Week I: Introduction

Hakan Dogan

Department of Mechanical Engineering

Hacettepe University

MMU604(MMU703) Analytical Methods in Engineering (II)
24-25 Spring

Syllabus and helpful information

- Textbook
- Course outline
- Marking
- Attendance

Definition

- **What is a Differential Equation (DE)?**
  An equation that contains derivative(s) of unknown function.

$$y'' + 2y' = 3y \tag{1}$$

Definition

- **What is a Differential Equation (DE)?**
  An equation that contains derivative(s) of unknown function.

$$y'' + 2y' = 3y \tag{1}$$

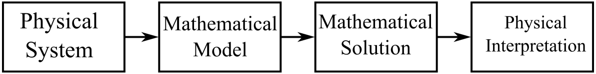
- **Why is it important to solve one?**
  They are really helpful for modeling and understanding how things work.

Definition

- **What is a Differential Equation (DE)?**
  An equation that contains derivative(s) of unknown function.

$$y'' + 2y' = 3y \tag{1}$$

- **Why is it important to solve one?**
  They are really helpful for modeling and understanding how things work.

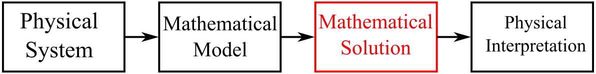


Definition

- **What is a Differential Equation (DE)?**
  An equation that contains derivative(s) of unknown function.

$$y'' + 2y' = 3y \tag{1}$$

- **Why is it important to solve one?**
  They are really helpful for modeling and understanding how things work.

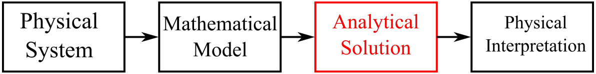


Definition

- **What is a Differential Equation (DE)?**
  An equation that contains derivative(s) of unknown function.

$$y'' + 2y' = 3y \tag{1}$$

- **Why is it important to solve one?**
  They are really helpful for modeling and understanding how things work.



DEs in engineering

- Failing stone

$$y'' = g$$

- Skydiving

$$mv' = mg - bv^2$$

- Vibrating mass on a spring

$$my'' + ky = 0$$

- Current I in an RCL circuit

$$LI'' + RI' + \frac{1}{C}I = E'$$

- Deformation on a beam

$$EIy^{IV} = f(x)$$

- Pendulum

$$L\theta'' + gsin\theta = 0$$

DEs in life

MATHEMATICS MAGAZINE
VOL. 61, NO. 1, FEBRUARY 1988 35

Love Affairs and Differential Equations

STEVEN H. STROGATZ
Harvard University
Cambridge, MA 02138

The purpose of this note is to suggest an unusual approach to the teaching of some standard material about systems of coupled ordinary differential equations. The approach relates the mathematics to a topic that is already on the minds of many college students: the time-evolution of a love affair between two people. Students seem to enjoy the material, taking an active role in the construction, solution, and interpretation of the equations.

The essence of the idea is contained in the following example.

Juliet is in love with Romeo, but in our version of this story, Romeo is a fickle lover. The more Juliet loves him, the more he begins to dislike her. But when she loses interest, his feelings for her warm up. She, on the other hand, tends to echo him: her love grows when he loves her, and turns to hate when he hates her.

A simple model for their ill-fated romance is

$$dr/dt = -aj, \qquad dj/dt = br,$$

where

$$r(t) = \text{Romeo's love/hate for Juliet at time } t$$
$$j(t) = \text{Juliet's love/hate for Romeo at time } t.$$

Positive values of r, j signify love, negative values signify hate. The parameters a, b are positive, to be consistent with the story.

The sad outcome of their affair is, of course, a neverending cycle of love and hate; their governing equations are those of a simple harmonic oscillator. At least they manage to achieve simultaneous love one-quarter of the time.

Types of DE

- *Ordinary Differential Equation (ODE):* has one or several derivatives of an unknown function of a **single** variable.
  1. $y' = cosx$
  2. $y'' + 9y = e^{-2x}$
  3. $y'y''' - \frac{3}{2}y'^2 + 5 = 0$

Types of DE

- *Ordinary Differential Equation (ODE):* has one or several derivatives of an unknown function of a **single** variable.
- *Partial Differential Equation (PDE):* involves partial derivatives of an unknown function of **two or more** variables.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

PDEs>ODEs

Linear DEs

The most general linear ODE of the n_{th} order is:

$$\frac{d^n y}{dx^n} + a_0(x)\frac{d^{n-1} y}{dx^{n-1}} + \ldots + a_{n-1}(x)\frac{dy}{dx} + a_n(x)y = h(x) \qquad (2)$$

No power or no product of dependent variable(s) or derivative(s).

Linear DEs

- No power or no product of dependent variable(s) or derivative(s).
  1. $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y^2 = 0$
  2. $\frac{d^2y}{dx^2} + 5\left(\frac{dy}{dx}\right)^3 + 6y = 0$
  3. $\frac{d^2y}{dx^2} + 5y\frac{dy}{dx} + 6y = 0$
  4. $\left(\frac{d^2y}{dx^2}\right)^2 = \frac{\partial^2 y}{\partial x^2}$

Linear DEs

- No power or no product of dependent variable(s) or derivative(s).
  1. $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y^2 = 0$
  2. $\frac{d^2y}{dx^2} + 5\left(\frac{dy}{dx}\right)^3 + 6y = 0$
  3. $\frac{d^2y}{dx^2} + 5y\frac{dy}{dx} + 6y = 0$
  4. $\left(\frac{d^2y}{dx^2}\right)^2 = \frac{\partial^2 y}{\partial x^2}$

  Nonlinear DEs

Linear DEs

- No power or no product of dependent variable(s) or derivative(s).
  1. $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y^2 = 0$
  2. $\frac{d^2y}{dx^2} + 5\left(\frac{dy}{dx}\right)^3 + 6y = 0$
  3. $\frac{d^2y}{dx^2} + 5y\frac{dy}{dx} + 6y = 0$
  4. $\left(\frac{d^2y}{dx^2}\right)^2 = \frac{\partial^2 y}{\partial x^2}$

  Nonlinear DEs
- Linear DEs
  1. $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0$ (Constant coefficients)
  2. $\frac{d^4y}{dx^4} + x^2\frac{d^3y}{dx^3} + x^3\frac{dy}{dx} = xe^x$ (Variable coefficients)

- **Solutions:** Function(s), *not a value or set of values*
- Analytical solutions $\rightarrow$ a certain group of DEs
- Fast and more sophisticated