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144
Elevation

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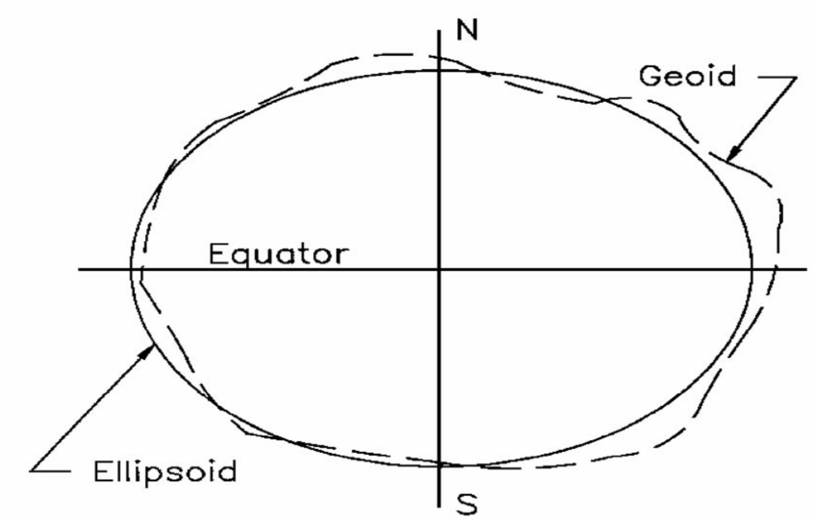
Elevation is the distance above or below a specified reference surface. In engineering and surveying the surface most commonly used to reference elevation is the geoid. The geoid is defined as an equipotential surface that closely approximates **mean sea level**. However, the geoid and mean sea level surfaces are not coincident. They may be separated by a meter or more at any specific location.

By definition, the potential due to gravity is equal at all points on the geoid, and the force of gravity is perpendicular to the geoid at all points. The geoid is an undulating, irregular surface that is affected by density variations within the earth. The geoid is not readily defined by mathematical equations. The mathematical ellipsoid used as a **datum** surface for geodetic position can approximate the geoid, but the ellipsoid and geoid will be separated by up to 100 meters or more for a mean global fit. Figure 144.1 illustrates the general relationship between the geoid and some approximating geodetic ellipsoid.

Figure 144.1 Relationship between geoid and approximating geodetic ellipsoid.



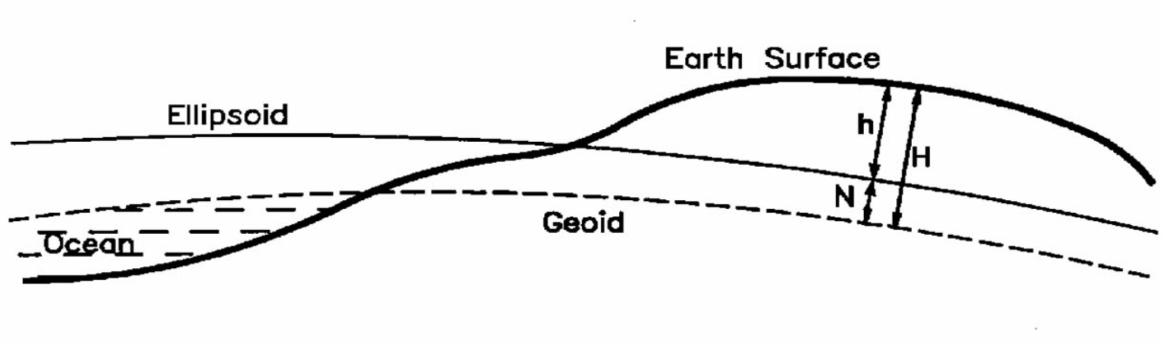
144.1 Measures of Elevation and Height

The vertical distance referenced to the geoid is called an orthometric **height** (elevation), H . Orthometric height is measured along the plumb line. A height referenced to the ellipsoid is called an *ellipsoidal height*, *h*. Ellipsoidal height is measured along the normal to the ellipsoid. Geoid height, *N*, is the distance between the geoid and the ellipsoid measured along the normal to the ellipsoid. Neglecting the deviation between the plumb line and the ellipsoidal normal, the geoid height is related to the orthometric and ellipsoidal heights by the equation

$$h = H + N$$

Thus, Fig. 144.2 shows a negative geoid height, which is typical in the U.S.

Figure 144.2 Illustration of negative geoid height.



Equipotential surfaces (level surfaces) are not parallel to another. The distance between the surfaces decreases toward the earth's north pole. Each level surface has a different gravity potential relative to the geoid that can be expressed as a geopotential number measured in terms of geopotential units: 1 geopotential unit (gpu) = 1000 gal-meters. As an expression of height, the geopotential number (a potential) is constant for a given level surface, whereas the orthometric height decreases for a given level surface proceeding toward the pole. The change in orthometric height can be calculated using the formula found at the end of this chapter. Geopotential numbers can be converted to distance units of height. One possible type of height is defined by dynamic height, H_d, given as

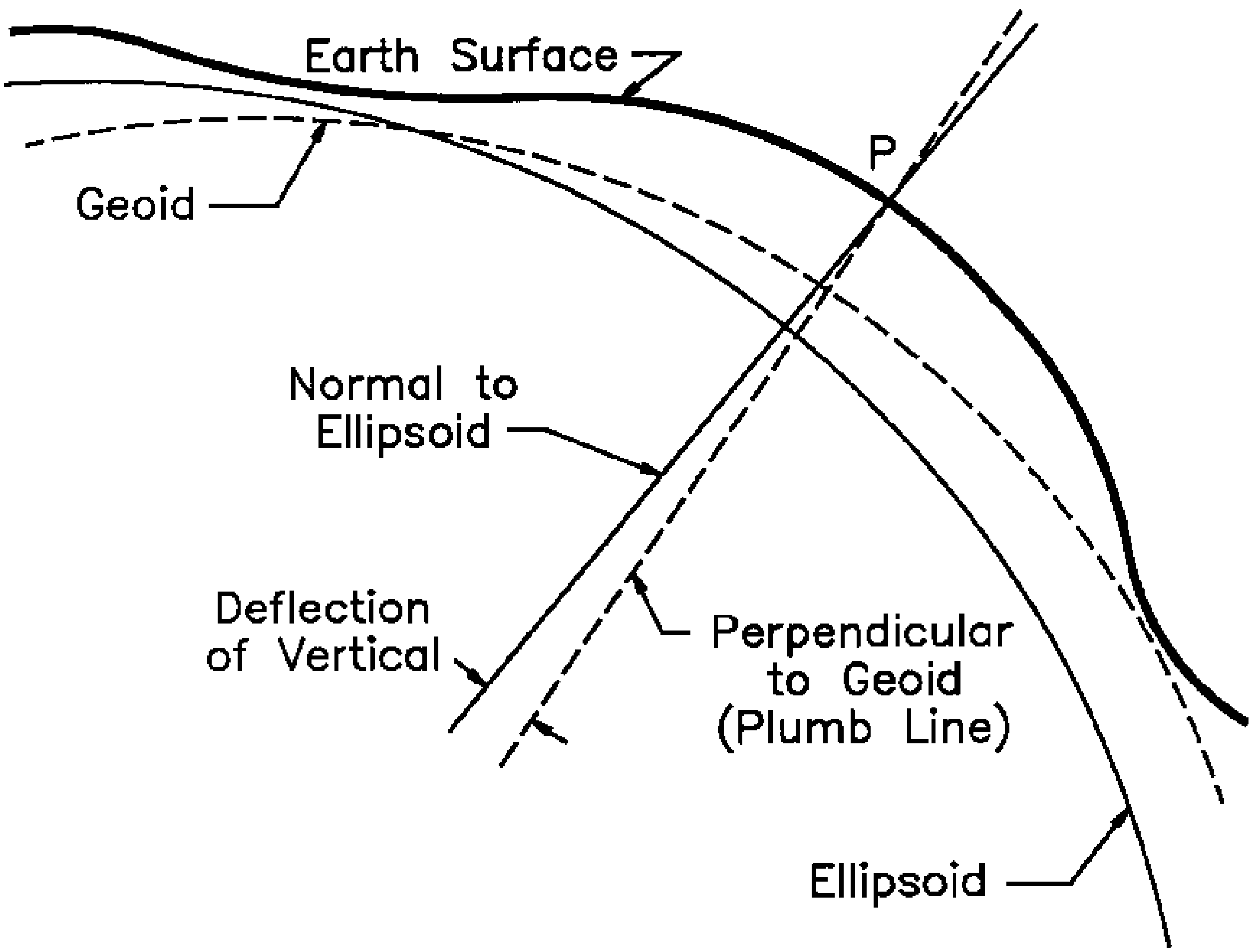
$$H_d = \frac{\mathrm{GPN}}{g_n}$$

where GPN is the geopotential number of the point and g_n is the value of gravity calculated on the ellipsoid at $45°$ latitude using a standard gravity formula.

144.2 Deflection of the Vertical

A **vertical** or plumb line is perpendicular to the geoid and therefore parallel to the direction of gravity. A normal line is perpendicular to the ellipsoid. Because the geoid and ellipsoid are generally not parallel, a vertical line and a normal line are not coincident for most points on the earth's surface. The deflection of the vertical is the angle between these two lines at a point on the earth's surface. Figure 144.3 illustrates the definition of the deflection of the vertical. The deflection of the vertical is generally resolved into two components: ξ in the N-S direction and η in the E-W direction.

Figure 144.3 Definition of the deflection of the vertical.



The geoid has been modeled regionally for the U.S. by the National Geodetic Survey (NGS). The model is known as GEOID93, and it is distributed as a computer program by NGS. The program estimates geoid height differences with a reported accuracy of 10 cm (one standard deviation) over lengths of approximately 100 km.

144.3 Vertical Datums

Vertical datums are defined for the purpose of specifying the elevation of points with respect to the geoid. There are two relevant continental vertical datums within the U.S.: the National Geodetic Vertical Datum of 1929 (NGVD 29) and the North American Vertical Datum of 1988 (NAVD 88).

The National Geodetic Vertical Datum of 1929 (NGVD 29) is a long-used reference for mean sea level in the U.S. NGVD 29 was the product of a 1929 general adjustment of the U.S. and Canadian vertical control networks. The 1929 adjustment was based, in part, upon the assumption that the local mean sea level at the tide stations used in the adjustment was equal (same equipotential surface). This is not a valid assumption since the elevation of mean sea level varies from the Atlantic coast to the Pacific coast of the U.S. This distortion of the vertical datum caused the official name of the 1929 datum to change from "Sea Level Datum of 1929" to "National Geodetic Vertical Datum of 1929" in 1976. Other distortions, including those from upheaval and subsidence of the earth's crust, are present in the NGVD 29; however, it remains a datum of reference for the U.S.

A new adjustment of the vertical datum for North America has been completed recently. This project, known as the North American Vertical Datum of 1988 (NAVD 88), is a least-squares readjustment of over 600 000 **benchmarks** across the North American continent, resulting in a better approximation of the geoid. The project includes re-leveling of approximately 83 000 km of first-order vertical control within the U.S. Leveling data from Canada, Mexico, and Central America is included in the adjustment. The result of the NAVD 88 adjustment is a computer database of vertical control stations and elevations across the U.S. This improved model of the geoid is beneficial for the determination of orthometric heights using Global Positioning System (GPS)–derived heights above the ellipsoid.

The change in elevations from NGVD 29 to NAVD 88 varies, depending upon the area of the country involved. The relative elevation between existing benchmarks will change only by a few millimeters. The absolute elevation of benchmarks may change by as much as a few decimeters. An elevation correction constant between the two datums will suffice for most project areas.

144.4 Elevation Measurement

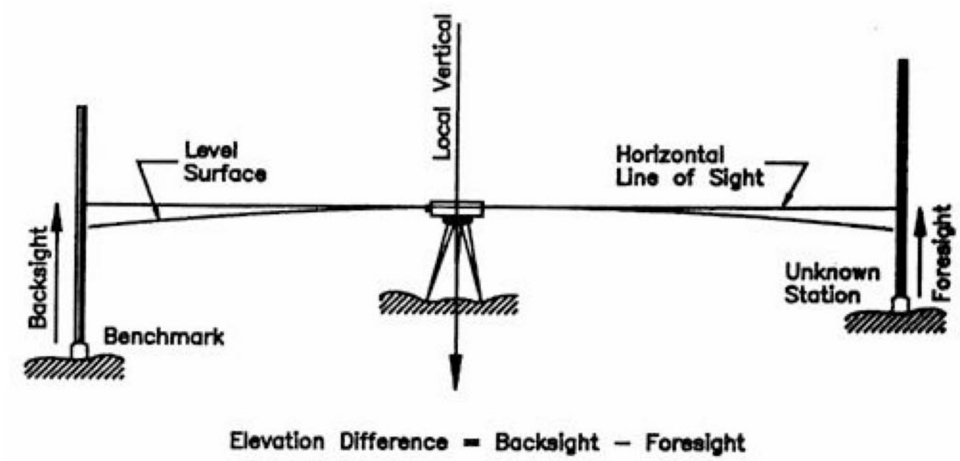
Elevation may be measured by several methods. Some of these methods measure elevation directly, for example, GPS satellite ranging, photogrammetric aerotriangulation, inertial surveying methods, and barometric altimetry. Some of these methods measure the difference in elevation from a reference benchmark to the point to be determined, for example, ordinary and precise differential leveling and trigonometric leveling. It should be noted that the direct methods typically must also be referenced to benchmarks so that translations and rotations can be performed to establish the proper relationship to the elevation datum. Thus the difference in elevation is, fundamentally, the important value to be measured.

Ordinary Differential Leveling

Differential leveling is a very simple process based on the measurement of vertical distances from a horizontal line. Elevations are transferred from one point to another through the process of using a leveling instrument to read a rod held vertically on, first, a point of known elevation and, then, on the point of unknown elevation.

A single-level setup is illustrated in Fig. 144.4. The known elevation of the backsight point is transferred vertically to the line of sight by adding the known elevation and the backsight rod reading. The elevation of the line of sight is the height of instrument, HI. By definition, the line of sight generates a horizontal plane at the instrument location when the telescope is rotated on the vertical axis. The line-of-sight elevation is transferred down to the unknown elevation point by turning the telescope to the foresight, subtracting the rod reading from the height of instrument. Note that the difference in elevation from the backsight station to the foresight station is determined by subtracting the foresight rod reading from the backsight rod reading.

Figure 144.4 Using differential leveling to measure elevation.



A level route consists of several level setups, each one carrying the elevation forward to the next foresight using the differential leveling method. A level route is typically checked by closing on a second known benchmark or by looping back to the starting benchmark. At the closing benchmark, closure = computed elevation − known elevation. Since differential leveling is usually performed with approximately equal setup distances between turning points, the level route is adjusted by distributing the closure equally to each setup:

$$\text{Adjustment} = \left(-\frac{\text{Closure}}{n}\right) \text{ per setup}$$

where n is the number of setups in the route. When a network of interconnected routes is surveyed, a least-squares adjustment is warranted.

Precise Leveling

Precise leveling methods are used when the highest accuracy is required for engineering and surveying work. Instruments used in precise leveling are specifically designed to obtain a high degree of accuracy in leveling. Improved optics in the telescope, improved level sensitivity, and carefully calibrated rod scales are all incorporated into the differential leveling process.

Typically when performing precise leveling, a method of leveling called *three-wire leveling* is used. This involves reading the center cross hair as well as the upper and lower "stadia" cross hairs.

The basic process of leveling is the same as ordinary differential leveling, except that the three cross hair readings are averaged to improve the precision of each backsight and foresight value.

Another method that can be used to improve the precision of the level rod reading is to use an optical micrometer on the telescope. The optical micrometer is a rotating parallel-plate prism attached in front of the objective lens of the level. The prism enables the observer to displace the line of sight parallel with itself and set the horizontal cross hair exactly on the nearest rod graduation. The observer adds the middle cross hair rod reading and the displacement reading on the micrometer to obtain a precise rod reading to the nearest 0.1 millimeter.

Trigonometric Leveling

Trigonometric leveling is a method usually applied when a total station is used to measure the slope distance, S, and the vertical angle, α, to a point. Assuming the total station is set up on a station of known elevation and the height of instrument, HI, and reflector, HR, are measured, the elevation of the unknown station is

$$\begin{aligned} V &= S \sin \alpha \\ P_{\mathrm{Elev}} &= A_{\mathrm{Elev}} + \mathrm{HI} + V - \mathrm{HR} \end{aligned}$$

The precision of trigonometric elevations is determined by the uncertainty in the vertical angle measurement and the uncertainty caused by atmospheric refraction effects. For long lines the effects of earth curvature and atmospheric refraction must be included.

Instruments

Altimeters

Surveying altimeters are precise aneroid barometers that are graduated in feet or meters. As the altimeter is raised in elevation, the barometer senses the atmospheric pressure drop. The elevation is read directly on the face of the instrument. Although the surveying altimeter may be considered to measure elevation directly, best results are obtained if a difference in elevation is observed by subtracting readings between a base altimeter kept at a point of known elevation and a roving altimeter read at unknown points in the area to be surveyed. The difference in altimeter readings is a better estimate of the difference in elevation, since the effects of local weather changes, temperature, and humidity that affect altimeter readings are canceled in the subtraction process. By limiting the distance between base and roving altimeters, accuracies of 3 to 5 feet are possible. Other survey configurations utilizing low and high base stations or leap-frogging roving altimeters can yield good results over large areas.

Level Bubble Instruments

Level bubble instruments, or spirit levels, contain a level vial with a bubble that must be centered to define a horizontal plane. Field instruments consist of three main components: a telescope to define a line of sight and magnify the object sighted, a level vial attached to the telescope to define the orientation of the instrument with respect to gravity, and a leveling head to tilt and orient the instrument.

All level bubble instruments are designed around the same fundamental relationships. These relationships are as follows:

- The axis of the level bubble (or compensator) should be perpendicular to the vertical axis.
- The line of sight should be parallel to the axis of the level bubble.

When these instrument adjustment relationships are true and the instrument is properly set up, the line of sight will sweep out a horizontal plane that is perpendicular to gravity at the instrument location.

Instruments that use a level bubble to orient the axes to the direction of gravity depend on the bubble's sensitivity for accuracy. Level bubble sensitivity is defined as the central angle subtended by an arc of one division on the bubble tube. The smaller the angle subtended is, the more sensitive the bubble is to dislevelment. A bubble division is typically 2 millimeters long, and bubble sensitivity typically ranges from 60 seconds to 1 second.

Builder's Level

The builder's level typically is less precise than other instruments in this category, but it is one of the most inexpensive and versatile instruments that is used by field engineers for construction layout. In addition to being able to perform leveling operations, it can be used to turn angles, and the scope can be tilted for inclined sights.

Transit

Although the primary functions of the transit are for angle measurement and layout, it can also be used for leveling because it has a bubble attached to the telescope. However, the field engineer should be aware that the transit may not be as sensitive and stable as a quality level.

Dumpy Level

The engineer's dumpy-type level has been the workhorse of leveling instruments for more than 150 years. Even with advancements in other leveling instruments, such as the automatic level and the laser, the dumpy may still be the instrument of choice in a construction environment because of its stability.

Automatic Compensator Levels

Compensators were developed about 50 years ago and incorporated into field levels. The compensator is a free-swinging pendulum arrangement in the optical path that maintains a fixed relationship between the line of sight and the direction of gravity. If the instrument is in adjustment, the line of sight will be maintained as a horizontal line. Compensator instruments are extremely fast to set up and level.

Laser Levels

A laser level uses a laser beam directed at a spinning optical reflector. The reflector is oriented so that the rotating laser beam sweeps out a horizontal reference plane. The level rod is equipped with a sensor to detect the rotating beam. By sliding the detector on the rod, a vertical reading can be obtained at the rod point. Laser levels are especially useful on construction sites. The spinning optics can also be oriented to produce a vertical reference plane.

Digital Levels

Digital levels are electronic levels that can be used to more quickly obtain a rod reading and make the reading process more reliable. The length scale on the level rod is replaced by a bar code. The digital level senses the bar code pattern and compares it to a copy of the code held in its internal memory. By matching the bar code pattern, a rod reading length can be obtained. Digital levels are available for ordinary and precise leveling applications.

Level Rods

In addition to the chosen leveling instrument, a level rod is required to be able to transfer elevations from one point to another. The level rod is a graduated length scale affixed to a rod and held vertically on a turning point or benchmark. The scale is read to obtain the vertical distance from the point to the line of sight.

Level rods are graduated in feet, inches, and fractions; feet, tenths, and hundredths; or meters and centimeters. Rods used in ordinary leveling may be multipiece extendable rods with graduations marked directly on the rod material or on a metal strip affixed to the rod for support. Rods used in precise leveling are one-piece rods with a stable invar metal graduated scale supported under constant tension by the rod. A precise rod can be calibrated for changes in length caused by temperature.

Accurate field leveling work is also aided by the use of rod targets, rod levels, and stable turning point pins when required.

144.5 Systematic Errors

Earth Curvature

The curved shape of the earth results in the equipotential surface through the telescope departing from the horizontal plane through the telescope as the line of sight proceeds to the horizon. This effect makes actual level rod readings too large by the following approximate relation,

$$C = 0.0239D^2$$

where D is the sight distance in thousands of feet.

Atmospheric Refraction

The atmosphere refracts the horizontal line of sight downward, making the level rod reading smaller. The typical effect of refraction is equal to about 14% of the effect of earth curvature. Thus, the combined effect of curvature and refraction is approximately

$$(C - r) = 0.0206D^2$$

Instrument Adjustment

If the geometric relationships defined in the preceding discussion are not correct in the leveling instrument, the line of sight will slope upward or downward with respect to the horizontal plane through the telescope. The test of the line of sight of the level to ensure that it is horizontal is called the "two-peg test." If the line of sight is inclined, the difference in elevation obtained from the two setups will not be equal. Either the instrument must be adjusted, or the slope of the line of sight must be calculated. The slope is expressed as a collimation factor, *C*, in terms of rod reading correction per unit of sight distance. It may be applied to each sight by the following:

$$\text{Corrected rod reading} \quad = \text{Rod reading} \ + (C_{\text{Factor}} \cdot D_{\text{Sight}})$$

In ordinary differential leveling, these effects are canceled in the field procedure by always setting up so that the backsight distance and foresight distance are equal. The errors are canceled in the subtraction process. If long unequal sight distances are used, the rod readings should be corrected for curvature and refraction and for collimation error.

Orthometric Correction

When long, precise level routes are surveyed, it is necessary to account for the fact that the equipotential surfaces converge as the survey proceeds north. The correction to be applied for convergence of equipotential surfaces at different elevations can be calculated by

$$\text{Correction} \ = -0.0053 \sin 2\phi H \Delta\phi_{\text{rad}}$$

where ϕ is the latitude at the beginning point, *H* is the elevation at the beginning point, and $\Delta\phi$ is the change in latitude from the southerly station to the northerly station expressed in radians.

Defining Terms

Benchmark (BM): A benchmark is a permanent object having a mark of known elevation, for example, a cross chiseled on a boulder or a concrete monument with an embedded brass disk.

Datum: Any quantity or set of such quantities that may serve as a reference or basis for calculation of other quantities.

Datum sea level: An equipotential surface passing through a specified point at mean sea level that is used as a reference for elevations; a surface passing through mean sea level at certain specified points to which elevations determined by leveling are referred. Note that, in general, the latter surface is not an equipotential surface.

Elevation: The distance, measured along the direction of gravity (plumb line), between a point and a reference equipotential surface, usually the geoid.

Height: The distance, measured along a perpendicular, between a point and a reference surface, for example, the ellipsoidal height; the distance, measured along the direction of gravity, between a point and a reference surface of constant geopotential, for example, the orthometric height. Note that the term *elevation* is preferred when the geoid is used as the reference surface.

Mean sea level: The arithmetic mean of elevations (heights) of the water's surface observed hourly over a specific 19-year cycle.

Vertical: The direction in which gravity acts.

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Further Information

The material in this chapter is intended only as an overview of elevation reference systems and basic surveying methods. There are many textbooks dedicated completely to the various aspects of surveying. For a more complete presentation of surveying theory, consult *Surveying*, 9th edition, by Moffitt and Bouchard, HarperCollins Publishing, 1992, or *Elementary Surveying*, 9th edition, by Wolf and Brinker, HarperCollins College Publishers, 1994. For more detailed information on the capabilities of various instruments and software, Business News Publishing prepares the trade magazine *P.O.B.* (Business News Publishing Company, 755 W. Big Beaver Rd., Suite 1000, Troy, MI 48084), and American Surveyors Publishing Company prepares the trade magazine *Professional Surveyor* (American Surveyors Publishing Company, Inc., Suite 501, 2300 Ninth

Street South, Arlington, VA 22204). Each of these publications conducts annual reviews of surveying instruments and software. These listings allow the reader to keep up to date and compare "apples to apples" when analyzing equipment.

Survey control information, software, and many useful technical publications are available from the National Geodetic Survey (NGS). The address is

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