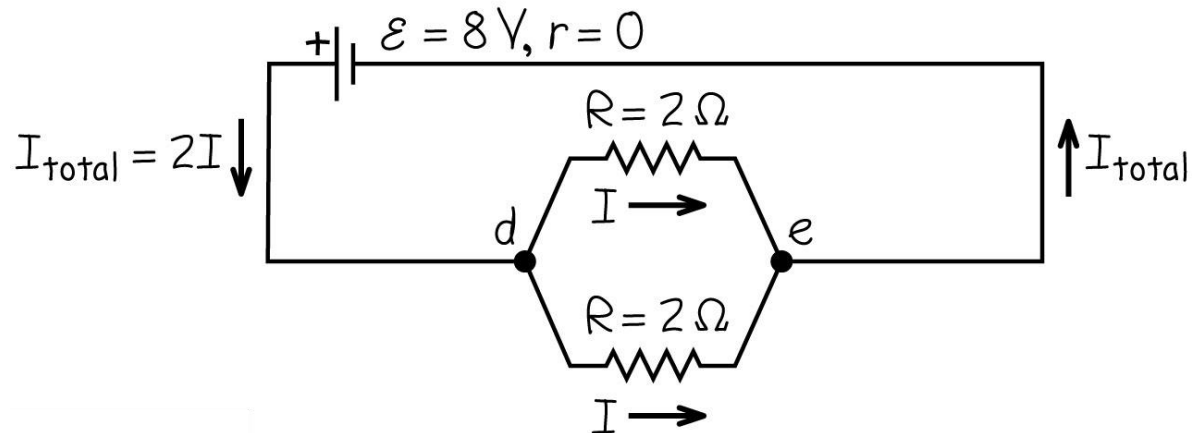
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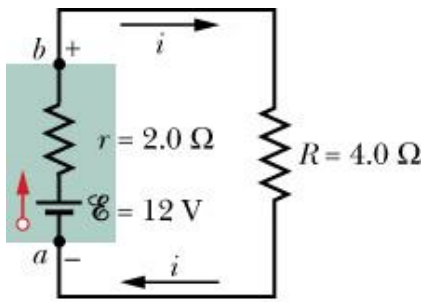
(a) Light bulbs in series



2

(b) Light bulbs in parallel



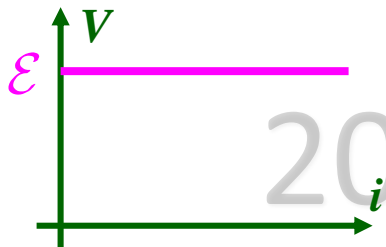


Ideal and Real Emf Devices

An emf device is said to be **ideal** if the voltage V across its terminals a and b does **not** depend on the current i that flows through the emf device: $V = \mathcal{E}$.

$$V = \mathcal{E}$$

Ideal emf device

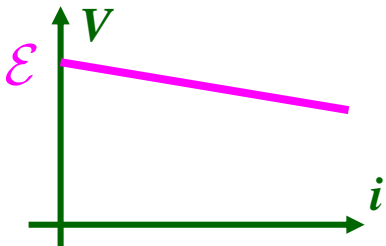


$$V = \mathcal{E} - ir$$

An emf device is said to be **real** if the voltage V across its terminals a and b **decreases** with current i according to the equation $V = \mathcal{E} - ir$.

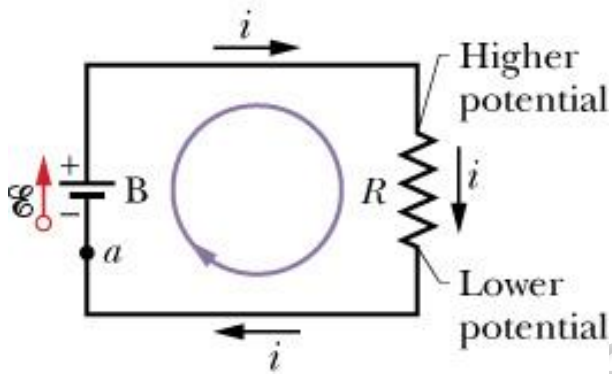
The parameter r is known as the "**internal resistance**" of the emf device.

Real emf device



Current in a Single - Loop Circuit

Consider the circuit shown in the figure. We assume that the emf device is ideal and that the connecting wires have negligible resistance. A current i flows through the circuit in the clockwise direction.



In a time interval dt a charge $dq = idt$ passes through the circuit. The battery is doing work $dW = e dq = e idt$. Using energy conservation we can set this amount of work equal to the rate at which heat is generated on R

$$e idt = Ri^2 dt \rightarrow ei = Ri \rightarrow ei - iR = 0.$$

Kirchhoff put the equation above in the form of a rule known as Kirchhoff's loop rule (KLR for short).

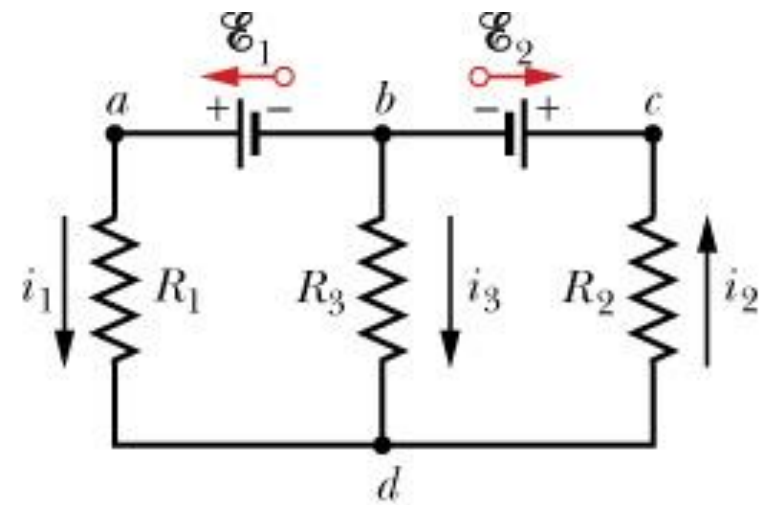
KLR: The algebraic sum of the changes in potential encountered in a complete traversal of any loop in a circuit is equal to zero.

The rules that give us the algebraic sign of the charges in potential through a resistor and a battery are given on the next page.

Multiloop Circuits

Consider the circuit shown in the figure. There are three branches in it: bad , bcd , and bd .

We assign currents for each branch and define the current directions arbitrarily. The method is self-correcting. If we have made a mistake in the direction of a particular current, the calculation will yield a negative value and thus provide us with a warning.



We assign current i_1 for branch bad , current i_2 for branch bcd , and current i_3 for branch bd . Consider junction d . Currents i_1 and i_3 arrive, while i_2 leaves.

Charge is conserved, thus we have: $i_1 + i_3 = i_2$. This equation can be formulated as a more general principle known as Kirchhoff's junction rule (KJR).

KJR : The sum of the currents entering any junction is equal to the sum of the currents leaving the junction.

KLR : The algebraic sum of the changes in potential encountered in a complete traversal of any loop in a circuit is equal to zero.

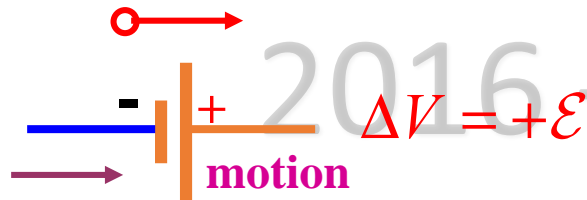


Resistance Rule:

For a move through a resistance in the direction of the current, the change in the potential is $DV = -iR$

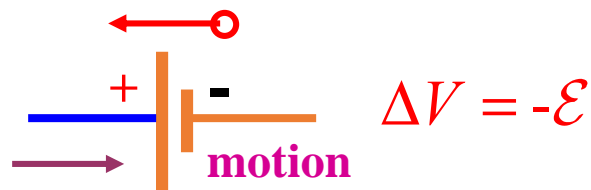


For a move through a resistance in the direction opposite to that of the current, the change in the potential is $DV = +iR$



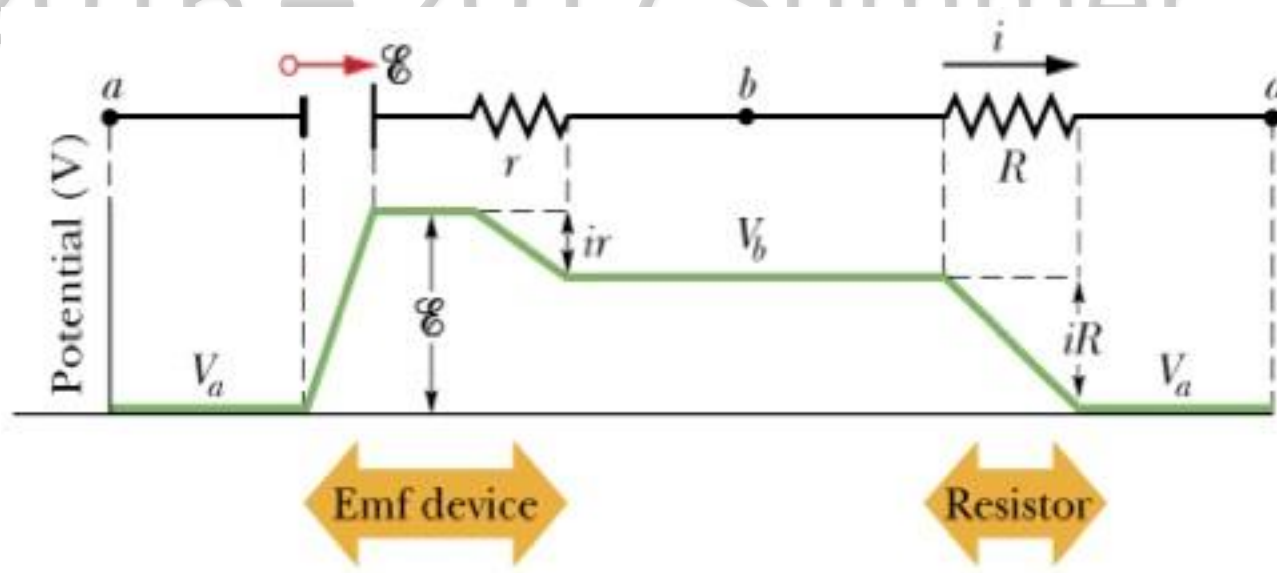
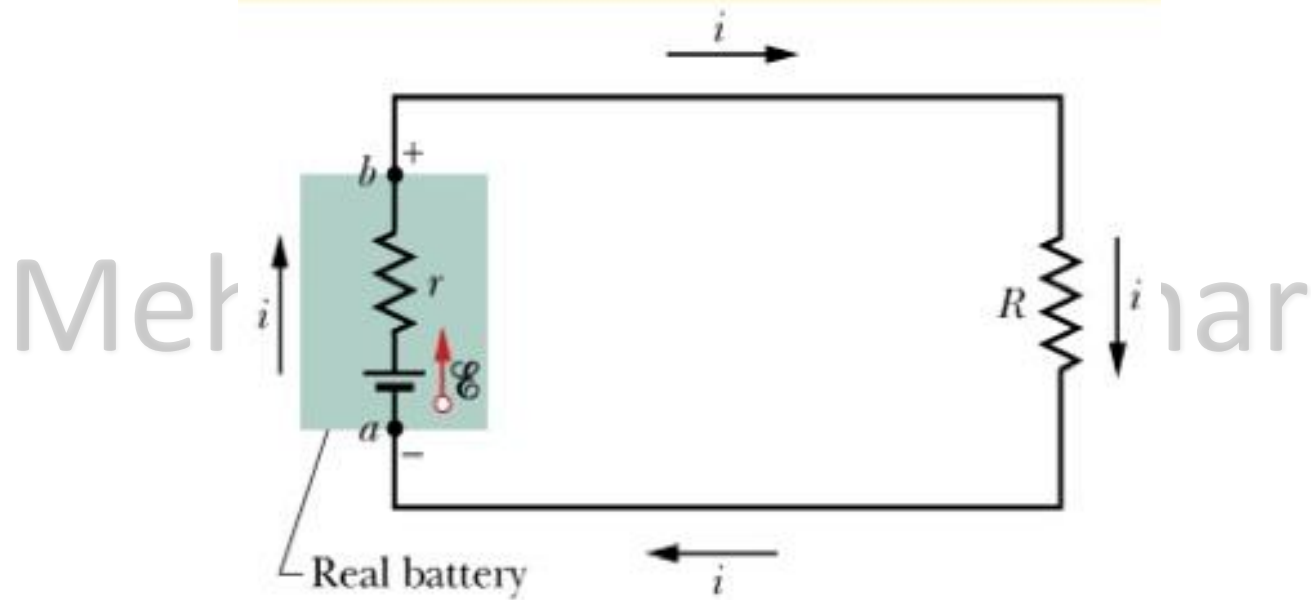
EMF Rule:

For a move through an ideal emf device in the direction of the emf arrow, the change in the potential is $\Delta V = +\mathcal{E}$.



For a move through an ideal emf device in a direction opposite to that of the emf arrow, the change in the potential is $\Delta V = -\mathcal{E}$.

Example



Example

Find the currents I_1 , I_2 , and I_3 in the circuit shown in Figure

Solution

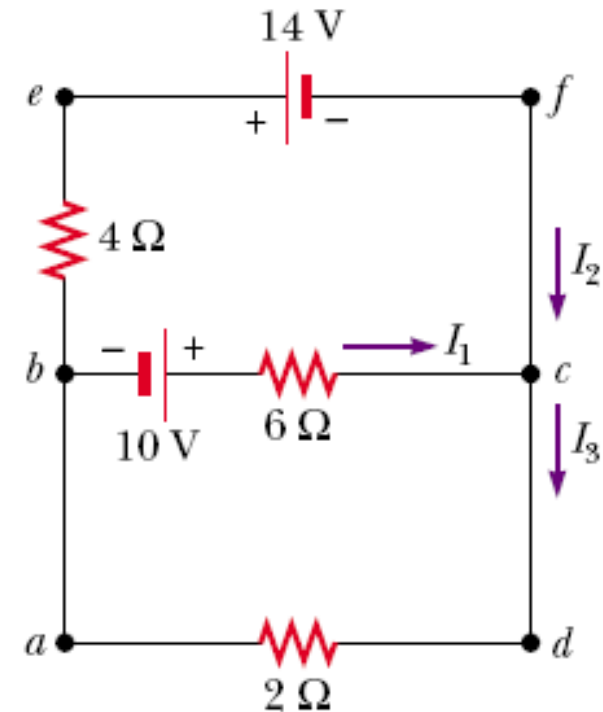
Applying Kirchhoff's junction rule to junction c gives

$$(1) \quad I_1 + I_2 = I_3$$

Applying Kirchhoff's loop rule to loops $abcda$ and $befcb$ and transferring these loops clockwise We obtain the expressions

$$(2) \quad abcda \quad 10 \text{ V} - (6 \Omega)I_1 - (2 \Omega)I_3 = 0$$

$$(3) \quad befcb \quad -14 \text{ V} + (6 \Omega)I_1 - 10 \text{ V} - (4 \Omega)I_2 = 0$$



Expressions (1), (2), and (3) represent three independent equations with three unknowns. Substituting Equation (1) into Equation (2) gives

$$10 \text{ V} - (6 \Omega)I_1 - (2 \Omega)(I_1 + I_2) = 0$$

$$(4) \quad 10 \text{ V} = (8 \Omega)I_1 + (2 \Omega)I_2$$

Dividing each term in Equation (3) by 2 and rearranging gives

$$(5) \quad -12 \text{ V} = -(3 \Omega)I_1 + (2 \Omega)I_2$$

Subtracting Equation (5) from Equation (4) eliminates I_2 , giving

$$22 \text{ V} = (11 \Omega)I_1$$

$$I_1 = 2 \text{ A}$$

Using this value of I_1 in Equation (5) gives a value for I_2 :

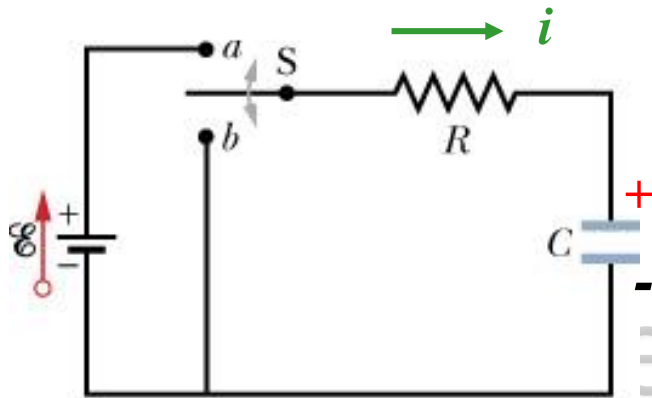
$$(2 \Omega)I_2 = (3 \Omega)I_1 - 12 \text{ V} = (3 \Omega)(2 \text{ A}) - 12 \text{ V} = -6 \text{ V}$$

$$I_2 = -3 \text{ A}$$

Finally,

$$I_3 = I_1 + I_2 = -1 \text{ A}$$

RC Circuits : Charging of a Capacitor



Consider the circuit shown in the figure. We assume that the capacitor is initially uncharged and that at $t = 0$ we throw the switch S from the middle position to position a . The battery will charge the capacitor C through the resistor R .

Our objective is to examine the charging process as a function of time.

We will write KLR starting at point b and going in the counterclockwise direction:

$E - iR - \frac{q}{C} = 0$. The current $i = \frac{dq}{dt} \rightarrow E - \frac{dq}{dt} R - \frac{q}{C} = 0$. If we rearrange the terms

we have: $\frac{dq}{dt} R + \frac{q}{C} = E$. This is an inhomogeneous, first order, linear differential

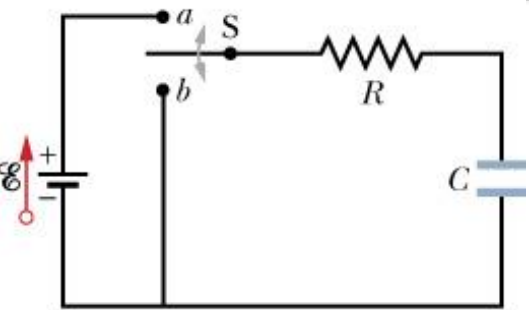
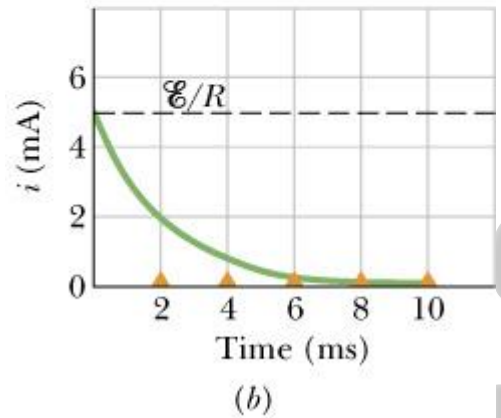
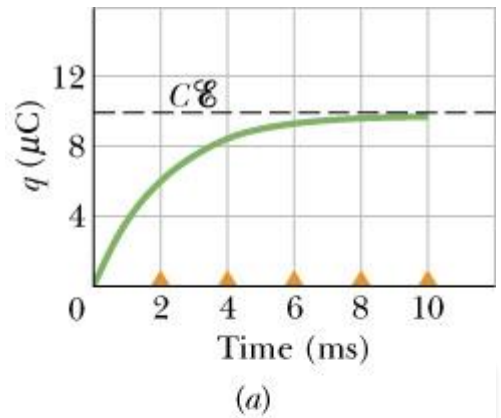
equation with initial condition $q(0) = 0$. This condition expresses the fact that at $t = 0$ the capacitor is uncharged.

$$\tau = RC$$

Differential equation: $\frac{dq}{dt}R + \frac{q}{C} = E$

Initial condition: $q(0) = 0$

Solution: $q = CE(1 - e^{-t/\tau})$ Here: $\tau = RC$



The constant τ is known as the "time constant" of the circuit. If we plot q versus t we see that q does not reach its terminal value CE but instead increases from its initial value and reaches the terminal value at $t = \infty$. Do we have to wait for an eternity to charge the capacitor? In practice, no.

$$q(t = \tau) = (0.632) CE$$

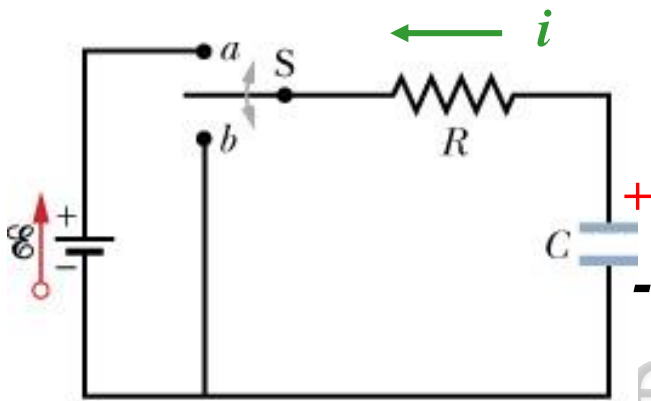
$$q(t = 3\tau) = (0.950) CE$$

$$q(t = 5\tau) = (0.993) CE$$

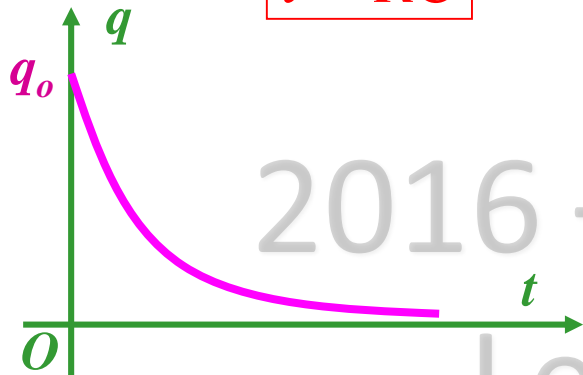
If we wait only a few time constants the charge, for all practical purposes, has reached its terminal value CE .

The current $i = \frac{dq}{dt} = \left(\frac{E}{R}\right) e^{-t/\tau}$. If we plot i versus t

we get a decaying exponential (see fig. *b*).



$$\tau = RC$$



RC Circuits : Discharging of a Capacitor

Consider the circuit shown in the figure. We assume that the capacitor at $t = 0$ has charge q_0 and that at $t = 0$ we throw the switch S from the middle position to position b . The capacitor is disconnected from the battery and loses its charge through resistor R .

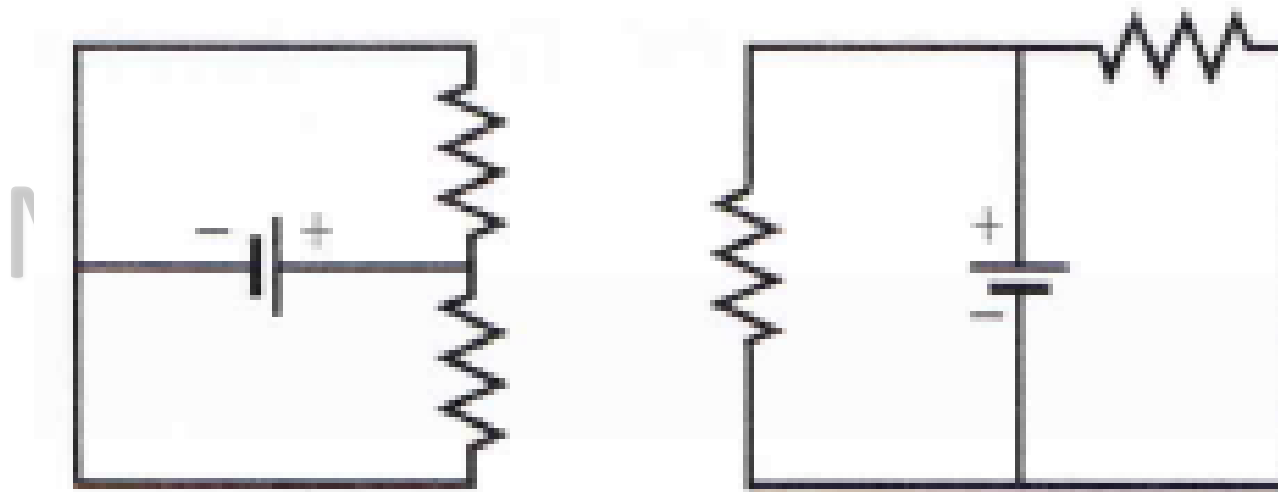
We will write KLR starting at point b and going in the counterclockwise direction: $-\frac{q}{C} - iR = 0$.

Taking into account that $i = \frac{dq}{dt}$ we get: $\frac{dq}{dt} R + \frac{q}{C} = 0$.

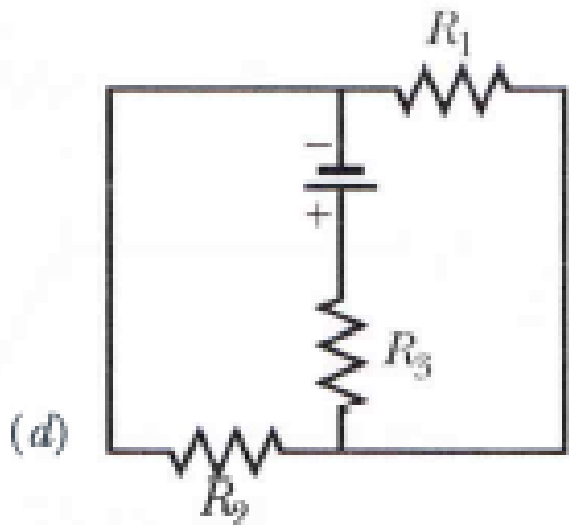
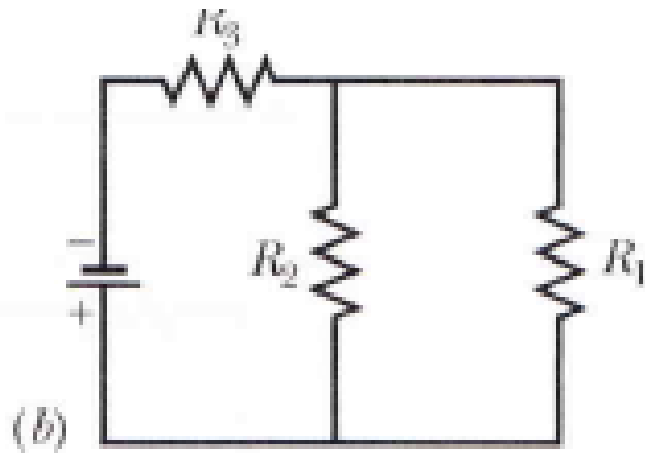
This is a homogeneous, first order, linear differential equation with initial condition

$q(0) = q_0$ The solution is: $q = q_0 e^{-t/\tau}$, where $\tau = RC$. If we plot q versus t we get a decaying exponential. The charge becomes zero at $t = \infty$. In practical terms we only have to wait a few time constants:

$$q(\tau) = (0.368)q_0, \quad q(3\tau) = (0.049)q_0, \quad q(5\tau) = (0.007)q_0.$$



2016 – 2017 Summer Lecture Notes



Burak Kaynar

138

2017 Summer

Notes

••15 A 10-km-long underground cable extends east to west and consists of two parallel wires, each of which has resistance $13 \Omega/\text{km}$. An electrical short develops at distance x from the west end when a conducting path of resistance R connects the wires (Fig. 27-32). The resistance of the wires and the short is then 100Ω when measured from the east end and 200Ω when measured from the west end. What are (a) x and (b) R ?

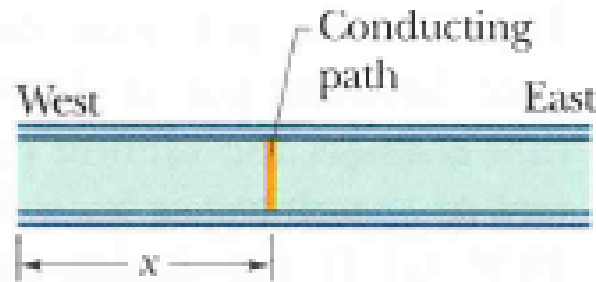


FIG. 27.32 Problem 15.

2016 – 2017 Summer

Lecture Notes

••17 In Fig. 27-34, $R_1 = 6.00 \Omega$, $R_2 = 18.0 \Omega$, and the ideal battery has emf $\mathcal{E} = 12.0 \text{ V}$. What are the (a) size and (b) direction (left or right) of current i_1 ? (c) How much energy is dissipated by all four resistors in 1.00 min?

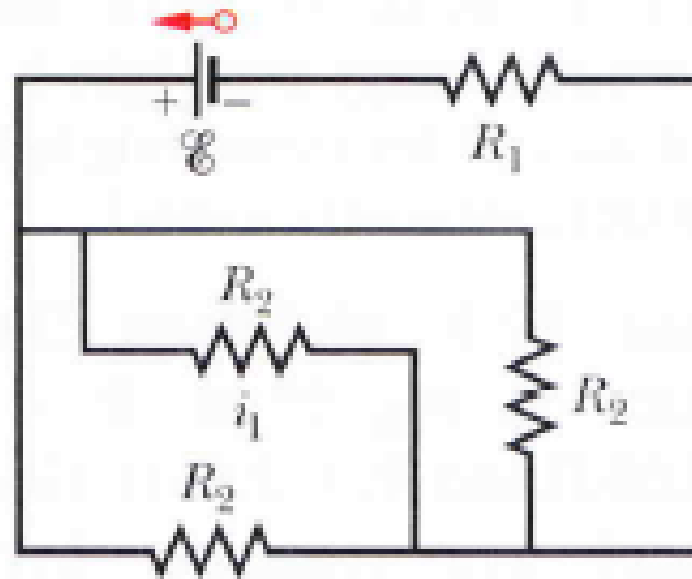


FIG. 27-34 Problem 17.

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