Chapter 28
Magnetic Fields

Magnetism

First observation was made 2500 years ago around a city called Magnesia (Manisa, Turkiye)

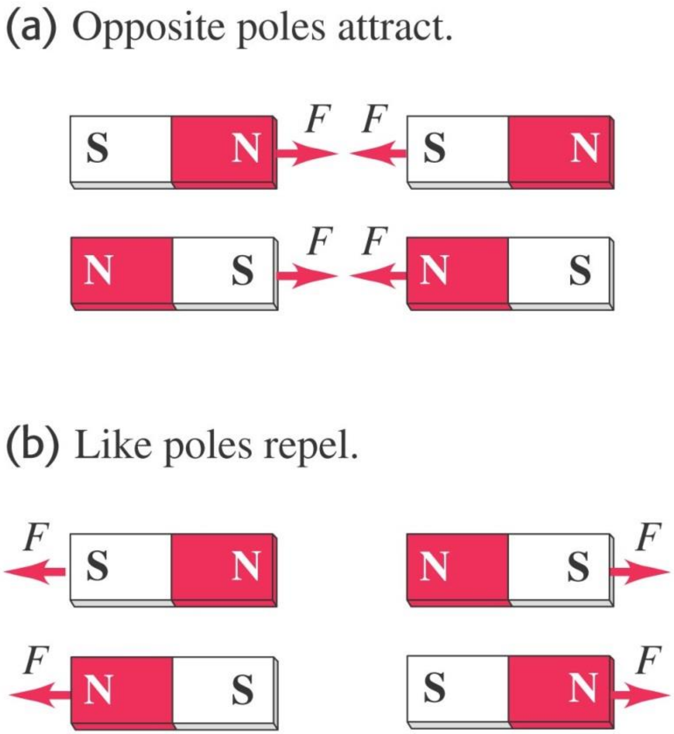
Permanent magnet

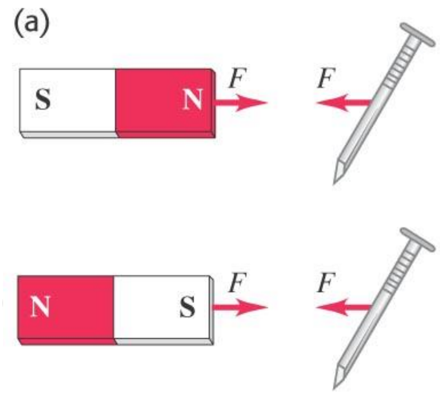
Material that creates its own magnetism without an external agent.

Magnetic material

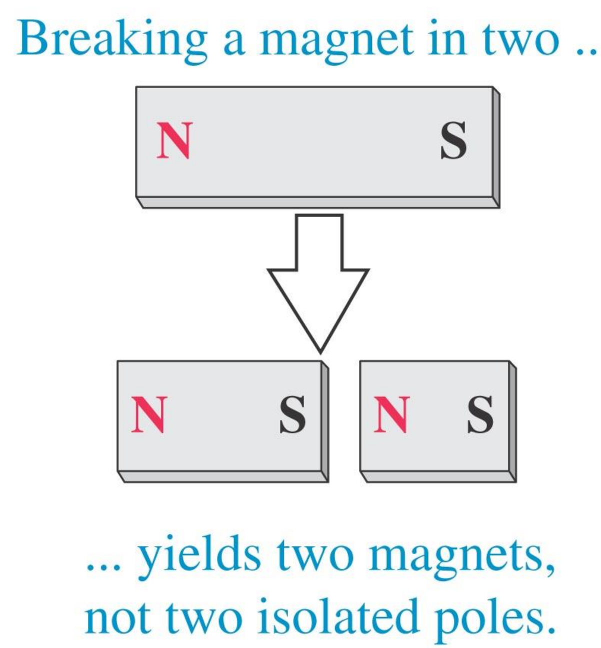
Material that gains magnetism by means of an external agent.

Magnetism

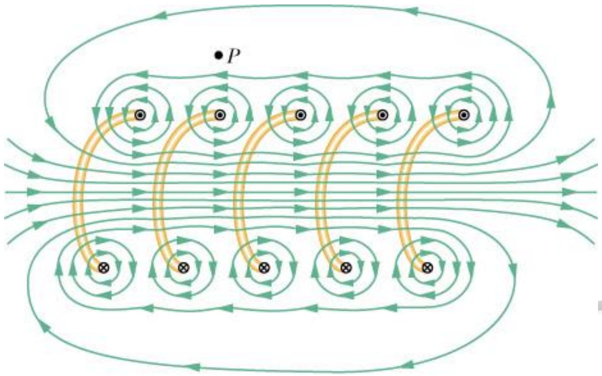




In contrast to electric charges, magnetic poles always come in pairs and can't be isolated.



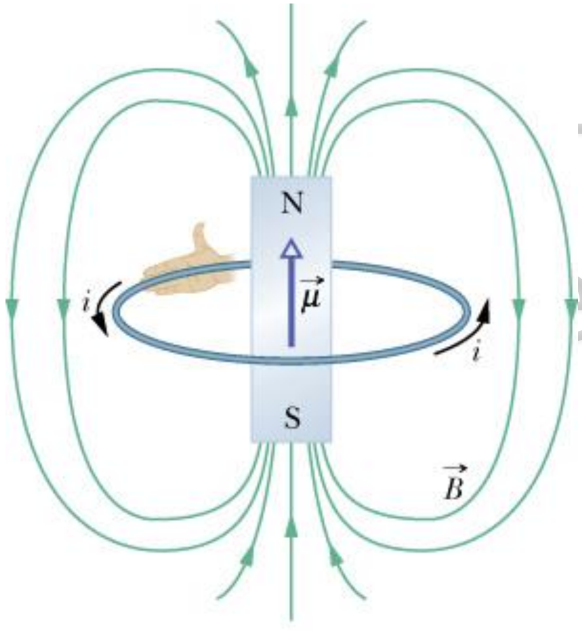
What Produces a Magnetic Field



One can generate a magnetic field using one of the following methods:

Pass a current through a wire and thus form what is known as an "electromagnet."

Use a "permanent" magnet.

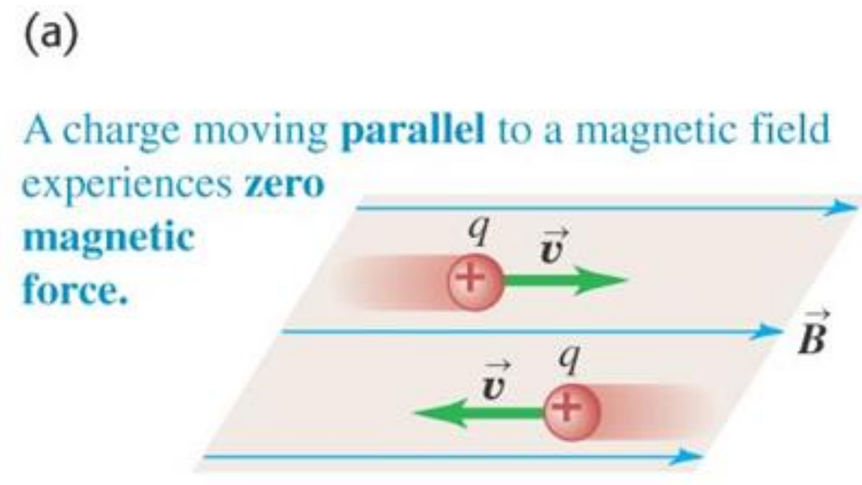


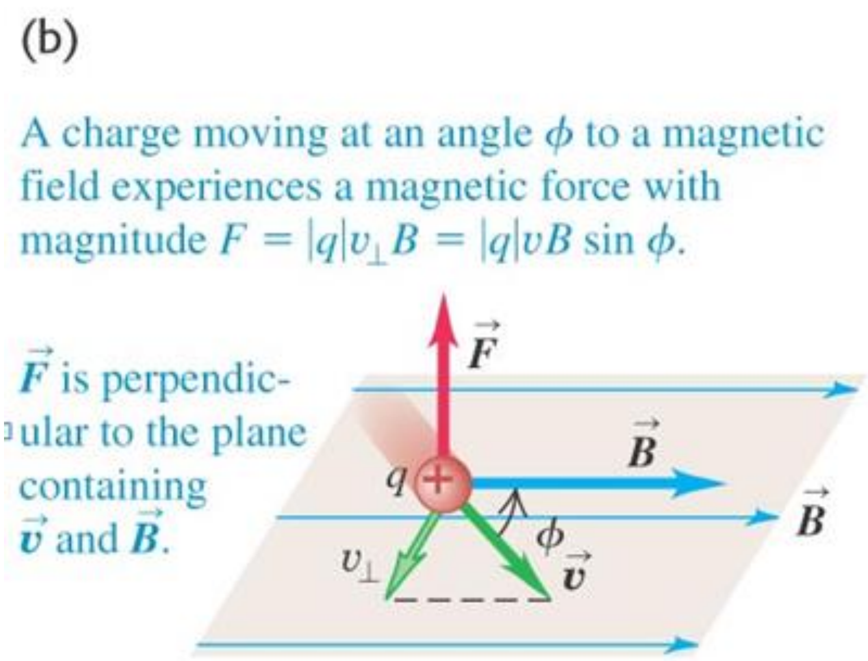
Empirically we know that both types of magnets attract small pieces of iron. Also, if suspended so that they can rotate freely they align themselves along the north-south direction. We can thus say that these magnets create in the surrounding space a "**magnetic field**" $\vec{B}$, which manifests itself by exerting a magnetic force $\vec{F}_B$.

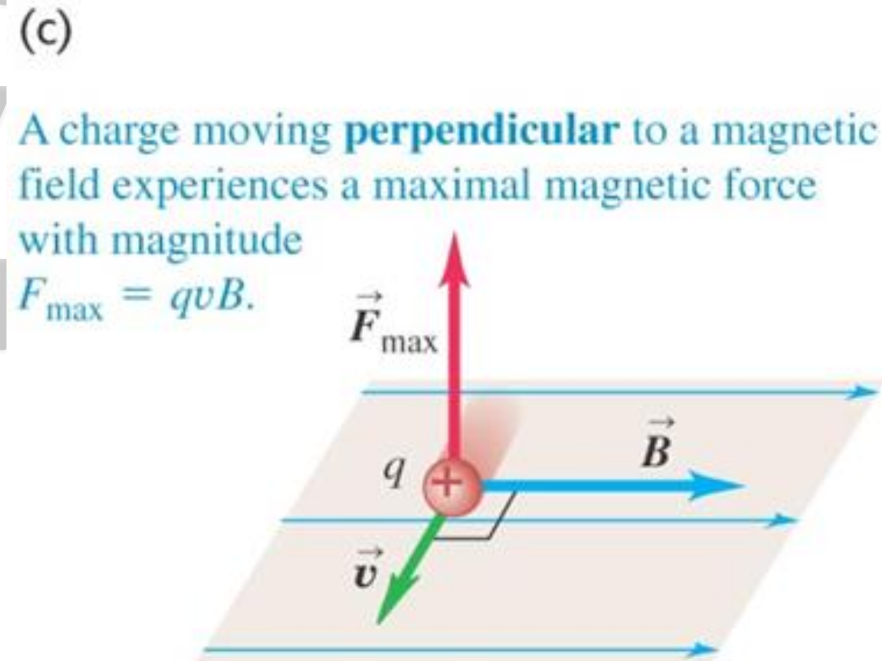
We will use the magnetic force to define precisely the magnetic field vector $\vec{B}$.

The magnetic force on a moving charge

- The magnetic force on a charge, q is perpendicular to *both* the velocity of q and the magnetic field, B.
- The magnitude of the magnetic force is $F = |q|vB \sin\varphi$.



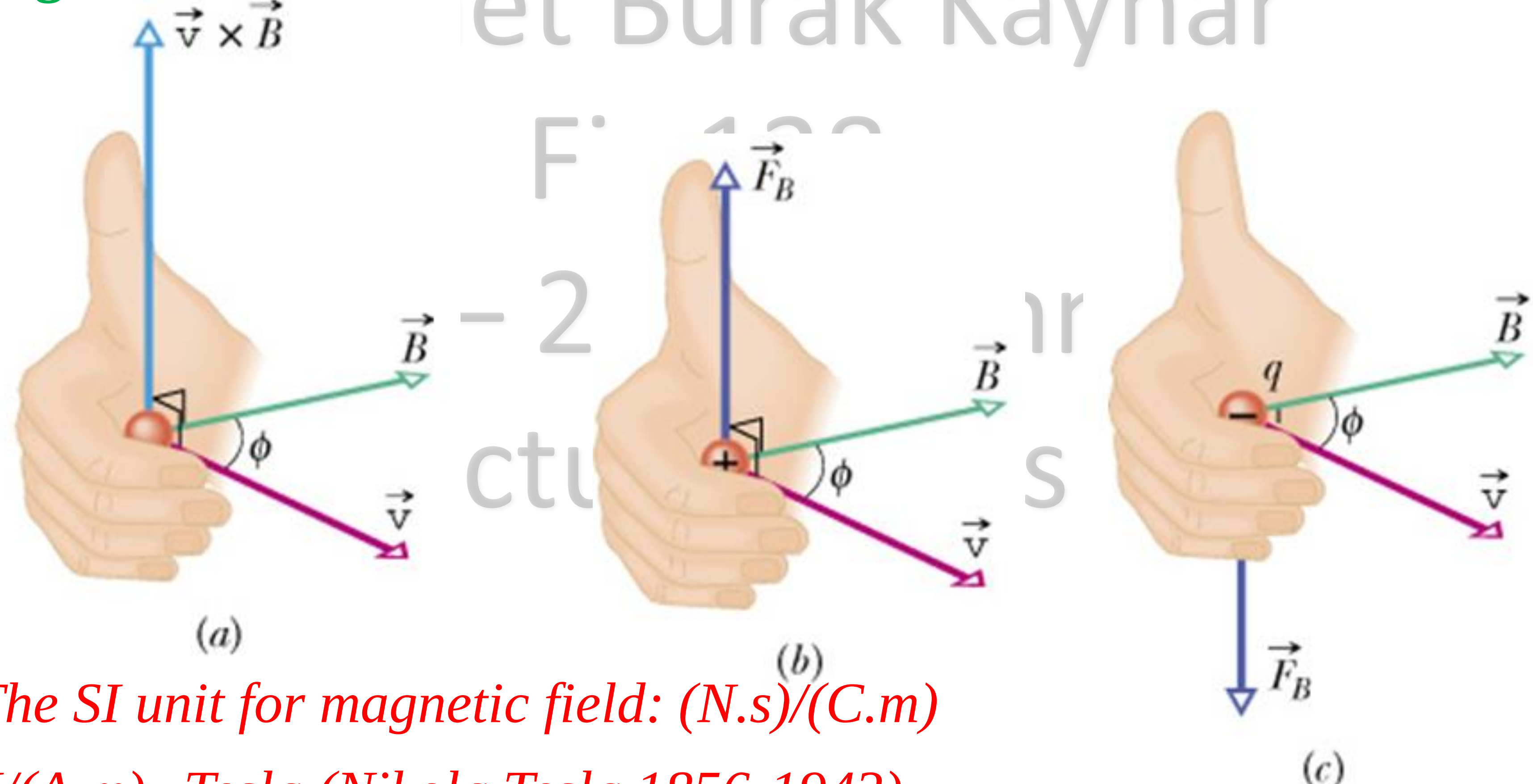




Magnetic force as a vector product

- We can write the magnetic force as a vector product $\vec{F}_B = q\vec{v} \times \vec{B}$
- The right-hand rule gives the direction of the force on a *positive* charge.

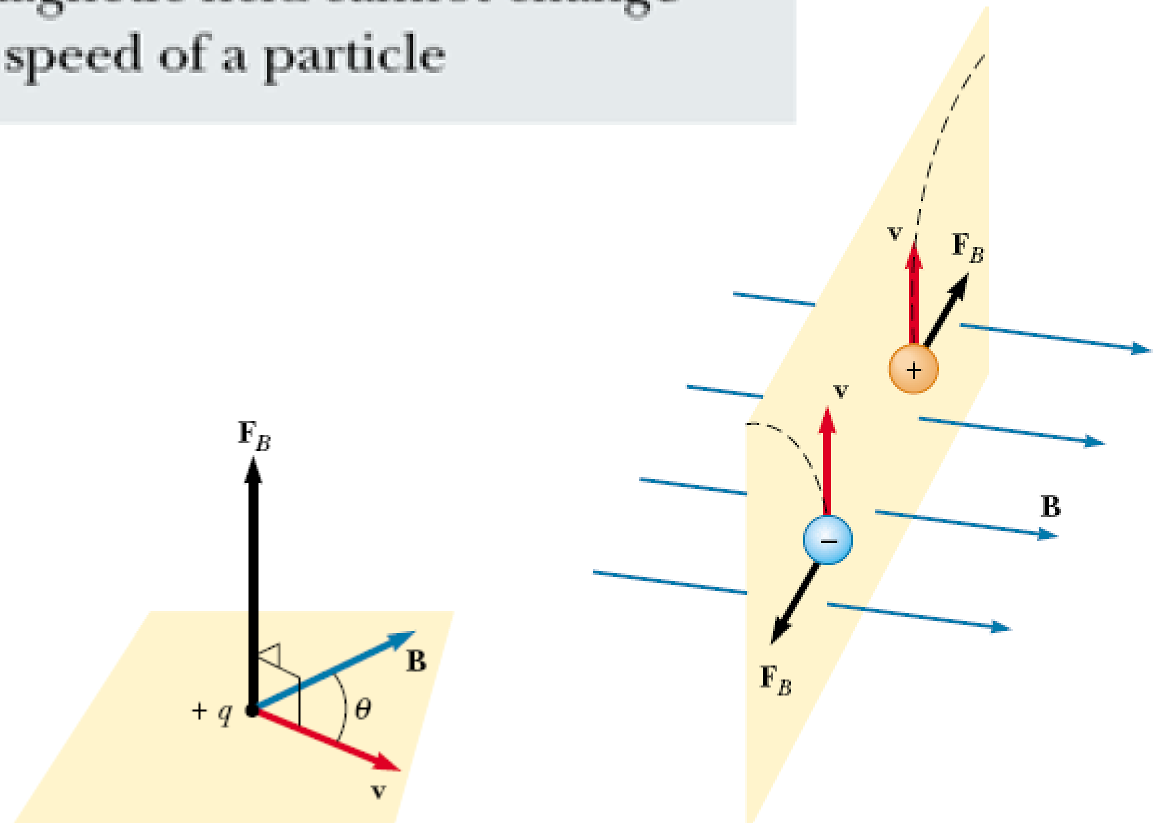
Right hand rule **Positive Charge** **Negative Charge**



The SI unit for magnetic field: (N.s)/(C.m)

N/(A.m)=Tesla (Nikola Tesla 1856-1943)

A magnetic field cannot change the speed of a particle

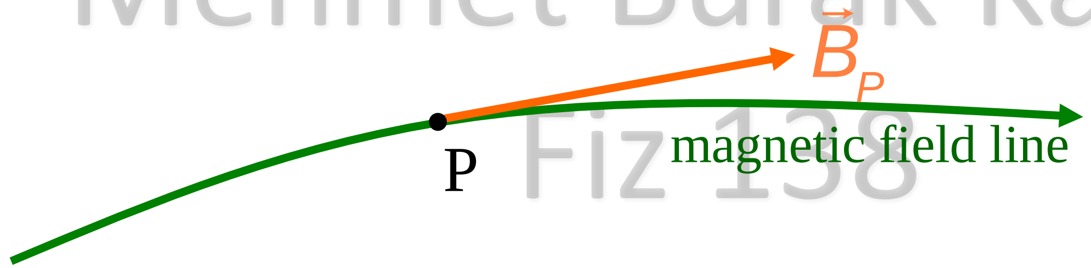


when a charged particle moves with a velocity **v** through a magnetic field, the field can alter the direction of the velocity vector but cannot change the speed or kinetic energy of the particle.

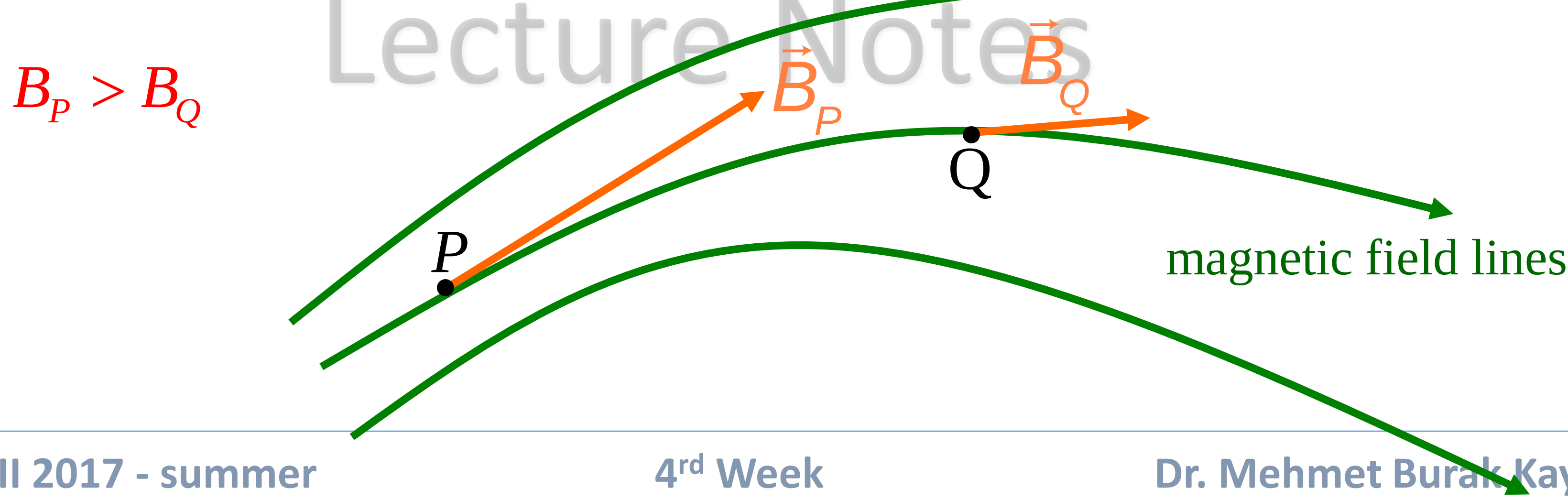
Magnetic Field Lines: In analogy with the electric field lines we introduce the concept of magnetic field lines, which help visualize the magnetic field vector $\vec{B}$ without using equations.

In the relation between the magnetic field lines and $\vec{B}$:

1. **At any point *P* the magnetic field vector $\vec{B}$ is tangent to the magnetic field lines.**

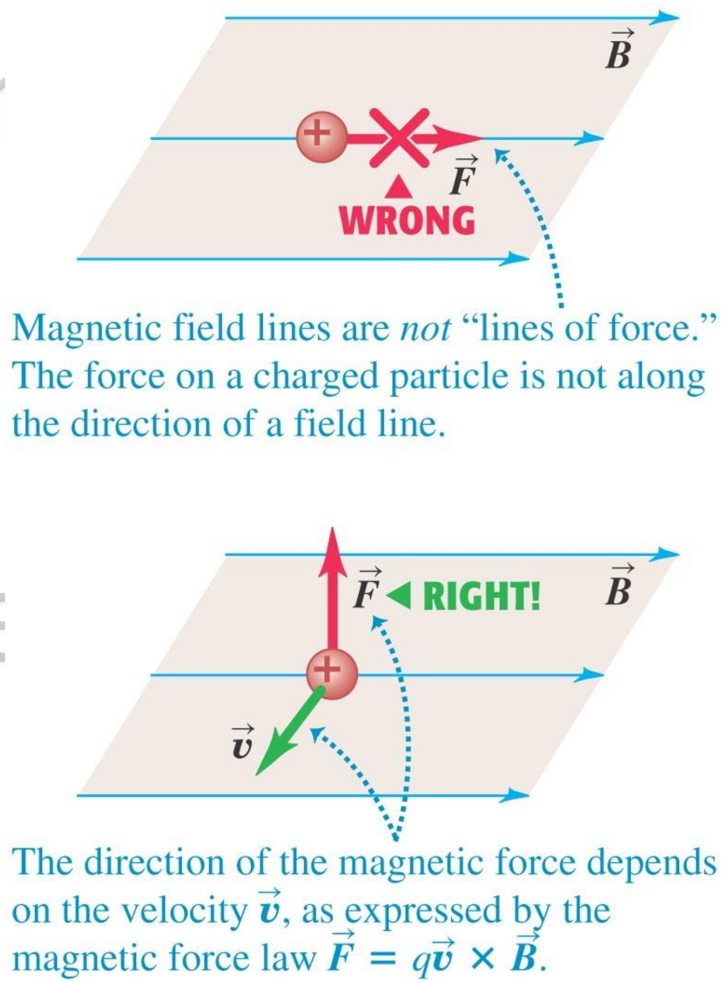


2. **The magnitude of the magnetic field vector $\vec{B}$ is proportional to the density of the magnetic field lines.**

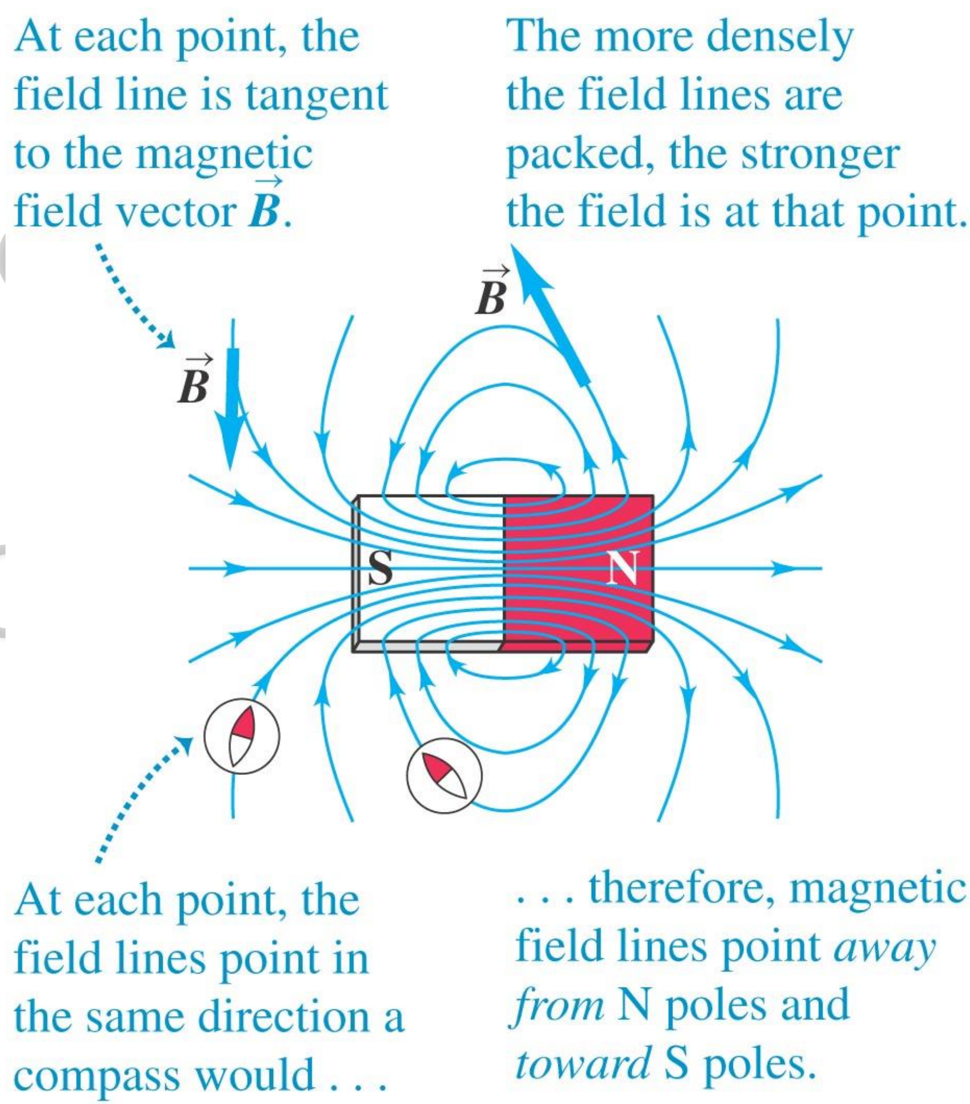


Magnetic field lines are *not* lines of force

- It is important to remember that magnetic field lines are *not* lines of magnetic force

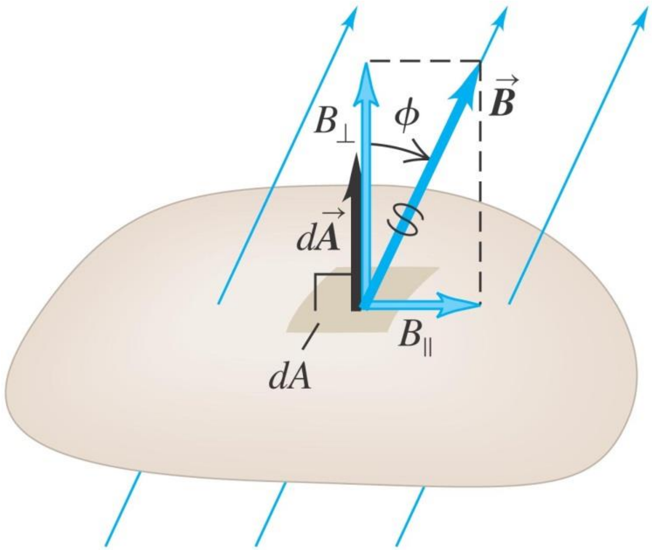


Magnetic field lines of a permanent magnet



Magnetic flux and Gauss's Law for Magnetism

- We define the *magnetic flux* through a surface just as we define electric flux.



$$d\Phi_B = B_\perp\, dA = B\cos\phi\, dA = \vec{B}\cdot d\vec{A}$$

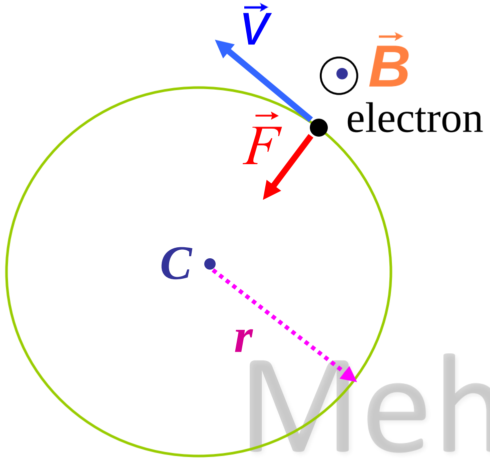
Total magnetic flux is the sum of the contributions from the individual area elements.

$$\Phi_B = \int \vec{B}\cdot d\vec{A}$$

The SI unit for magnetic flux: T.m^2=Weber

By analogy with the electric flux, magnetic flux through a closed surface would be proportional with magnetic monopoles. Since magnetic monopoles do not exist magnetic flux through a closed surface is always ZERO!!!

$$\oint \vec{B}\cdot d\vec{A} = 0$$



Motion of a Charged Particle in a Uniform Magnetic Field

(also known as ***cyclotron motion***)

A particle of mass m and charge q, when injected with a speed v at right angles to a uniform magnetic field $\vec{B}$, follows a circular orbit with uniform speed. The centripetal force required for such motion is provided by the magnetic force

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

$$r = \frac{mv}{|q|B} \qquad \omega = \frac{|q|B}{m}$$

The circular orbit of radius r for an electron is shown in the figure. The magnetic force

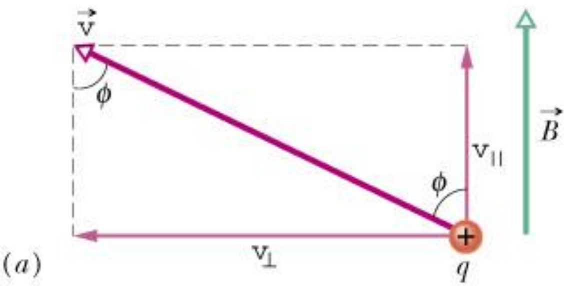
$F_B = |q|vB = ma = m\frac{v^2}{r} \rightarrow r = \frac{mv}{|q|B}$. The period is $T = \frac{2\pi r}{v} = \frac{2\pi mv}{|q|Bv} = \frac{2\pi m}{|q|B}$.

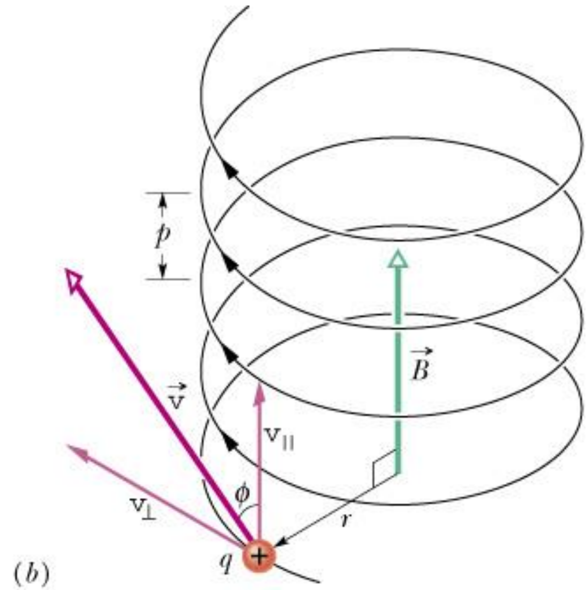
The corresponding frequency is $f = \frac{1}{T} = \frac{|q|B}{2\pi m}$. The angular frequency is $\omega = 2\pi f = \frac{|q|B}{m}$.

Note1: The cyclotron period does not depend on the speed v. All particles of the same mass complete their circular orbit during the same time T regardless of speed.

Note2: Fast particles move on larger-radius circular orbits, while slower particles move on smaller-radius orbits. All orbits have the same period T.

$$r = \frac{mv_\perp}{|q|B}$$ $$T = \frac{2\pi m}{|q|B}$$





Helical Paths

We now consider the motion of a charge in a uniform magnetic field $\vec{B}$ when its initial velocity $\vec{v}$ forms an angle ϕ with $\vec{B}$. We decompose $\vec{v}$ into two components.

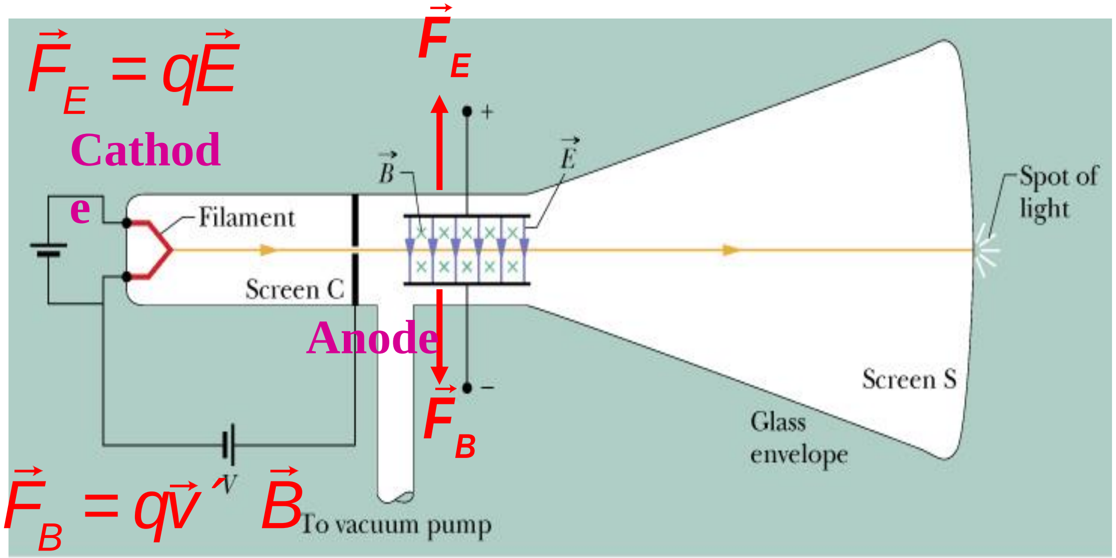
One component $(v_\parallel)$ is parallel to $\vec{B}$ and the other $(v_\perp)$ is perpendicular to $\vec{B}$ (see fig. *a*):

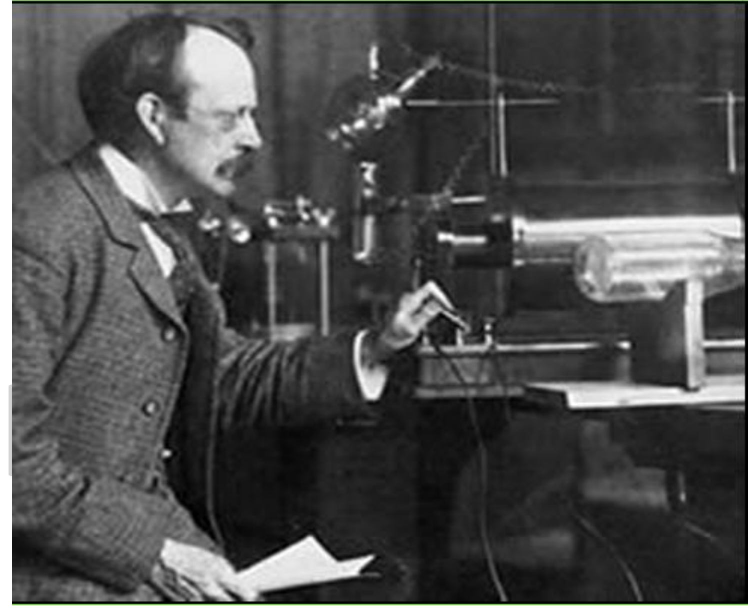
$v_\parallel = v\cos\phi \qquad v_\perp = v\sin\phi$ The particle executes two independent motions.

One, the cyclotron motion, is in the plane perpendicular to $\vec{B}$ that we have analyzed on the previous page. Its radius is $r = \dfrac{mv_\perp}{|q|B}$. Its period is $T = \dfrac{2\pi m}{|q|B}$.

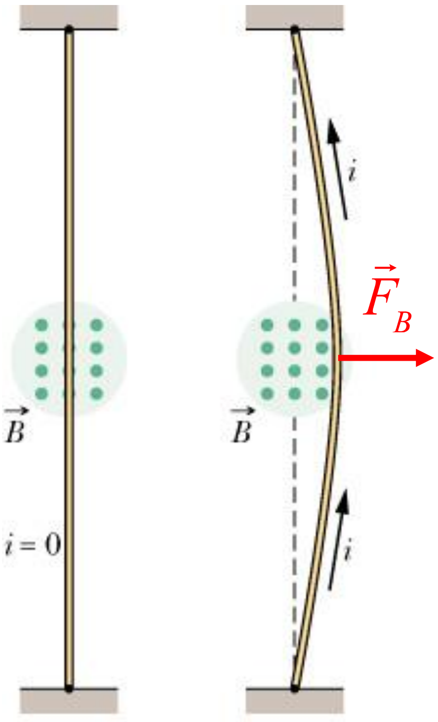
The second motion is along the direction of $\vec{B}$ and it is linear motion with constant speed $v_\parallel$. The combination of the two motions results in a helical path (see fig. *b*).

The pitch p of the helix is given by $p = Tv_\parallel = \dfrac{2\pi m v\cos\phi}{|q|B}$.

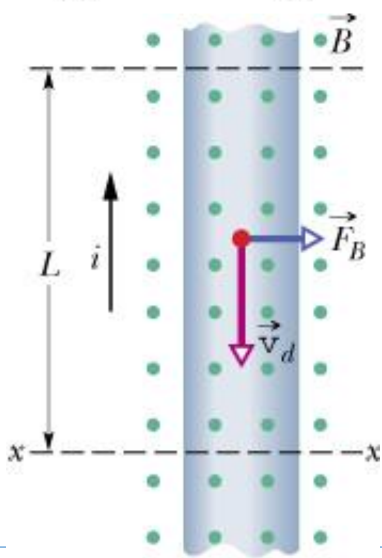




Discovery of the Electron: A cathode ray tube is shown in the figure. Electrons are emitted from a hot filament known as the "cathode." They are accelerated by a voltage V applied between the cathode and a second electrode known as the "anode." The electrons pass through a hole in the anode and they form a narrow beam. They hit the fluorescent coating of the right wall of the cathode ray tube where they produce a spot of light. J.J. Thomson in 1897 used a version of this tube to investigate the nature of the particle beam that caused the fluorescent spot. He applied constant electric and magnetic fields in the tube region to the right of the anode. With the fields oriented as shown in the figure the electric force $\vec{F}_E$ and the magnetic force $\vec{F}_B$ have opposite directions. By adjusting B and E, Thomson was able to have a zero net force.



(a) (b)



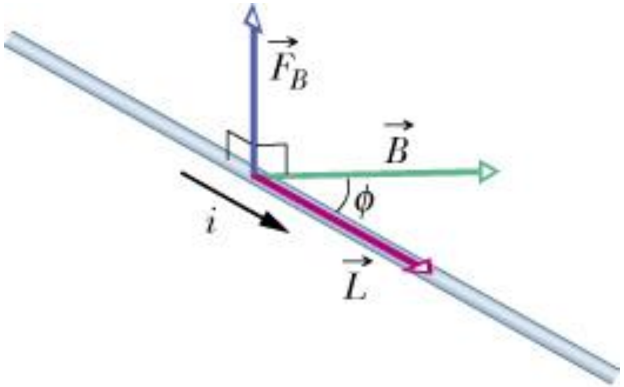
Magnetic Force on a Current - Carrying Wire

Consider a wire of length L that carries a current i as shown in the figure. A uniform magnetic field B is present in the vicinity of the wire. Experimentally it was found that a force $\vec{F}_B$ is exerted by $\vec{B}$ on the wire, and that $\vec{F}_B$ is perpendicular to the wire. The magnetic force on the wire is the vector sum of all the magnetic forces exerted by $\vec{B}$ on the electrons that constitute i. The total charge q that flows through the wire in time t is given by

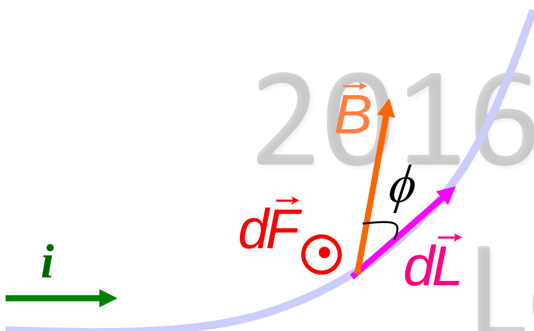
$q = it = i\dfrac{L}{v_d}$. Here v_d is the drift velocity of the electrons in the wire.

The magnetic force is $F_B = qv_dB\sin 90° = i\dfrac{L}{v_d}v_dB = iLB.$

$$F_B = iLB$$



$$\vec{F}_B = i\vec{L} \times \vec{B}$$



$$d\vec{F}_B = i\,d\vec{L} \times \vec{B}$$

$$\vec{F}_B = i\int d\vec{L} \times \vec{B}$$

Magnetic Force on a Straight Wire in a Uniform Magnetic Field

If we assume the more general case for which the magnetic field $\vec{B}$ forms an angle ϕ with the wire the magnetic force equation can be written in vector form as $\vec{F}_B = i\vec{L} \times \vec{B}$. Here $\vec{L}$ is a vector whose magnitude is equal to the wire length L and has a direction that coincides with that of the current. The magnetic force magnitude is $F_B = iLB\sin\phi$.

Magnetic Force on a Wire of Arbitrary Shape Placed in a Nonuniform Magnetic Field

In this case we divide the wire into elements of length dL, which can be considered as straight. The magnetic force on each element is $d\vec{F}_B = i\,d\vec{L} \times \vec{B}$. The net magnetic force on the wire is given by the integral $\vec{F}_B = i\int d\vec{L} \times \vec{B}$.

Sample Problem 28-1

A uniform magnetic field $\vec{B}$, with magnitude 1.2 mT, is directed vertically upward throughout the volume of a laboratory chamber. A proton with kinetic energy 5.3 MeV enters the chamber, moving horizontally from south to north. What magnetic deflecting force acts on the proton as it enters the chamber? The proton mass is 1.67×10^{-27} kg. (Neglect Earth's magnetic field.)

•3 An electron that has velocity

$$\vec{v} = (2.0 \times 10^6 \text{ m/s})\hat{\text{i}} + (3.0 \times 10^6 \text{ m/s})\hat{\text{j}}$$

moves through the uniform magnetic field $\vec{B} = (0.030 \text{ T})\hat{\text{i}} - (0.15 \text{ T})\hat{\text{j}}$. (a) Find the force on the electron due to the magnetic field. (b) Repeat your calculation for a proton having the same velocity.

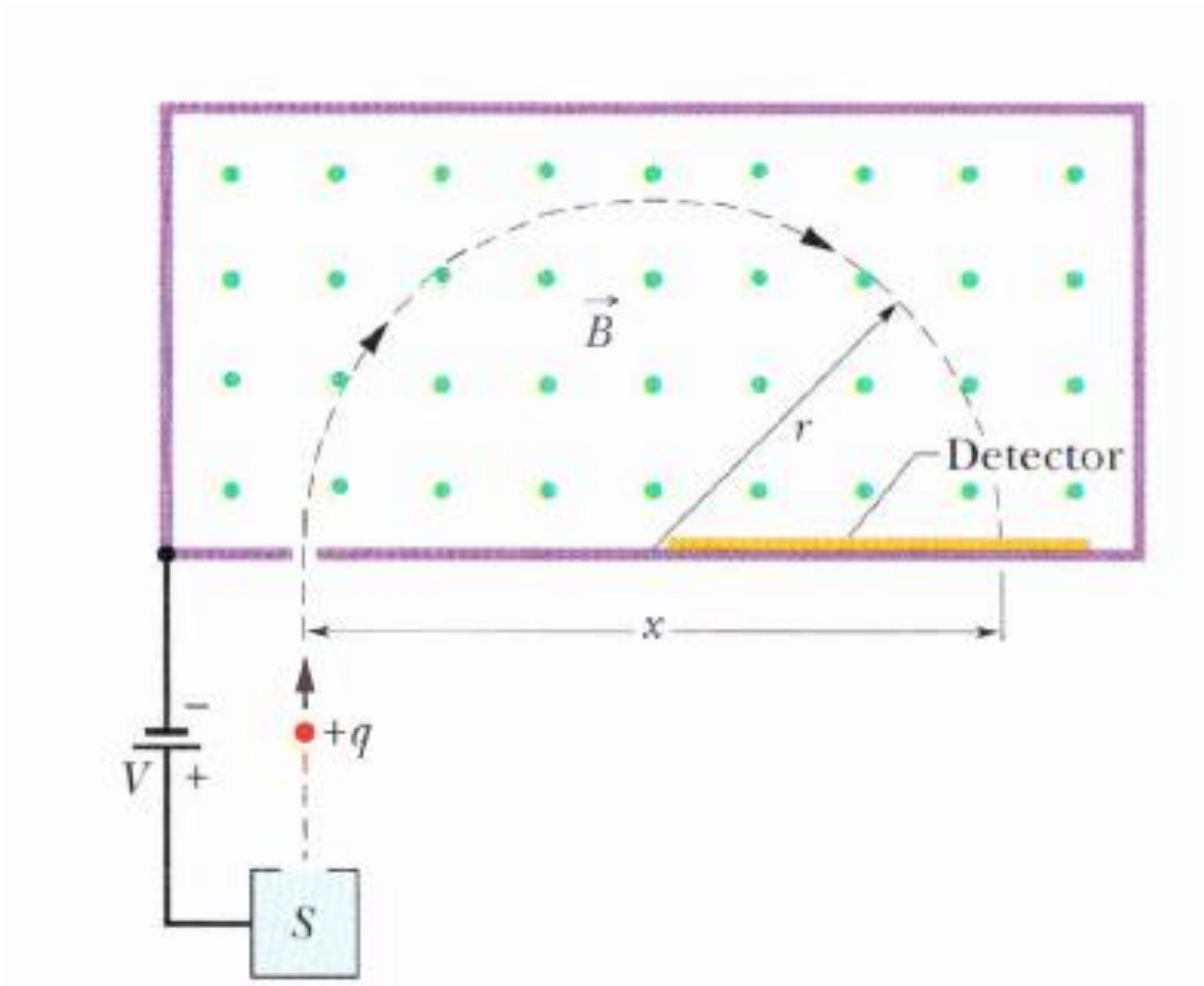
•7 An electron has an initial velocity of $(12.0\hat{j} + 15.0\hat{k})$ km/s and a constant acceleration of $(2.00 \times 10^{12}\ \text{m/s}^2)\hat{i}$ in a region in which uniform electric and magnetic fields are present. If $\vec{B} = (400\ \mu\text{T})\hat{i}$, find the electric field $\vec{E}$.

Sample Problem 28-2 Build your skill

Figure 28-9 shows a solid metal cube, of edge length d = 1.5 cm, moving in the positive y direction at a constant velocity $\vec{v}$ of magnitude 4.0 m/s. The cube moves through a uniform magnetic field $\vec{B}$ of magnitude 0.050 T in the positive z direction.

(a) Which cube face is at a lower electric potential and which is at a higher electric potential because of the motion through the field?

(b) What is the potential difference between the faces of higher and lower electric potential?



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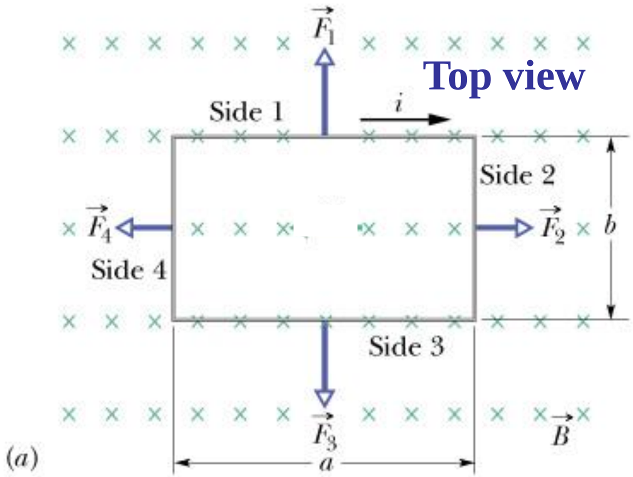
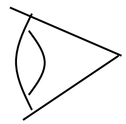
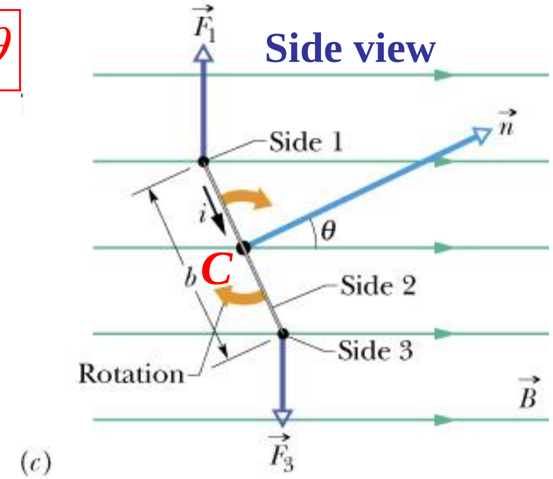




$$\tau_{\text{net}} = iAB\sin\theta$$

$$F_{\text{net}} = 0$$



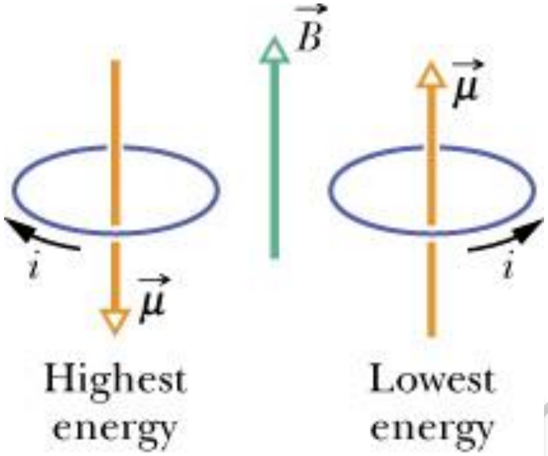
Magnetic Torque on a Current Loop

Consider the rectangular loop in fig. a with sides of lengths a and b and that carries a current i. The loop is placed in a magnetic field so that the normal $\hat{n}$ to the loop forms an angle θ with $\vec{B}$. The magnitude of the magnetic force on sides 1 and 3 is $F_1 = F_3 = iaB\sin 90° = iaB$. The magnetic force on sides 2 and 4 is $F_2 = F_4 = ibB\sin(90-\theta) = ibB\cos\theta$. These forces cancel in pairs and thus $F_{\text{net}} = 0$. The torque about the loop center C of F_2 and F_4 is zero because both forces pass through point C. The moment arm for F_1 and F_3 is equal to $(b/2)\sin\theta$. The two torques tend to rotate the loop in the same (clockwise) direction and thus add up. The net torque $\tau = \tau_1 + \tau_3 = (iabB/2)\sin\theta + (iabB/2)\sin\theta = iabB\sin\theta = iAB\sin\theta$.

Magnetic Dipole Moment

$\vec{\tau} = \vec{\mu} \times \vec{B}$

$U = -\vec{\mu} \cdot \vec{B}$



$U = \mu B$ $\quad\quad$ $U = -\mu B$

The torque of a coil that has N loops exerted by a uniform magnetic field B and carries a current i is given by the equation $\tau = NiAB$.

We define a new vector $\vec{\mu}$ associated with the coil, which is known as the magnetic dipole moment of the coil.

The magnitude of the magnetic dipole moment is $\mu = NiA$.

Its direction is perpendicular to the plane of the coil.

The sense of $\vec{\mu}$ is defined by the right-hand rule. We curl the fingers of the right hand in the direction of the current. The thumb gives us the sense. The torque can be expressed in the form $\tau = \mu B \sin\theta$ where θ is the angle between $\vec{\mu}$ and $\vec{B}$.

In vector form: $\vec{\tau} = \vec{\mu} \times \vec{B}$.

The potential energy of the coil is: $U = -\mu B \cos\theta = -\vec{\mu} \cdot \vec{B}$.

U has a minimum value of $-\mu B$ for $\theta = 0$ (position of **stable** equilibrium).

U has a maximum value of μB for $\theta = 180°$ (position of **unstable** equilibrium).

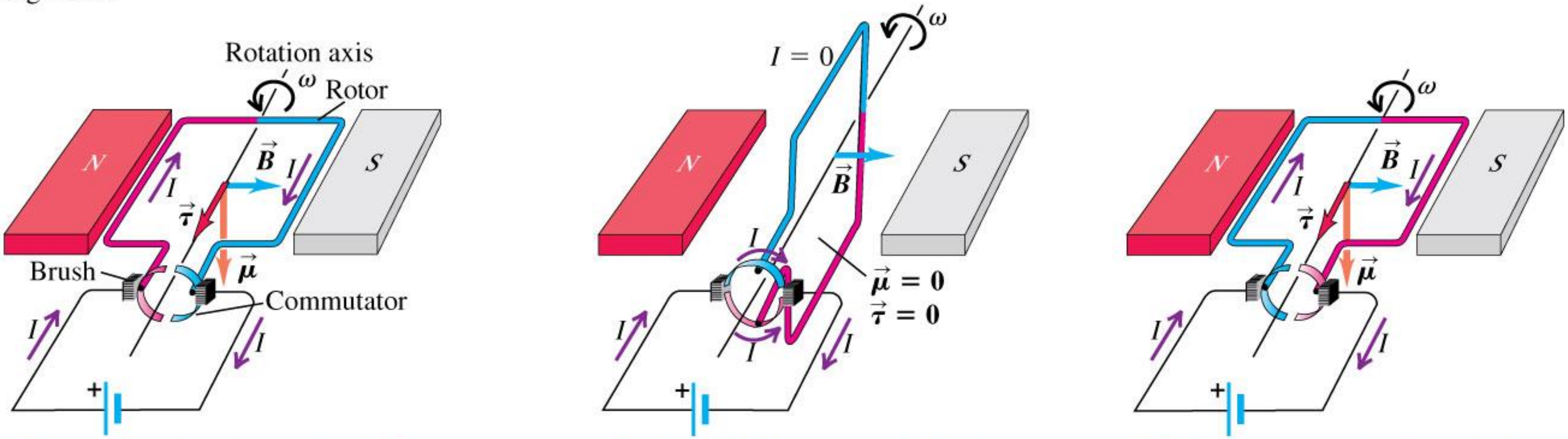
Note: For both positions the net torque is $\tau = 0$.

An Application: The direct-current motor

(a) Brushes are aligned with commutator segments.

(b) Rotor has turned 90°.

(c) Rotor has turned 180°.



(a)
- Current flows into the red side of the rotor and out of the blue side.
- Therefore the magnetic torque causes the rotor to spin counterclockwise.

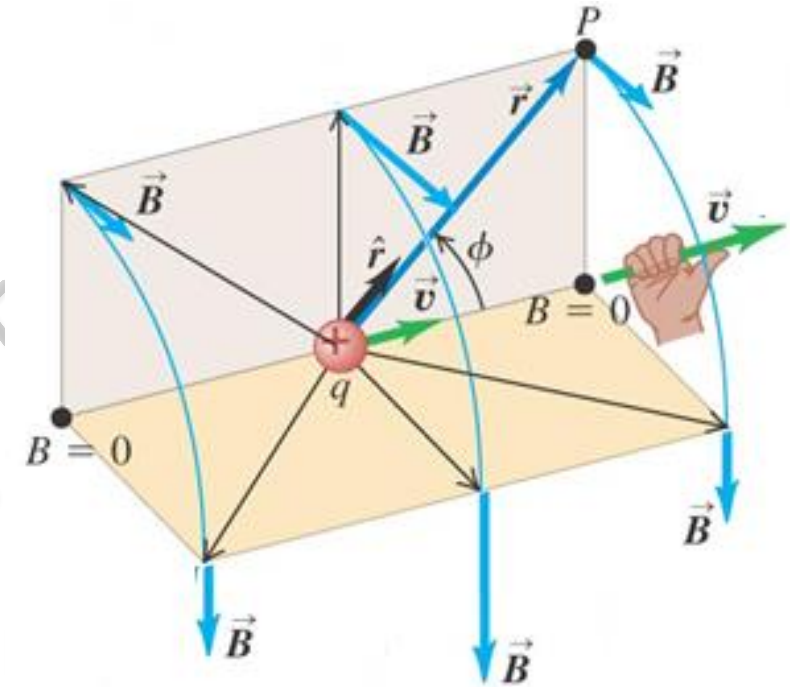
(b)
- Each brush is in contact with both commutator segments, so the current bypasses the rotor altogether.
- No magnetic torque acts on the rotor.

(c)
- The brushes are again aligned with commutator segments. This time the current flows into the blue side of the rotor and out of the red side.
- Therefore the magnetic torque again causes the rotor to spin counterclockwise.

Chapter 29
Magnetic Fields Due to Currents

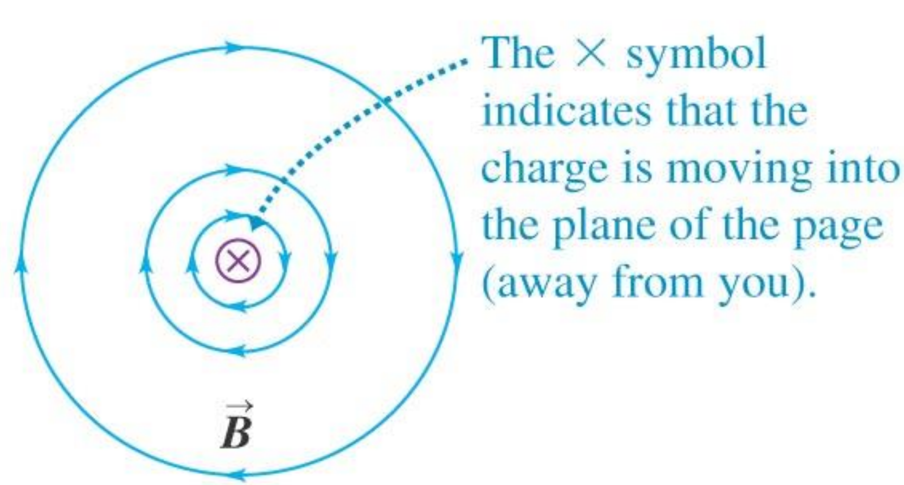
The magnetic field of a moving charge

- A moving charge generates a magnetic field that depends on the velocity of the charge.



E and *B* fields posses similiar dependence on charge and the distance to the source, however *B* is not directed along the line from the source to the point of interest.

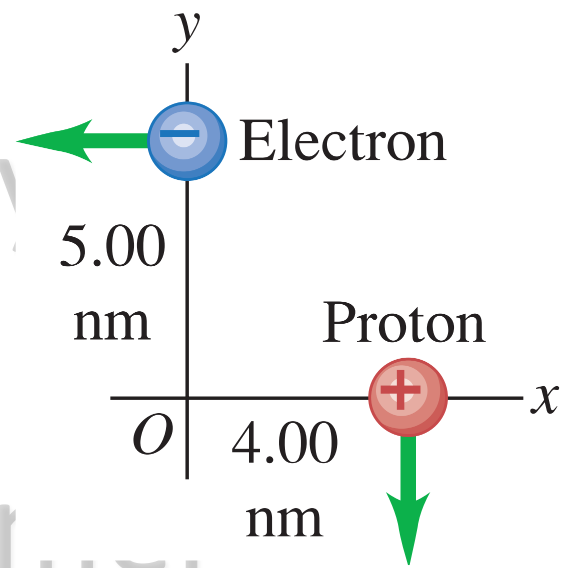
View from behind the charge



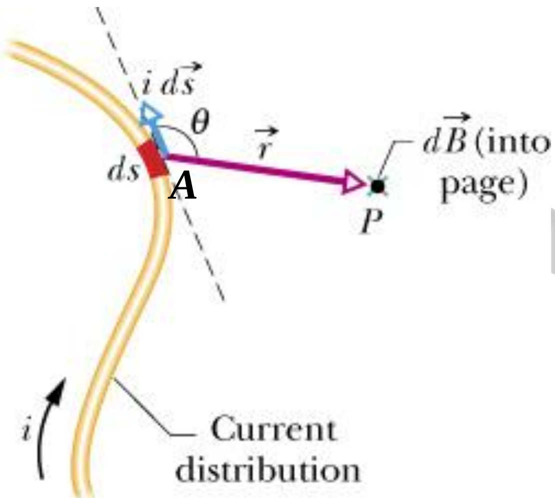
Experimental Observations } $B \propto \frac{qv\sin\phi}{r^2}$

28.8 •• An electron and a proton are each moving at 845 km/s in perpendicular paths as shown in Fig. E28.8. At the instant when they are at the positions shown in the figure, find the magnitude and direction of (a) the total magnetic field they produce at the origin; (b) the magnetic field the electron produces at the location of the proton; (c) the total electric force and the total magnetic force that the electron exerts on the proton.

Figure **E28.8**



$$d\vec{B} = \frac{\mu_0 i}{4\pi}\frac{d\vec{s} \times \vec{r}}{r^3}$$



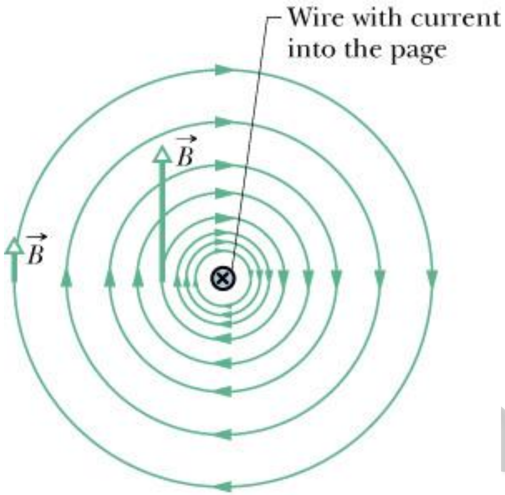
The Law of Biot - Savart

This law gives the magnetic field $d\vec{B}$ generated by a wire segment of length ds that carries a current i. Consider the geometry shown in the figure. Associated with the element ds we define an associated vector $d\vec{s}$ that has magnitude equal to the length ds. The direction of $d\vec{s}$ is the same as that of the current that flows through segment ds.

The magnetic field $d\vec{B}$ generated at point P by the element $d\vec{S}$ located at point A is given by the equation $d\vec{B} = \frac{\mu_0 i}{4\pi}\frac{d\vec{s} \times \vec{r}}{r^3}$. Here $\vec{r}$ is the vector that connects point A (location of element ds) with point P at which we want to determine $d\vec{B}$.

The constant $\mu_0 = 4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A} = 1.26 \times 10^{-6}\ \text{T}\cdot\text{m/A}$ and is known as the "***permeability constant.***" The magnitude of $d\vec{B}$ is $dB = \frac{\mu_0 i}{4\pi}\frac{ds \sin\theta}{r^2}$.

Here θ is the angle between $d\vec{s}$ and $\vec{r}$.

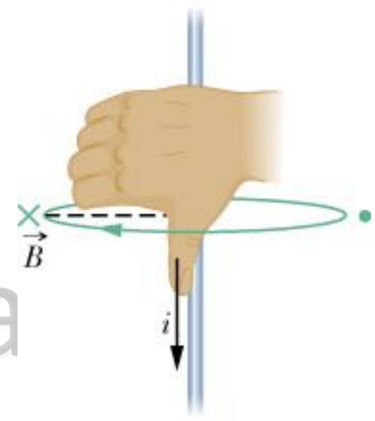


Magnetic Field Generated by a Long Straight Wire

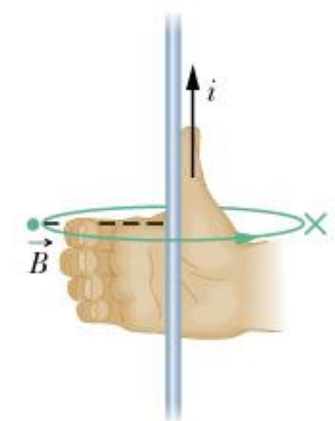
The magnitude of the magnetic field generated by the wire at point P located at a distance R from the wire is given by the equation

$$B = \frac{\mu_0 i}{2\pi R}$$

$$B = \frac{m_0 i}{2pR} \, .$$



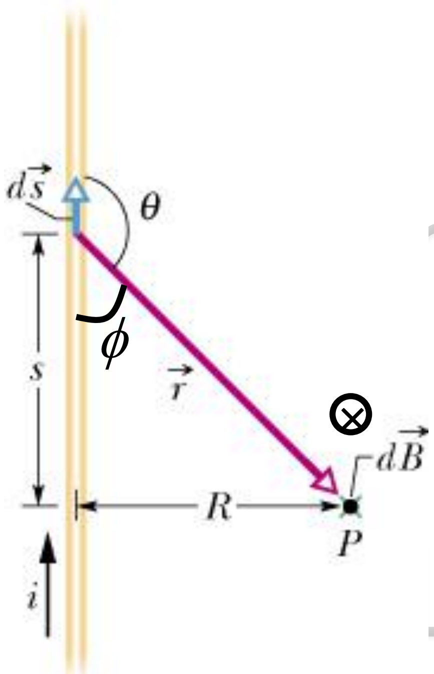
(a)



(b)

The magnetic field lines form circles that have their centers at the wire. The magnetic field vector $\vec{B}$ is tangent to the magnetic field lines. The sense for $\vec{B}$ is given by the **right - hand rule**. We point the thumb of the right hand in the direction of the current. The direction along which the fingers of the right hand curl around the wire gives the direction of $\vec{B}$.

$$B = \frac{\mu_0 i}{2\pi R}$$



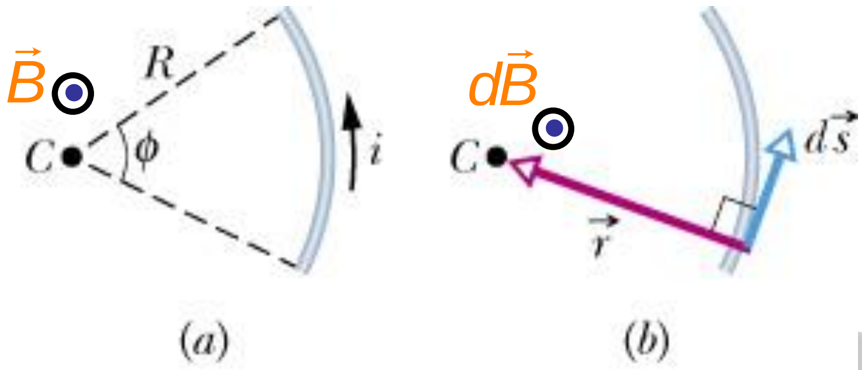
Consider the wire element of length ds shown in the figure. The element generates at point P a magnetic field of magnitude $dB = \frac{\mu_0 i}{4\pi}\frac{ds\sin\theta}{r^2}$. Vector $d\vec{B}$ is pointing **into** the page. The magnetic field generated by the whole wire is found by integration:

$$B = \int_{-\infty}^{\infty} dB = 2\int_0^{\infty} dB = \frac{\mu_0 i}{2\pi}\int_0^{\infty}\frac{ds\sin\theta}{r^2}$$

$$r = \sqrt{s^2 + R^2} \qquad \sin\theta = \sin\phi = R/r = R/\sqrt{s^2 + R^2}$$

$$B = \frac{\mu_0 i}{2\pi}\int_0^{\infty}\frac{R\,ds}{\left(s^2 + R^2\right)^{3/2}} = \frac{\mu_0 i}{2\pi R}\left[\frac{s}{\sqrt{s^2 + R^2}}\right]_0^{\infty} = \frac{\mu_0 i}{2\pi R}$$

$$\int\frac{dx}{\left(x^2 + a^2\right)^{3/2}} = \frac{x}{a^2\sqrt{x^2 + a^2}}$$



Magnetic Field Generated by a Circular Wire Arc of Radius *R* at Its Center *C*

$$B = \frac{\mu_0 i \phi}{4\pi R}$$

A wire section of length ds generates at the center C a magnetic field $d\vec{B}$.

The magnitude $dB = \frac{\mu_0 i}{4\pi}\frac{ds \sin 90°}{R^2} = \frac{\mu_0 i}{4\pi}\frac{ds}{R^2}$. The length $ds = R d\phi$

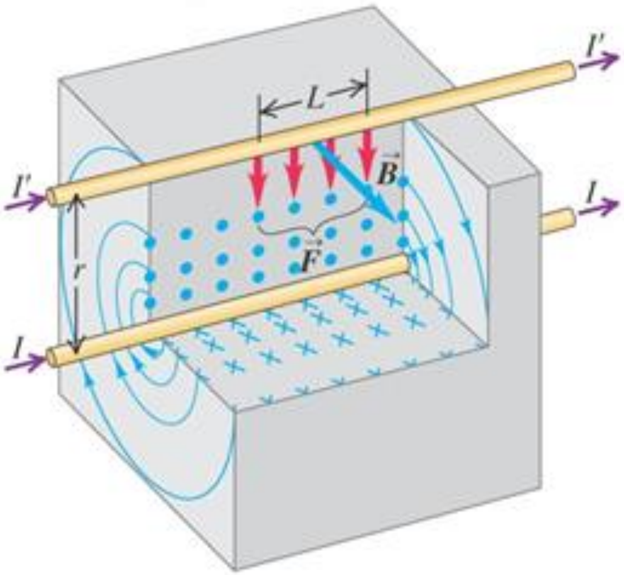
$\rightarrow dB = \frac{\mu_0 i}{4\pi R} d\phi$.   Vector $d\vec{B}$ points **out** of the page.

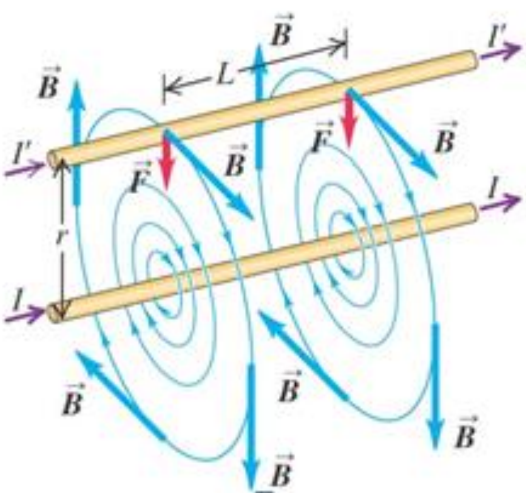
The net magnetic field $B = \int dB = \int_0^{\phi} \frac{\mu_0 i}{4\pi R} d\phi = \frac{\mu_0 i \phi}{4\pi R}$.

Note: The angle ϕ must be expressed in radians.

For a circular wire, $\phi = 2\pi$. In this case we get: $B_{\text{circ}} = \frac{\mu_0 i}{2R}$.

Force Between Parallel Conductors





One of the wires is assumed to be the source of the field and the other current carrying wire is being affected in that field

Lower wire $$B = \frac{\mu_0 I}{2\pi r}$$

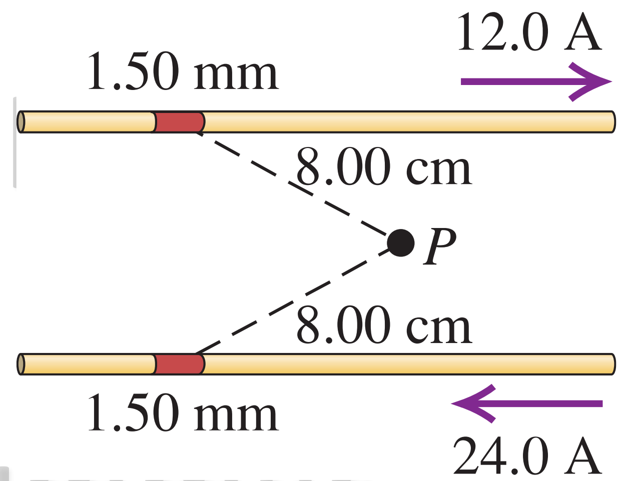
Force experienced by the upper wire due to the field of the lower one is,

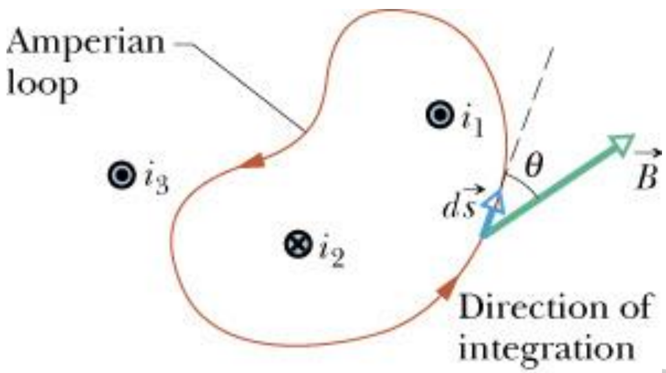
$$F = I'LB = I'L\left(\frac{\mu_0 I}{2\pi r}\right)$$

$$\frac{F}{L} = \frac{\mu_0 I I'}{2\pi r}$$: length

28.14 •• Two parallel wires are 5.00 cm apart and carry currents in opposite directions, as shown in Fig. E28.14. Find the magnitude and direction of the magnetic field at point P due to two 1.50-mm segments of wire that are opposite each other and each 8.00 cm from P.

Figure **E28.14**





$$\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{enc}$$

Ampere's Law

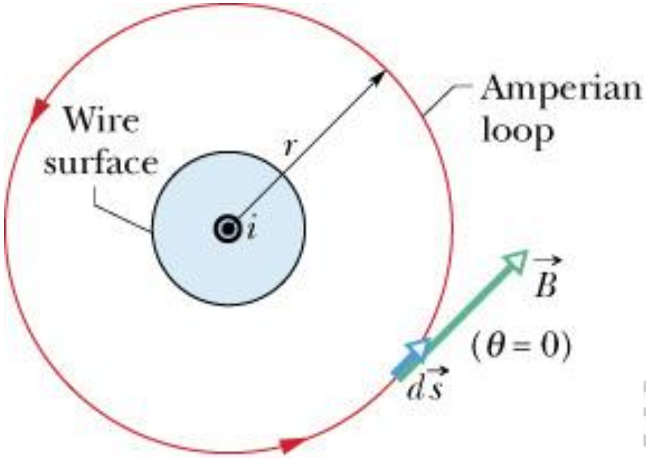
French Scientist

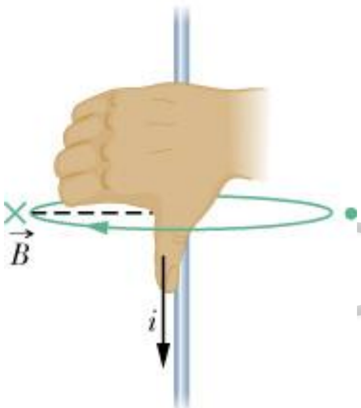
The law of Biot-Savart combined with the principle of superposition can be used to determine $\vec{B}$ if we know the distribution of currents. In situations that have high symmetry we can use Ampere's law instead, because it is simpler to apply.

Ampere's law can be derived from the law of Biot-Savart, with which it is mathematically equivalent. Ampere's law is more suitable for advanced formulations of electromagnetism. It can be expressed as follows:

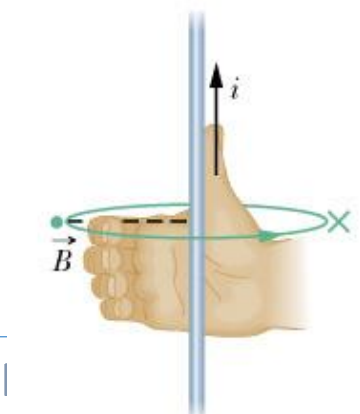
The line integral of the magnetic field B along any closed path is equal to the total current enclosed inside the path multiplied by μ_0

The closed path used is known as an "**Amperian loop.**" In its present form Ampere's law is not complete. A missing term was added by Clark Maxwell.





(a)



Magnetic Field Outside a Long Straight Wire

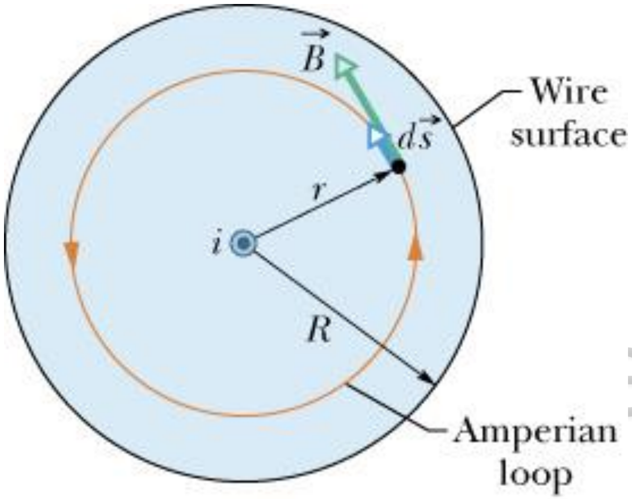
We already have seen that the magnetic field lines of the magnetic field generated by a long straight wire that carries a current i have the form of circles, which are concentric with the wire.

We choose an Amperian loop that reflects the cylindrical symmetry of the problem. The loop is also a circle of radius r that has its center on the wire. The magnetic field is tangent to the loop and has a constant magnitude B:

$$\oint \vec{B} \cdot d\vec{s} = \oint B\,ds\cos 0 = B\oint ds = 2\pi r B = \mu_0 i_{enc} = \mu_0 i$$

$$\rightarrow B = \frac{\mu_0 i}{2\pi r}$$

Note: Ampere's law holds true for any closed path. We choose to use the path that makes the calculation of $\vec{B}$ as easy as possible.



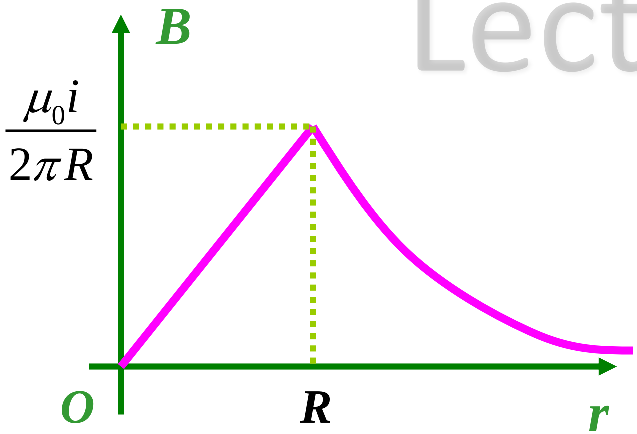
Magnetic Field Inside a Long Straight Wire

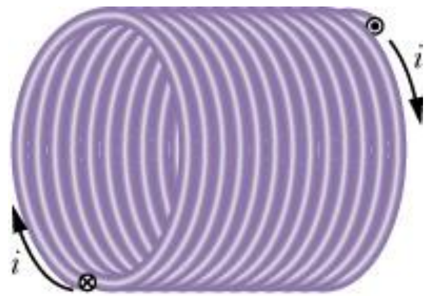
We assume that the distribution of the current within the cross-section of the wire is uniform. The wire carries a current i and has radius R. We choose an Amperian loop that is a circle of radius r ($r < R$) with its center on the wire. The magnetic field is tangent to the loop and has a constant magnitude B:

$$\oint \vec{B} \cdot d\vec{s} = \oint B\, ds \cos 0 = B \oint ds = 2\pi r B = \mu_0 i_{enc}$$

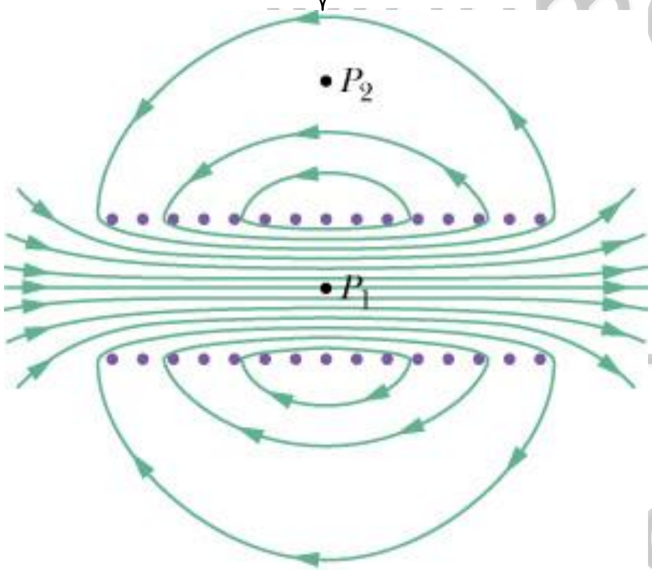
$$i_{enc} = i \frac{\pi r^2}{\pi R^2} = i \frac{r^2}{R^2}$$

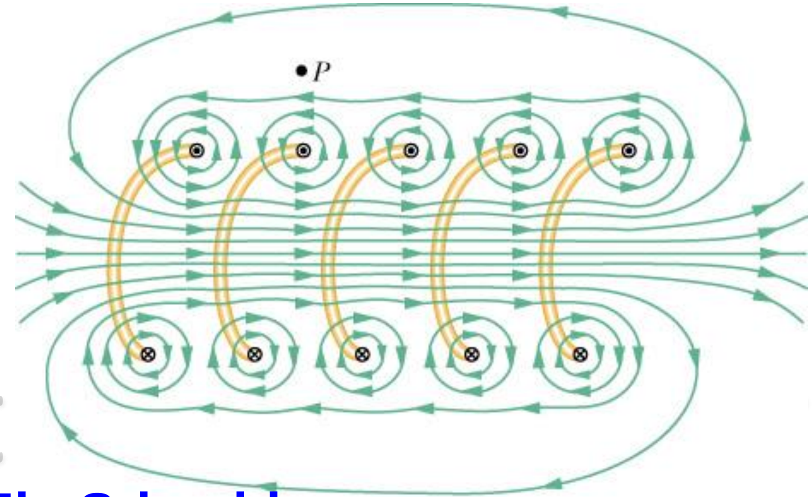
$$2\pi r B = \mu_0 i \frac{r^2}{R^2} \rightarrow B = \left(\frac{\mu_0 i}{2\pi R^2} \right) r$$





Outside field is almost zero

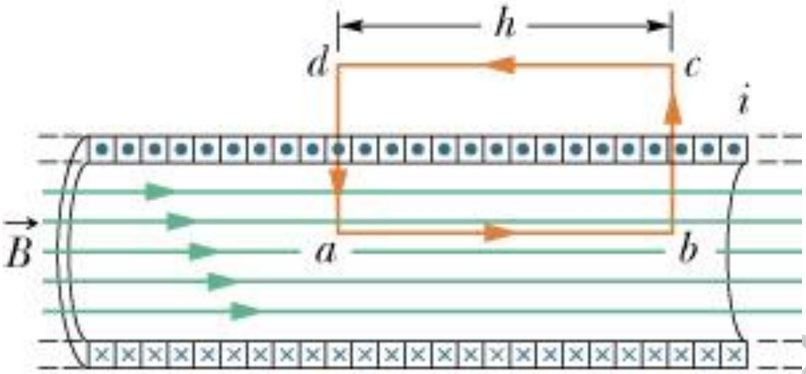




The Solenoid

The solenoid is a long, tightly wound helical wire coil in which the coil length is much larger than the coil diameter. Viewing the solenoid as a collection of single circular loops, one can see that the magnetic field inside is approximately uniform.

The magnetic field inside the solenoid is parallel to the solenoid axis. The direction of $\vec{B}$ can be determined using the right-hand rule. We curl the fingers of the right hand along the direction of the current in the coil windings. The thumb of the right hand points along $\vec{B}$. The magnetic field outside the solenoid is much weaker and can be taken to be approximately zero.



$$B = \mu_0 n i$$

We will use Ampere's law to determine the magnetic field inside a solenoid. We assume that the magnetic field is uniform inside the solenoid and zero outside. We assume that the solenoid has *n* turns per unit length.

We will use the Amperian loop *abcd*. It is a rectangle with its long side parallel to the solenoid axis. One long side (*ab*) is inside the solenoid, while the other (*cd*) is outside:

$$\oint \vec{B}\cdot d\vec{s} = \int_a^b \vec{B}\cdot d\vec{s} + \int_b^c \vec{B}\cdot d\vec{s} + \int_c^d \vec{B}\cdot d\vec{s} + \int_d^a \vec{B}\cdot d\vec{s}$$

Magnetic field outside the solenoid is assumed to be ZERO!!!

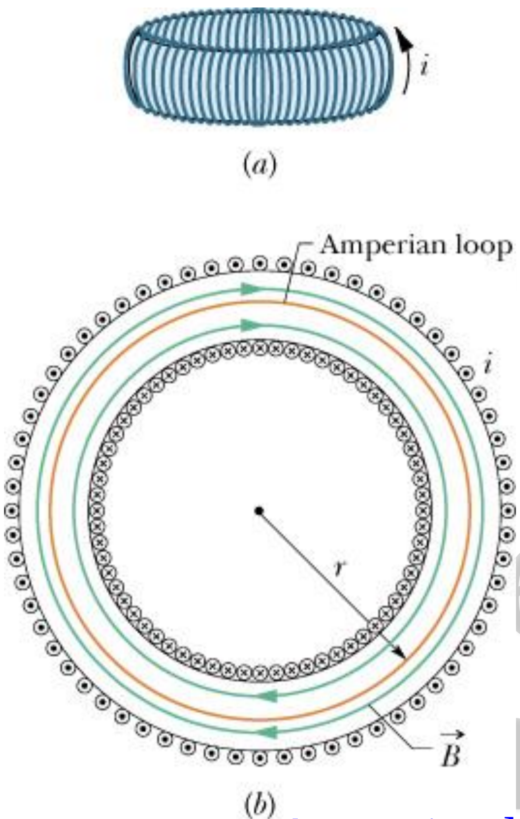
$$\int_a^b \vec{B}\cdot d\vec{s} = \int_a^b B\,ds\cos 0 = B\int_a^b ds = Bh \qquad \int_b^c \vec{B}\cdot d\vec{s} = \int_c^d \vec{B}\cdot d\vec{s} = \int_d^a \vec{B}\cdot d\vec{s} = 0$$

$$\rightarrow \oint \vec{B}\cdot d\vec{s} = Bh$$

The enclosed current $i_{enc} = nhi$.

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{enc} \rightarrow Bh = \mu_0 nhi \rightarrow B = \mu_0 ni$$

$$B = \frac{\mu_o N i}{2\pi r}$$



We use an Amperian loop that is a circle of radius r (orange circle in the figure):

$\oint \vec{B} \cdot d\vec{s} = \oint B\,ds\cos 0 = B\oint ds = 2\pi r B.$ The enclosed current $i_{enc} = Ni$.

Thus: $2\pi r B = \mu_0 N i \rightarrow B = \dfrac{\mu_0 N i}{2\pi r}$.

Note: The magnetic field inside a toroid is not uniform.