ÖDEV VI Cevapları

1) $$pf_X(r) = \begin{cases} \frac{1}{4} e^{-r/4}, & r \geq 0 \\ 0, & \text{diğer} \end{cases}$$

a) $$P(X \geq 5) = \int_5^{\infty} pf_X(r)\,dr$$

$$= \int_5^{\infty} \frac{1}{4} e^{-r/4}\,dr = e^{-5/4}$$

b) $$P(X \geq 15 \mid X \geq 10) = \frac{P(X \geq 15 \text{ ve } X \geq 10)}{P(X \geq 10)}$$

$$= \frac{\int_{15}^{\infty} \frac{1}{4} e^{-r/4}\,dr}{\int_{10}^{\infty} \frac{1}{4} e^{-r/4}\,dr} = \frac{e^{-15/4}}{e^{-10/4}} = e^{-5/4}$$

2) X hipergeometrik dağılıma sahiptir

$N = 25 \quad a = 5, \; n = 3$

$$pf_X(r) = \frac{\binom{a}{r}\binom{N-a}{n-r}}{\binom{N}{n}}$$

$$= \frac{\binom{5}{r}\binom{20}{3-r}}{\binom{25}{3}}$$

$$E(X) = \sum_{r=0}^{3} r\, pf_X(r)$$

$$= pf_X(1) + 2pf_X(2) + 3pf_X(3)$$

$$= \frac{1}{\binom{25}{3}} \left[\binom{5}{1}\binom{20}{2} + 2\binom{5}{2}\binom{20}{1} + 3\binom{5}{3}\binom{20}{0} \right]$$

$$= \frac{1}{\frac{25!}{3!\,22!}} \left[5 \cdot \frac{20!}{2!\,18!} + 2\frac{5!}{3!\,2!} \cdot 20 + 3\frac{5!}{3!\,2!} \cdot 1 \right]$$

$$= \frac{1}{\frac{25 \cdot \cancel{24}^{4} \cdot 23}{\cancel{3} \cdot \cancel{2}}} \left[5 \; \frac{20 \cdot 19}{2} + 2\frac{5 \cdot 4}{2}\; 20 + 3\frac{5 \cdot 4}{2} \right]$$

$$= \frac{1}{25 \cdot 23 \cdot 4} \left[190 \cdot 5 + 2 \cdot 200 + 3 \cdot 10 \right] = 0{,}6$$

$$E(X^2) = \sum_{r=0}^{3} r^2\, pf_X(r)$$

$$= \frac{1}{\binom{25}{3}} \left[\binom{5}{1}\binom{20}{2} + 4\binom{5}{2}\binom{20}{1} + 9\binom{5}{3}\binom{20}{0} \right]$$

$$= \frac{1}{25 \cdot 23 \cdot 4} \left[190 \cdot 5 + 4 \cdot 200 + 9 \cdot 10 \right] = 0.8$$

$$Var(X) = E(X^2) - (E(X))^2 = 0.44$$

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3) X geometrik dağılıma sahiptir.

$$pf_X(r) = q^{r-1} p = 0{,}75\,(0{,}25)^{r-1} \qquad r=1,2,\dots$$

$$E(X) = \sum_{r=1}^{\infty} r\, pf_X(r)$$

$$= 0{,}75 \cdot \sum_{r=1}^{\infty} \frac{r}{4^{r-1}} = 0{,}75 \left(\frac{1}{1-\cancel{x}}\right)'\Bigg|_{x=0{,}25}$$

$$= 0{,}75 \left(\frac{1}{(1-x)^2}\right)\Bigg|_{x=0{,}25} = \frac{0{,}75}{(0{,}75)^2}$$

$$= 4/3$$

$$E(X^2) = \sum_{r=1}^{\infty} r^2 pf_X(r) = \sum_{r=1}^{\infty} r^2 \frac{1}{4^{r-1}}$$

$$1 + x + x^2 + \dots = \frac{1}{1-x}, \quad |x|<1$$

$$\Rightarrow 1 + 2x + 3x^2 + \dots = \frac{1}{(1-x)^2}, \quad |x|<1$$

$$\Rightarrow x + 2x^2 + 3x^3 + \dots = \frac{x}{(1-x)^2}, \quad |x|<1$$

$$\Rightarrow 1 + 2^2 x + 3^2 x^2 + \dots = \frac{(1-x)^2 + x\,2(1-x)}{(1-x)^4}$$

$$\Rightarrow \sum_{r=1}^{\infty} r^2 x^{r-1} = \frac{1+x}{(1-x)^3}, \quad |x|<1.$$

$$\mathrm{Var}\,X = \frac{0,25}{(0,75)^2} = \frac{1/4}{(3/4)^2} = \frac{4}{9}$$

a) $K = \text{Kazanç} = 100 - 10X$

$$E(K) = 100 - 10\,E(X)$$
$$= 100 - 10\,\frac{4}{3} = \frac{260}{3}$$

$$\mathrm{Var}(K) = \mathrm{Var}(100) - \mathrm{Var}(10.X)$$
$$= 100\ \mathrm{Var}(X) = \frac{400}{9}$$

b) Oyunun dürüst olması için kazancın beklenen değeri sıfır olmalıdır.

$$K = a - 10X \quad , E(K) = 0$$
$$E(K) = a - 10\,E(X)$$
$$0 = a - 10 . 4/3 \Rightarrow a = \frac{40}{3}\ \text{TL}$$

olmalıdır.

4) $P(180 < S_{400} < 200)$

$$\approx F_{N_{0,1}}\left(\frac{200 - 400.0.5}{\sqrt{100}}\right) - F_{N_{0,1}}\left(\frac{180 - 400\ 0.5}{\sqrt{100}}\right)$$

$$= F_{N_{0,1}}(0) - F_{N_{0,1}}(-2) = 0.5 - 1 + F_{N_{0,1}}(2)$$
$$= 0,4772$$

5) $\operatorname{Var} X = 1$, $\operatorname{Var} Y = \dfrac{(1-(-2))^2}{12} = \dfrac{9}{12} = \dfrac{3}{4}$

$EX = -2$, $EY = \dfrac{1+(-2)}{2} = -1/2$

$$\operatorname{Var} XY = 1 \frac{3}{4} + 1\left(-\frac{1}{2}\right)^2 + \frac{3}{4}(-2)^2$$

$$= \frac{3}{4} + \frac{1}{4} + \frac{12}{4} = \frac{16}{4} = 4$$