

Ödev 3 Cevapları

1) $S = \{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq 2\} \subseteq \mathbb{R}^2$ olsun

$$X_1: S \rightarrow \mathbb{R}$$

$$(x, y) \mapsto xy + 1$$

$$X_2: S \rightarrow \mathbb{R}$$

$$(x, y) \mapsto xy - 1$$

surekli fonksiyonlardır. Böylece ölçülebilirlerdir
ve $\vec{X} = (X_1, X_2)$ rastgele vektör olur.

$$X(S) = \{(x, y) : 1 \leq x \leq 3, -1 \leq y \leq 1\}$$

$$F(r_1, r_2) = P(X \leq r)$$

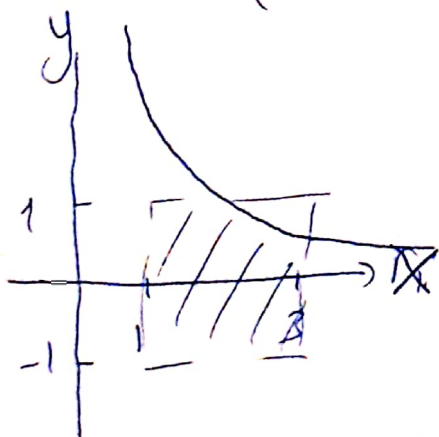
$$= P(\{s \in S : X_1(s) \leq r_1 \wedge X_2(s) \leq r_2\})$$

$$= P(\{s \in S : xy + 1 \leq r_1 \wedge xy - 1 \leq r_2\})$$

$$= P(\{s \in S : y \leq \frac{r_1 - 1}{x} \wedge y \leq \frac{r_2 + 1}{x}\})$$

$$= P(\{s \in S : y \leq \frac{1}{x} \min\{r_1 - 1, r_2 + 1\}\})$$

$$= \text{Alan}(\text{Torel. bölge})$$



$$F(r_1, r_2) = \begin{cases} 0, & r_1 \leq 1 \text{ veya } r_2 \leq -1 \\ \text{Alan}, & \text{diğer} \\ 1, & r_1 \geq 3 \text{ ve } r_2 \geq 1 \end{cases}$$

Ölçülebilirlik değilim fonksiyonu

$$P: \mathcal{B}(\mathbb{R}^2) \rightarrow [0, 1]$$

$$A \mapsto P(A) = P(\{s \in S : (xy + 1, xy - 1) \in A\})$$

$$\textcircled{2} S = \{ (0, 2, 2), (1, 1, 2), (1, 2, 1), (2, 0, 2), (2, 2, 0), (2, 1, 1), (3, 0, 1), (3, 1, 0) \}$$

a)

$$\Sigma \sim \mathcal{P}(S)$$

$$1 = \sum_{r \in R} p(r) = c \left[\binom{3}{0} \binom{2}{2} \binom{2}{2} + \binom{3}{1} \binom{2}{1} \binom{2}{2} + \right.$$

$$\left. \binom{3}{1} \binom{2}{2} \binom{2}{1} + \binom{3}{2} \binom{2}{0} \binom{2}{2} + \binom{3}{2} \binom{2}{2} \binom{2}{0} \right.$$

$$\left. + \binom{3}{2} \binom{2}{1} \binom{2}{1} + \binom{3}{3} \binom{2}{0} \binom{2}{1} + \binom{3}{3} \binom{2}{1} \binom{2}{0} \right]$$

$$= c [1 + 6 + 6 + 3 + 3 + 2 + 2 + 2]$$

$$= 35c \Rightarrow c = 1/35$$

$$b) S_{X_i} = \{0, 1, 2, 3\}$$

$$p(0) = \frac{1}{35} \left[\binom{3}{0} \binom{2}{2} \binom{2}{2} \right]$$

$$p(1) = \frac{1}{35} \left[\binom{3}{1} \binom{2}{1} \binom{2}{2} + \binom{3}{1} \binom{2}{2} \binom{2}{1} \right]$$

$$p(2) = \frac{1}{35} \left[\binom{3}{2} \binom{2}{0} \binom{2}{2} + \binom{3}{2} \binom{2}{2} \binom{2}{0} + \binom{3}{2} \binom{2}{1} \binom{2}{1} \right]$$

$$p(3) = \frac{1}{35} \left[\binom{3}{3} \binom{2}{0} \binom{2}{1} + \binom{3}{3} \binom{2}{1} \binom{2}{0} \right]$$



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$\textcircled{2}$

$$P_{T_X}(r_i) = P_X(r_i) = \begin{cases} \frac{1}{35}, & r_i = 0 \\ \frac{12}{35}, & r_i = 1 \\ \frac{18}{35}, & r_i = 2 \\ \frac{4}{35}, & r_i = 3 \\ 0, & \text{difer} \end{cases}$$

$$c) S_{(X_1, X_2)} = \{ (0, 2), (1, 1), (1, 2), (2, 0), (2, 2), (2, 1), (3, 0), (3, 1) \}$$

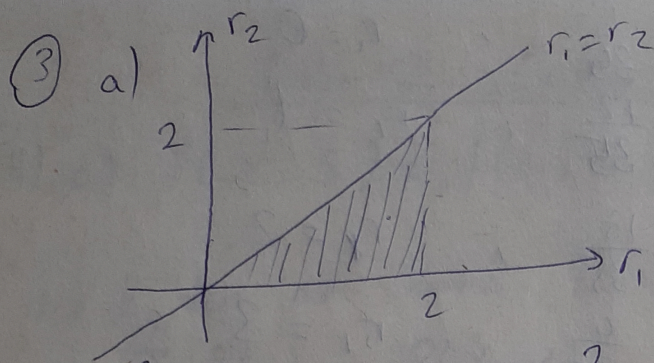
$$P_{T_{(X_1, X_2)}}(r_1, r_2) = P_{(X_1, X_2)}(r_1, r_2) = \begin{cases} \frac{1}{35}, & (r_1, r_2) = (0, 2) \\ \frac{6}{35}, & (r_1, r_2) = (1, 1) \\ \frac{6}{35}, & (r_1, r_2) = (1, 2) \\ \frac{3}{35}, & (r_1, r_2) = (2, 0) \\ \frac{3}{35}, & (r_1, r_2) = (2, 2) \\ \frac{12}{35}, & (r_1, r_2) = (2, 1) \\ \frac{2}{35}, & (r_1, r_2) = (3, 0) \\ \frac{2}{35}, & (r_1, r_2) = (3, 1) \\ 0, & \text{difer} \end{cases}$$



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(3)



$$1 = \int_0^2 \int_0^{r_1} a \, dr_2 \, dr_1 = \int_0^2 a r_1 \, dr_1 = a \frac{r_1^2}{2} \Big|_0^2 = 2a$$

$$\Rightarrow \boxed{a = 1/2}$$

$$b) \quad p_{f_{X_1}}(r_1) = \begin{cases} \int_0^{r_1} \frac{1}{2} \, dx_2 & , 0 \leq r_1 \leq 1 \\ 0, & \text{diper} \end{cases}$$

$$= \begin{cases} \frac{r_1}{2} & , 0 \leq r_1 \leq 1 \\ 0, & \text{diper} \end{cases}$$

$$p_{f_{X_2}}(r_2) = \begin{cases} \int_{r_2}^2 \frac{1}{2} \, dx_1 & , 0 \leq r_2 \leq 1 \\ 0, & \text{diper} \end{cases}$$

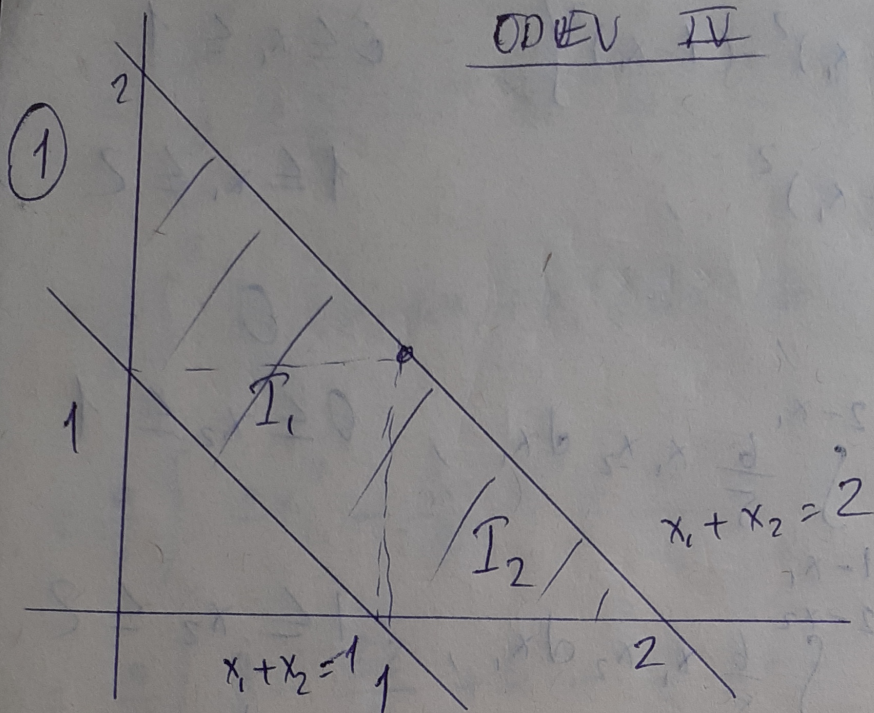
$$= \begin{cases} 1 - \frac{r_2}{2} & , 0 \leq r_2 \leq 1 \\ 0, & \text{diper} \end{cases}$$



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④



$$I = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} a x_1 x_2 dx_1 dx_2$$

$$= \int_0^1 \int_{1-x_1}^{2-x_1} a x_1 x_2 dx_2 dx_1 + \int_1^2 \int_0^{2-x_1} a x_1 x_2 dx_2 dx_1$$

$$= \int_0^1 \left(a x_1 \left[\frac{x_2^2}{2} \right]_{1-x_1}^{2-x_1} \right) dx_1 + \int_1^2 \left(a x_1 \left[\frac{x_2^2}{2} \right]_0^{2-x_1} \right) dx_1$$

$$= \int_0^1 \frac{a x_1}{2} \left[(2-x_1)^2 - (1-x_1)^2 \right] dx_1 + \int_1^2 \frac{a x_1}{2} (2-x_1)^2 dx_1$$

$$= \begin{cases} \frac{3}{5} x_1 \left[(2-x_1)^2 - (1-x_1)^2 \right], & 0 \leq x_1 \leq 1 \\ \frac{3}{5} x_1 (2-x_1)^2, & 1 \leq x_1 \leq 2 \\ 0 & \text{diğer} \end{cases}$$

$$p_{f_{x_2}}(x_2) = \begin{cases} \int_{2-x_1}^{2-x_2} \frac{6}{5} x_1 x_2 dx_1, & 0 \leq x_2 \leq 1 \\ \int_{1-x_2}^{2-x_2} \frac{6}{5} x_1 x_2 dx_1, & 1 \leq x_2 \leq 2 \\ 0 & \text{diğer} \end{cases}$$

ve $p_{f_{x_2}} \neq p_{f_{x_1}} \cdot p_{f_{x_2}}$ bağımsız değildir

$$2) p_{f_{x_1}}(x_1) = \begin{cases} \frac{9}{36}, & x_1 = 1 \\ \frac{12}{36}, & x_1 = 2 \\ \frac{15}{36}, & x_1 = 3 \\ 0 & \text{diğer} \end{cases}$$

$$p_{f_{x_2}}(x_2) = \begin{cases} \frac{9}{36}, & x_2 = -3 \\ \frac{12}{36}, & x_2 = -2 \\ \frac{15}{36}, & x_2 = -1 \\ 0 & \text{diğer} \end{cases}$$



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$$p_{f_{x_1}}(1) \neq \frac{2}{36} \neq \frac{9}{36} \cdot \frac{15}{36} \quad (3)$$

$$= p_{f_{x_1}}(1) p_{f_{x_2}}(1)$$

$$\begin{aligned}
&= \int_0^2 \frac{ax_1}{2} (2-x_1)^2 dx_1 - \int_0^1 \frac{ax_1}{2} (1-x_1)^2 dx_1 \\
&= \frac{a}{2} \int_0^2 x_1 (4-4x_1+x_1^2) dx_1 - \frac{a}{2} \int_0^1 x_1 (1-2x_1+x_1^2) dx_1 \\
&= \frac{a}{2} \left[2x_1^2 - \frac{4}{3}x_1^3 + \frac{x_1^4}{4} \right]_0^2 - \left[\frac{x_1^2}{2} - \frac{2}{3}x_1^3 + \frac{x_1^4}{4} \right]_0^1 \\
&= \frac{a}{2} \left[8 - \frac{32}{3} + 4 - \frac{1}{2} + \frac{2}{3} - \frac{1}{4} \right] \\
&= \frac{a}{2} \left[\frac{4}{3} + \frac{2}{3} - \frac{3}{4} \right] = \frac{a}{2} \cdot \frac{5}{3} = 1 \Rightarrow \boxed{a = \frac{6}{5}}
\end{aligned}$$

$$b) p_{f_{x_1}}(x_1) = \begin{cases} \int_{1-x_1}^{2-x_1} \frac{6}{5} x_1 x_2 dx_2, & 0 \leq x_1 \leq 1 \\ \int_0^{2-x_1} \frac{6}{5} x_1 x_2 dx_2, & 1 \leq x_1 \leq 2 \\ 0, & \text{diger} \end{cases}$$

$$= \begin{cases} \frac{3}{5} x_1 x_2^2 \Big|_{1-x_1}^{2-x_1}, & 0 \leq x_1 \leq 1 \\ \frac{3}{5} x_1 x_2^2 \Big|_0^{2-x_1}, & 1 \leq x_1 \leq 2 \\ 0, & \text{diger} \end{cases}$$



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3) İlk önce (X_1, X_3) 'ün marginal yoğunluk fonksiyonunu bulalım

$$f_{(X_1, X_3)}(x_1, x_3) = \begin{cases} \int_0^1 (x_1 + x_2) e^{-x_3} dx_2, & 0 < x_1 < 1, x_3 > 0 \\ 0, & \text{diğer} \end{cases}$$

$$= \begin{cases} (x_1 + \frac{1}{2}) e^{-x_3}, & 0 < x_1 < 1, x_3 > 0 \\ 0, & \text{diğer} \end{cases}$$

ve

$$f_{X_2|X_1=\frac{1}{3}, X_3=3}(x_2) = \begin{cases} \frac{f(\frac{1}{3}, x_2, 3)}{f_{(X_1, X_3)}(\frac{1}{3}, 3)}, & 0 < x_2 < 1 \\ 0, & \text{diğer} \end{cases}$$

$$= \begin{cases} \frac{(\frac{1}{3} + x_2) e^{-3}}{\frac{5}{6} e^{-3}}, & 0 < x_2 < 1 \\ 0, & \text{diğer} \end{cases}$$

b) $P\left(X \in \left[\frac{1}{2}, \frac{3}{4}\right]\right) = \int_{\frac{1}{2}}^{\frac{3}{4}} \frac{6}{5} \left(\frac{1}{3} + x_2\right) dx_2$

$$= \frac{6}{5} \left(\frac{1}{3} x_2 + \frac{x_2^2}{2} \right) \Big|_{\frac{1}{2}}^{\frac{3}{4}}$$



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