Buckner, R. B. “Distance Measurements”
The Engineering Handbook.
Ed. Richard C. Dorf
Boca Raton: CRC Press LLC, 2000

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Distance Measurements

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When the word **distance** is used in surveying without qualification, it is usually construed to mean *horizontal distance* ¾ that is, a distance either measured directly along a horizontal line or measured along a slope and then mathematically projected to the horizontal. If linear measurements along other alignments are being discussed, they are given specific designations, such as *slope distance* or *vertical distance* (elevation difference).

Since ancient times, direct distance measurements have been made using lines, cords, ropes, rods, chains, tapes, and other such devices. Distances have also been measured indirectly, employing stadia and other tacheometric methods, as well as trigonometrically, using a combination of distance and angle measurements and calculations. Technological advances during the second half of the 20th century have created changes in the methods used to measure many distances. During the 1960s and 1970s most surveyors made a gradual shift from the chain and tape to electronic distance systems that measure distances with light waves or microwaves. During the 1980s this trend continued into using *total station* systems, which measure both distances and angles electronically and process the combined measurements into a Cartesian coordinate form for the survey points. Thus, many distances are determined by computations from the coordinate positions rather than by direct measurements. Most recently, there has been a trend toward using the global positioning system (GPS) for precise determination of positions of points, and these positions are then used to indirectly determine the distances between survey points. Eventually, GPS may replace both the tape and electronic distance instruments (including the total station) for most surveying measurements, including angles, elevations, and distances. GPS is covered in **Chapter 149**.

An overview of the concepts of direct and indirect distance measurements will be given here. The geometric, electronic, and other physical principles of the measurement systems will not be discussed—only the basic measuring procedures and the appropriate corrections needed to achieve optimum accuracy.

145.1 Fundamentals of Distance Measurement

The Need for Corrections

Measurements, whether of distance or any other quantity, are but estimates of the magnitude of the quantity. Few initial readings (observations) in surveying contain the accuracy desired without some correction for systematic errors. Much of the consideration regarding measurements is not so much of how to operate the instruments and perform the calculations to reduce the data, but of how to identify and evaluate the magnitude of the corrections that must be applied to the readings. The "true value," theoretically, is always equal to the reading plus the corrections. That is, $T = R + \sum C$. If the true value is known and the reading is needed to achieve this, $R = T - \sum C$. Application of this basic concept will be illustrated subsequently. The following sections discuss the typical methods of determining survey distances.

Taped Measurements

Most **taped** measurements are made using a steel tape, usually of 100 ft length. The tape is laid along a line between points and stretched tight. If the distance desired is longer than the length of the tape, more than one tape length is employed, the ends of the tape being marked with taping pins (sometimes called "arrows"). Alignment of the ends of the tape, to keep the tape on a straight line, is achieved either by hand signals from a rear tapeman or by a transit or theodolite centered over one of the two ends of the line.

The tape is either (1) allowed to rest on the ground surface ("fully supported"), or (2) suspended throughout its length ("end supported"), with the distance mark on the tape being transferred to the ground using plumb bobs at one or both ends. In the former case corrections for ground slope must be made, the slope having been measured using other instruments, such as hand levels, clinometers, or theodolites. In the latter case the two ends of the tape are usually held near the same elevation as determined by hand levels or clinometers. End-supported taping requires corrections for the sag of the tape, but not the slope. In either type of taping, corrections must usually be made (depending on the accuracy desired) for calibrated length, changes from calibration temperature to field temperature, and tension (if different from that used when calibrated).

Steel tapes are made in various lengths other than 100 feet, for example, 25 meters, 50 meters, 200 feet, and so forth. The principles in the use and correction of readings is much the same for various tapes, regardless of length.

Tapes are also made of other materials, including woven fiber and invar (a nickel-steel alloy). The fiber ("cloth") tapes are used only for rough measurements, and there is usually no attempt to apply corrections to the readings. In contrast, invar tapes are used for precise surveys, such as the establishment of calibration base lines for electronic distance measurements, and all applicable corrections must be made. The use of these more precise tapes and the less precise "cloth" tapes will not be covered here.

Tacheometric Measurements

The most common method in the **tacheometry** category uses the stadia. Such measurements are made through the telescope of a transit, theodolite, or level instrument. The measurement is achieved by observing where the two horizontal stadia hairs, viewed through the telescope, strike a graduated rod held vertically on a survey point. The difference between the two readings is a function of the separation of the stadia hairs, the focal length of the telescope, and the distance between the instrument and the rod. When the line of sight is other than horizontal, the vertical or zenith angle must also be considered. Stadia distances are generally accurate to only about one or two feet for normal sight distances, and thus there is little need to be concerned about corrections to the data.

The subtense bar is another tacheometric instrument. The principle employs the measurement of a precise horizontal angle between two distinct targets at the two ends of the bar. The bar is erected on a tripod to lie horizontally and perpendicular to the line of sight, centered over a survey point. The theodolite is centered over the other end of the line. The separation between the two targets defining the bar is a known length (commonly two meters). Using this known length and the measured angle, the horizontal distance is computed by trigonometry. For relatively short distances (up to perhaps 150–200 feet) this method can exceed the accuracy of both taping and electronic measurements, and thus it is very useful for measurements across busy streets or other small, inaccessible places. Because this method lacks error due to slope corrections, it is useful for measuring to high places, such as to tops of buildings. Its application is probably overlooked nowadays because of the ease of using electronic instruments—even though it is more accurate for short measurements, has advantages similar to those of electronic distance measurement, and avoids slope corrections.

Electronic Distance Measurements

The most common of **electronic distance instruments** (EDMs) employs a visible light beam reflected off a system of reflectors called *retroprisms.* The light beam is reflected onto the instrument for interpretation of the wavelengths and partial wavelengths comprising the double-slope distance between the instrument and the reflector. Older EDMs are individual units, measuring distance only. The most modern version is part of the aforementioned total station system, which also measures angles. Whether an individual unit or part of a more complete survey system, the principle of EDM operation is much the same.

A common fallacy, particularly when using total station systems, is that the measurements are free of errors. As has been mentioned, there is error in any measuring system. When measuring with an EDM, the surveyor must be concerned with calibration, just as with any distance-measuring system. For example, the electronic center of the instrument may not be located precisely along the same vertical line as the geometric center plumbed over the ground station. The instrument will usually have this small, constant instrument error, as well as an error that is proportional to the distance measured (often called the *parts per million* or *PPM correction*). Also, the reflector has a "constant," and the optical plummets in the tribrach mounting systems can be out of adjustment. The magnitude and sign of these errors should be determined by field tests for best accuracy. EDMs are also affected by variations in atmospheric pressure and temperature. Microwave measurements are also affected by humidity.

All measurements made with EDMs are "slope" measurements between the center of the EDM and the reflector, and thus must be corrected to horizontal using measured vertical or zenith angles, and also instrument heights.

Indirect Computational Methods

Distances are commonly determined by **indirect measurement** using trigonometric principles. The most common of these methods are *intersection* and the coordinate *inverse.* Using intersection, a distance is measured (or computed from other measurements) and angles measured to a common point from the two ends of this *base line,* which forms an oblique triangle. The unknown sides of the triangle are solved using the sine law. Using the inverse, the computation starts with determination of the departure (difference in x coordinates) and the latitude (difference in y coordinates) of the line defined by the two coordinate points. The distance is the hypotenuse of a right triangle whose sides are the departure and latitude of the line; it is determined using the Pythagorean theorem. Many variations of these indirect methods occur in practice and usually involve some variation of an oblique triangle solution or the coordinate inverse. Some of these concepts are discussed in section 145.2.

Photogrammetry is another such method for determining distances. After proper orientation of a stereomodel, coordinates can be read from it, from which distances can be determined by the same inverse computation as discussed earlier. The science of photogrammetry is briefly explained in **Chapter 147**.

Approximate Methods

As mentioned, all measurements are estimates of an unknown quantity. The method used to determine a distance is chosen as dictated by the accuracy needed. The above methods are typical for measurements where accuracies of a few millimeters to a fraction of a meter are desired. If less accuracy is needed, a range finder, a measuring wheel, pacing, the odometer on a bicycle or vehicle, map scaling, digitizing coordinates from maps (with subsequent "inverse" computations), or even visual estimation can be used. Range finders are optical instruments that might be considered under the classification of tacheometric methods. These instruments and the calibrated measuring wheel or an odometer on a bicycle have accuracy comparable to the stadia method. A car odometer can yield an accuracy of perhaps 50 to 100 feet in a mile, if calibrated. The accuracies of the other approximate methods depend on several factors, particularly the map scale and the map accuracy. It must be emphasized that digitizing from maps cannot yield high accuracy of positions or distances. Even for a fairly large-scale map, the accuracy of such methods is seldom better than 10 feet.

145.2 Applications and Calculations

Taping

The variables affecting a horizontal distance using a tape, with their correction equations, are as follows:

1. *Calibration.* $C_l = l_t - l_r$. The correction per tape length is the actual ("true") length of the tape minus the nominal length (reading between end marks). The actual length is determined by calibration at some observed temperature, tension, and support condition, comparing the tape's length with an accurate base line.
2. *Temperature.* $C_t = K_t l (t_f - t_s)$. Temperature correction is given by the coefficient of thermal expansion multiplied by the nominal length of the tape times the difference between the field temperature and the calibration temperature. The correction is positive when $t_f > t_s$, since the tape expands when the temperature is warmer. The value for K_t is 0.00000645 units per unit per $^\circ\mathrm{F}$ (the constant is 0.0000116 if $^\circ\mathrm{C}$ is used).
3. *Pull* (tension). $C_p = (P_f - P_s) l \div AE$. This is the correction for tension if the field tension is different from the standardization tension. In this equation *A* is the cross-sectional area of the tape and E is the modulus of elasticity, which is 29000000 lb/in.^{2}. Care should be taken that the field tension appears first in the equation, subtracting the standardization tension from it.
4. *Sag.* $C_s = -(w^2 l)/(24P^2)$. This is the correction for sag, where *w* and *l* are the weight and length of the portion of the tape suspended between supports. The pull, *P*, is the actual tension on the tape. It has no relationship to the standardization pull.
5. *Slope.* $C_g = -(v^2)/(2l)$. This is the correction for grade or slope, which, when added to the slope length, yields the horizontal distance. This correction is applied when the tape ends are not at the same elevation. In this equation *s* is the slope distance being corrected and *v* is the elevation difference between the two tape ends at this length. This equation is inexact but is accurate for slopes less than about 10%. The Pythagorean theorem can be used in any case, solving the equation $h^2 + v^2 = s^2$ for the horizontal distance, *h*.

Slope corrections can also be made trigonometrically if the vertical angle (slope angle) is measured with a clinometer or theodolite. For a line of any length $H = S\cos\gamma$, where *H* is the horizontal distance, *S* is the slope distance, and γ is the vertical angle. ($H = S\sin\gamma$ if the zenith angle is used.)

The alignment error is generally not corrected in practice, but instead rendered negligible by the process of careful tape alignment.

For many errors the correction can usually be made for one tape length and then multiplied by the number of tape lengths, as long as the condition causing the error does not vary between tape lengths. This generally always applies to the calibration, temperature, and tension errors, and sometimes to the sag and slope errors.

Taping problems are of two types: (1) calculation of the horizontal distance between two established points, and (2) calculation of the reading to be observed to establish a given distance. The theory of systematic errors is applicable in making taping corrections. Solving for the true value $T = R + \sum C$ is the first type of problem. If "layout" is required, then the reading $R = T - \sum C$ is solved from the given value and the corrections.

The following problems will utilize a calibrated 100 ft tape, found to be 99.992 feet long at $70^\circ\mathrm{F}$, 15 lb tension, fully supported. It has a cross-sectional area of 0.006 in.^{2} and weighs 2.2 lb. In the solutions TL is the number of tape lengths.

Example 145.1. A reading of 458.97 feet is observed between two points when the field temperature is $40°\mathrm{F}$, along a 4% slope. Find the correct horizontal distance.
Solution. There are three systematic errors to consider: calibration, temperature, and slope.

$$C_L = (l_t - l_r)\mathrm{TL} = (99.992 - 100.000)4.59 = -0.037 \text{ ft}$$

$$C_t = K_t l(t_f - t_s)\mathrm{TL} = 0.000\,006\,45(100)(40 - 70)4.59 = -0.089 \text{ ft}$$

$$C_g = -\frac{v^2}{2l}\mathrm{TL} = [4^2 \div (2 \times 100)](4.59) = -0.367 \text{ ft}$$

$\sum C = -0.493 \text{ ft}$, from which $T = 458.97 - 0.49 = 458.48 \text{ ft}$.

Example 145.2. A distance of 200.00 feet is to be laid out along a horizontal alignment. The tape must be suspended for 60 feet of one of its tape lengths. A tension of 30 lb is used for this portion of the layout; otherwise, 15 lb is used. Temperature is $70°\mathrm{F}$.
Solution. There are three systematic errors to consider: calibration, tension, and sag.

$$C_L = (l_t - l_r)\mathrm{TL} = (99.992 - 100.000)2.00 = -0.016 \text{ ft}$$

$$C_p = (P_f - P_s)l \div AE = (30 - 15)60 \div (0.006 \times 29 \times 10^6) = 0.005 \text{ ft}$$

$$C_s = -\frac{w^2 l}{24P^2} = -(2.2 \times 0.6)^2 60 \div (24 \times 30^2) = -0.005 \text{ ft}$$

$$\sum C = -0.016 \text{ ft} \quad \text{and} \quad R = 200.00 - (-0.016) = 200.016 = 200.02 \text{ ft}$$

It is seen in this example that the added tension for the end-supported part of the taping compensated for the sag effect.

Stadia

Stadia hairs or lines are placed in most telescopes so that the *stadia interval factor, K*, equals 100. This makes it convenient to measure distance, merely subtracting the two stadia hair readings and multiplying by 100 to get the distance between the rod and the theodolite. Since the stadia hairs are each read with a precision of approximately ± 0.01 feet at ordinary distances, the precision of a stadia distance is no better than ± 1.0 feet, with variation according to how clearly the rod is seen and how carefully it is read.

Old *external focusing* telescopes had a *stadia constant, C*, of about one foot. The instruments of recent generations, however, being *internally focusing,* eliminate this constant.

For a horizontal sighting $H = KI + C$, where *K* and *C* are as defined earlier and *I* is the rod intercept (difference between the upper and lower rod readings). When vertical angles are involved, $H = KI\cos^2\gamma + C\cos\gamma$, where γ is the vertical angle.

Example 145.3. A theodolite has a stadia interval factor of 100. The reading on the upper stadia hair is 7.54 ft and on the lower hair it is 3.66 ft. The zenith angle to the center crosshair is $96^\circ 36' 30''$. What is the horizontal distance, to the nearest foot?
Solution. Assuming the stadia constant to be zero and converting the zenith angle to vertical angle, the appropriate values in the preceding equation are as follows:

$$H = 100(7.54 - 3.66)\cos^2(-6^\circ 36' 30'') = 383 \text{ ft}$$

Subtense Bar

After the horizontal angle is measured between the two end targets on the bar, the horizontal distance is computed from $H = \frac{1}{2} b \cot \alpha/2$, where b is the bar length and α is the horizontal angle. Using a bar of the usual 2-meter length, the value of *H* is in meters, since half the bar length is 1 meter.

Note that the distance is *always* horizontal, since the horizontal angle is the same regardless of the relative elevation of the two points. Thus, no slope corrections are ever required.

In practice, a $1''$ theodolite is generally used and several angles are measured, in order to achieve adequate precision.

Example 145.4. A $1''$ theodolite is used to measure the angle between the targets on the ends of a 2-meter subtense bar. The mean of six independent readings of the angle is $0^\circ 45' 46''$. Compute the horizontal distance between the theodolite and the bar.

$$H = \tfrac{1}{2} b \cot \alpha/2 = \tfrac{1}{2}(2 \text{ meters }) \cot(0^\circ 45' 46''/2) = 75.110 \text{ m}$$

Electronic Distance Measurements

Although the instrument constant is in practice, usually adjusted to zero whenever the instrument is serviced, calibration can discover a small instrument constant. Similarly, the reflector constant is usually keyed into the instrument by the surveyor and thus compensated, but a field test of reflector constants can discover slight discrepancies between what the manufacturer states the constant to be and what it actually is. Likewise, the atmospheric errors are generally keyed into the instrument after reading the temperature and pressure but are sometimes overlooked, and an old setting remains in the instrument. The surveyor should be aware of these possible error sources. The following example assumes that the atmospheric errors and reflector constant have been handled properly.

Example 145.5. An EDM has been calibrated using a four-station NGS base line, and errors are found as follows: $C = +0.003$ meters and $P = +0.000\,004\,56$, where *C* is the constant correction and *P* is the "scale" correction, which may be expressed as 4.56 PPM. The zenith angle along the line is $88^\circ 34' 42''$. The observed slope distance is 1789.783 meters. What is the corrected horizontal distance?

Solution. The PPM correction is +0.00000456(1789.8 m) = +0.0082 m. The constant correction is +0.003 m. The corrected slope distance is

$$\begin{aligned} 1789.783 + 0.008 + 0.003 &= 1789.794 \text{ m} \\ H = 1789.794 \sin 88^\circ 34' 42'' &= 1789.243 \text{ m} \end{aligned}$$

An additional correction might be necessary if the reflector is not set at the same height as the EDM because, if so, the measured slope angle does not correspond to the slope of a line connecting the two ground points. The correction involves adjusting the measured zenith angle before calculating the horizontal distance as done previously.

Defining Terms

Distance: A linear value, either measured or computed. Unless qualified otherwise, it is understood to lie along the horizontal. The observed distance is considered inaccurate until corrected for systematic errors caused by instruments, nature, or other sources.

Electronic distance instrument: An instrument that measures distances using reflected light waves or microwaves.

Indirect measurement: A measurement that has been computed from other measurements, usually employing trigonometric principles.

Tacheometry: A distance-measuring procedure that involves measuring intervals between cross hairs on a rod or angles subtended between marks on a bar of known length, or a similar indirect method.

Tape: A surveying instrument used to measure distance by stretching it, end for end, along a line between points.

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Further Information

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