Notes: i) Use maximum 3 digits for decimal parts of numbers (e.g., 25.127)
ii) Analytical solutions are required for all questions. You can use graphics to control your calculations.
(25 p) 1. Calculate the area of the following ABCD polygon by using triangulation method.
Hint: Area of a triangle $=1 / 2\left[S_{1} \cdot S_{2} \cdot \operatorname{Sin}\left(\alpha_{1}-\alpha_{2}\right)\right]$


## SOLUTION:

Area of $I A B=1 / 2\left[120 \mathrm{~m}^{*} 70 \mathrm{~m} * \operatorname{Sin}(320-260)\right]=3637.3 \mathrm{~m}^{2}$
Area of IAD $=1 / 2\left[120 \mathrm{~m}^{*} 55 \mathrm{~m} * \operatorname{Sin}(260-210)=2527.9 \mathrm{~m}^{2}\right.$
Area of IBC $=1 / 2\left[60 \mathrm{~m}^{*} 70 \mathrm{~m} * \operatorname{Sin}(320-285) \quad=1204.5 \mathrm{~m}^{2}\right.$
Area of ICD $=1 / 2\left[60 \mathrm{~m}^{*} 55 \mathrm{~m} * \operatorname{Sin}(285-210)=1593.8 \mathrm{~m}^{2}\right.$
Area of $A B C D$ Polygon $=I A B+I A D-I B C-I C D=6165.2-2798.3=3366.9 \mathbf{m}^{2}$
(15 p) 2. Determine the bearing and the azimuth angles of the lines $A B$ in each of the following figures. (Show your drawings and calculations)

(a)

(b)

(c)

## SOLUTION:

a) Azimuth $\mathrm{AB}=180^{\circ}+138^{\circ}=318^{\circ}$

Bearing (NW quadrant) $=\mathrm{N}\left(360^{\circ}-318^{\circ}\right) \mathrm{W}=\mathrm{N} 42^{\circ} \mathrm{W}$
b) Azimuth $\mathrm{AB}=270^{\circ}-30^{\circ}=240^{\circ}$
c) Azimuth $\mathrm{AB}=360^{\circ}-\left(90^{\circ}-70^{\circ}\right)=340^{\circ}$

Bearing (SW quadrant) $=\mathrm{S}(90-30) \mathrm{W}=\mathrm{S} 60^{\circ} \mathrm{E}$
Bearing (NW quadrant) $=\mathrm{N}\left(360^{\circ}-340^{\circ}\right) \mathrm{W}=\mathrm{N} 20^{\circ} \mathrm{W}$
(10 p) 3. If the elevations of two points are known as well as the horizontal distance between them, determine the slope (in degree) from point $A$ to $B$.


## SOLUTION:

Elevation difference $=241.65-265.32=-23.67 \mathrm{~m}$ (since point $B$ is lower)
Distance $=327-150=177 \mathrm{~m}$
Slope (Gradient) $=(-23.67 / 177)^{\star} 100=-13.37 \%$ or $\tan ^{-1}(23.67 / 177)=-7.6$ degree
(20 p) 4. Interior angles of a triangle are obtained as follows. If bearing of side $A B$ is $N 30^{\circ} \mathrm{W}$, determine azimuths and bearings of each side (move counterclockwise direction) after correcting the angles.

$$
\begin{array}{ll}
\text { Interior angle at point A } & : 70^{\circ} 12^{\prime} \\
\text { Interior angle at point B } & : 98^{\circ} 58^{\prime} \\
\text { Interior angle at point C } & : 10^{\circ} 38^{\prime}
\end{array}
$$

## SOLUTION:

Sum of interior angles $(A+B+C)=178^{\circ} 108^{\prime}=179^{\circ} 48^{\prime}$
Error $=179^{\circ} 48^{\prime}-179^{\circ} 60^{\prime}=-12^{\prime}$
Correction (equal) $=12 / 3=4$ ' for each angle
Corrected angle $A=70^{\circ} 12^{\prime}+4^{\prime}=70^{\circ} 16^{\prime}$
Corrected angle $B=98^{\circ} 58^{\prime}+4^{\prime}=99^{\circ} 02^{\prime}$
Corrected angle $C=10^{\circ} 38^{\prime}+4^{\prime}=10^{\circ} 42^{\prime}$
Bearing $A B=N 30^{\circ} \mathrm{W}$ (given)


Azimuth $A B=360^{\circ}-30^{\circ}=330^{\circ}$
A
Azimuth $B A=330^{\circ}-180^{\circ}=150^{\circ}$
Azimuth $B C=150^{\circ}+99^{\circ} 02^{\prime}=\mathbf{2 4 9}^{\circ} \mathbf{0 2}$
Bearing $\mathrm{BC}=\mathrm{S}\left(249^{\circ} 02^{\prime}-180^{\circ}\right) \mathrm{W}=\mathbf{S} 69^{\circ} 02^{\prime} \mathrm{W}$
Azimuth $\mathrm{CB}=69^{\circ} 02^{\prime}$
Azimuth CA $=69^{\circ} 02^{\prime}+10^{\circ} 42^{\prime}=79^{\circ} 44^{\prime}$
Bearing CA $=$ N79 ${ }^{\circ} 44^{\prime} \mathrm{E}$
Azimuth $A C=79^{\circ} 44^{\prime}+180^{\circ}=259^{\circ} 44^{\prime}$
Azimuth $\mathrm{AB}=259^{\circ} 44^{\prime}+70^{\circ} 16^{\prime}=330^{\circ}$ (check)
( 10 p ) 5. The side lengths of a rectangle on a plan are 5 cm and 10 cm . It represents a property with an area of $50 \mathrm{~m}^{2}$. Determine the scale of the plan.

## SOLUTION:

$f / F=1 / M^{2} \rightarrow M=(F / f)^{1 / 2}=\left[500000 \mathrm{~cm}^{2} /\left(5^{\star} 10 \mathrm{~cm}^{2}\right)\right]^{1 / 2}=[100000]^{1 / 2}=100$ then, scale $=1: 100$
(20 p) 6. Following measurements (Azimuth angles and lengths) have been taken from an open traverse measurement. By using given data, determine the distance and azimuth angle of the line AD. Complete the table.

| Course |  | Azimuth <br> (degree) |  | Length <br> (meter) |
| :---: | :---: | :---: | :---: | :---: |
|  |  | 220 |  | 78 |
| BC |  | 80 |  | 152 |
| CD |  | 170 |  | 90 |

## SOLUTION:

Assume point $\mathrm{A}=0 ; 0$
Assume a scale $=1 / 2000$ to draw.
20 meter $=1 \mathrm{~cm}$.
Departure of a side
$\Delta \mathrm{E}=$ Length*sin(Azimuth)
Latitude of a side
$\Delta \mathrm{N}=$ Length* $\cos ($ Azimuth $)$
Easting $\mathrm{B}=$ Easting $\mathrm{A}+\Delta \mathrm{E}$ and so on
Northing $B=$ Northing $A+\Delta N$ and so on

$\Delta E_{A B}=78^{*} \sin 220^{\circ}=-50.14 m$
$E_{B}=0.00+(-50.14)=-50.14 \mathrm{~m}$
$\Delta \mathrm{N}_{\mathrm{AB}}=78^{*} \cos 220^{\circ}=-59.75 \mathrm{~m}$
$N_{B}=0.00+(-59.75)=-59.75 \mathrm{~m}$
$\Delta \mathrm{E}_{\mathrm{BC}}=152^{*} \sin 80^{\circ}=149.69 \mathrm{~m} \quad \Delta \mathrm{~N}_{\mathrm{BC}}=152^{*} \cos 80^{\circ}=26.39 \mathrm{~m}$
$E_{C}=-50.14+149.69=99.55 m \quad N_{C}=-59.75+26.39=-33.36 m$
$\Delta \mathrm{E}_{\mathrm{CD}}=90^{*} \sin 170^{\circ}=15.63 \mathrm{~m} \quad \begin{aligned} & \quad \Delta \mathrm{N}_{\mathrm{CD}}=90^{*} \cos 170^{\circ}=-88.63 \mathrm{~m} \\ & \mathrm{E}_{\mathrm{D}}=99.55+15.63=115.18 \mathrm{~m} \\ & \mathrm{~N}_{\mathrm{D}}=-33.36+(-88.63)=-121.99 \mathrm{~m}\end{aligned}$
Distance $A D=\left[E_{D}{ }^{2}+N_{D}{ }^{2}\right]^{1 / 2}=\left[115.18^{2}+121.99^{2}\right]^{1 / 2}=(28148)^{1 / 2} \quad \Rightarrow$ Distance AD=167.77 meter
$\alpha_{A E}=\tan ^{-1}\left[E_{D} / N_{D}\right]=\tan ^{-1}[115.18 /(-121.99)]=\tan ^{-1}(-0.944)=-43.35^{\circ}$
$\Rightarrow$ Azimuth AD=180+(-43.35) $=136.65^{\circ}$

| Pnt | Side | Length <br> $(\mathrm{m})$ | Azimuth <br> $($ degree $)$ | $\Delta \mathrm{E}(\mathrm{m})$ | $\Delta \mathrm{N}(\mathrm{m})$ | Easting <br> $(\mathrm{m})$ | Northing <br> $(\mathrm{m})$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A |  |  |  |  |  | 0.00 | 0.00 |
|  | AB | 78 | 220 | -50.14 | -59.75 |  |  |
| B |  |  |  |  |  | -50.14 | -59.75 |
|  | BC | 152 | 80 | 149.69 | 26.39 |  |  |
| C |  |  |  |  |  | 99.55 | -33.36 |
|  | CD | 90 | 170 | 15.63 | -88.63 |  |  |
| D |  |  |  |  |  | 115.18 | -121.99 |

