HACETTEPE UNIVERSITY
MINING ENGINEERING DEPARTMENT

MAD256 – SURVEYING

2013-14 Spring Semester

COURSE COORDINATOR : ***Dr. MEHMET ALİ HİNDİSTAN***

- ***Course*** **: 4 hours/week**
- ***Course Time and Room***

THURSDAY	**: 09:00-12:45 (Sec.01)**	**...... Y306 + Computer Class**
THURSDAY	**: 13:00-16:45 (Sec.02)**	**...... Y306 + Computer Class**

Course Objective:
The course aims to give the student the required knowledge to read any map and to draw some simple plans. A student who successfully finishes the course is expected to organize and supervise surveying measurements in both open-pit and underground mining activities.

Course Notes and Solved Problems :
http://yunus.hacettepe.edu.tr/~hmali/surveying.htm

Web Pages for Additional Information:
- http://www.geom.unimelb.edu.au/collier/451-102.html
- http://www.koeri.boun.edu.tr/jeodezi/links.html
  (Many links to domestic and abroad sites)
- http://www.geod.nrcan.gc.ca/index_e/links_e/links_e.html
  (Many links to domestic and abroad sites)
- http://www.google.com.tr
  *(search for **surveying, geodesy, geomatics, GPS, GIS, etc.**)*

Grading:

Two Midterm Exams ……...	**50 %**
Final Exam ..…………………..	**40 %**
Lab applications & report ...	**10 %**

COURSE SCHEDULE - 2013-14 SPRING TERM

Week	Sect. 01	Sect. 02	Activity
1	Febr. 20	Febr. 20	Introduction, Fundamental concepts
2	Febr. 27	Febr. 27	Distances, angles and directions
3	March 06	March 06	Coordinate system
4	March 13	March 13	Area measurements and calculation
5	March 20	March 20	Traversing
6	March 27	March 27	Leveling, dip and volume
7	April 03		1st MIDTERM – at 10 o'clock
8	April 10	April 10	
9	April 17	April 17	COMPUTER APPLICATION (Details will be given)
10	April 24	April 24	
11	May 01	May 01	
12	May 08	May 08	
13	May 15	May 15	
14	May 22		2nd MIDTERM – at 14 o'clock

Course Requirements:

Class tools:

- A scientific calculator
- A ruler (min.20 cm with mm divisions)
- A compasses (good quality)
- A protractor (with degree and grad divisions)
- A milimetric paper block (at least 10 pages with A4 size)
- A rubber, a 0.5 pencil, etc.

Attendance: The students are responsible to attend the computer applications. The student who doesn't attend two of these classes will get F3 grade.

References:

- ♦ Bary F.KAVANAGH and S.J.Glenn BIRD, "SURVEYING-Principles and applications", Fifth Edition, Prentiec-Hall, 2000. **(Metu Library TA545-K37)**
- ♦ Paul R.WOLF and Russell C.BRINKER, "Elementary Surveying", Eighth Edition, 1989. **(Metu Library TA545-W77)**
- ♦ W.SCHOFIELD, "Engineering Surveying", Fourth Edition, 1993. **(Metu Library TA545-S263)**
- ♦ W.Randolph WILLIAMS, "Mine Mapping and Layout", 1983. **(Beytepe Library TN273-W48)**
- ♦ W.WHYTE&R.PAUL, "Basic Surveying", Fourth Edition,1997. **(Metu Library TA545-W69)**
- ♦ Şenol KUŞÇU, "Madenlerde Ölçme ve Değerlendirme", Filiz Kitabevi, İstanbul, 1997.
- ♦ Cevat İNAL, Ali ERDİ ve Ferruh YILDIZ, "Topoğrafya-Ölçme Bilgisi", Atlas Kit., Konya, 1996.
- ♦ Gündoğdu ÖZGEN, "Mühendis ve Mimarlar İçin Topoğrafya-Ölçme Bilgisi", İTÜ İnşaat Fakültesi Matbaası, İstanbul, 1993.

GLOSSARY (Terms related to Surveying)

Accuracy : The closeness to the truth of measurement data or station coordinates.

Adjustments : Since all real measurements are imperfect, some amount of error will accumulate in the course of a survey. That error can be logically distributed throughout the survey by various adjustment procedures. Adjustments can and should be done with any set of measurements for which error can be assessed. Adjustment procedures do not correct the errors in the measurements. They simply produce a set of data that is self-consistent (e.g., the starting and ending points of a closed-loop leveling circuit have the same elevation - which is physical reality). The adjusted values are not necessarily true - they are just likely to be closer to the true values than the original measurements.

Azimuth : The horizontal direction of a line clockwise from a reference plane, usually the meridian.

Azimuth Angle : The angle less than 180° between the plane of the celestial meridian and the vertical plane with the observed object, reckoned from the direction of the elevated pole. In geodetic work, it is the horizontal angle between the celestial pole and the observed terrestrial object.

Backsight : A backsight is a reading taken on a position of known coordinate(s). Since a survey progresses from a point of known position to points of unknown position, a backsight is a reading looking "backward" along the line of progress.

Bearing : The direction of a line with respect to the meridian described by degrees, minutes, and seconds within a quadrant of the circle. Bearings are measured clockwise or counterclockwise from north or south, depending on the quadrant.

Benchmark : A survey mark made on a monument having a known location and elevation, serving as a reference point for surveying.

Booking : Booking means entering the field data in the field book.

Breakpoints : A breakpoint is a point where a change in some parameter of interest occurs. In surveying, breakpoints are usually associated with changes in slope.

Collimation : A physical alignment of a survey target or antenna over a mark or to a reference line.

Contour Line : An imaginary line on the ground, all points of which are at the same elevation above or below a specified datum.

Contour Interval : A predetermined difference in elevation (vertical distance) at which contour ines are drawn. The contour interval is usually the same for maps of the same scale.

Contour Map : A map that portrays relief by means of contour lines.

Control Points : Control Points are fixed points of known coordinates. Such information can give only elevation or can include all coordinates. Control points are determined by high-accuracy surveys.

Coordinates : Linear or angular quantities, or both, which designate the position of a point in relation to a given reference frame.

Datum : A reference point from which other points in a survey are measured from. A datum may be a known point within an existing survey grid or it may be any arbitrary point.

Electronic Distance Measurement (EDM) : EDM is a technique that is used to measure distances. EDM is based on the idea that light (and radio waves) travel at a finite velocity and by measuring how long a signal takes travel back and forth between two points and knowing the speed of light, the distance can be measured.

Elevation : The elevation of a point is its vertical distance above or below a given level reference surface.

Error : An error is the difference between the true value of a quantity and the measured value of the same quantity.

Face Left : Face Left refers to the position of a theodolite when the vertical circle is situated to the left hand side of the observer's face.

Face Right : Face Right refers to the position of a theodolite when the vertical circle is situated to the right hand side of the observer's face.

Field Books : Field books are standard forms for recording of survey data as it is collected.

Foresight : A foresight is a reading taken on a position of unknown coordinate(s). Since a survey progresses from a point of known position to points of unknown position, a foresight is a reading looking "forward" along the line of progress.

Geodesy : Determination of the time-varying size and figure of the earth by such direct measurements as triangulation, leveling and gravimetric observations.

Geodetic Surveying : Geodetic surveying is that branch of surveying wherein all distances and horizontal angles are projected onto the surface of the reference spheroid that represents mean sea level on the earth.

Geoid : An equipotential surface of the gravity field approximating the earth's surface and corresponding with mean sea level in the oceans and its extension through the continents.

GPS : Global Positioning System. A surveying technology using specialized radio receivers tuned to signals from military navigation satellites to position survey stations.

Horizontal Angle : A horizontal angle is the angle formed in a horizontal plane by two intersecting vertical planes.

Interior Angle : An angle between adjacent sides of a closed figure and lying on the inside of the figure. The three angles within a triangle are interior angles.

Interpolation Method : Determination of a intermediate value between given values using a known or assumed rate of change of the values between the given values.

Landmark : A survey mark made on a 'permanent' feature of the land such as a tree, pile of stones, etc.

Leveling : Leveling is the operation in surveying performed to determine and establish elevations of points and to determine differences in elevation between points.

Map : A conventional representation, usually on a plane surface and at an established scale, of the physical features (natural, artificial, or both) of a part or whole of the Earth's surface by means of signs and symbols and with the means of orientation indicated.

Map Scale : The ratio of a specified distance on a map to the corresponding distance in the mapped object.

Meridian : A meridian is one of the imaginary lines (longitudes) joining the North and South Poles at right angles to the Equator, measured by degrees from 0° at Greenwich to 180°.

Mistake : A mistake is not an error, but is a blunder on the part of the observer.

Monument : A permanently placed survey marker such as a stone shaft sunk into the ground.

Nadir : The nadir is that part of the celestial sphere that is directly below the observer. For a theodolite, it is the point directly below the vertical axis of the instrument.

Occupied Point : The physical point over which the instrument (level, transit, total station, etc.) is set up. It is the point from which any measurements taken while at that point are reckoned.

Parallax : A change in the position of the image of an object with respect to the telescope cross hairs when the observer's eye is moved. This can be practically eliminated by careful focusing.

Plane Surveying : Plane surveying is that branch of surveying wherein all distances and horizontal angles are assumed to be projected onto one horizontal plane.

Plumb Bob : A plumb bob is carefully machined, pointed weight that is suspended with a string. It is used to indicate a (local) vertical line through the point of suspension. Plumb bobs are commonly used for locating an instrument precisely over a fixed point or to project a vertical line between a tape and a point on the ground.

Point of Beginning : The starting point of the survey.

Position : The coordinates, in a horizontal reference system, of station mark or feature.

Precision : The repeatability of a measurement. (The amount by which a measurement deviates from its mean.)

Random Errors : A random error is one the magnitude and sign of which cannot be predicted.

Rod (Ranging Pole) : A surveying rod is used to sight on a target. A simple rod fitted with a sharp-pointed, shoe of steel and usually painted alternately in red and white bands at 1-foot intervals.

Set-up : In general, the situation in which a surveying instrument is in position at a point from which observations are made.

Sideshot or Intermediate Foresight : A shot onto an unknown point which is not a station on the traverse. Sideshots (or intermediate foresights) are booked as regular foresights onto traverse stations, but are not included in the calculations for vertical error of closure.

Sketch : A good sketch is invaluable. It will help to explain the job and show the orientations of various important features. It is definitely worth taking a few minutes to produce a good sketch. When you arrive at the job site, size up the whole thing. How is the job situated with respect to permanent features in the area (roads, buildings, fences, trees, etc.)? Begin by noting the permanent (or nearly so) features around the perimeter (and within) the job site. Drawing these provides a "frame" for the rest of the sketch. Then draw in the details of the job site. Always make the sketch with North at (or near) the top (or left side) of the page. Put an arrow with an "N" on the sketch to indicate North. Include the scale of the drawing. Be reasonably precise!!

Staff (Leveling Rod) : A staff is a measuring bar which has marked with gradations.

Surveying : The purpose of surveying is to locate the positions of points on the surface of the earth.

Systematic Errors : An error that, as long as conditions are unchanged, will always have the same magnitude and the same algebraic sign.

Tape : A tape is a flexible device used for measuring linear distances. There are tapes made of many materials, such as steel, invar. The most common tape used by surveyors is the steel tape.

Total Station : Electronic surveying instrument that combines angle and distance measuring capabilities in a single unit.

Traverse : A traverse is a series of consecutive line segments whose lengths and directions are determined by field measurements. A **closed traverse** either closes back upon its starting point, or begins and ends on stations of known positions. An **open traverse** does not close on either itself or a station of known position. An open traverse does not provide any means for checking for errors and mistakes.

Tripod : The three-legged stand upon which surveying instruments and targets are mounted during use.

Vertical Angle : A vertical angle is an angle measured in a vertical plane.

Zenith : The zenith is that point of the celestial sphere that is directly overhead from the observer. For a theodolite, it is the point directly above the vertical axis of the instrument.

Zenith Angle : Measured in a positive direction downwards from the observer's zenith to the observed target.

SURVEYING TERMS

Adjustment: Dengeleme

Arc: Yay

Benchmark: Röper noktası, seviye işareti

Backsight Reading: Geri okuma

Circular Level: Küresel düzeç

Clamping Screw: Bağlama vidası

Closed Traverse: Kapalı poligon

Coefficient: Katsayı

Coincide: Çakıştırmak

Collimator: Gez göz arpacık

Contour Line: Eşyükselti eğrisi

Cooking (Cheating): Sabunlama

Cross Hairs: Gözleme çizgileri

Cross-section: Enkesit

Deflection: Sapma açısı

Direction: Doğrultu

Dropping A Perpendicular: Dik düşmek

Eccentricity: Dışmerkezlik

Erecting A Perpendicular: Dik çıkmak

Fence: Tahta çit

Foresight Reading: İleri okuma

Hedge: Çit

Horizontal Axis: Yatay eksen

Horizontal Circle Reading: Yatay açı okuması

Horizontal Clamp Screw: Yatay hareket bağlama vidası

Horizontal Slow-Motion (Tangent) Screw: Yatay az hareket vidası

Inclined: Eğik

Intermediate-sight Reading: Orta okuma

Leg: Kenar (poligonda)

Leveling: Nivelman

Leveling Instrument: Nivo

Leveling Screw: Tesviye vidası

Light Reflecting Mirror: Aydınlatma aynası

Line of Sight: Gözleme çizgisi

Open Traverse: Açık poligon

Optical Plummet: Optik çekül

Plumb-bob: Çekül

Ranging A Line: Doğrultuya girmek

Reading Eyepiece: Okuma oküleri

Reconnaissance: Keşif, istikşaf

Reduction: İndirgeme

Residual: Arta kalan

Ranging Rod (Pole): Jalon

Rough Orientation: Kaba yöneltme

Route: Güzergah

Sketch: Kroki

Staff: Mira

Steel Tape: Çelik şerit

Telescope Eyepiece: Dürbün oküleri

Telescope Focussing Screw: Dürbün görüntü netleştirme vidası

Tribrach: Alidat

Tribrach Fixing Screw: Üçayak bağlama vidası

Tribrach Locking Lever: Alidat Bağlama Vidası

Traverse: Poligon

Tripod: Üçayak

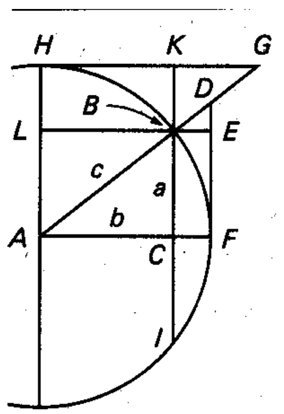
Tubular Level: Silindirik düzeç

Vertical Slow-Motion (Tangent) Screw: Düşey az hareket vidası

Volume: Hacim

Zenith: Başucu

TRIGONOMETRIC EQUATIONS



Let A = angle BAC = arc BF, and let radius $AF = AB = AH = 1$. Then,

$\sin A = BC$	$\csc A = AG$
$\cos A = AC$	$\sec A = AD$
$\tan A = DF$	$\cot A = HG$
$\operatorname{vers} A = CF = BE$	$\operatorname{covers} A = BK = LH$
$\operatorname{exsec} A = BD$	$\operatorname{coexsec} A = BG$
$\operatorname{chord} A = BF$	$\operatorname{chord} 2A = BI = 2\,BC$

In the right-angled triangle ABC, let $AB = c$, $BC = a$, $CA = b$. Then

1. $\sin A = \dfrac{a}{c}$

2. $\cos A = \dfrac{b}{c}$

3. $\tan A = \dfrac{a}{b}$

4. $\cot A = \dfrac{b}{a}$

5. $\sec A = \dfrac{c}{b}$

6. $\csc A = \dfrac{c}{a}$

7. $\operatorname{vers} A = 1 - \cos A = \dfrac{c-b}{c} = \operatorname{covers} B$

8. $\operatorname{exsec} A = \sec A - 1 = \dfrac{c-b}{b} = \operatorname{coexsec} B$

9. $\operatorname{covers} A = \dfrac{c-a}{c} = \operatorname{vers} B$

10. $\operatorname{coexsec} A = \dfrac{c-a}{a} = \operatorname{exsec} B$

11. $a = c \sin A = b \tan A$

12. $b = c \cos A = a \cot A$

13. $c = \dfrac{a}{\sin A} = \dfrac{b}{\cos A}$

14. $a = c \cos B = b \cot B$

15. $b = c \sin B = a \tan B$

16. $c = \dfrac{a}{\cos B} = \dfrac{b}{\sin B}$

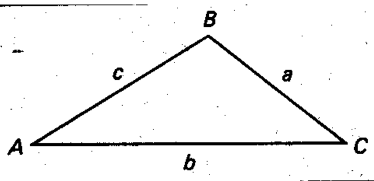
17. $a = \sqrt{c^2 - b^2} = \sqrt{(c-b)(c+b)}$

18. $b = \sqrt{c^2 - a^2} = \sqrt{(c-a)(c+a)}$

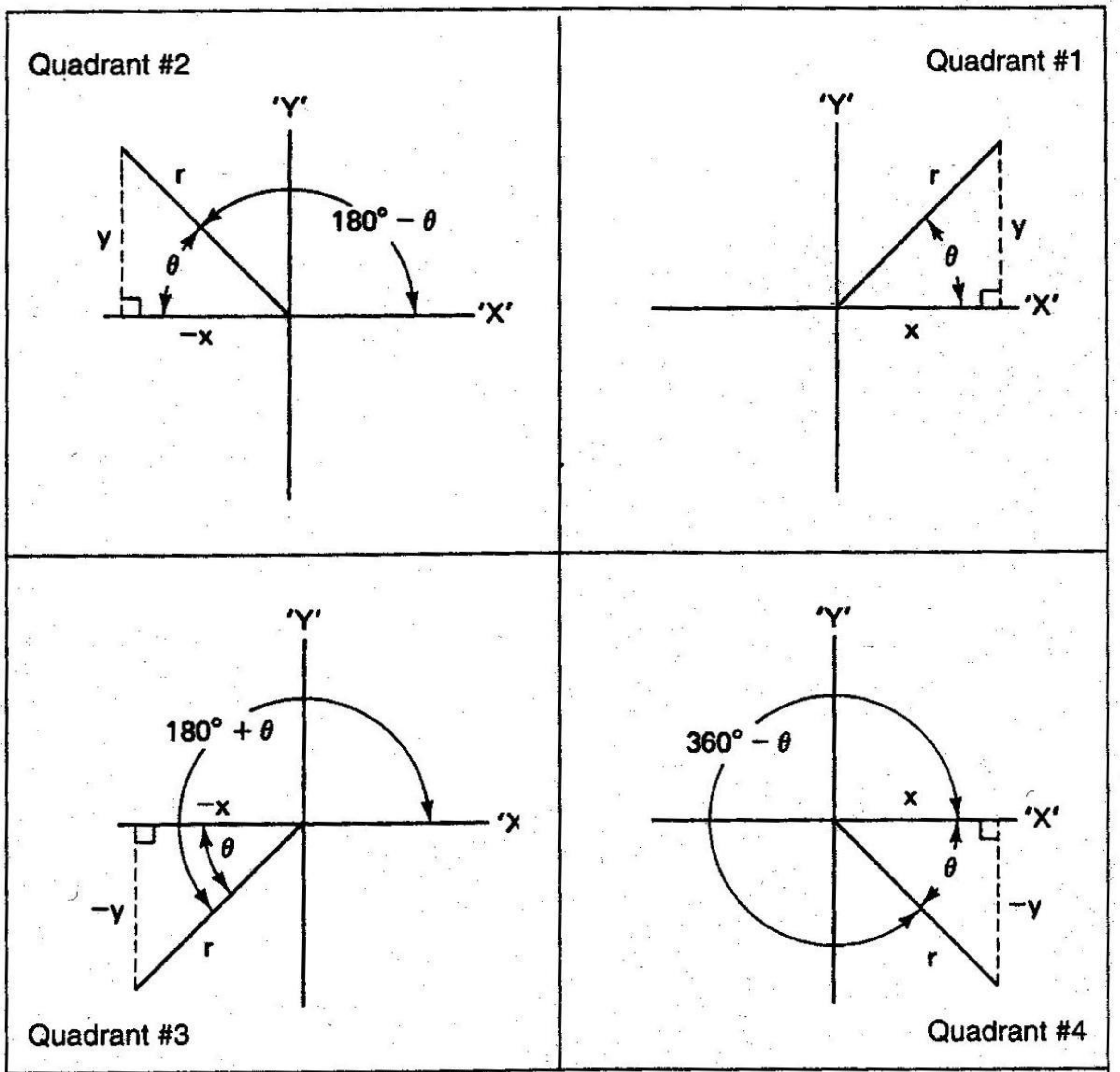
19. $c = \sqrt{a^2 + b^2}$

20. $C = 90^\circ = A + B$

21. area $= \frac{1}{2}ab$



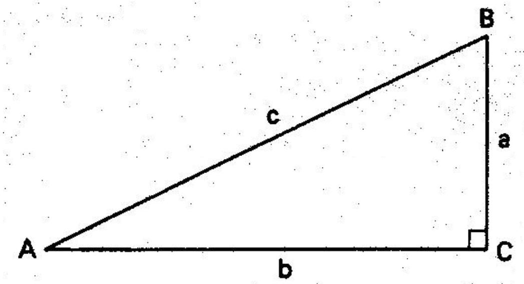
NO.	GIVEN	SOUGHT	FORMULA
22	A, B, a	C, b, c	$C = 180° - (A + B)$
			$b = \dfrac{a}{\sin A} \times \sin B$
			$c = \dfrac{a}{\sin A} \times \sin (A + B) = \dfrac{a}{\sin A} \times \sin C$
		Area	$\text{Area} = \tfrac{1}{2}ab \sin C = \dfrac{a^2 \sin B \sin C}{2 \sin A}$
23	A, a, b	B, C, c	$\sin B = \dfrac{\sin A}{a} \times b$
			$C = 180° - (A + B)$
			$C = \dfrac{a}{\sin A} \times \sin C$
		Area	$\text{Area} = \tfrac{1}{2}ab \sin C$
24	$C, a, b,$	c	$c = \sqrt{a^2 + b^2 - 2ab \cos C}$
25		$\tfrac{1}{2}(A + B)$	$\tfrac{1}{2}(A + B) = 90° - \tfrac{1}{2}C$
26		$\tfrac{1}{2}(A - B)$	$\tan \tfrac{1}{2}(A - B) = \dfrac{a - b}{a + b} \times \tan \tfrac{1}{2}(A + B)$
27		A, B	$A = \tfrac{1}{2}(A + B) + \tfrac{1}{2}(A - B)$ $B = \tfrac{1}{2}(A + B) - \tfrac{1}{2}(A - B)$
28		c	$c = (a + b) \times \dfrac{\cos \tfrac{1}{2}(A + B)}{\cos \tfrac{1}{2}(A - B)} = (a - b) \times \dfrac{\sin \tfrac{1}{2}(A + B)}{\sin \tfrac{1}{2}(A - B)}$
29		Area	$\text{Area} = \tfrac{1}{2}ab \sin C$
30	a, b, c	A	$\text{Let } s = \dfrac{a + b + c}{2}$
31			$\sin \tfrac{1}{2}A = \sqrt{\dfrac{(s - b)(s - c)}{bc}}$
			$\cos \tfrac{1}{2}A = \sqrt{\dfrac{s(s - a)}{bc}}$
			$\tan \tfrac{1}{2}A = \sqrt{\dfrac{(s - b)(s - c)}{s(s - a)}}$
32			$\sin A = \dfrac{2\sqrt{s(s - a)(s - b)(s - c)}}{bc}$
			$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$
33		Area	$\text{Area} = \sqrt{s(s - a)(s - b)(s - c)}$



Quadrant		
#1	All Ratios Positive:	$\sin\theta = y/r$ $\cos\theta = x/r$ $\tan\theta = y/x$
#2	sin is the Positive Ratio:	$\sin(180 - \theta) = y/r = \sin\theta$ $\cos(180 - \theta) = -x/r = -\cos\theta$ $\tan(180 - \theta) = y/-x = -\tan\theta$
#3	tan is the Positive Ratio:	$\sin(180 + \theta) = -y/r = -\sin\theta$ $\cos(180 + \theta) = -x/r = -\cos\theta$ $\tan(180 + \theta) = -y/-x = \tan\theta$
#4	cos is the Positive Ratio:	$\sin(360 - \theta) = -y/r = -\sin\theta$ $\cos(360 - \theta) = x/r = \cos\theta$ $\tan(360 - \theta) = -y/x = -\tan\theta$

Right triangles

Basic function definitions



$$\sin A = \frac{a}{c} = \cos B \qquad \cos A = \frac{b}{c} = \sin B$$

$$\tan A = \frac{a}{b} = \cot B \qquad \cot A = \frac{b}{a} = \tan B$$

$$\sec A = \frac{c}{b} = \operatorname{cosec} B \qquad \operatorname{cosec} A = \frac{c}{a} = \sec B$$

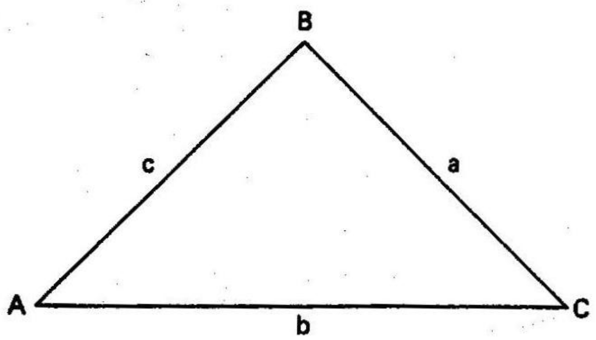
Derived relationships

$$a = c \sin A = c \cos B = b \tan A = b \cot B = \sqrt{c^2 - b^2}$$

$$b = c \cos A = c \sin B = a \cot A = a \tan B = \sqrt{c^2 - a^2}$$

$$c = \frac{a}{\sin A} = \frac{a}{\cos B} = \frac{b}{\sin B} = \frac{b}{\cos A} = \sqrt{a^2 + b^2}$$

Oblique triangles



Sine law

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine law

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Given	Required	Formulas
A, B, a	C, b, c	$C = 180 - (A + B)$; $b = \frac{a}{\sin A} \sin B$; $c = \frac{a}{\sin A} \sin C$
A, b, c	a	$a^2 = b^2 + c^2 - 2bc \cos A$
a, b, c	A	$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$
a, b, c	Area	Area $= \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a + b + c)$
C, a, b	Area	Area $= \frac{1}{2} ab \sin C$

General trigonometric formulas

$$\sin A = 2\sin\tfrac{1}{2}A\cos\tfrac{1}{2}A = \sqrt{1-\cos^2 A} = \tan A\cos A$$

$$\cos A = 2\cos^2\tfrac{1}{2}A - 1 = 1 - 2\sin^2\tfrac{1}{2}A = \cos^2\tfrac{1}{2}A - \sin^2\tfrac{1}{2}A = \sqrt{1-\sin^2 A}$$

$$\tan A = \frac{\sin A}{\cos A} = \frac{\sin 2A}{1+\cos 2A} = \sqrt{\sec^2 A - 1}$$

Addition and subtraction identities

$$\sin(A \pm B) = \sin A\cos B \pm \sin B\cos A$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

$$\sin A + \sin B = 2\sin\tfrac{1}{2}(A+B)\cos\tfrac{1}{2}(A-B)$$

$$\sin A - \sin B = 2\cos\tfrac{1}{2}(A+B)\sin\tfrac{1}{2}(A-B)$$

$$\cos A + \cos B = 2\cos\tfrac{1}{2}(A+B)\cos\tfrac{1}{2}(A-B)$$

$$\cos A - \cos B = 2\sin\tfrac{1}{2}(A+B)\sin\tfrac{1}{2}(A-B)$$

Double-angle identities

$$\sin 2A = 2\sin A\cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1$$

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$

Half-angle identities

$$\sin\frac{A}{2} = \sqrt{\frac{1-\cos A}{2}}$$

$$\cos\frac{A}{2} = \sqrt{\frac{1+\cos A}{2}}$$

$$\tan\frac{A}{2} = \frac{\sin A}{1+\cos A}$$

Trigonometric Relations

$\cos\theta = \cos(-\theta) = \sin(\pi/2 - \theta)$
$\sin\theta = -\sin(-\theta) = \cos(\pi/2 - \theta)$
$\tan\theta = -\tan(-\theta) = \cot(\pi/2 - \theta)$

$\sin 2\theta = 2\cos\theta\sin\theta$
$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$
$\tan 2\theta = 2\tan\theta/(1 - \tan^2\theta)$

$\sin\theta/2 = \pm[(1 - \cos\theta)/2]^{1/2}$
$\cos\theta/2 = \pm[(1 + \cos\theta)/2]^{1/2}$
$\tan\theta/2 = \sin\theta/(1 + \cos\theta)$

$\cos^2\theta + \sin^2\theta = 1$
$\sec^2\theta - \tan^2\theta = 1$
$\mathrm{cosec}^2\theta - \cot^2\theta = 1$

$\sin^2\theta = ½(1 - \cos 2\theta)$
$\cos^2\theta = ½(1 + \cos 2\theta)$
$\tan^2\theta = (1 - \cos 2\theta)/(1 + \cos 2\theta)$

$\sin(A + B) = \sin A\cos B + \cos A\sin B$
$\sin(A - B) = \sin A\cos B - \cos A\sin B$
$\cos(A + B) = \cos A\cos B - \sin A\sin B$
$\cos(A - B) = \cos A\cos B + \sin A\sin B$
$\tan(A + B) = (\tan A + \tan B)/(1 - \tan A\tan B)$
$\tan(A - B) = (\tan A - \tan B)/(1 + \tan A\tan B)$

$\sin A + \sin B = 2\sin ½(A + B)\cos ½(A - B)$
$\sin A - \sin B = 2\cos ½(A + B)\sin ½(A - B)$
$\cos A + \cos B = 2\cos ½(A + B)\cos ½(A - B)$
$\cos A - \cos B = -2\sin ½(A + B)\sin ½(A - B)$
$\tan A + \tan B = \sin(A + B)/(\cos A\cos B)$
$\tan A - \tan B = \sin(A - B)/(\cos A\cos B)$

$\sin^2 A + \sin^2 B = 1 - \cos(A + B)\cos(A - B)$
$\sin^2 A - \sin^2 B = \sin(A + B)\sin(A - B)$
$\cos^2 A + \sin^2 B = 1 - \sin(A + B)\sin(A - B)$
$\cos^2 A - \sin^2 B = \cos(A + B)\cos(A - B)$
$\cos^2 A + \cos^2 B = 1 + \cos(A + B)\cos(A - B)$
$\cos^2 A - \cos^2 B = -\sin(A + B)\sin(A - B)$

For a triangle with sides *a, b, c,* and angles *A,B,C* opposite sides *a, b* and *c* respectively, the following relations hold.

$a^2 = b^2 + c^2 - 2bc\cos A$
$a/\sin A = b/\sin B = c/\sin C.$
$(a - b)/(a + b) = \tan ½(A - B)/\tan ½(A + B)$

Conversion factors for systems of measurements

This unit	Multiplied by	Equals this unit
Feet (ft)	12	Inches (in)
Feet (ft)	0.305	Meters (m)
Yards (yd)	3	Feet (ft)
Yards (yd)	0.914	Meters (m)
Inches (in)	2.54	Centimeters (cm)
Rod, pole or perch	16.5	Feet (ft)
Miles (mi), statute	1.609	Kilometer (km)
Miles, nautical	1.852	Kilometer (km)
Kilometers (km)	1 000	Meters (m)
Square inches (in^2)	6.45	Square centimeters (cm^2)
Square feet (in^2)	0.0929	Square meters (m^2)
Square yards (yd^2)	0.836	Square meters (m^2)
Square inches (in^2)	6.45	Square centimeters (cm^2)
Square miles (mi^2)	2.590	Square kilometers (km^2)
Ares (a)	100	Square meters (m^2)
Acres	0.40469	Hectares
Acres	43560	Square feet (ft^2)
Hectares	10 000	Square meters (m^2)
Cubic inches (in^3)	16.4	Cubic centimeters (cm^3)
Cubic feet (ft^3)	0.0283	Cubic meters (m^3)
Cubic yards (yd^3)	0.7646	Cubic meters (m^3)
Gallons, US (gal)	3.785	Liters (L), Cubic decimeters (dm^3)
Gallons, UK (gal)	4.546	Liters (L), Cubic decimeters (dm^3)
Fluid ounce (oz)	28.41	Cubic centimeters (cm^3)
Ounces (oz)	28.4	Grams (g)
Pounds (lb)	0.454	Kilograms (kg)
Tons (short)	0.9072	Tons, metric
Tons (long)	1.016	Tons, metric
Stones	6.35	Kilograms (kg)
Quintals	100	Kilograms (kg)
Feet per minute (ft/min)	0.3048	Meters per minute (m/min)
Feet per second (ft/s)	0.3048	Meters per second (m/s)
Miles per hour (mi/h)	1.609	Kilometers per hour (km/h)
Pounds per square inch, psi (lb/in^2)	0.0703	Kilograms per square centimeter (kg/cm^2)
Pounds per square inch, psi (lb/in^2)	6.895	Kilopascal (kPa)
Bars (bar)	100 000	Pascal (Pa)
Atmospheres (atm)	1.01	Bars (bar)
Pounds per cubic feet (lb/ft^3)	16.026	Kilograms per cubic meter (kg/m^3)
Horsepower (hp)	0.746	Kilowatts (kW)
British thermal units (Btu)	0.252	Calories (cal)
Equals this unit	**Divided by**	**This unit**

Metric prefixes		
tera-	T	10^{12}
giga-	G	10^{9}
mega-	M	10^{6}
kilo-	k	10^{3}
hecto-	h	10^{2}
deca-	da	10^{1}
deci-	d	10^{-1}
centi-	c	10^{-2}
milli-	m	10^{-3}
micro-	μ	10^{-6}
nano-	n	10^{-9}
pico-	p	10^{-12}
femto-	f	10^{-15}
atto-	a	10^{-18}

Greek alphabet			
alpha	α	nu	ν
beta	β	xi	ξ
gamma	γ	omicron	ο
delta	δ or Δ	pi	π
epsilon	ε	rho	ρ
zeta	ζ	sigma	σ or Σ
eta	η	tau	τ
theta	θ	upsilon	υ
iota	ι	phi	φ
kappa	κ	chi	χ
lambda	λ	psi	ψ
mu	μ	omega	ω

Basic units of measurements

length .... meter (in MKS system), centimeter (in CGS system)
mass .... kilogram (in MKS system), gram (in CGS system)
time .... second (in MKS system), second (in CGS system)
volume .... liter (for liquids)
area .... square meter (produced unit of length for 2-dimensions)
volume .... cubic meter (produced unit of length for 3-dimensions)

Some important numbers in surveying

0.000 000 1 (or 2)	= coefficient of expansion, Invar tape, per 1°F
0.000 004 848	= sin 1" = tan 1"
0.000 006 45	= coefficient of expansion, steel tape, per 1°F
0.000 290 89	= approx. sin 1' = tan 1'
0.017 45	= approx. sin 1° = tan 1° = about 0.01¾
0.6745	= coefficient for 50% standard deviation
1.6449	= coefficient for 90% standard deviation
1.9599	= coefficient for 95% standard deviation
1.15 miles	= 1 minute (1') of latitude ≅ 1 nautical mile
69.1 miles	= 1° latitude (=1.15 miles*60 min/hr)
101 ft	= 1 second (1") of latitude
15° longitude	= width of one time zone = 360°/24 hr
1° longitude	= 4 minute
3.141 592 654	= π
66 ft	= length of Gunter's chain = 100 links (lk) = 20 m
20°C	= standard temperature (Celcius) in taping = 68°F
57° 17' 44.6"	= 1 radian (rad) = 57.295 779 51° = 180°/π in sec = 206 264.806 25 sec
400 grads (gons)	= 360°
10 000 km	= dictance from equator to pole (basis for length of meter)
6 356 752.3 m	= earth's polar semi-axis (GRS80 ellipsoid)
6 378 137.0 m	= earth's equatorial semi-axis (GRS80 ellipsoid)
6 371 000 m	= mean radius of earth = 20 902 000 ft

1. INTRODUCTION

1.1 Definition of Surveying

Surveying has traditionally been defined as the science, art and technology of determining relative positions of points above, on, or beneath the surface of the earth, or establishing such points. The survey of land, together with the natural (river, mountain, forest, etc.) and/or man-made (bridge, building, dam, mining area, etc.) features on or adjacent to the surface of the Earth, usually requires two-dimensional drawings of three-dimensional objects. Such drawings are usually produced on either horizontal or vertical plane.

A drawing on a horizontal plane is known as a plan or map. A ***plan*** is a true-to-scale representation, while a ***map*** may contain features which are represented by conventional signs or generalized symbols, these not being true scale representations.

Drawings in a vertical plane are known as sections, cross-sections or elevations. An ***elevation*** is a side or end view of an object. A ***section*** is a vertical 'plan' of a line through a building, or a line of a proposed road, etc. A long section such as along a proposed road or rail route is known as a ***longitudinal section***, while sections taken at right angles to the longitudinal line are known as ***cross-sections***.

In a more general sense surveying can be regarded as that discipline which encompasses all methods for gathering and processing information about the physical earth and environment. Conventional ground systems are now supplemented by aerial and satellite surveying methods, which evolved through the defense and space programs.

1.2 Objects of Surveying

Surveying techniques may be considered to be used for three distinct purposes as follows.

i. Surveying for the preparation of maps, plans, etc.

The determination of the relative positions of natural and artificial features so that they may be correctly represented on maps, plans or sections.

ii. Setting out

The setting out upon or under the ground of proposed construction or engineering works.

iii. Computations such as areas and volumes

The execution of calculations for land areas, for earthworks volumes, etc., either based on field measurements or on measurements abstracted from maps, plans and sections.

1.3 Classifications of Surveying

Surveying operations have been classified in a great variety of ways, depending upon the purpose of the work or alternatively the equipment or methods actually used.

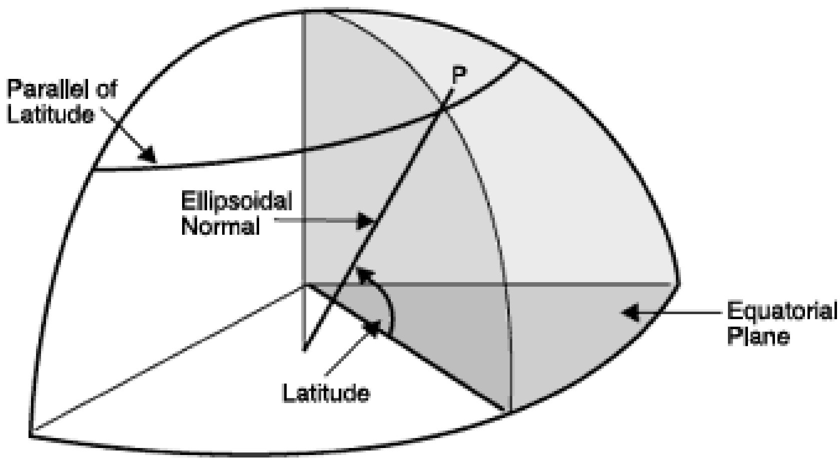
1.3.1 Plane surveying or geodetic surveying

Plane surveying is that type of surveying in which the surface of the earth is considered to be a plane for all X and Y dimensions. All Z dimensions (height) are referenced to the mean spherical surface of the earth (mean sea level). Most engineering and property surveys are plane surveys.

Geodetic surveying is that type of surveying in which the surface of the earth is considered to be spherical (actually an ellipsoid of revolution) for X and Y dimensions. As in plane surveying, the Z dimensions (height) are referenced tho the mean surface of the earth (mean sea level). Traditional geodetic surveys were very precise surveys of great magnitude (e.g., national boundaries, control networks, etc.). Modern surveys (data gathering, control and layout) utilizing the Global Positioning System (satellite surveying) are also based on the geometric shape of the earth.

It is evident that the earth's surface can not be accurately represented on a plane. In ordinary geometry, the angles of a triangle always add up to 180°, but on the surface of a sphere the angles of a triangle add up to more than 180°.

In geodetic surveying, the curved surface of the earth is considered by performing the computations on an ellipsoid (curved surface approximating the size and shape of the earth). It is now becoming common to do geodetic computations in a three-dimensional, earth-centered Cartesian coordinate system. The calculations involve solving equations derived from spherical trigonometry and calculus.



We use something called a "Geodetic coordinate system" to identify the positions on our nice, easy to use mathematical model, described earlier. This system allows positions on the earth's surface to be described in terms of latitude, longitude and height.

In plane surveying, except for leveling, the refrence base for field work and computations is assumed to be a flat horizontal surface. The direction of a plumb line (and thus gravity) is considered parallel throughout the survey region, and all measured angles are presumed to be plane angles. In general, algebra, plane and analytic geometry, and plane trigonometry are used in plane surveying calculations.

Geodetic Surveying: If surveying area is greater than 50 km^2 ;

- 50 – 5000 km^2 ==➔ Earth's shape is assumed **sphere**
- > 5000 km^2 ==➔ Earth's shape is assumed **ellipsoid**

Plane Surveying: If surveying area is less than 50 km^2.

1.3.2 Classification according to purpose or use

Many types of surveys are so specialized that a person proficient in aparticular discipline may have little contact with the ather areas. Some important classifications are described briefly here.

The ***preliminary survey*** is the gathering of data (distances, position, and angles) to locate physical features (e.g., trees, rivers, roads, structures, or property markers) so that the data can be plotted to scale on a map or plan. Preliminary surveys also include the determination of differences in elevation (vertical distances) so that elevations and contours may also be plotted.

Layout surveys involve marking on the ground (using wood stakes, iron bars, monuments, nails, spikes, etc.) the features shown on a design plan. The layout can be for property lines, as in subdivision surveying, or it can be for a wide variety of engineering works (e.g., roads, pipelines, bridges); the latter group is known as construction surveys.

Control surveys are used to reference both preliminary and layout surveys. Horizontal control can be arbitrarily placed, but it is usually tied directly to property lines, roadway center lines, or coordinated control stations. Vertical control is a series of benchmarks-permanent points whose elevation above mean sea level have been carefully determined.

1. *Topographic surveys*: preliminary surveys used to tie in the natural and man-made surface features of an area. The features are located relative to one another by tying them all into the same control lines or control grid.
2. *Hydrographic surveys*: preliminary surveys that are used to tie in underwater features to a surface control line. Usually shorelines, marine features, and water depths are shown on the hydrographic map.
3. *Route surveys*: preliminary, layout, and control surveys that range over a narrow, but long strip of land. Typical projects that require route surveys are highways, railroads, transmission lines, and channels.
4. *Property surveys*: preliminary, layout, and control surveys that are involved in determining boundary locations or in laying out new property boundaries (also known as *cadastral* or *land surveys*).
5. *Aerial surveys*: preliminary and final surveys using aerial photography. Photogrametric techniques are employed to convert the aerial photograph into scale maps and plans.
6. *Mine surveys*: are performed above and below ground to guide tunneling and other operations associated with mining, including geophysical surveys for mineral and energy resource exploration.
7. *Construction (engineering) surveys*: layout surveys for an gineering works.
8. *Final ("as built") surveys*: similar to preliminary surveys. Final surveys tie in features that have just been constructed to provide a final record of the construction and to check that the construction has proceeded according to design plans.

1.4 Some Definitions in Surveying

Vertical Line is the line passing through a point, say A, in the direction of gravity at that point (Figure 1).

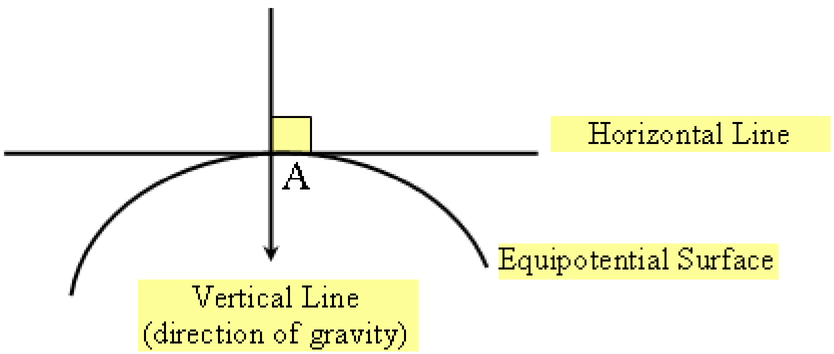


Figure 1. Vertical and Horizontal lines.

Horizontal Line is the line passing through the point A and perpendicular to the vertical line at that point as shown in the Figure.

Vertical Plane is the plane containing the vertical line at A.

Horizontal Plane is the plane passing through A and perpandicular to the vertical line at A.

Horizontal Distance is the distance between two given points measured on a horizontal plane as shown in Figure 2.

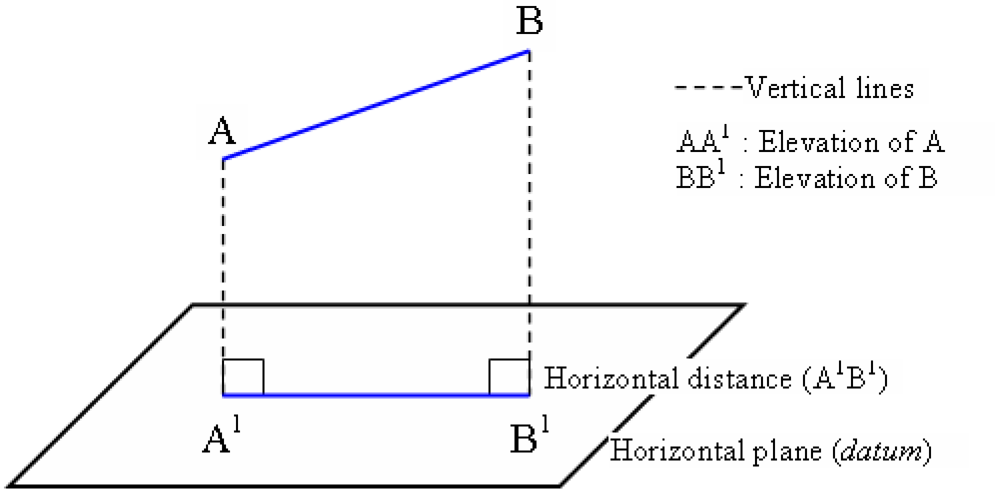


Figure 2. Horizontal distance and elevation.

Elevation is the vertical distance (measuring along the vertical line) from a predifined datum as shown in Figure 2.

Horizontal Angle is the angle measured in a horizontal plane between two vertical planes.

Vertical Angle (V) is the angle in a vertical plane measured from a horizontal plane as shown in Figure 3. It's also known as elevation (or inclination) angle. If it is measured downward from a horizontal plane it is called a *depression angle (d)*.

Zenith Angle (Z) is the angle on a vertical plane measured downward from the upward direction as shown in Figure 3. The relation between vertical angle and the zenith angle is

Zenith angle (Z) = 90° - Vertical angle (V)

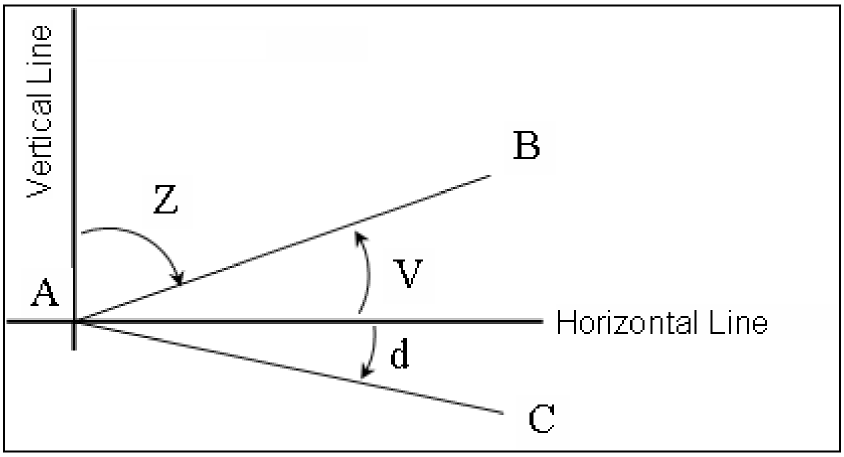


Figure 3. Vertical angle, depression angle and zenith angle..

Mean Sea Level (Datum) is the average level of the ocean surface halfway between the highest and lowest levels recorded. We use mean sea level as a plane upon which we can reference or describe the heights of features on, above or below the ground.

What is Geodesy?
Quite simply 'Geodesy' is the study of the shape and size of the earth. **Geodesy** is the discipline that deals with the measurement and representation of the earth, its gravity field and geodynamic phenomena (polar motion, earth tides, and crustal motion) in three-dimensional time varying space. Geodesy is primarily concerned with positioning and the gravity field and geometrical aspects of their temporal variations.

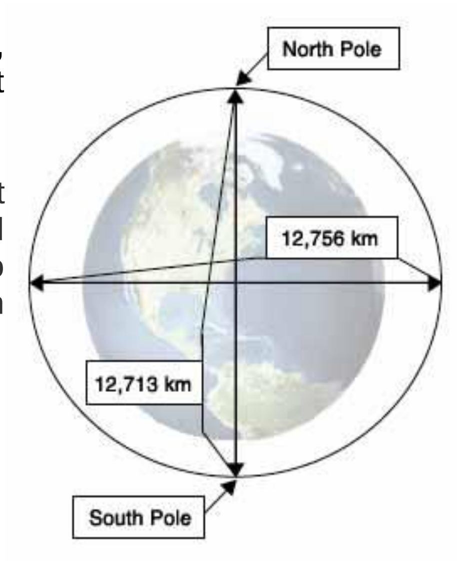
Why do we need maps?

There's an old saying that goes something like, "You can't tell where you're going unless you know where you've been". After all, if you don't know where you are, how can you figure out which way to go, much less find your way home?

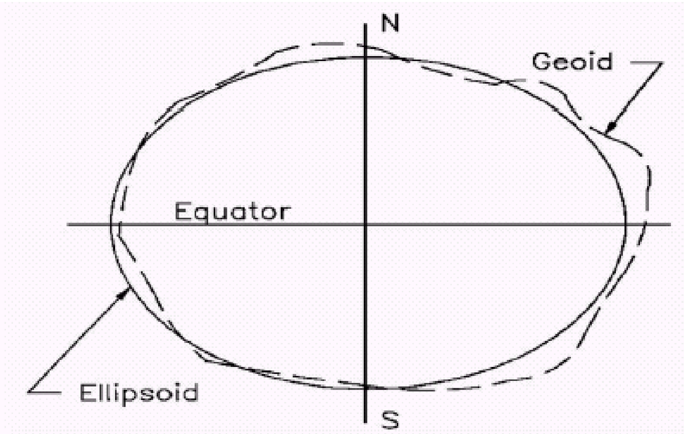
Maps are just like pictures. Have you ever heard the phrase, "A picture is worth a thousand words"? Well, a good map can help us to make sense of all kinds of information and these days, any thing we can do to help us sort out the mountains of information we're faced with is a good thing! Pictures, or maps, make these large amounts of information much easier to understand.

Maps play a big role in our life. For example - when you invite a friend over for the first time, you'll probably need to give them directions so they can find your house. You might tell them to follow a certain street, go past the convenience store and turn right at the playground to get to your house. In other words, you use 'landmarks' to describe to your friend where you live. Landmarks are places, buildings, roads etc. that are easy to identify. They give your friend a helping hand to find your house. If you make a picture of the landmarks and how to use them to find your house, well, you've made a map. A map is easy for you to draw, because you know the turns and landmarks near your house and it's easy for your friend to follow, because they can just follow the pictures.

Well, the earth is almost round, but not quite. And, because the earth's not quite round, we need to know just what shape it is, so we can make accurate maps.



Geoid is esentially the real shape of the earth, without accounting for the topographic features. It is an idealized equilibrium surface. The geoid, unlike the ellipsoid, is too complicated to serve as the computational surface on which to solve geometrical problems like point position.



Relationship between geoid and approximating geodetic ellipsoid.

The geoid is theoretical only. You can't see it, touch it or even dig down to find it. Simply put, the geoid is the natural extension of the mean sea level surface under the landmass. The Geoid model contributes to the vertical component of the reference system so that ellipsoidal GPS heights can be converted to orthometric elevations for practical uses.

- Topography - The surface of the Earth
- Ellipsoid - GPS heights are referenced to this mathematical surface
- Geoid - The natural surface extension of mean sea level

What is geomatics?

The mathematics of the earth; the science of the collection, analysis, and interpretation of data, especially instrumental data, relating to the earth's surface. *(Oxford English Dictionary)*

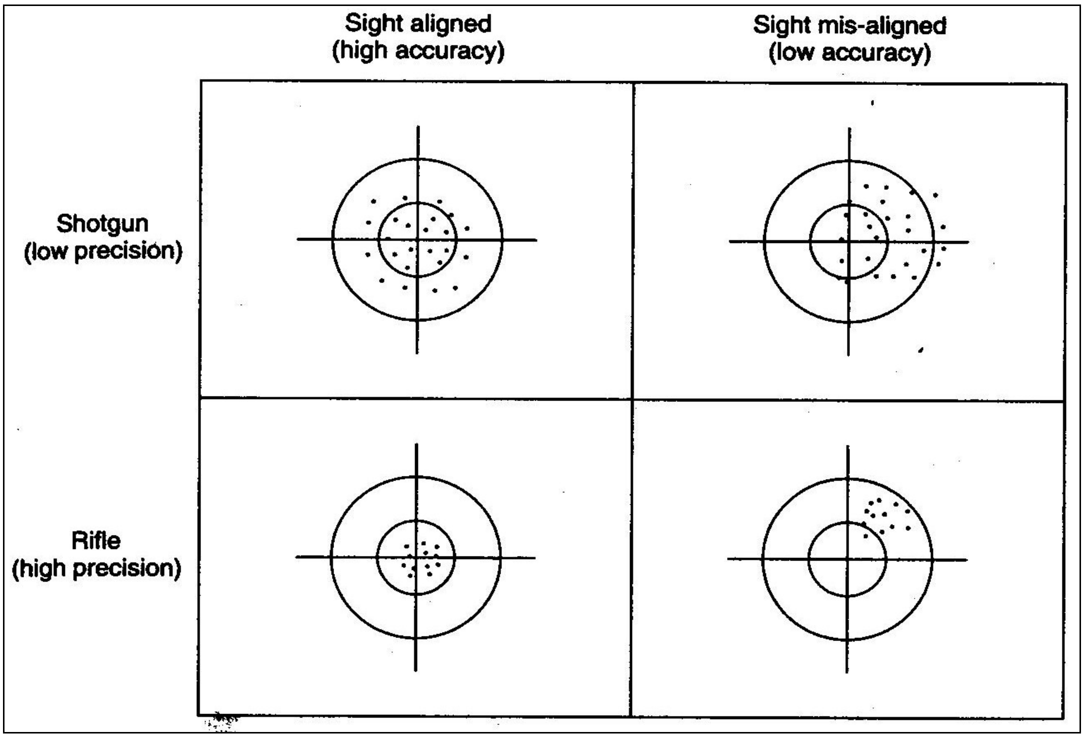
Geomatics is the science and technology of gathering, analyzing, interpreting, distributing and using geographic information. Geomatics encompasses a broad range of disciplines that can be brought together to create a detailed but understandable picture of the physical world and our place in it. These disciplines include:

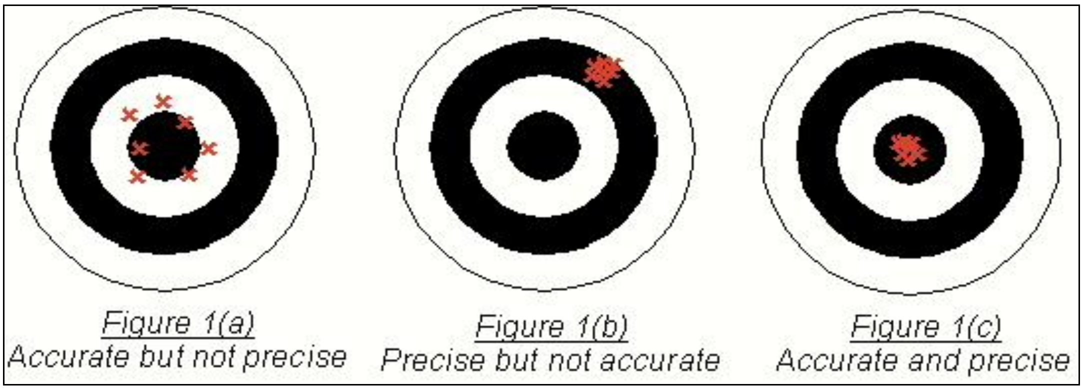
- surveying
- mapping
- remote sensing
- geographic information systems (GIS)
- global positioning system (GPS)

Geomatics Engineering is a modern discipline, which integrates acquisition, modelling, analysis, and management of spatially referenced data, i.e. data identified according to their locations. Based on the scientific framework of geodesy, it uses terrestrial, marine, airborne, and satellite-based sensors to acquire spatial and other data. It includes the process of transforming spatially referenced data from different sources into common information systems with well-defined accuracy characteristics.

2. ACCURACY AND PRECISION

Accuracy is the relationship between the value of a measurement and the "true" value of the dimension being measured . *Precision* describes the refinement of the measuring process and the ability to repeat the same measurement with consistently small variations in the measurements. The following figure figure shows targets with hit marks for both a rifle and a shotgun, which illustrates the concepts of precision and accuracy.





The concepts of accuracy and precision are also illustrated in the following example: A building wall that is known to be 157,22 ft long is measured by two methods. In the first case the wall is measured very carefully using a fiberglass tape graduated to the closest 0,1 ft. The result of this operation is a measurement of 157,2 ft. In the second case the wall is measured wit the same care, but with a more precise steel tape graduated to the closest 0,01 ft. The result of this operation is a measurement of 157,23 ft. In this example, the more precise method (steel tape) resulted in the more accurate measurement.

	"True" distance	Measured distance	Error
Cloth tape	157,22	157,2	0,02
Steel tape	157,22	157,23	0,01

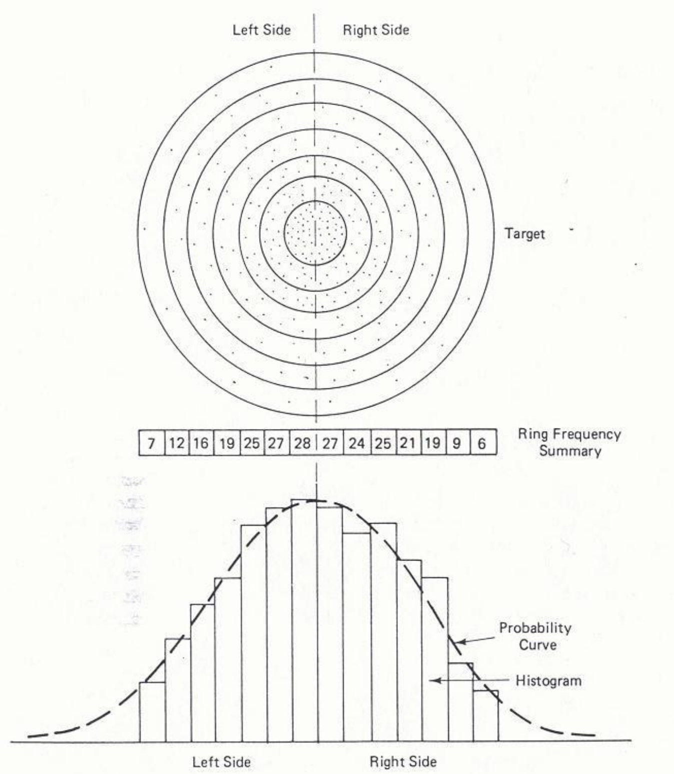
3. MEASURE OF ACCURACY AND PRECISION

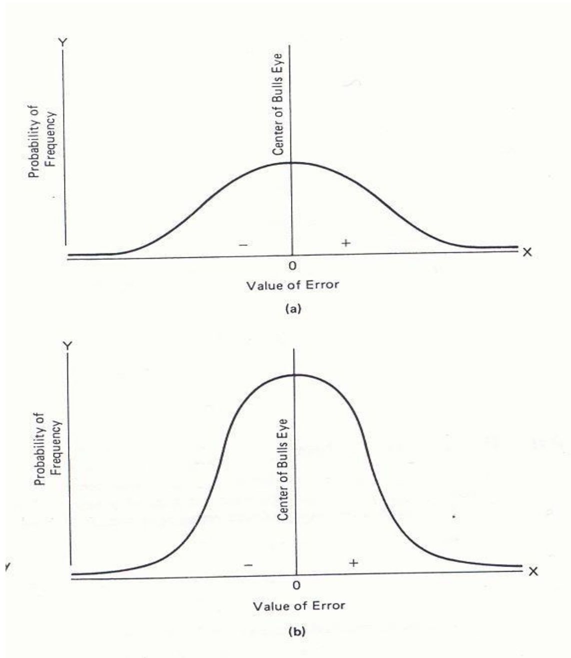
3.1 Probability Curve

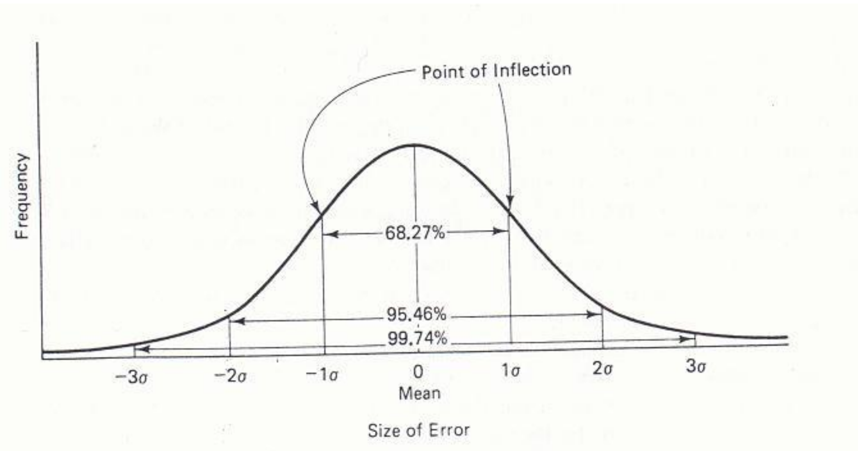
The probability curve of normal distribution has the equation

$$y = \frac{1}{\sigma\sqrt{2\pi}} e^{-v^2/2\sigma^2}$$

where “y” is the ordinate value of a point on the curve; “v” is the size of the residual; “e” is the base of natural logarithms (2,718); and “σ” is a constant known as the standard deviation or standard error, a measure of precision. Since σ is associated with an infinitely large sample size, the term standard error (SE) will be used when analyzing finite survey repetitions.







3.2 Most Probable Value

A residual is the difference between the true value or location (e.g., bull’s-eye center) and the value or location of one occurence or measurement. It is the amount of error for individual measurements.

The true value (never known), but that for the purpose of calculating errors, the *arithmetic mean* is taken to be the true, or *most probable*, value.

Since we do not have large (infinite) numbers of repetitive measurements in surveying, the arithmetic mean;

Mean: $$\overline{X} = \frac{\sum x}{n}$$

where $\sum x$ is the sum of the individual (x) measurements, and n is the number of individual measurements. Then residuals of measurements;

$v_i = x_i - \overline{X}$ where i=1,2,3, ..., n

3.3 Standard Error

In statistical theory, precision is measured by the *standard deviation* (also called *standard error* or *root mean square error*). Theoretically:

$$\sigma = \pm\sqrt{\frac{\sum v^2}{n}}$$

where σ is the standard deviation of a very large sample, v is the true residual, and n is the very large sample size. Practically,

$$SE = \pm\sqrt{\frac{\sum v^2}{n-1}}$$

where SE is the standard error of a set of repetitive measurements; v is the error ($x - \overline{X}$), and n is the number of repetitions. Since the use of $\overline{X}$ instead of the true value always results in an underestimation of the standard deviation, (n-1) is used in place of n. The term (n-1) is known in statistics as *degree of freedom* and represents the number of extra measurements taken. That is, if a line were measured 10 times, it would have 9 (10-1) degrees of freedom. Obviously, as the number of repetitions increases, the difference between n and (n-1) becomes less significant.

4. MEASUREMENTS AND ERRORS

It can be said that no measurement (except for counting) can be free of error. In practice every measurement is subject to errors due to the imperfection of the human being and instruments used. Therefore exactness is not possible through measurements. However, by using precise instruments and through repeated measurements we can achieve a high degree of refinement.

For purposes of calculating errors, the "true" value is determined statistically after repeated measurements. In the simplest case, the true value for a distance is taken as the mean value for a series of repeated measurements.

Errors in measurements are catagorised as:

a) Blunders (mistakes): They are unacceptable errors caused by the observer or bed instrument, etc. In order to avoid these errors one should work carefully and conduct control measurements. Examples of mistakes include transporting figures (recording a tape value of 68 as 86), miscounting the number of full tape lengths in a long measurement, measuring to or from the wrong point, and the like. As a rule, every measurement is immediately checked or repeated. This immediate repetition enables the surveyor to eliminate most mistakes and, at the same time, improve the precision of the measurement.

b) Systematic errors: These are defined as those errors whose magnitude and algebraic sign can be determined. The fact that these errors can be determined allows the surveyor to eliminate them from the measurements and thus improve the accuracy. An error due to the effects of temperature on a steel tape is an example of a systematic error. If the temperature is known, the shortening or lengthening effects on a steel tape can be precisely determined.

c) Random errors: They are associated with the skill of the surveyor. Random (also known as *accidential*) errors are introduced into each measurement mainly because no human being can perform perfectly. The sign (+ or -) of random errors is not known, therefore they can not be added to or subtracted from the measured quantity. However we can minimize these errors by having redundant (more than enough) observations.

We see that we can correct for blunders and systematic errors. But we can not correct for random errors. However we can minimize random errors and get the best estimation for the parameters.

Accuracy Ratio: The *accuracy ratio* of a measurement or series of measurements is the ratio of error of closure to the distance measured. The *error of closure* is the difference between the measured location and the heoretically correct location. The theoretically correct location can be determined from repeated measurements or mathematical analysis.

To illustrate, a distance was measured and found to be 250,56 m. The distance was previously known to be 250,50 m. The error is 0,06 m in a distance of 250,50 m.

Accuracy ratio = 0,06/250,50 = 1/4175 = 1/4200

The accuracy ratio is expressed as a fraction whose nomerator is unity and whose denominator is rounded to the closest 100 units.

Many land and engineering surveys have in the past performed at 1/5000 or 1/3000 levels of accuracy. With the trend to polar layouts from coordinated control, accuracy ratios on the order of 1/10000 and 1/20000 are now often specified. It should be emphasized that for each of these specified orders of accuracy, the techniques and instrumentation used must also be specified.

Correction, Tolerance : Error and correction have opposite signs. For instance; sum of interior angles of a triangle is found $180^o42''$. Thus error (v) is 42" ($=180^o42''–180^o$). Then total correction value will be –42". In this case, the error is distributed equally to the angles and 14"(=42"/3) is substracted from each angle. This process is named as "distribution of error" or "balancing the measurements". To perform this process, the amount of error should be within an acceptable limits. This limit value is named as **"tolerance"**. This value depends on accuracy of job and size of measurement. If this error is obtained bigger than tolarable value then the measurement must be repeated.

Example : A length is measured 10 times and the results are obtained as given in the following table. Determine errors for this trial.

	i=1	i=2	i=3	i=4	i=5	i=6	i=7	i=8	i=9	i=10	Sum
Measurements, x (m)	180,57	180,62	180,63	180,55	180,56	180,62	180,57	180,61	180,62	180,55	1805,9
Residuals, v (cm)	-2	3	4	-4	-3	3	-2	2	3	-4	0
v^2 (cm^2)	4	9	16	16	9	9	4	4	9	16	96

Aritmetic mean, $\overline{X} = \dfrac{x_1 + x_2 + ... + x_{10}}{10}$ ➔ $\overline{X}$ =180,59 m

Residuals are obtained by $v_i = x_i - \overline{X}$

Standard error, $SE = \pm\sqrt{\dfrac{v_1^2 + v_2^2 + ... + v_{10}^2}{10-1}} = \sqrt{\dfrac{96}{9}} = \pm 3{,}3$ cm

Probable error, is determined after ordering absolute values of residuals;

2 2 2 3 **3** **3** 3 4 4 4 ➔ r = (3+3)/2 = ±3 cm is found.

Accuracy ratio (Relative error) = 3,3 cm / 18059 cm = 1/5472 is determined.

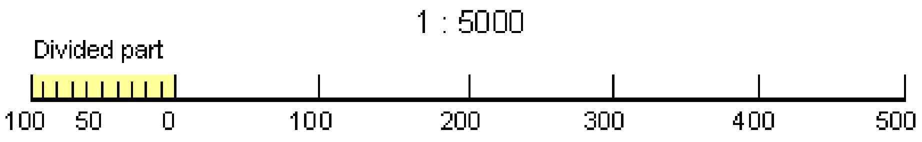
5. SCALES

1) Linear scale : Ratio between drawing length and true length.
M (scale) = Drawing length / True length

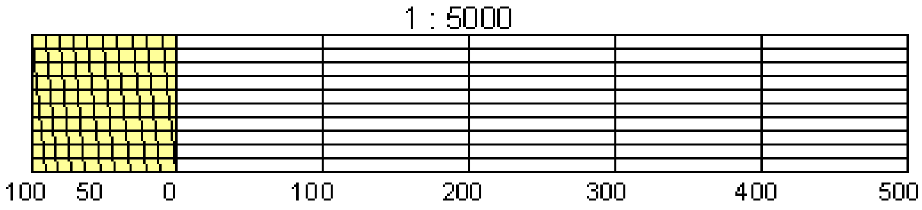
Three types of scale are used

i) Numerical scale : e.g., 1/500 (1:500), 1/1000 (1:1000), 1/5000 (1:5000)

ii) Graphic scale :



iii) Geometric scale : Similar to graphic scale, but more precise.



2) Area scale : If we take a rectangle with sides **a** and **b** on a map, then the area of rectangle is determined as f=a*b. To detrmine the corresponding true field area (F);

$$F = (a*M)(b*M) = a*b*M^2 .$$

$$\frac{f}{F} = \frac{1}{M^2} \qquad \text{(M=scale of the map)}$$

Example-1 : Distance between two points on a map is measured 64,2 mm. If the scale of the map is 1/2000 determine the true distance between the points;

a=64,2 mm   If a/A = 1/M then A=64,2*2000 = 128400 mm = 128,4 m

A=?

Example-2 : Field distance between two points is 292 m. Determine map scale if map distance between the points is measured 58,4 mm.

a/A = 1/M   M=292000 mm/58,4 mm = 5000

Then Scale, M=1/5000 is found.

Example-3 : The area of a rectangle is determined 1225 mm^2 on a map which has a scale of 1/2000. Determine the true area of this land in m^2 and hectare.

$f/F = 1/M^2$ $F=1225\ mm^2*2000^2 = 4900*10^6\ mm^2$ is found. Then;

$F = 4900\ m^2 = 0,49$ hektar

ANGLES

Degree System

1° (degree) is the angle between two radii of a circle bordering an arc equals 1/360 of the circumference. i.e. $\Delta c/c = 1/360°$

1° = 60' (arc minutes) = 3600" (arc seconds)

1' = 60" (arc seconds)

Example : 16° 18' 37" = 16 + 18/60 + 37/3600 = 16 + 0,3 + 0,010278 = **16°,310278**

Example : 10,125° = $10^o,(\frac{125}{1000})*60 = 10^o 7',5 = 10^o 7'(\frac{5}{10})*60 =$ **10° 7' 30"**

Centesimal System

1^g (grade) is the angle between two radii of a circle bordering an arc equals 1/400 of the circumference. i.e. $\Delta c/c = 1/400^g$

$1^g = 100^c$ (centigrades) $= 10000^{cc}$ (decimiligrades)

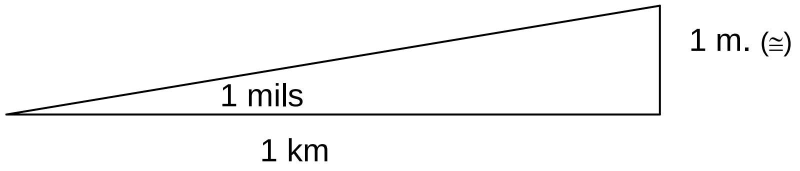
1^c (centigrades) $= 100^{cc}$ (decimiligrades)

Example : $56^g\ 27^c\ 52^{cc} = 56 + 27/100 + 52/10000 =$ $\mathbf{56^g{,}2752}$

Example : $26^g{,}1278 = 26^g{,}(\frac{1278}{10000})*100 = 26^g 12^c(\frac{78}{100})*100 =$ $\mathbf{26^g\ 12^c\ 78^{cc}}$

Mils System (military use)

1 mils is the angle between two radii of a circle bordering an arc equals 1/6400 of the circumference. i.e. $\Delta c/c = 1/6400$ mils

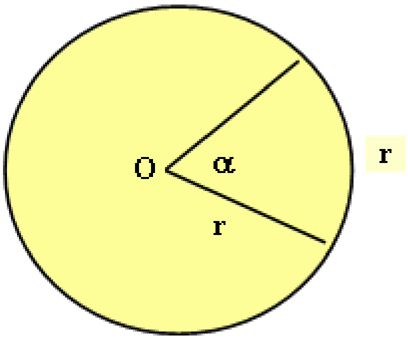


ARC

RADIAN: is the angle subtended by an arc of a circle having a length equal to the circle's radius.

Therefore 2π rad = 360°, 1 rad = 57°17'44.8" = 57,2958°

1° = 0,01745 rad



Conversions between the angles and arc unit

$$\frac{D}{360} = \frac{G}{400} = \frac{M}{6400} = \frac{R}{2\pi}$$

Example: Conversion factors for Centesimal angles.

$$\rho^g = \frac{200}{\pi} \cong 63^g.662 \qquad \rho^\circ = \frac{180}{\pi} \cong 57^\circ.2958 \qquad \rho^{=} = \frac{3200}{\pi} \cong 1018.59\overset{=}{2}$$

Example: $0{,}45\ \ rad = 28^g.6479 = 25^\circ.7831 = 458.366\overset{=}{4}$

4. TAPE MEASUREMENTS

4.1 Methods of Linear Measurement

Throughout recorded history, people have always had some method of measuring distances. Measuring techniques are direct, such as applying a graduated tape against the marks to be measured, or indirect, such as measuring related parameters (e.g., the phase differences for light waves to be reflected to a source as in EDM work), and then computing the required (horizontal) distance.

4.2 Types of Measurement

4.2.1 Pacing

Pacing is a very useful (although imprecise) form of measurement. Surveyors can determine the length of a pace, which, for them, can be comfortably repeated. Pacing is particularly useful when looking for survey markers in the field; plan distance from a found marker to another marker can be paced off so that the marker can be located. Pacing is also useful as a rough check on construction layouts.
When performed on horizontal or uniformly sloping land, pacing can be performed at an accuracy level of 1/50 to 1/100. The accuracy of pacing cannot be relied upon when pacing up or down steep hills. Paces shorten on inclines and lengthen on declines.

4.2.2 Odometer

Automobile odometer readings can be used to measure from one fence line to another adjacent to a road. Odometer readings are precise enough to enable the surveyor to differentiate fence lines and assist in identification of property lines. This type of measurement is useful when beginning a surveyor collecting information in order to begin a survey.
Measuring wheels (with attached odometers) are used by surveyors from assessment offices to measure and check lot frontages. Police officers also use these measuring wheels to map out automobile accident scenes.

4.2.3 Electronic Distance Measuring (EDM) Instruments

EDM instruments function by sending light waves or microwaves along the path to be measured and measuring the phase differences between transmitted and received signals, as with microwaves, or in measuring the phase differences between transmitted and received signals in returning the reflecting light wave to source.

4.2.4 Stadia

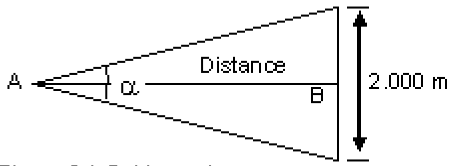
Stadia is a form of tacheometry that uses a telescopic cross-hair configuration to assist in determining distances. A series of rod readings is taken with a theodolite, and the resultant intervals are used to determine distances.

4.2.5 Tacheometry

Tacheometry involves the measurement of a related distance parameter either by means of a fixed-angle intercept (see stadia), by means of phase-differenced light-wave signal reflection (see EDM), or by means of a measured angle to a fixed base. The latter type of measurement is obtained with a wide variety of range-finding devices employed by the military. The most popular range-finding device used in surveying is the subtense bar. Subtense bar targets are held at 2.000 m apart (regardless of fluctuations in temperature) by invar wires under slight tension. The subtense bar, which is mounted on a tripod, is centered and leveled over point *B* (see Figure 2.1) and aligned roughly toward a theodolite set up over point *A.* The theodolite (I-second capability) can be used to align the bar exactly perpendicular to the line *AB* by noting the sighting device on the subtense bar.

(The sighting mark can only be clearly seen when the bar is perpendicular to the line of sight). The accuracy of this technique over short distances is comparable to careful measuring with a steel tape.

Since the horizontal angle between the subtense bar targets is independent of any vertical angle (the *a* angle is being measured between the vertical planes containing the targets), the distance given by cot (α/2) m is always the horizontal distance. This feature made the subtense bar a valuable aid to surveyors working in hilly and even mountainous country. However, the severe limitations on accuracy for long sights, together with the influx of EDM equipment, have resulted in very limited use of this technique in modem North American practice.



Distance AB = $\text{Cot}\,\frac{\alpha}{2}$ ....meters

Figure 2.1 Subtense bar.

4.3 Gunter's Chain

The measuring device in popular use during the time of the settlement of North America was the Gunter's chain, which is 66 ft long, subdivided into 100 links. The length of 66 ft was chosen because of its relationship to other units in the foot system of measurements:

80 chains = 1 mile

10 square chains = 1 acre ($10*66^2 = 43{,}560$ ft^2)

4 rods = 1 chain

The original surveys for most of Canada and the United States were performed by surveyors using Gunter's chains. Most of North America's early legal plans and records contain dimensions in chains and links. The dimensions of the original lots reflect the Gunter's chain (e.g., 20 chains*100 chains = 200 acres), and in some areas the allowance for future roads was routinely set at I chain (66 ft) in width.

■ **EXAMPLE 2.1**

An old plan shows a dimension of 5 chains 32 links. Convert this value to (a) feet and (b) meters.

Solution

(a) 5.32*66 = 351.12 ft

(b) 5.32*66*0.3048 = 107.021 m

The chain, which is awkward to use and is relatively imprecise when compared to modern steel tapes, is apparently still being used in England (metric versions);

4.4 Fiberglass Tapes

For measurements that do not require the precision available with steel tapes, fiberglass tapes are routinely used. These 1/2-in. wide tapes are sturdy enough to withstand the rigors of both natural field conditions and construction sites. The tapes illustrated in Figure 2.2 are nonwoven PVC-coated fiberglass and come graduated in feet, feet/inches, and meters in lengths of 50, 100, 165, 200, and 300 feet or 20, 30, 50, and 100 meters.

Woven tapes made of linen, Dacron, and the like can have fine-gauge copper strands interwoven to provide strength and to limit deformation due to long use and moisture. These so called metallic tapes can conduct electricity and should not be used near electric installations. Work near electric installations should involve use of dry nonmetallic tapes.

All these tapes come in various lengths, the 100-ft (30 m) tape being the most popular, and are used for many types of measurements where high precision is not required. All woven types should be periodically compared with a steel tape to determine their levels of precision.

Many tapes are now manufactured with foot units on one side and metric units on the reverse side. Foot units are in feet, tenths of a foot, and 0.05 ft; or in feet, inches and quarter-inches. Metric units are in meters, centimeters, and half-centimeters (0.005m)

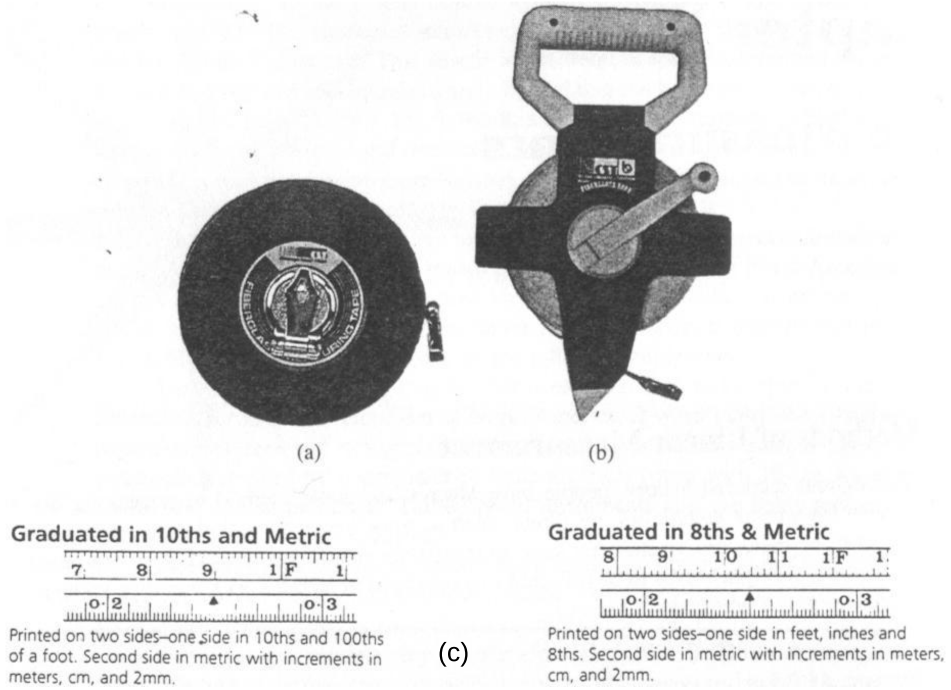


Figure 2.2 Fiberglass tapes. (a) Closed case. (b) Open reel. (c) Tape graduations.

4.5 Steel Tapes

Steel tapes are manufactured in both foot and metric units and come in various lengths, markings, and unit weights (Figure 2.3). In foot units, the steel tapes are manufactured in many lengths, but the 100-, 200-, and 300-ft tapes are the most common, with the 100-ft length by far the most popular.

In metric units, steel tapes are manufactured in 20-, 30-, 50-, and 100-m lengths. Although metric practice in North America is fairly recent, the 30-m length is already established as the most popular length. It most closely resembles the popular 100-ft length and can be used with a comfortable "normal" tension.

Steel tapes come in two prevalent cross sections: heavy duty, 8 mm*0.45 mm (5/16 in.*0,18 in.) or a normal cross section of 6 mm*0.30 mm (1/4 in.*0.012 in.), Very lightweight tapes are manufactured in the longer (300 ft or 100 m) lengths, which can the of value for their easier handling characteristics.

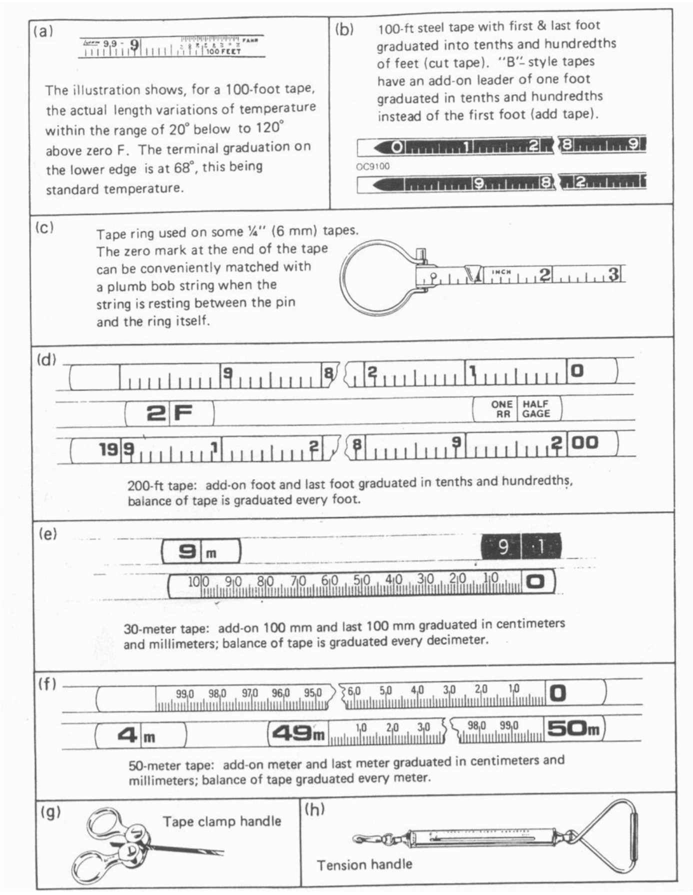


Figure 2.3 Lufkin steel tapes and accessories.

Generally, the heavy-duty tapes (drag tapes) are used in route surveying (e.g., highways. railways) and are designed for use of the reel. Leather thongs are tied through the eyelets at both ends of the tape to aid in measuring. The lighter-weight tapes can be used on or off the reel and are found in structural and municipal work.
Invar tapes are composed of 35% nickel and 65% steel. This alloy has a very low co-efficient of thermal expansion, making the tapes useful in precise work.
Steel tapes are sometimes referred to as chains-a throwback to early survey practice.

4.5.1 Types of Readouts
Steel tapes are normally marked in one of three ways:

1. Graduated throughout in feet and hundredths (0.01) of a foot or in meters and mil-limeters (see Figures 2.3 and 2.4).
2. The *cut* tape is marked throughout in feet, with the first and last foot graduated in tenths and hundredths of a foot (see Figure 2.3b). The metric cut tape is marked throughout in meters and decimeters, with the first and last decimeters graduated in millimeters (see Figure 2.3e). Some metric cut tapes have the first and last meters graduated in centimeters and millimeters. A measurement is made with the cut tape by one surveyor holding that even foot (decimeter) mark, which will allow the other surveyor to read a distance on the first foot graduated in hundredths of a foot (millimeters). For example, the distance from A to *B* in Figure 2.4a is determined by holding 39 ft at *B* and reading 0.18 ft at A. Distance *AB* = 38.82 ft (i.e., 39 ft "cut" 0.18 = 38.82 ft). Each measurement involves this cut subtraction from the even foot (meter) mark being held at the far end. The cut tapes have been with us for generations and are accepted as adequate by many surveyors. However, these tapes, even in the hands of experienced surveyors, can lead to serious blunders. The mental subtraction required for each measurement is sooner or later going to result in an undetected mistake and cost someone considerable expense.

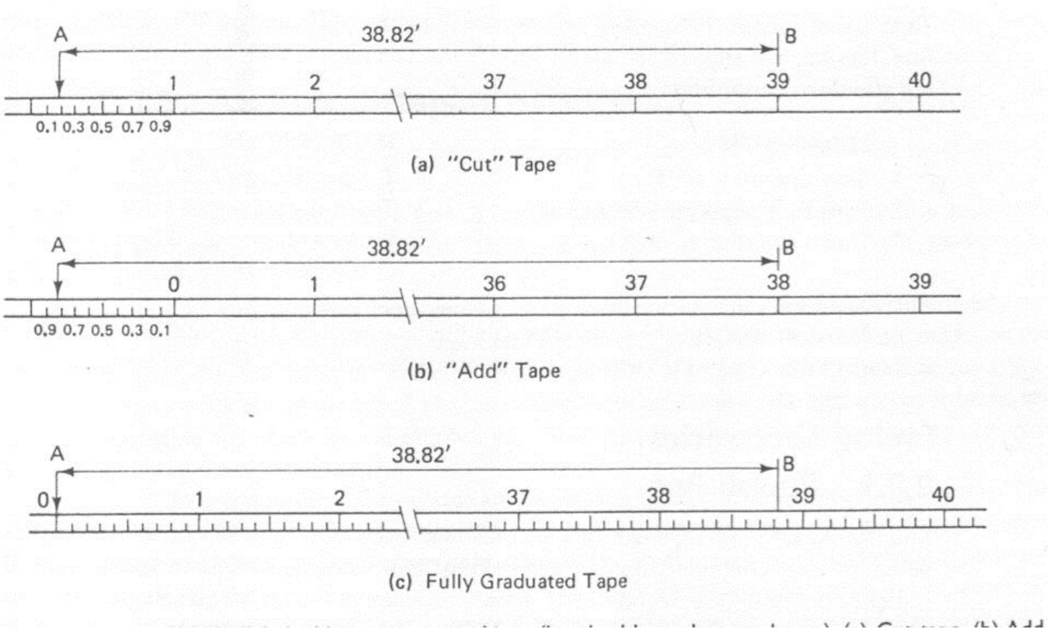


Figure 2.4 Various tape marking. (a) Cut tape. (b) Add tape. (c) Fully graduated tape.

3. The *add* tape is also marked throughout in feet (meters and decimeters) with the last foot (meter or decimeter) being graduated to tenths and hundredths of a foot (cm and mm). An additional graduated foot (decimeter or meter) is included prior to the zero mark (Figure 2.3d, e, and f). The distance from *A* to *B* in Figure 2.4b is determined by holding 38 ft at *B* and reading 0.82 ft at *A.* Distance *AB* is 38.82 ft (i.e., 38 "add" 0.82 = 38.82 ft).

As noted, the cut tapes have the disadvantage of creating opportunities for subtraction mistakes. The add tapes have the disadvantage of forcing the surveyor to adopt awkward measuring stances when measuring from the zero mark. The full meter add tape is almost impossible to use correctly consistently. The hand holding the leather thong at the end of the tape must be fully extended to allow the surveyor to properly position the zero tape mark over the ground mark. It is the authors' belief, based on many years of field experience, that there is no valid reason for employing either cut or add tapes. The small extra cost of fully graduated tapes is a small price to pay to remove the confusion and mistakes associated with add and particularly cut tapes. Tape manufacturers will supply fully graduated tapes in both reel-use tapes and off-the-reel (drag) tapes (Figure 2.4c).

4.6 Standard Conditions for Use of Steel Tapes

Tape manufacturers, noting that steel tapes will behave differently under various temperature, tension, and support situations, specify the accuracy of their tapes under the following **standard conditions:**

FOOT SYSTEM	**METRIC SYSTEM**
1. Temperature = 68°F.	1. Temperature = 20°C.
2. Tape fully supported throughout.	2. Tape fully supported throughout.
3. Under a tension of 10 lbs.	3. Under a tension of 50 N (newtons) (1 lb force = 4.448 N).

If one or more of these standard conditions cannot be met, suitable corrections or techniques must be used to account for the errors that will result from nonstandard use.

4.7 Taping Accessories

4.7.1 Plumb Bob

Plumb bobs are normally made of solid brass and weigh from 8 to 18 oz, with 10 and 12 oz being the most commonly used. When held by their strings, plumb bobs point toward the center of the earth (see Section 12.13 for a discussion of the *geoid)* and are used in taping to transfer from tape to ground (and vice versa) when the tape is being held off the ground \0 maintain its horizontal alignment (see Figure 2.5).

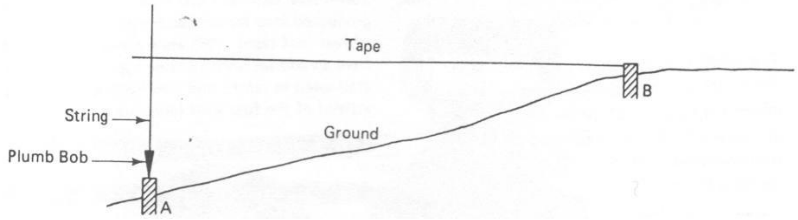


Figure 2.5 Use of a plumb bob

4.7.2 Hand Level

The hand level (see figure 2.6) can be used to keep the steel tape horizontal when measuring distances. The hand level is taken by the surveyor at the lower elevation and a sight is taken back at the higher-elevation surveyor (see Figure 2.6b). For example, if the surveyor with the hand level is sighting horizontally on his or her partner’s waist, and if both are roughly the same height, then the surveyor with the hand level is lower by the distance given from his or her eye to waist. The low end of the tape is held that distance off the ground (using a plumb bob) with the high end of the tape held on the mark.

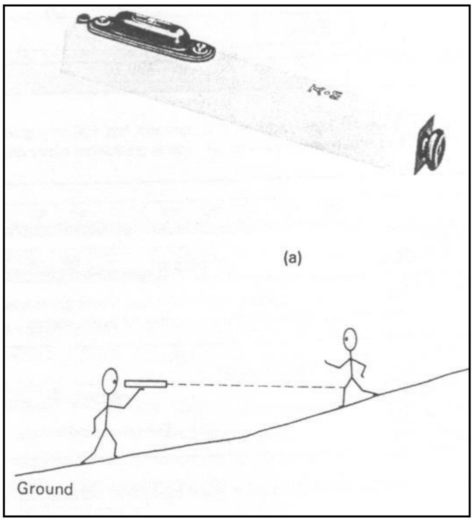
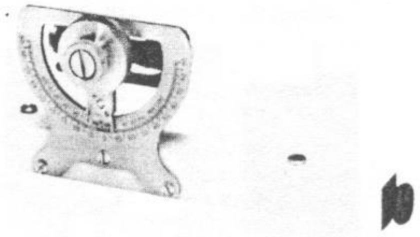
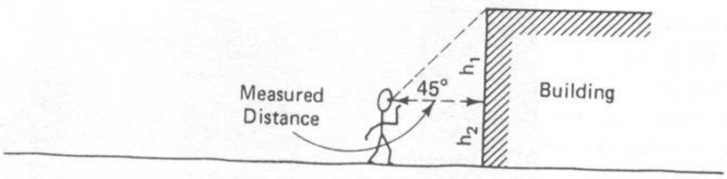
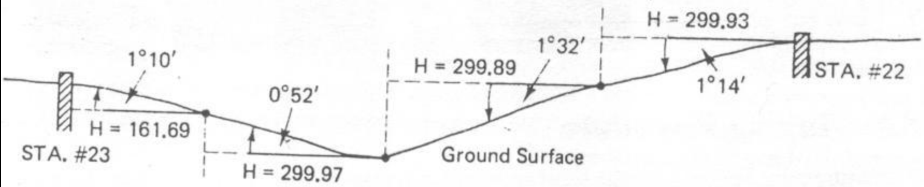


Figure 2.6 (a) Hand level. (b) It's application.

Also shown is an Abney hand level (clinometer) (see Figure 2.7a). This is a hand level that allows for rough vertical angle determination by means of a graduated scale and a movable vernier scale with a level vial reference. This instrument can be used as a hand level, and it can also be used for vertical angle and slope determination.







Course	Slope angle line 22-23	Horizontal distance
300	−1°14′	299.93
300	−1°32′	299.89
300	+0°52′	299.97
161.72	+1°10′	161.69
	Line 22–23 =	1061.48

Figure 2.7 (a) Abney hand level; scale graduated in degrees with a vernier reading to 10 minutes.
(b) Abney hand level application in height determination.
(c) Abney hand level typical application in taping.

■ **EXAMPLE 2.2**

For height determination a value of 45° is placed on the scale. The surveyor moves forward or back until he or she is sighting the point that is being located. The height of the building would be the measured distance that is equal to hi (h/measured distance = tan 45° = 1), plus the distance from the eye to the ground *(h_2)* or from a mark on the building (or observer) to the ground *(h_2* is also measured) (see Figure 2.7b).

This technique is particularly useful in determining the height (clearance) of overhead electrical power lines. In that case, one surveyor can line up under the power line by holding up a plumb bob and lining up the power line with the string line. The other surveyor, holding the other end of the tape, backs away until the 45° setting on the clinometer allows sighting of the power line. The height of the power line is hi + *h2; h2* can be determined by setting the clinometer to zero (level) and sighting on the surveyor under the power line, and then measuring from that sighted point to the ground.

The Abney hand level is also useful when working on route surveys where extended length (300 ft or 100 m) tapes are being used. The long tape can be used in a slope position (mostly supported) under proper tension. The clinometer can be used to record the slope angle, which will later be used to compute the appropriate horizontal distances (see Figure 2.7c).

4.7.3 Additional Taping Accessories

The *clamp handle* (Figure 2.3g) helps grip the tape at any intermediate point without bending or distorting the tape.

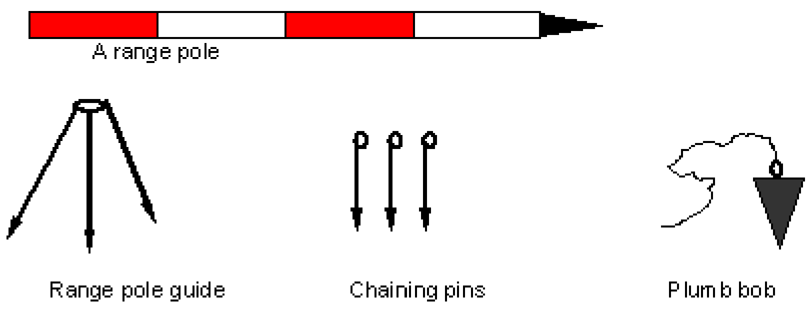
Tension handles (Figure 2.3h) are used in precise work to ensure that the appropriate tension is being applied. They are graduated to 30 lb in 1/2-lb graduations (50 N=11.24 lb).

Chaining pins (marking arrows) come in sets of 11. They are painted alternately red and white and are 14 to 18 in. long. Chaining pins are used to set intermediate marks on the ground; they are set at 45° to the ground, perpendicular to the tape alignment. In route surveying work the whole set of 11 pins is used to measure out "center line", the rear surveyor being responsible for checking the number of whole tape lengths used by keeping an accurate count of the pins collected.

Tape repair kits are available so that broken tapes can be put back into service. The repair kits come in three main varieties: (1) punch pliers and repair eyelets, (2) steel punch block and rivets, and (3) tape repair sleeves. The steel punch block and rivets type is the only method that will give lasting repair. The block and rivets are simple to use, although great care must be exercised to ensure that the repair is precisely accomplished and the integrity of the tape is maintained.

Range poles are 6-ft wood or metal poles with steel points. The poles are usually painted alternately red and white in 1-ft (1/2 meter) sections. Range poles are used in taping and transit work to provide alignment sights (see Figure 2.8).

Plumb bob targets are designed for use with the plumb bob. The plumb bob string is threaded through the upper and lower notches, so that the target center line is superimposed on the plumb bob string. The target can be adjusted up or down to aid in sighting.



4.8 Taping Methods

Taping (also known as chaining) is a common method for determining either the distance between existing field points or for establishing points in the field at prescribed distances.

Taping is normally performed with the tape held horizontally, If the distance to be measured is to be across smooth level land, the tape can be simply laid on the ground, properly aligned and tensioned, and then the end mark on the tape can be marked on the ground. If the distance is to be measured is across sloping or uneven land, then at least one end of the tape must be raised off the ground to keep the tape horizontal. The raised end of the tape is referenced to the ground mark with the aid of a plumb bob (see figure 2.10). Normally, the only occasion when both ends of the tape are plumbed is when the ground rises, or obstacles exists, between the two surveyors (see figure 2.11)

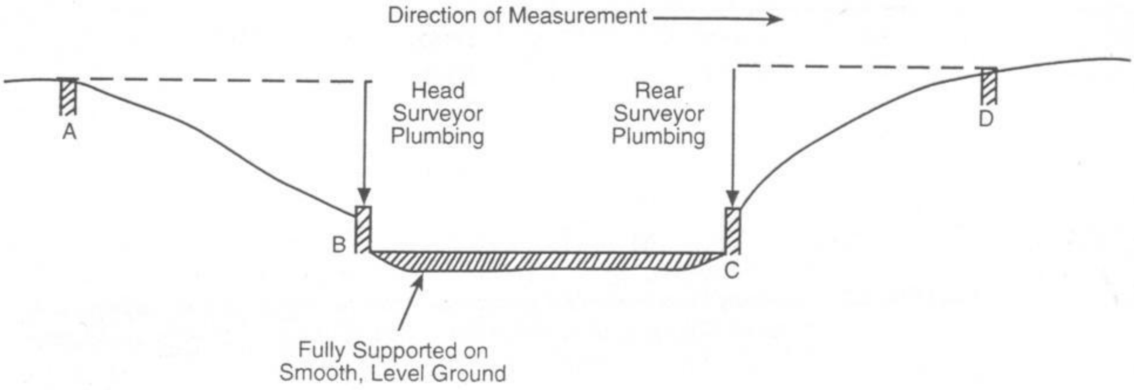


Figure 2.10 Horizontal taping; plumb bob used at one end.

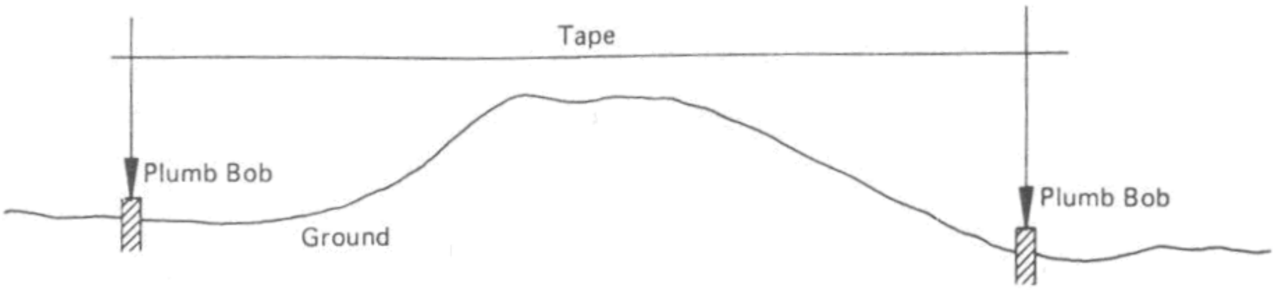


Figure 2.11 Horizontal taping; plumb bob used at both ends.

4.8.1 Taping Procedure

The measurement begins with the head surveyor carrying the zero end of the tape forward toward the final point. This continues until the tape has been unwound, at which point the rear surveyor calls "tape" to alert the head surveyor to stop walking and to prepare for measuring.

If a drag tape is being used, the tape is removed from the reel and a leather thong is attached to the reel end to facilitate measuring. If the tape is not designed to come off the reel, the winding handle is folded to the lock position and the reel is used to help hold the tape. The head surveyor is put on line by the rear surveyor, who is sighting forward to a range pole or other target that has been erected at the final mark. In precise work, the intermediate marks can be aligned by transit. The rear surveyor holds the appropriate graduation (e.g., 100.00 ft or 30.000 m) against the mark from which the measurement is being taken. The head surveyor, ensuring that the tape is straight, slowly increases tension to the proper amount and then marks the ground with a chaining pin or other marker. Once the mark has been made, both surveyors repeat the measuring procedure to check the measurement. If necessary, adjustments are made and the check procedure repeated.

The ground is not level (determined by estimation or by use of a hand level) one or both surveyors must use a plumb bob. When plumbing, the tape is usually held at waist height, although any height between the shoulders and the ground is common. Holding the tape above shoulder height creates more chance for error, as the surveyor must move his or her eyes up and down to include the ground mark and tape graduation in the field of view.

The plumb bob string is usually held on the tape with the left thumb (right-handed people), taking care not to completely cover the graduation mark. As the tension is increased, it is common for the surveyor to take up some of the tension with the left thumb, causing it to slide along the tape; if the graduations have been covered with the left thumb, the surveyor is often not aware that the thumb (and string) has moved, resulting in an erroneous measurement. When plumbing, it is advisable to hold the tape close to the body in order to provide good leverage for applying or holding tension and to accurately transfer from tape to ground, and vice versa

If the rear surveyor is using a plumb bob, he or she shouts out "tape," "mark," or some other sign at that instant when the plumb bob is steady and over the mark. If the head surveyor is also using a plumb bob, he or she must wait until both plumb bobs are simultaneously over their respective marks. The student will discover that plumbing is a difficult aspect of taping. The student will encounter difficulty in holding the pluI1;lb bob steady over the point and at the same time applying appropriate tension. To help steady the plumb bob, the plumb bob is held only a short distance above the mark and is continuously touched down. This momentary touching down will dampen the plumb bob oscillations and generally steady the plumb bob. The student is cautioned against allowing the plumb bob to actually rest on the point, as this will result in an erroneous measurement.

REAR SURVEYOR

1. Aligns the head surveyor by sighting to d range pole or other target placed at the forward station.
2. Holds the tape on the mark, either directly or with the aid of a plumb bob. If a plumb bob is being used, the rear surveyor will call out "tape," "mark," or similar, signifying to the head surveyor that, for that instant in time, the plumb bob (tape mark) is precisely over the station.
3. Calls out the station and tape reading for each measurement and listens for verification from the head surveyor.
4. Keeps a count of all full tape lengths included in each overall measurement.
5. Maintains the equipment (e.g., wipes the tape clean at the conclusion of the day's work or as conditions warrant).

HEAD SURVEYOR

1. Carries the tape forward, ensuring that the tape is free of loops, which could lead to tape breakage.
2. Prepares the ground surface for the mark (e.g., clears away grass, leaves, etc.).
3. Applies proper tension, after first ensuring that the tape is straight.
4. Places marks (chaining pins, wood stakes, iron bars, nails, rivets, cut crosses, etc.).
5. Takes and records measurements of distances (also temperature and other factors).

4.9 Taping Corrections

Section 2.6 outlined the standard conditions that must be met for a steel tape to give precise results. The standard conditions referred to specific temperature and tension and to a condition of full support.

In addition to satisfying the standard conditions, the surveyors must also be concerned with horizontal versus slope distances and with ensuring that their techniques are sufficiently precise.

4.9.1 Taping Errors

The surveyor must make corrections for all significant systematic taping errors and must use techniques and equipment that will satisfactorily reduce random errors.

Systematic Errors	**Random Errors**
1. Slope	1. Slope
2. Erroneous tape length	2. Temperature
3. Temperature	3. Tension and sag
4. Tension	4. Alignment
5. Sag	5. Marking and plumbing

The reason for including some factors in both systematic and random categories is that even when correcting for systematic errors in slope and temperature, there still exists the possibility of error when determining the correction parameters (i.e., the actual temperature or slope angle, etc.).

4.10 Slope Corrections

Survey distances can be measured either horizontally or on a slope. Since survey measurements are normally shown on a plan, if the measurements were taken an a slope, they then must be converted to their horizontal equivalents before they can be plotted. To convert slope distances, either the slope angle (or zenith angle) or the vertical distance must also be known:

H (horizontal)/S (slope) = $\cos\theta$ or $H = S\cos\theta$

$H/S = \sin(90° - \theta)$ or $H = S\sin(90° - \theta)$

where θ is the angle of inclination, and $(90° - \theta)$ is the zenith angle.

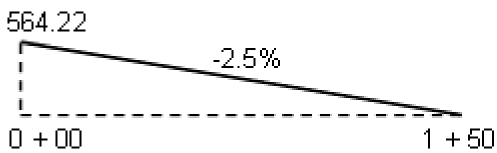
$H^2 = S^2 – V^2$ or $H = (S^2 – V^2)^{1/2}$

where V is the difference in elevation [see Example 2.3, part (c)].

Slope can also be defined as *gradient*, or *rate of grade*. The gradient is expressed as a ratio of the vertical distance over the horizontal distance; this ratio is multiplied by 100 to give a percentage gradient. For example, if the ground rises 2 ft (m) in 100 ft (m). it is said to have a 2% gradient (i.e., 2/100*100 = 2); or if the ground rises 2 ft (m) in 115 ft (m), it is said to have a 1.74% gradient (i.e., 2/115*100 = 1.739).

If the elevation of a point on a gradient is known, the elevation of any other point on that gradient can be calculated as follows:

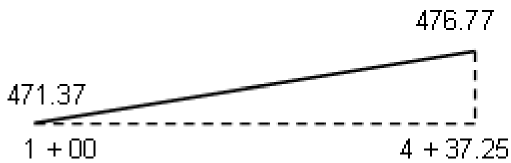
Station	Elevation
0 +00	564.22 ft
(gradient= -2.5%)	
1 +50	Required



Difference in elevation = 150*(-2.5/100) = -3.75
Elevation at 1+50 = 564.22 - 3.75 = 560.47 ft

If the elevations of two points are known as well as the distance between them, the gradient between can be calculated as follows:

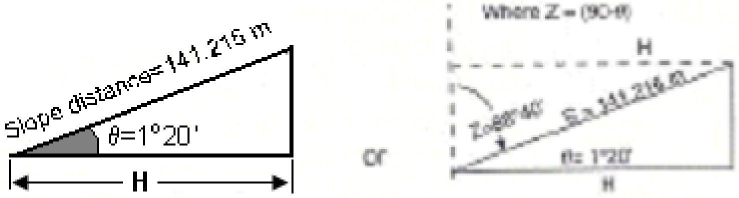
Station	Elevation
1 +00	471.37 ft
4 +37.25	476.77 ft



Elevation difference = 476.77 – 471.37 = 5.40 ft
Distance = 437.25-100.00 = 337.25 ft
Gradient = (+5.40/337.25)*100 = +1.60%

■ **EXAMPLES 2.3** *Slope corrections*

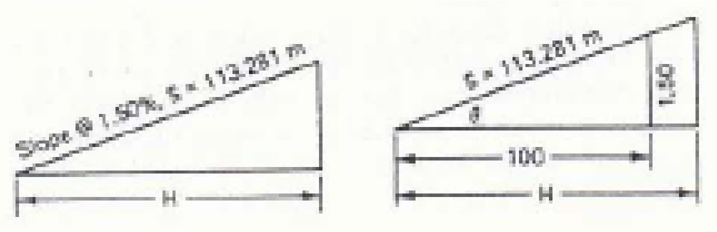
<u>(a) Slope angle</u>: Given the slope distance (S), and slope angle (θ), or Zenith angle (90°-θ)



H (horizontal)/S (slope) = cos θ
H = S cos θ = 141.216 * cos 1°20'
H = 141.178 m

H/S = sin Z where Z = 88°40'
H = S*sin Z = 141.216*sin 88°40'
H = 141.178 m

<u>(b) Slope gradient</u>: Given the slope distance and gradient (slope)

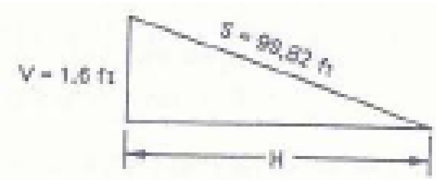


1.50/100 = tan θ
θ = 0.85937°
H/113.281 = cos 0.85937°
H = 113.268 m

<u>(c) Slope and vertical distance</u>: Given the slope distance and difference in elevation(V)

$H^2 = S^2 – V^2$
$H = (S^2 – V^2)^{1/2}$
$= (253.101^2 – 3.721^2)^{1/2}$
$= 253.074$ m

<u>(d) Given the slope distance and difference in elevation</u>



$H = (S^2 – V^2)^{1/2}$
$= (99.82^2 – 1.6^2)^{1/2}$
$= 99.807$ ft

In practice, most measurements are taken with the tape held horizontally. If the slope is too great to allow an entire tape length to be employed, shorter increments will be measured until all the required distance has been measured. This operation is known as breaking tape (see Figure 2.12). The sketch shows distance AB, composed of increments AL, LM. and MB.

The exception to the foregoing occurs when preliminary route surveys (e.g., hydroelectric transmission lines) are performed using a 300-ft(100-m) steel tape. It is customary to measure slope distances, which allows the surveyors to keep this relatively heavy tape more or less fully supported on the ground. In order to allow for reduction to horizontal, each tape length is accompanied by its slope angle, usually determined by using a clinometer (Abney hand level) (see Section 2.7.2).

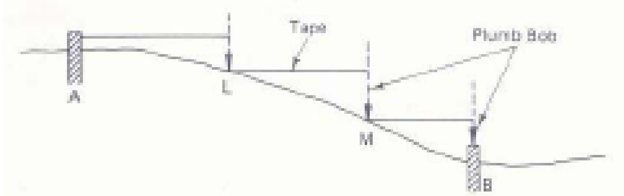


Figure 2.11 Breaking tape.

4.11 Erroneous Tape Length Corrections

For all but precise work, tapes as supplied by the manufacturer are considered to be correct under standard conditions. Through extensive use, tapes do become kinked, stretched, and repaired. The length can become something other than that specified. When this occurs, the tape must be corrected, or the measurements taken with the erroneous tape must be corrected.

■ **EXAMPLE 2.4**

A measurement was recorded as 171.278 m with a 30-m tape that was only 29.996 m under standard conditions. What is ihe corrected measurement?

Solution

Correction per tape length = -error = -(30.000 - 29.996) = -0.004 m
Number of limes the tape was used = 171.278/30
Total correction = -0.004 * 171.278/30 = -0.023 m
Corrected distance = 171.278 - 0.023 = 171.255 m

or

Corrected distance = (29.996/30) * 171.278 = 171.255 m

■ **EXAMPLE 2.5**

It is required to lay out the front corners of a building, a distance of 210.08 ft. The tape to be used is known to be 100,02 ft under standard conditions.

Solution

Correction per tape length = -error = -(100.00 - 100.02) = 0.02 ft
Number of times that the tape is to be used = 2.1008
Total correction = 0,02 * 2.1008 = +0.04 ft

When the problem involves a layout distance, the sign of the correction must be reversed before being applied to the layout measurement. We must find that distance that, when corrected by +0,04, will give 210.08 ft.

210.08 - 0.04 = 210.04 ft

This is the distance to be laid out with that tape (100.02 ft) so that the corner points will be exactly 210.08 ft apart.

The student will discover that four variations of this problem are possible: correcting a measured distance while using (1) a long tape or (2) a short tape, or precorrecting a layout distance using (3) a long tape or (4) a short tape. To minimize confusion as to the sign of the correction, the student is urged to consider the problem with the distance reduced to only one tape length (100 ft or 30 m).

In Example 2.4, a recorded distance of 171.278 m was measured with a tape only 29.996 m long. The total correction was found to be 0.023 m. If doubt exists as to the sign of 0.023, ask yourself what the procedure would be for correcting only one tape length. In this example, after one tape length had been measured, it would have been recorded that 30 m had been measured. If the tape were only 29.996 m long, then the field book entry of 30 m must be corrected by -0.004 m.

The magnitude of the tape error is determined by comparing the tape with a tape that has been certified (National Bureau of Standards, Gaithersburg, Maryland; or the National Research Council, Ottawa, Ontario, Canada). In practice, tapes that require corrections for ordinary work are discarded.

4.12 Temperature Corrections

Section 2.6 notes the conditions under which tape manufacturers specify the accuracy of their tapes. One of these standard conditions is that of temperature. In the United States and Canada, tapes are standardized at 68°F, or 20°C. Temperatures other than standard result in an erroneous tape length.

The thermal coefficient of expansion of steel is 0.00000645 per unit length per degree

Fahrenheit (^oF), or 0.0000116 per unit length per degree Celsius, °C.

$C_t = a(T - T_s)L$ general formula

Foot units

$C_t = 0.00000645(T - 68)L$

where C_t = correction due to temperature, in feet

T = temperature of tape (°F) during measurement

L = distance measured, in feet

Metric units

$C_t = 0.0000116(T - 20)L$

where C_t = correction due to temperature, in meters

T = temperature of tape (°C) during measurement

L = distance measured, in meters

■ **EXAMPLE 2.6**

A distance was recorded as being 471.37 ft at a temperature of 38°F.

$C_t = 0.00000645*(38 - 68)*471.37 = -0.09$

Corrected distance = 471.37 - 0.09 = 471.28 ft

■ **EXAMPLES 2.7**

It is required to lay out two points in the field that will be exactly 100.00 m apart. Field conditions indicate the temperature of the tape will be 27°C. What distance will be laid out?

Solution

$C_t = 0.0000116*(27 - 20)*100.00 = +0.008$ m

Since this is a layout (precorrection) problem, the correction sign must be reversed (i.e., we are looking for the distance that, when corrected by +0,008, will give us 100.000 m):

Layout distance = 100.00 – 0.008 = 99.992 m

For most survey, accuracy requirements don't demand precision in determining the actual temperature of the tape. Usually, it is sufficient to estimate air temperature. However, for more precise work (say 1/10,000 and higher), care is required in determining the actual temperature of the tape, which can be significantly different than the temperature of the air. **Invar Steel tapes** High-precision surveys require the use of steel tapes that have a low coefficient of thermal expansion. Such a tape is composed of a nickel-steel alloy having a thermal expansion ranging from 0.0000002 to 0.00000055 per degree Fahrenheit ($3.60*10^{-7}$ to $5.50*10^{-7}$ per degree Celsius). Since the temperature of the tape can be significantly different from that of the surrounding air, it is customary to attach thermometers directly to the invar tapes. Electronic distance measurement (EDM) systems have largely replaced invar tapes for precise distance measurements.

4.13 Tension and Sag Corrections

The three conditions under which tapes are normally standardized are given in Section 2.6. If a tension other than standard is applied, a *tension* (pull) error exists. The tension correction formula is

$C_p = [(P - P_s)L] / AE$

If a tape has been standardized while fully supported and is being used without full support, an error called sag will occur.

The force of gravity pulls the center of the unsupported section downward in the shape of a catenary, thus creating an error B'B. The sag correction formula is

$$C_s = \frac{-w^2L^3}{24P^2} = \frac{-W^2L}{24P^2}$$

Table 2,1 defines the terms in these two formulas. Referring to Table 2.1, 1 newton is the force required to accelerate a mass of 1 kg by 1 meter/s^2.

Force = mass * acceleration

Weight = mass * acceleration due to gravity (g) g = 32.2 ft/s^2 = 9.807 m/s^2

In SI units, a mass of 1 kg has a weight of 1 * 9.807 kg.m/s^2 = 9.807 N. That is,

1 kg(f) = 9.807 N

Table 2.1 CORRECTION FORMULA TERMS DEFINED (FOOT, METRIC [OLD], AND METRIC[SI] UNITS)

Unit	Description	Foot	Metric (old)	Metric (SI)
C_p	Correction due to tension per tape length	ft	m	m
Cs	Correction due to sag per tape length	ft	m	m
L	Length of tape under consideration	ft	m	m
Ps	Standard tension Typical standard tension	lb(force) 10 lb(f)	kg(force) 4.5-5 kg(f)	N(newtons) 50 N
P	Applied tension	lb(f)	kg(f)	N
A	Cross-sectional area	in^2	cm^2	m^2
E	Average modulus of elasticity of steel tapes Average modulus of elasticity of invar tapes	$29*10^6$ lb(f)/in^2 $21*10^6$ lb(f)/in^2	$21*10^5$ kg(f)/cm^2 $14.8*10^5$ kg(f)/cm^2	$20*10^{10}$ N/m^2 $14.5*10^{10}$ N/m^2
w	Weight of tape per unit length	lb(f)/ft	kg(f)/m	N/m
W	Weight of tape	lb(f)	kg(f)	N

Since some tension spring balances are graduated in kilograms and since standard tensions are given in newtons by tape manufactures, some students must be prepared to work in both old metric and SI units.

4.13.1 Examples of Tension Corrections

■ **EXAMPLE 2.8**

Given a standard tension of a 10-lb force for a 100-ft steel tape that is being used with a 20-lb force pull. If the cross-sectional area of the tape is 0.003 in^2 what is the tension error for each tape length used?

Solution

C_p = [(20 - 10)*100] / [29,000,000 * 0.003] = +0.011 ft

If a distance of 421.22 ft had been recorded, the total correction would be 4.2122 * 0.011 = +0.05 ft. The corrected distance would be 421.27 ft.

■ **EXAMPLE 2.9**

Given a standard tension of 50 N for a 30-m tape that is being used with a 100-N force. If the cross-sectional area of the tape is 0.02 cm^2, what is the tension error per tape length?

Solution

C_p = [(100-50)*30] / [0.02*21*10^5*9.807] = +0.0036 m

If a distance of 182.716 m had been measured under these conditions,

Total correction = (182.716/30) * 0.0036 = +0.022 m

The corrected distance would be 182.738 m.

4.13.2 Notes on Tension Corrections

The cross-sectional area of the tape can be measured wilh a micrometer, taken from manufacturer's specifications, or it can be determined by using the following expression:

Tape length*tape area*specific weight of tape steel = weight

or Tape area = weight/(length*specific weight)

■ **EXAMPLE 2.10**
A tape is weighed and found to be 1.95 lb. The overall length of the 100-ft tape (end to end) is 102 ft. The specific weight of steel is 490 lb/ft^3.

[102 ft*12 in*area (in^2)]*490 lb/ft^3 / 1728 in^3 = 1.95 lb

Area =(1.95*1728) / (102*12*490) = 0.0056 in^2

Tension errors are usually quite small and as such have, relevance only for very precise surveys. Even for precise surveys it is seldom necessary to calculate tension corrections, as availability of a tension-spring balance allows the surveyor to apply standard tension and thus eliminate the necessity of calculating a correction.

4.13.3 Sag Corrections

■ **EXAMPLE 2.11**

$$C_s = \frac{-w^2L^3}{24P^2} = \frac{-W^2L}{24P^2} \qquad \text{where } W^2 = w^2L^2$$

w = weight of tape per unit length
W = weight of tape between supports
L = length of tape between supports

A 100-ft steel tape weighs 1.6 lb and is supported only at the ends with a force of 10 lb. What is the sag correction?

Solution

$$C_s = \frac{-1.6^2 * 100}{24 * 10^2} = \text{-0.11 ft}$$

If the force were increased to 20 lb. the sag is reduced to $C_s = \frac{-1.6^2 * 100}{24 * 20^2} = \text{-0.03 ft}$

■ **EXAMPLE 2.12**
Calculate the length between two supports if the recorded length is 42.071 m, the mass of this tape is 1.63 kg, and the applied tension is 100 N.

Solution

$$C_s = \frac{-(1.63 * 9.807)^2 * 42.071}{24 * 100^2} = \text{-0.045 m}$$

Therefore, the length between supports = 42,071 - 0.045 = 42.026 m.

4.13.4 Normal Tension

The error in a measurement due to sag can be eliminated by increasing the tension. Although sag cannot be entirely eliminated, the tape can be stretched to compensate for the residual sag.

Tension that will eliminate sag errors is known as normal tension. It ranges from 19 lb (light 100-ft tapes) to 31 lb (heavy 100-ft tapes).

$$P_n = \frac{0.204W\sqrt{AE}}{\sqrt{P_n - P_s}}$$

This formula will give a value for P_n that will eliminate the error caused by sag. The formula is solved by making successive approximations for P_n until the equation is satisfied. This is not much used in practice due to uncertainties go with individual tape characteristics.

Experiment to determine normal tension Normal tension can be determined experimentally for individual tapes.

The most popular steel tapes (100 ft) require a normal tension of about 24 lb(f). For most 30-m tapes (lightweight), a normal tension of 90 N [20 lb(f) or 9.1 kg(f)] is appropriate.

■ **EXAMPLE 2.13** *Determination of a normal tension for a100-ft steel tape.*

1. Lay out the tape on a flat, horizontal surface; an indoor corridor is ideal.
2. Select (or mark) a well-defined point on the surface at which the 100-ft mark is held.
3. Attach a tension handle at the zero end of the tape, apply standard tension-say, 10 lb(f)-and mark the surface at 0.00 ft.
4. Repeat the process, switching personnel duties, ensuring that the two marks are in fact exactly 100.00 ft apart.
5. Raise the tape off the surface to a comfortable height (waist). While the surveyor at the 100-ft end holds a plumb bob over the point, the surveyor(s) at the zero end slowly increases tension (a third surveyor could perform this function) until the plumb bob is over the zero mark on the surface. The tension is then read off the tension handle. This process is repeated several times, until a set of consistent results is obtained (see Section 2.8.1).

4.14 Random Errors Associated With Systematic Taping Errors

As mentioned in Section 2.9.1, the opportunity exists for random errors to coexist with systematic errors. For example, when dealing with the systematic error caused by variations in temperature, the surveyor can determine the prevailing temperature in several ways:

1. The air temperature can be estimated.
2. The air temperature can be taken from a pocket thermometer.
3. The actual temperature of the tape can be determined by a tape thermometer held in contact with the tape.

For an error of 15°F in temperature, the error in a 100-ft tape would be

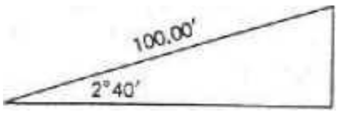
$$C_t = 0.00000645*(15)*100 = 0.01 \text{ ft}$$

Since 0.01/100 = 1/10,000, even an error of 15°F would be significant only for higher-order surveys. If metric equipment were being used, a comparable error would be 8½°C. That is,

$$C_t = 0.0000116*(8½)*30 = 0.003 \text{ m}$$

However, for precise work, random errors in determining temperature will be significant. Tape thermometers are recommended for precise work because of difficulties in estimating and because of the large differentials possible between air temperature and the actual temperature of the tape on the ground.

As a second example, consider the treatment of systematic errors dealing with slope versus horizontal dimensions. For a slope angle of 2°40' read with an Abney hand level (clinometer) to [he closest 10 minutes, we can say that an uncertainty of 5 minutes exists.



Horizontal

Horizontal = 100*cos 2°40' = 99.89 ft

In this ease an uncertainty of 5 min in the slope angle introduces an uncertainty of only 0.01 ft in the answer.

Horizontal = 100*cos 2°45' = 99.88 ft
= 100 cos 2°35' = 99.90 ft

Once again this error (1/10,000) would be significant for higher-order surveys.

A third example considers the treatment of systematic errors dealing with sag and tension. Consider the sag for a light 100-ft tape weighing 1 Ib, with A = 0.003 and E = 30,000,000. Normal tension would be

Trial 1: $$20 = \frac{0.204*1.0\sqrt{0.003*30000000}}{\sqrt{20-10}} = \frac{61.2}{3.16} = 19.35$$

Trial 2: $19.7 = \frac{61.2}{\sqrt{9.7}} = 19.7$ OK for normal tension

If a tension of 25 lb were exerted instead of 19.7 Ib. the following error would occur:

C_p = [(P – P_n)L] / AE = [(25-19.7)*100] / (0.003*30000000) = 0.006 ft

Once again, this error (1/16,700), by ilself, is not significant for ordinary taping.

4.15 Random Taping Errors

In addition to the systematic and random errors already discussed, there are random errors associated directly with the skill and care of the surveyors. These errors result from the inasbility of the surveyor to work with perfection in the areas of alignment, plumbing and marking, and estimating horizontal.

Alignment errors exist when the tape is inadvertently aligned off the true path (see Figure 2.13). Under ordinary surveying conditions, the rear surveyor can keep the head surveyor on line by sighting a range pole marking the terminal point. It would take an alignment error of about 1½ ft to produce an error of 0.01 ft in 100 ft. Since it is not difficult to keep the tape aligned by eye to within a few tenths of a foot (0.2 to 0.3 ft), alignment is not usually a major concern. It should be noted that although most random errors arc compensating, alignment errors are cumulative (misalignment can randomly occur on the left or on the right, but in both cases the result of the misalignment is to make the measured course too long). Alignment errors can be virtually eliminated on precise surveys by using a transit or theodolite to align all intermediate points.

Marking and plumbing errors are the most significant of all random errors. Even experienced surveyors must exercise great care to place a plumbed mark accurately to within 0.02 ft of true value over a distance of 100 ft. Horizontal measurements taken with the tape fully supported on the ground can be more accurately determined than measurements taken on slope requiring the use of plumb bobs, additionally, rugged terrain conditions that require many breaks in the taping process will cause errors to multiply significantly.

Errors are also introduced when surveyors **estimate a horizontal position** of a plumbed measurement. The effect of this error is identical to that of the alignment error previously discussed, although the magnitude of these errors is often larger than alignment errors. Skilled surveyors can usually estimate a horizontal position within 1 ft (0.3 m) over a distance of 100 ft (30 in); however, even experienced surveyors can be seriously in error when measuring across sidehills where one's perspective with respect to the horizon can be seriously distorted. These errors can be largely eliminated by using a hand level.

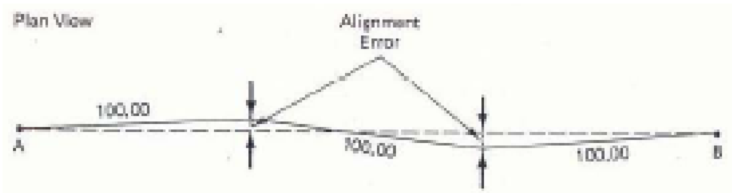


Figure 2.13 Alignment errors.

4.16 Techniques for Ordinary Taping Precision

Ordinary taping precision is referred to as being that which can result in 1/5000 accuracy. The techniques used for ordinary taping, once mastered, can easily be maintained. It is possible to achieve an accuracy level of 1/5000 with little more effort than is required to attain the 1/3000 level. Since the bulk of all surveying is either at the 1/3000 or 1/5000 level, experienced surveyors will often use 1/5000 techniques even for the 1/3000 level work. This practice permits good measuring work habits to be continually reinforced without appreciably increasing survey costs.

Because of the wide variety of field conditions that exist, absolute specifications cannot be prescribed. The specifications in Table 2.2 can be considered as typical for ordinary 1/5000 taping.

Table 2.2 SPECIFICATIONS FOR 1/5000 ACCURACY

	Maximum effect on one tape length	
Source of error	100 ft	30 m
Temperature estimated to closest T°P (4"C)	±0.005 ft	±0.0014 m
Care is taken 10 apply at least normal tension (lightweight tapes) and tension is known within 5 lb (20 N)	±0.006 ft	±0.0018 m
Slope errors are no larger than 1 ft/100 ft or 0.30m/30 m	±0.005 ft	±0.0015 m
Alignment erron are no larger than 0.5 ft/100 ft or 0.15 m/30 m	±0.001 ft	±0.0004 m
Plumbing and marking errors are at a maximum of 0.015 ft /100 ft or 0.0046 m/30 m	±0.015 ft	±0.0046 m
Length of tape is known within ±0.005 ft (0.0015 m)	±0.005 ft	±0.0015 m

To determine the total random error (Σe) in one tape length, take the square root of the sum of the squares of the individual maximum errors:

Foot		*Metric*	
	0.005^2		0.0014^2
	0.006^2		0.0018^2
	0.005^2		0.0015^2
	0.001^2		0.0004^2
	0.015^2		0.0046^2
	0.005^2		0.0015^2
	0.000337		0.000031

$\Sigma e = \sqrt{0.000337} = 0.018$ ft or $\sqrt{0.000031} = 0.0056$ m

Accuracy = 0.018/100 = 1/5400 or 0.0056/30 = 1/5400

In the foregoing example, it is understood that corrections due to systematic errors had already been applied.

4.17 Mistakes in Taping

If errors are associated with inexactness, mistakes must be thought of as blunders. Whereas errors can be analyzed and even, to some degree, predicted, mistakes are totally unpredictable: since just one undetected mistake can nullify the results of an entire survey, it is essential to perform tbe work in a manner that will minimize the opportunity for mistakes to develop and allow for verification of the results.

The opportunities for the occurrence of mistakes are minimized by setting up and then rigorously following a standard method of performing the measurement. The more standardized and routine the measurement manipulations, the more likely it is that the surveyor will immediately spot a mistake. The immediate double-checking of all measurement manipulations reduces the opportunities for mistakes to go undetected and at the same time increases the precision of the measurement. In addition to the immediate checking of all measurements, the surveyor is constantly looking for independent methods of verifying the survey results. Gross mistakes can often be detected by comparing the survey results with distances scaled from existing plans. The simple check technique of pacing can be an invaluable tool far rough verification of measured distances, especially construction layout distances. The possibilities for verification are limited only by the surveyor's diligence and imagination.

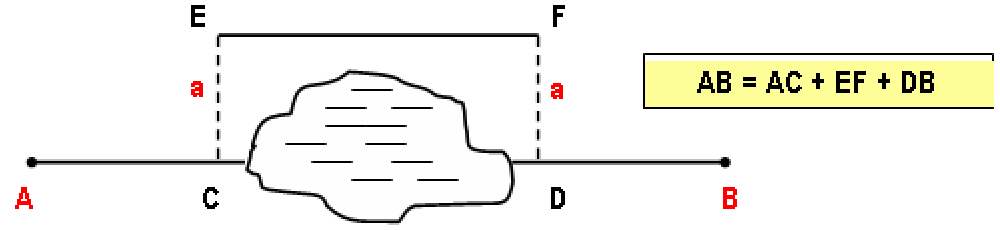
Common mistakes encountered in taping are:

1. *Measuring to and from the wrong marker.* All members of the survey crew must be vigilant to ensure that measurements begin or end at the appropriate permanent or temporary marker. Markers include legal bars, construction stakes or bars, nails and the like.
2. *Reading the tape incorrectly.* It sometimes happens that mistakes are made by reading. Transposing figures is a common mistake (e,g., reading 56 instead of 65).
3. *Losing proper count of the full tape lengths involved in a measurement.* The counting of full tape lengths is primarily the responsibility of the rear surveyor and can be as simple as counting the chaining pins that have been collected as the work progresses. If the head surveyor is also keeping track of full tape lengths, mistakes, such as failing to pick up all chaining pins, can easily be spotted and corrected.
4. *Recording the values in the notes incorrectly.* It sometimes happens that the note-keeper will hear the rear surveyor's call-out correctly but then transpose the figures as they are being entered in the notes. This mistake can be eliminated if the note-keeper calls out the value as it is recorded. The rear surveyor listens for this call-out to ensure that the values called out are the same as the data originally given.
5. *Calling out values ambiguously.* The rear surveyor can call out 20.27 as twenty (pause) two, seven. This might be interpreted as 22.7. To avoid mistakes, this value should be called out as twenty, decimal (point), two, seven.
6. *When using cloth, fiberglass or steel tapes, the zero point of the tape is often not identified correctly.* This mistake can be avoided if the surveyor checks unfamiliar tapes before use. The tape itself can be used to verify the zero mark,
7. *Arithmetic mistakes can exist in sums of dimensions and in error corrections* (e.g., for temperature and slope). These mistakes can be identified and corrected if each member of the crew is responsible for checking (and signing) all survey notes.

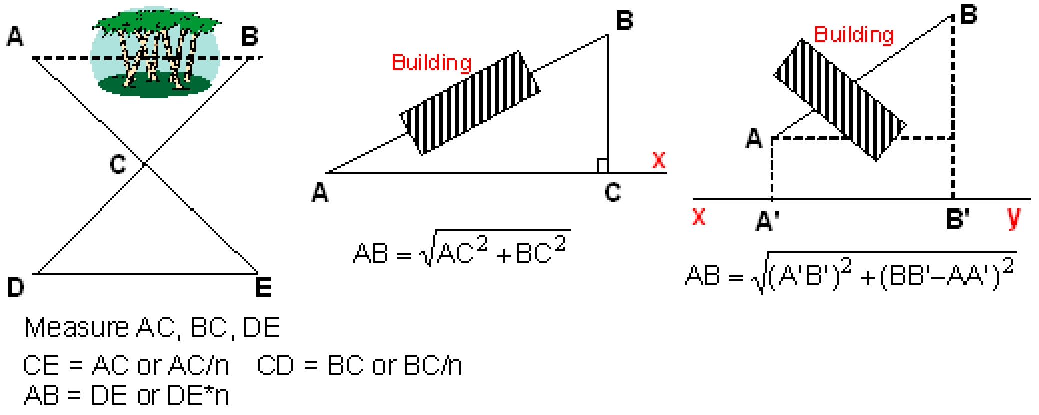
Indirect measurement of distances obstructed naturaly or artificialy:

In case the distance will be measured is hindered by a natural (e.g., lake, river) or an artificial (e.g., building) obstacle, it is done indirectly by using subsidiary lines.

Case A- Lake:

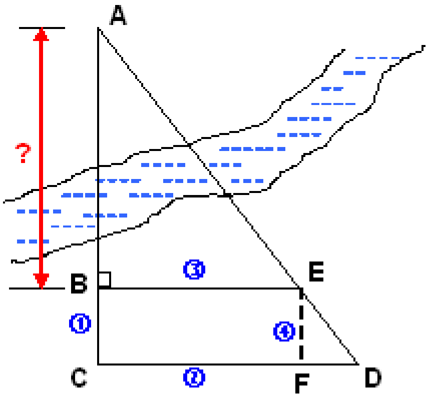


Case B- Points can not see each other :



Case C- River : To measure the distance between the points A and B which are on the opposite sides of a river, three different methods can be applied.

1st Method



① Extend AB line and mark point C.

② Lay out CD line perpendicular AC.

③ Find intersection point E by drawing BE perpendicular to AC at point B.

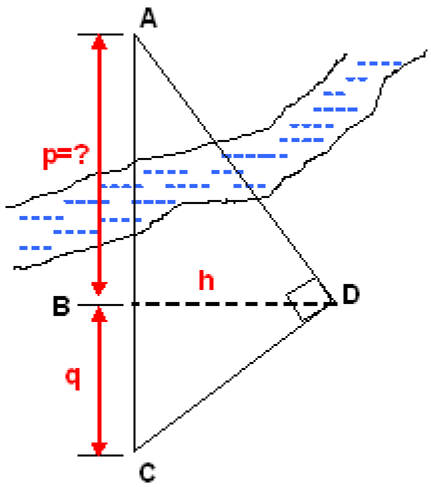
④ Trace EF vertical at point F.

Measure the courses BE, EF ve FD.

ABE triangle ≈ EFD triangle, then

$$\frac{AB}{EF} = \frac{BE}{FD} \quad ===> \quad \boxed{AB = EF * \frac{BE}{FD}}$$

2nd Method



① Mark D by laying out BD line vertical at B.

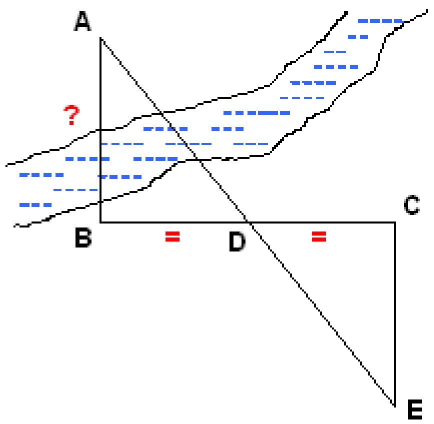
② Find point C intersection of CD line vertical at D and extension of AB line.

③ Measure h ve q.

ABD triangle ≈ DBC triangle, then

$$\frac{p}{h} = \frac{h}{q} \Rightarrow h^2 = p.q \Rightarrow p = AB = \frac{h^2}{q}$$

3rd Method



① Draw a vertical at point B and mark D.

② Extend BD to satisfy BD=DC and mark C.

③ Find intersection point E by drawing a line vertical at point C and extension of AD.

④ The triangles ABD and ECD are identical.

⑤ Then AB=CE. Measure CE.

Problems – Related with Tape Measurements *(Ref: B.F.Kavanagh Chapter.2)*

2.1 The following distances were measured with a Gunter's chain. Convert these distances to feet.
(a) 20 chains, 61 links (b) 2 chains, 18 links (c) 23.17 chains (d) I chain, 60 links

2.2 Give two examples each of suitable use of the following measuring techniques or instruments.
(a) Pacing (b) Odometer (c) EDM (d) Stadia (e) Subtense (t) Fiberglass (g) Steel tape

2.3 A 100-ft "cut" steel tape was used to measure between two property markers. The rear surveyor held 61 ft, while the head surveyor cut .31 ft. What was the distance between the markers?

2.4 The slope measurement between two points is 31.773 m and the slope angle is 1°58'. Compute the horizontal distance.

2.5 A distance of 278.17 ft was measured along a 2% slope. Compute the horizontal distance.

2.6 The slope distance between two points is 68.631 m and the difference in elevation between the points is 0.75 m. Compute the horizontal distance.

2.7 A 100-ft steel tape known to be only 99.97 ft long (under standard conditions) was used to record a measurement of 271.90 ft. What is the distance corrected for erroneous tape length?

2.8 A 30-m steel tape, known to be 30.006 m (under standard conditions) was used to record a measurement of 258.333 m. What is the distance corrected for erroneous tape length?

2.9 It is required to layout a rectangular commercial building 90.00 ft wide and 140.00 ft long. If the steel tape being used is 100.04 ft long (under standard conditions), what distances would be laid out?

2.10 A survey distance of 418.63 ft was recorded when the field temperature was 1000°F. What is the distance, corrected for temperature?

2.11 Station 13 + 21.78 must be marked in the field. If the steel tape to be used is only 99.98 (under standard conditions) and if the temperature will be 90°F at the time of the measurement, how far from the existing station mark at 11 + 73.26 will the surveyor have to measure to locate the new station?

2.12 The point of intersection of the center line of Elm Rd. with the center line of First St. was originally recorded as being at 34 + 98.761. How far from existing station mark 34 + 00 on First St. would a surveyor have to measure along the center line to reestablish the intersection point under the following conditions?
Temperature to be -8°C, with a tape that is 29.996 under standard conditions.

In Problems 2.13 through 2.17, compute the corrected horizontal distance.

	Temperature	Tape length	Slope data	Slope
2.13	-10°F	99.98 ft	Difference in elevation = 4.62 ft	211.10 ft
2.14	25°F	100.00 ft	Slope angle = 2°20'	106.07 ft
2.15	25°C	29.990 m	Slope angle = -3°42'	233.717 m
2.16	0°C	30.004 m	Slope @ 1.50%	250.00 m
2.17	100°F	100.03ft	Slope @ -0.80%	418.99 ft

In Problems 2.18 through 2.22, compute the required layout distance.

	Temperature	Tape length	Required horizontal distance
2.18	55°F	99.98 ft	204.48 ft
2.19	22°C	30.012 m	177.623 m
2.20	15°C	29.990 m	250.00 m
2.21	25°C	100.02 ft	600.00 ft
2.22	100°F	100.04ft	280.00 ft

2.23 A 50-m tape is used to measure between two points. The average weight of the tape per meter is 0.320 N. If the measured distance is 48.888 m, with the tape supported at the ends only and with a tension of 100 N, find the corrected distance.

2.24 A 30-m tape has a mass of 544 g and is supported only at the ends with a force of 80 N. What is the sag correction?

2.25 A 100-ft steel tape weighing 1.8 Ib and supported only at the ends with a tension of 24 Ib is used to measure a distance of 471.16 ft. What is the distance corrected for sag?

2.26 A distance of 72.55 ft is recorded using a steel tape supported only at the ends with a tension of 15 Ib and weighing 0.016 Ib per foot. Find the distance corrected for sag.

ANGLES AND DIRECTIONS

1. Introduction

Determining the locations of points and orientations of lines frequently depend on measurements of angles and directions. In surveying directions are given by ***bearings*** and ***azimuths***. Angles measured in surveying are classified as *horizontal* or *vertical*, depending on the plane in which they are measured. Horizontal angles are the basic measurements needed for determining bearings and azimuths. Vertical angles are used in trigonometric leveling, stadia, and for reducing measured slope distances to horizontal.

Angles are more often *directly* measured in the field by theodolite or transit, but a compass can also be used. An angle can be measured *indirectly* by the tape method and its value computed from the relationship of known quantities in a triangle (Figure 1-chord method) or other simple geometric figure (e.g. tangent method).

$$\sin\frac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{bc}} \qquad \text{where } s=(a+b+c)/2$$

or

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

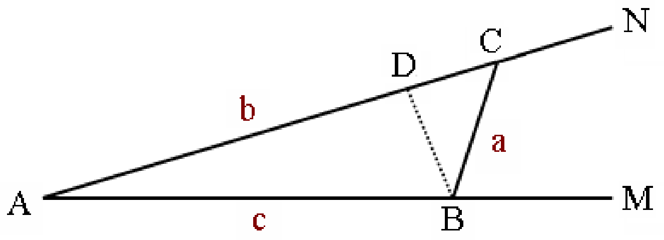
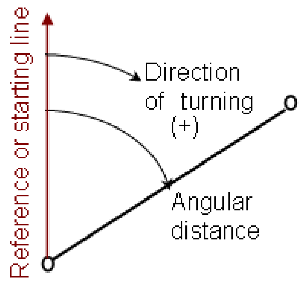


Figure 1. Measuring an angle with a tape by the chord method.

Three basic requirements determine an angle. As shown in Figure, they are;

1) reference or starting line
2) direction of turning
3) angular distance (value of the angle).



2. Units of angle measurement

The *sexadesimal* system used in the United States and many other countries is based on degrees, minutes, and seconds, with the last unit further divided decimally. In Europe the *grad* or *gon* is a standard unit. Radians may be more suitable in computations, and in fact are employed in electronic computers.

3. Reference directions for vertical angles

Vertical angles, which are used in slope distance corrections or in height determination, are referenced to (1) the horizon by plus (up) or minus (down) angles, (2) the zenith, or (3) the nadir (Figure 2).

Zenith and *nadir* are terms describing points on a celestial sphere, Figure 3 (i.e., a sphere or infinitely large radius with its center at the center of the earth). The zenith is directly above the observer and the nadir is directly below the observer; the zenith, nadir, and the observer are all on the same vertical line.

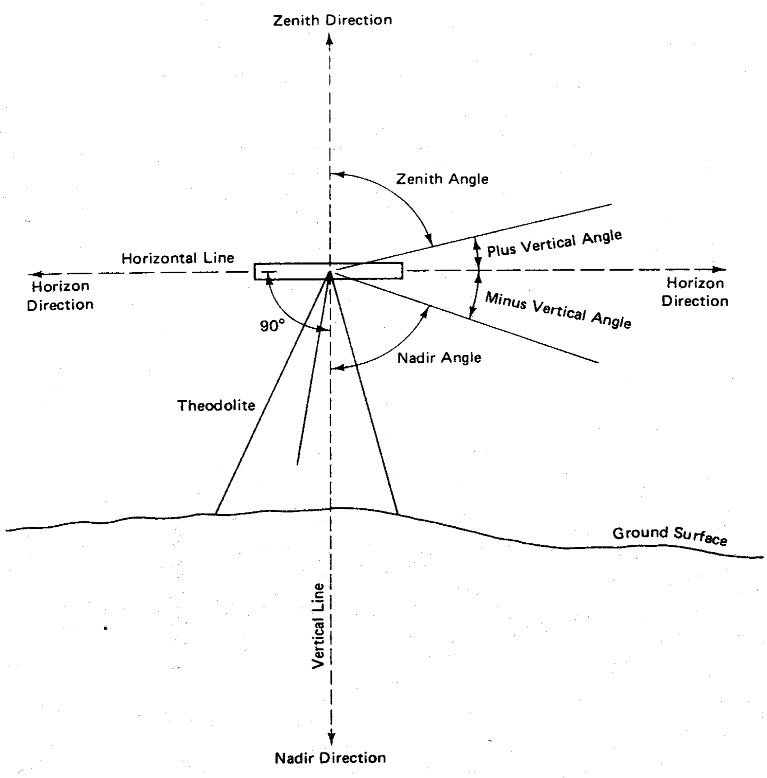


Figure 2. The three reference directions for vertical angles: horizontal, zenith and nadir.

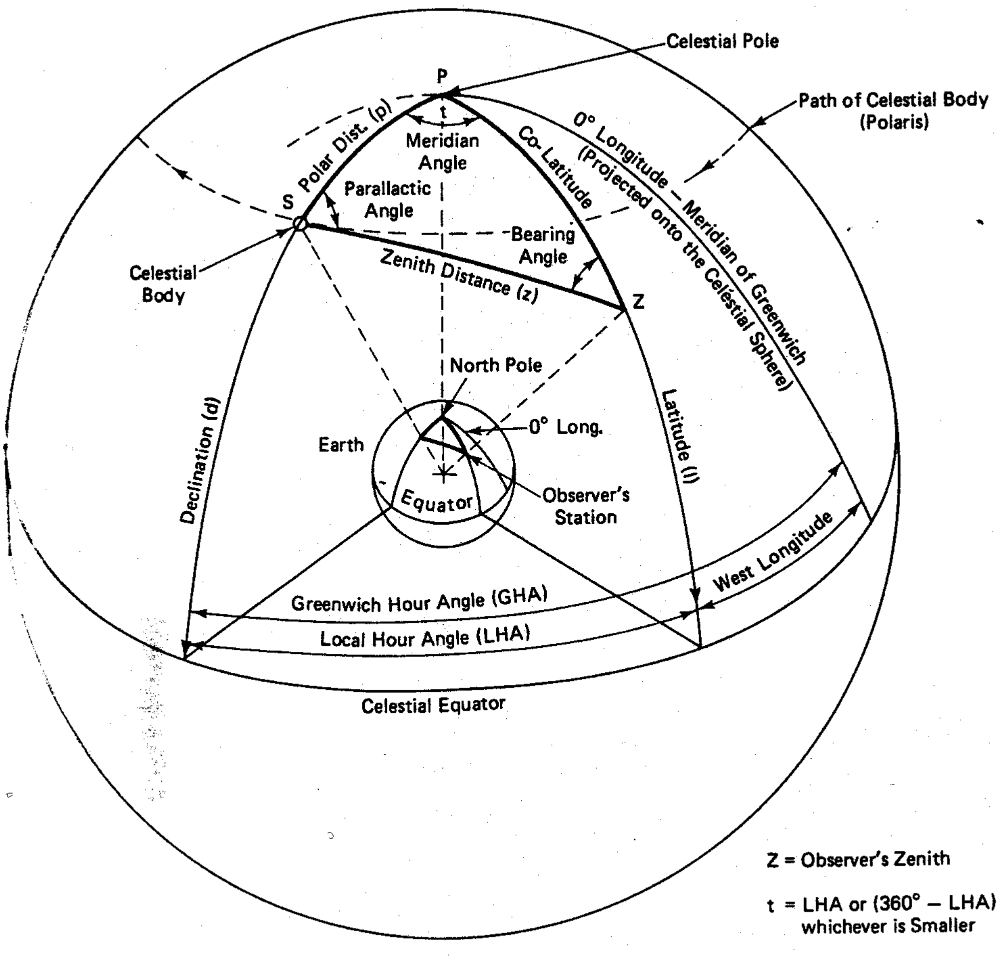


Figure 3. Celestial sphere.

4. Kinds of horizontal angles

The kinds of horizontal angles most commonly measured in surveying are (1) *interior angles,* (2) *angles to the right,* and (3) *deflection angles.*

Interior angles, shown in Figure 4, are on the inside of a closed polygon. *Exterior angles,* located outside a closed polygon, are explements of interior angles. They are only measured to serve as a check on the interior angle, since their sum at any station must equal 360^o. For all closed polygons of n sides, the sum of the interior angles will be equal $(n-2)180^o$; the sum of the exterior angles will be $(n+2)180^o$.

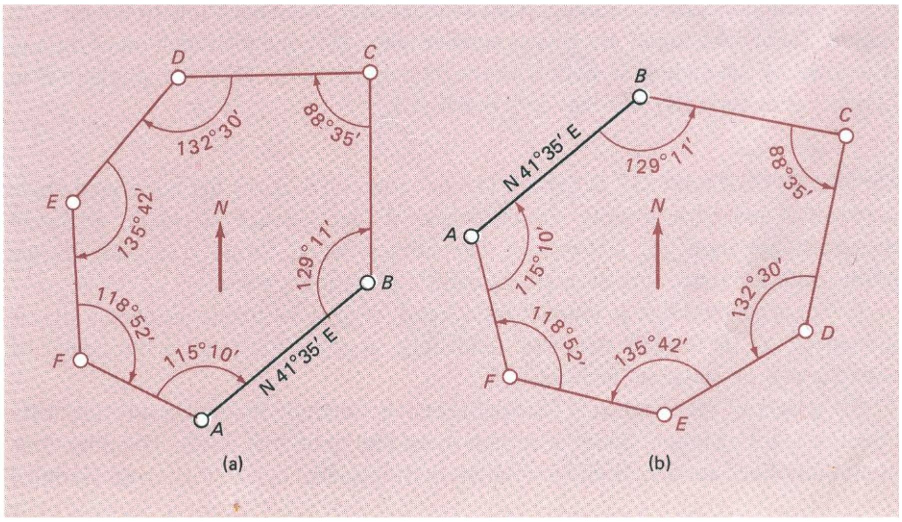


Figure 4. Closed polygon. (a) Clockwise interior angles (angles to the right),

(b) Counterclockwise interior angles (angles to the left).

As illustrated in Figure 4, interior angles can be turned clockwise (right) or counterclockwise (left). By definition, *angles to the right* are measured clockwise from the rear to the forward station. *Angles to the left,* turned counterclockwise from the rear station, are illustrated in Figure 4(b). Note that the polygons of Figure 4 are "right" and "left" –that is, similar in shape but turned over like the right and left hands. It is clear that a serious mistake that occurs if clockwise and counterclockwise angles are mixed. Therefore, a uniform procedure should be adopted, such as *measuring angles clockwise if possible*, and the direction of turning noted in the field book with a sketch.

Deflection angles (Figure 5) are measured right (clockwise, considered plus) of left (counterclockwise, minus) from an extension of the back line to the forward station. Deflection angles are always less than 180^o, and the direction of turning is defined by appending R or L to the numerical value.

It is also possible to measure the change in direction (Figure 5b) by directly sighting the back line and turning the angle left or right to the forward line.

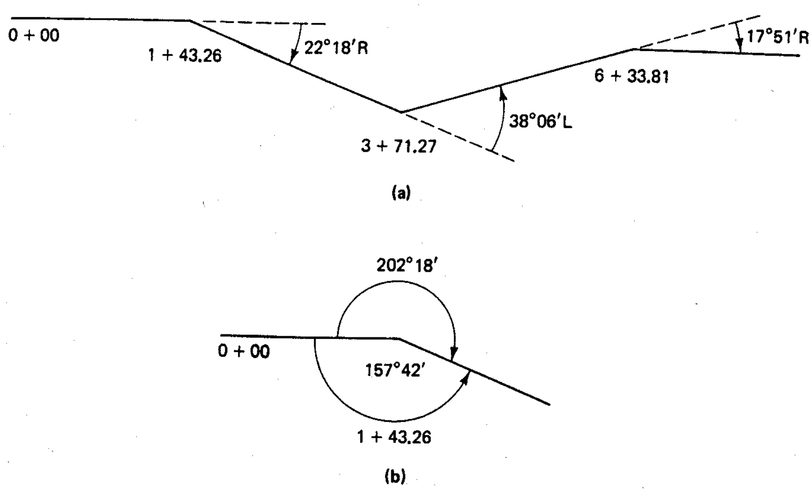


Figure 5. (a) Open traverse showing deflection angles, (b) Same traverse showing angle right (202^o 18") and angle left (157^o 42").

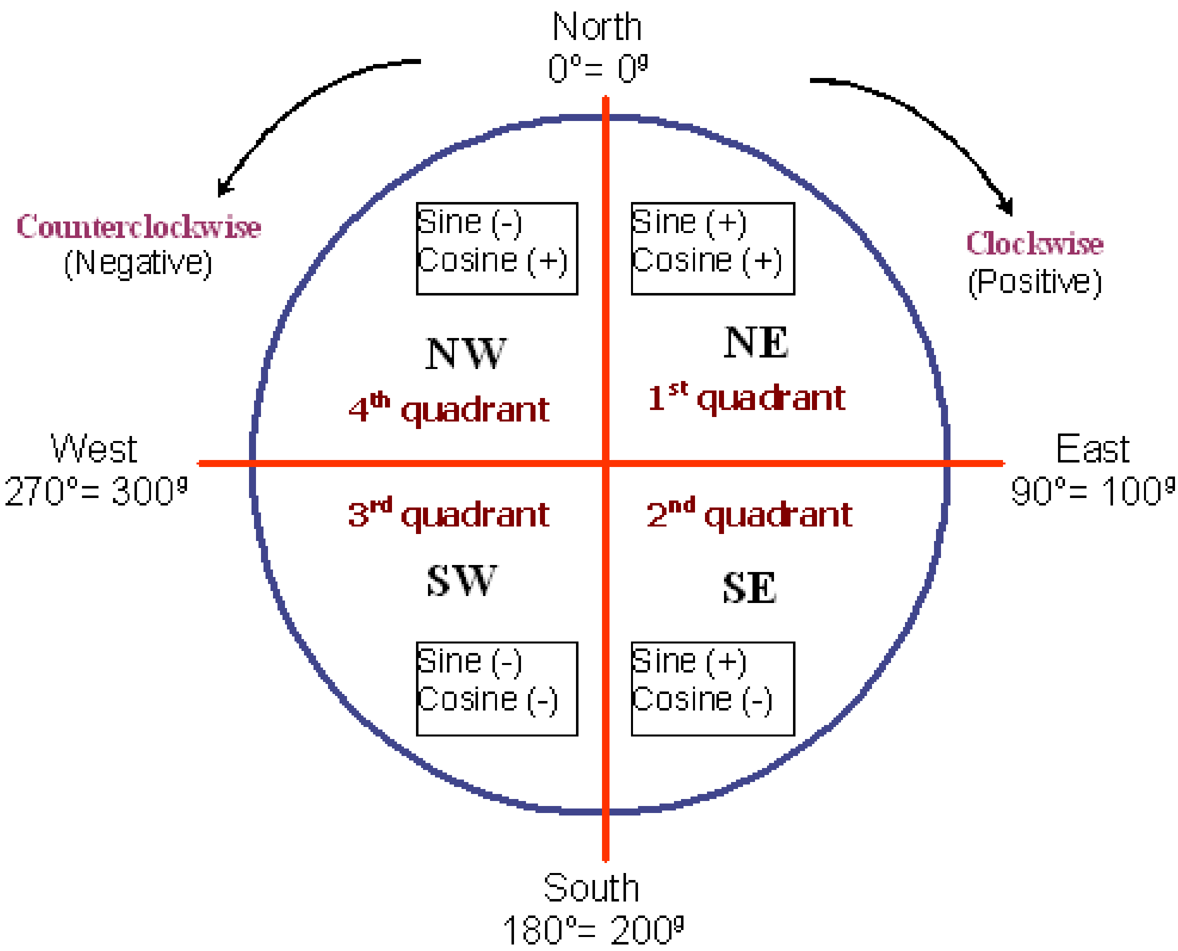
5. Direction of a line

The direction of a line is the horizontal angle between it and an arbitrarily chosen reference line called a *meridian*. Meridian is a line on the mean surface of the earth joining the north and south poles.

Different meridians (grid, astronomic, magnetic, guide, central, etc.) are used. Surveys based on a state or other plane coordinate system employ a grid meridian for reference. Grid meridians are lines that are parallel to a grid reference meridian (central meridian).

A map projection is a distorted rendition of a portion of the curved earth's surface on a surface that can be laid out flat. The projection has an inherent x; y coordinates system, with y in the general direction of north. In the projection any line parallel to the y axis is a grid meridian.

In mathematics, polar coordinates are often used to specify the position of a point. Here, the direction of the line from the origin to the point is based on the angle from the positive x axis, with counterclockwise angles being positive. In surveying, the direction of a line can be expressed either in terms of bearing or azimuth.



East-West direction : **Departure of a distance, Sine axis, Y axis.**

North-South direction : **Latitude of a distance, Cosine axis, X axis.**

5. Bearings and Azimuths

Bearing of a line is specified as an acute horizontal angle between the line and a meridian, along with letters specifying the proper quadrant. The angle is measured from either the north or south toward the east or west to give a reading smaller than 90^o. It is expressed in the form of the letter N or S, followed by an angle (less than or equal to 90^o), followed by the letter E or W.

True bearings are measured from the local astronomic or true meridian, magnetic bearings from the local magnetic meridian, assumed bearings from any adopted meridian, and grid bearings from the appropriate grid meridian. Magnetic bearings can be obtained in the field by observing the magnetic needle of a compass, and used along with measured angles to get computed bearings.

Azimuth is specified as the clockwise horizontal angle from the meridian to the line. The angular value is positive and less than 360^o . In plane surveying, azimuths are generally measured from north, though this is not a universally accepted standard. Some applications (astronomers, the military and the National Geodetic Survey) use azimuths that are referenced from south. Due to this possible ambiguity, one should clearly indicate whether a north or south reference is implied. Figure 6 gives examples of bearings and azimuths (from north) for selected lines.

Azimuths may be true, magnetic, grid, assumed, etc. depending on the reference meridian used. They may also be forward or back azimuths.

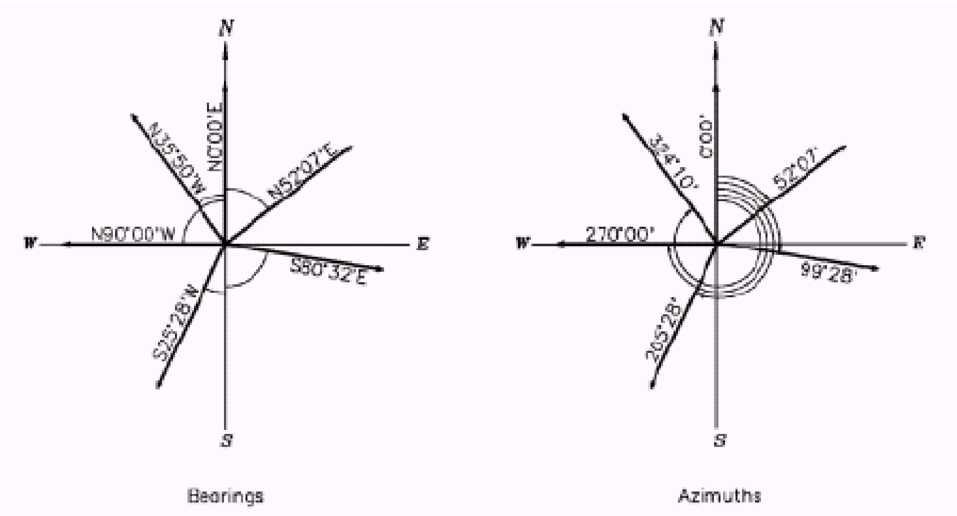


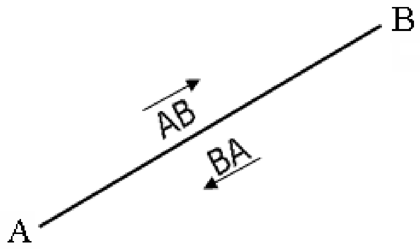
Figure 6. Examples of bearings of selected lines and their equivalent azimuths.

5.1 Comparison of Bearings and Azimuths

BEARINGS	AZIMUTHS
Vary from 0 to 90^o	Vary from 0 to 360^o
Require two letters and a numerical value	Require only a numerical value
May be true, magnetic, grid, assumed, forward, or back.	Same
Are measured clockwise and counterclockwise	Are measured clockwise only
Are measured from north and south	Measured from north only in any one survey, or from south only

5.2 Reverse directions

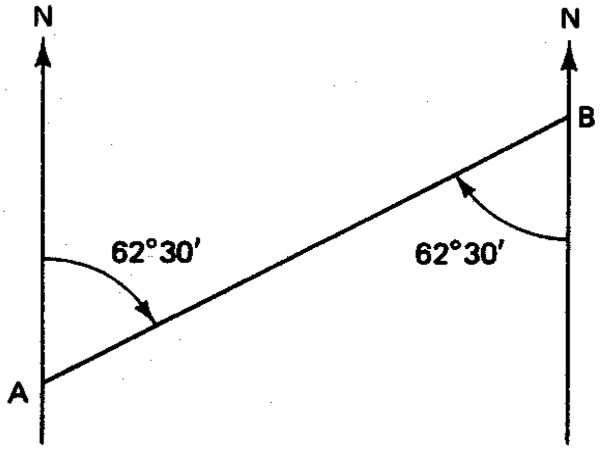
It can be said that every line has two directions. Referring to Figure, the line has direction AB or it has direction BA. In surveying, a direction is called *forward* if it is oriented in the direction of fieldwork or computation staging. If the direction is the reverse of that, it is called a *back* direction.

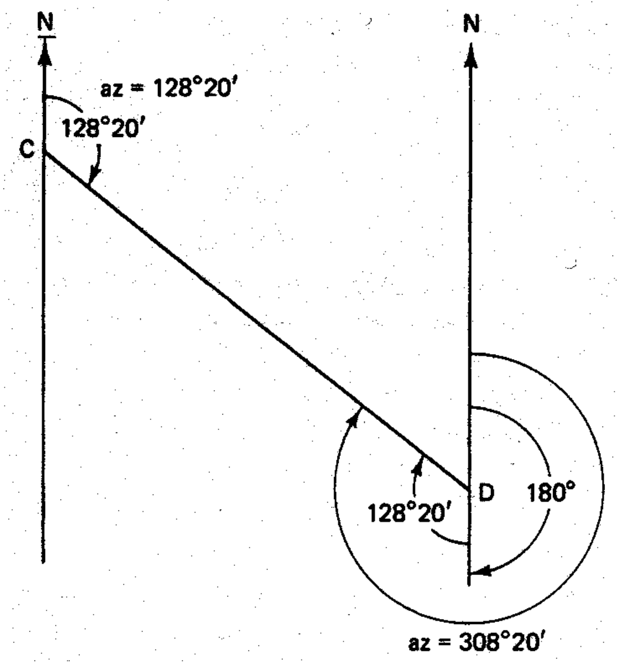


In Figure 7(a), the line AB has a bearing of N 62°30' E, whereas the line BA has a bearing of S 62°30' W. That is, to reverse a bearing, simply reverse the direction letters.

In Figure 7(b), the line CD has an azimuth of 128°20'; analysis of the sketch leads quickly to the conclusion that the azimuth of DC is 308°20'; that is, to reverse an azimuth, simply add 180° to the original direction. If the original azimuth is greater than 180°, 180° can be subtracted from it in order to reverse its direction. The key factor to remember is that a forward and back azimuth must differ by 180°.

Line	Bearing
AB	N 62° 30′ E
BA	S 62° 30′ W





Line	Azimuth
CD	128°20′
DC	308°20′

Figure 7. Reverse bearings and azimuths.

5.3 Relationship between Bearings and Azimuths

Quadrant	from azimuth to bearing	from bearing to azimuth
1. NE	bearing = azimuth	azimuth = bearing
2. SE	bearing = 180^o - azimuth	azimuth = 180^o - bearing
3. SW	bearing = azimuth - 180^o	azimuth = 180^o + bearing
4. NW	bearing = 360^o - azimuth	azimuth = 360^o - bearing

5.4 Computing Bearings and Azimuths

Computation of the bearing of a line is simplified by drawing a sketch similar to those in Figures 8 and 9, showing all data. In Figure 8, the bearing of line AB from Figure 4(a) is N41^{o}35'E, and the angle at B turned clockwise (to the right) from known line BA is 129^{o}11'. Then the bearing angle of line BC is 180^o-(41^{o}35'+129^{o}11') = 9^{o}14', and from the sketch, the bearing of BC is N9^{o}14'W.

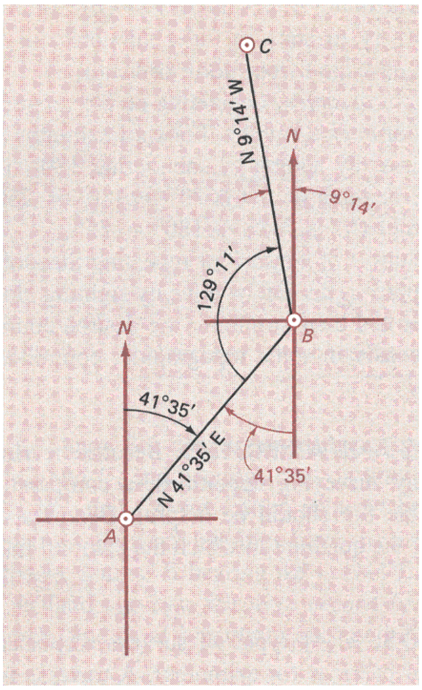


Figure 8. Computation of bearing BC of Figure 4(a).

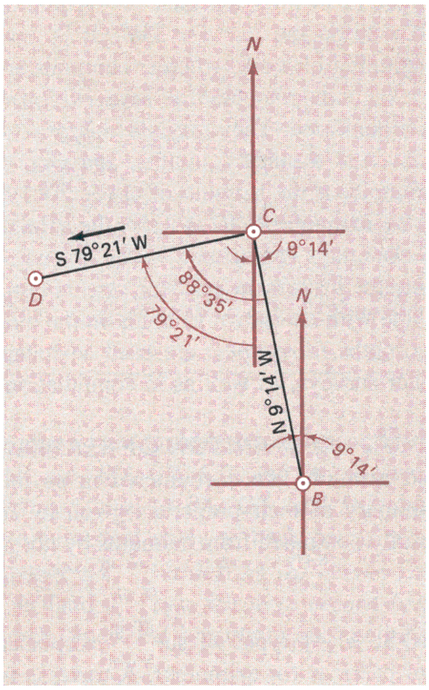


Figure 9. Computation of bearing CD of Figure 4(a).

In Figure 9, the clockwise angle at C from B top D was measured as 88^{o}35'. The bearing of CD is 88^{o}35'-9^{o}14' = S79^{o}21'W, Continuing this technique, the bearings in the following table have been determined for all lines in Figure 4(a).

Course	Bearing
AB	N41^{o}35'E
BC	N9^{o}14'W
CD	S79^{o}21'W
DE	S31^{o}51'W
EF	S12^{o}27'E
FA	S73^{o}35'E
AB	N41^{o}35'E (Check)

Azimuth calculations, like those for bearings, are best made with the aid of a sketch. Figure 10 illustrates the computations for azimuth BC in Figure 4(a). Azimuth BA is found by adding 180° to azimuth AB: 180° + 41°35’= 221°35’. Then clockwise angle B, 129°11’, is added to azimuth BA to get azimuth BC = 221°35’+129°11’ = 350°46’. The calculations are conveniently handled in tabular form. The following table lists the calculations for all azimuths on Figure 4(a).

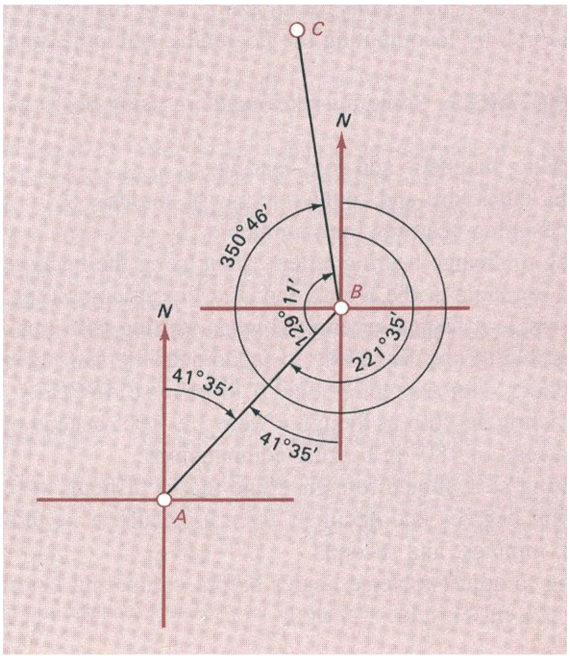


Figure 10. Computation of azimuth BC of Figure 4(a).

Computation of azimuths (Angles to the right – Figure 4a)

41°35’	= AB	31°51’	= ED
+ 180°00’		+ 135°42’	
221°35’	= BA	167°33’	= EF
+ 129°11’		+ 180°00’	
350°46’	= BC	347°33’	= FE
- 180°00’		+ 118°52’	
170°46’	= CB	466°25’	
+ 88°35’		- 360°00’ (*)	
259°21’	= CD	106°25’	= FA
- 180°00’		+ 180°00’	
79°21’	= DC	286°25’	= AF
+ 132°30’		+ 115°10’	
211°51’	= DE	401°35’	= AB
- 180°00’		- 360°00’ (*)	
31°51’	= ED	41°35’	= AB (Check)

(*) When a computed azimuth exceeds 360°, the correct azimuth is obtained by subtracting 360°.

REVERSE AZIMUTH AND BEARING

1. Azimuth Computations

Line	Bearing
AB	N 62° 30′ E
BA	S 62° 30′ W

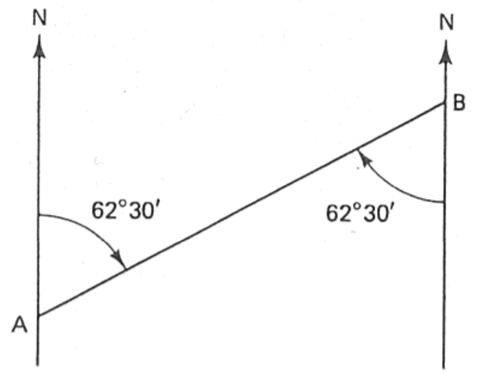
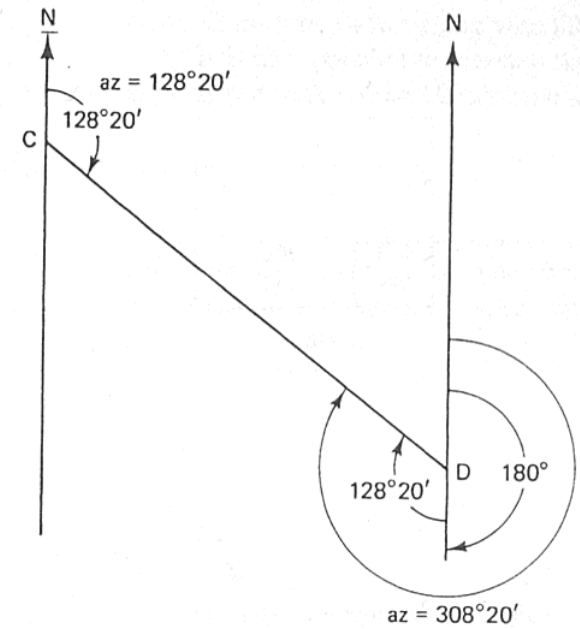


FIGURE 4.8 Reverse bearings.

In Figure 4.10 the five angles do add to 540°00′ and *AB* has a given azimuth of 330°00′. At this point a decision must be made as to the direction that the computation will proceed. Using the given azimuth and the angle at *B*, the azimuth of *BC* can be computed (counterclockwise direction); or using the given azimuth and the angle at *A*, the azimuth of *AE* can be computed (clockwise direction). Once a direction for solving the problem has been established, the computed directions must all be consistent with that general direction. It is strongly advised that a neat, well-labeled sketch accompany each step of the computation.

Analysis of the preceding azimuth computations gives the following observations:

1. If the computation is proceeding in a **counterclockwise** manner, **add the interior angle to the back azimuth of the previous course.**



Line	Azimuth
CD	128°20′
DC	308°20′

FIGURE 4.9 Reverse azimuths.

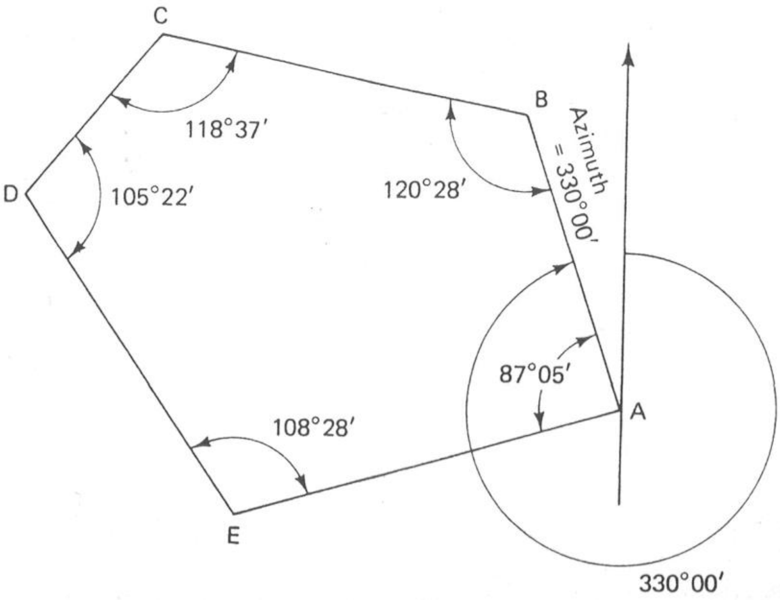
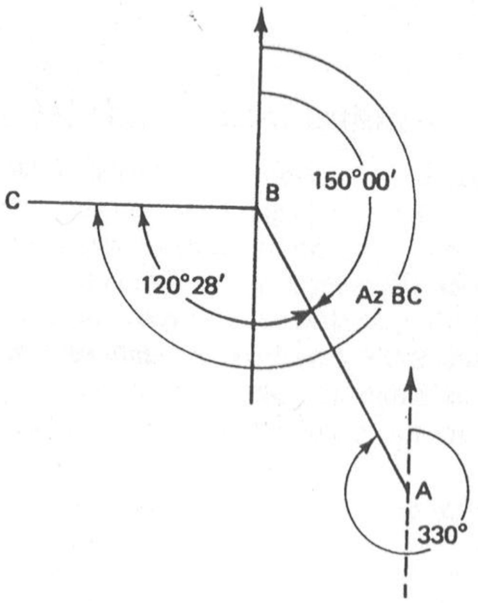


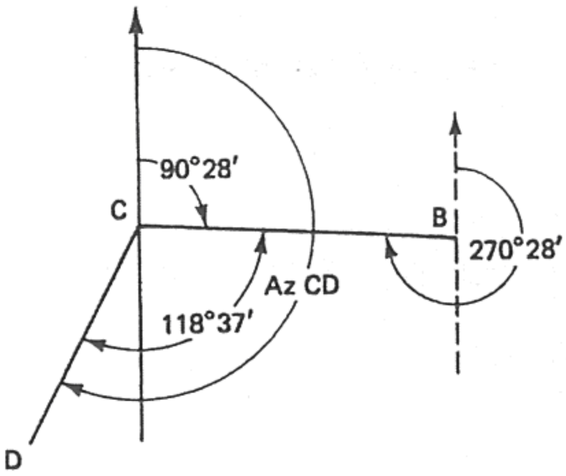
FIGURE 4.10 Sketch for azimuth calculations.

2. If the computation is proceeding in a **clockwise** manner, **subtract the interior angle from the back azimuth of the previous course.**
   If the bearings of the sides are also required, they can now be derived from the computed azimuths.

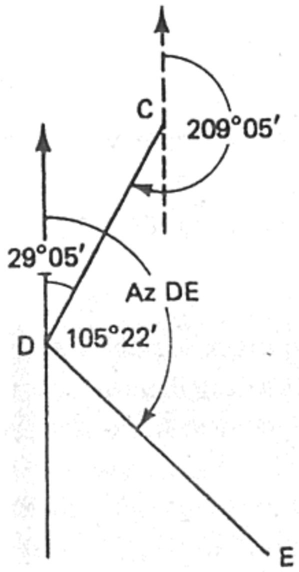
Counterclockwise solution



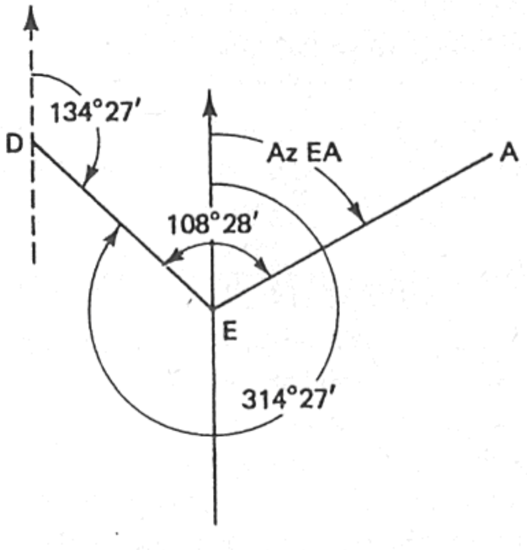
Az AB =	330°00′	Given
	− 180	
Az BA =	150°00′	
+ $<B$ =	120°28′	
Az BC =	270°28′	



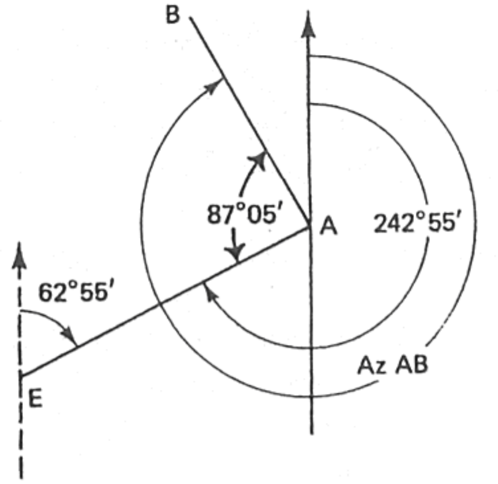
$$
\begin{aligned}
\text{Az } BC &= 270°28' \\
&\underline{-\ 180} \\
\text{Az } CB &= 90°28' \\
+ \angle C & \underline{118°37'} \\
\text{Az } CD &= 208°65' \\
\text{Az } CD &= 209°05'
\end{aligned}
$$



$$
\begin{aligned}
\text{Az } CD &= 209°05 \\
&\underline{-\ 180} \\
\text{Az } DC &= 29°05' \\
+ \angle D & \underline{105°22'} \\
\text{Az } DE &= 134°27'
\end{aligned}
$$

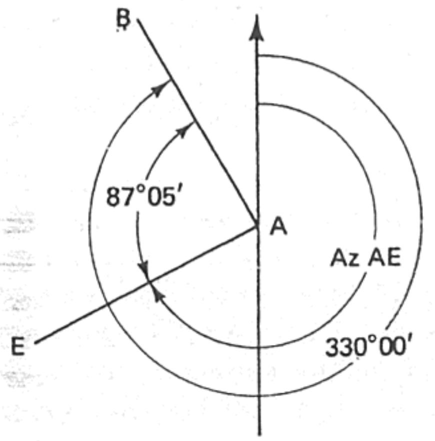


$$
\begin{aligned}
\text{Az } DE &= 134°27' \\
&\underline{+\ 180°} \\
\text{Az } ED &= 314°27' \\
+ \angle E & \underline{108°28'} \\
\text{Az } EA &= 422°55' \\
&\underline{-\ 360} \\
\text{Az } EA &= 62°55'
\end{aligned}
$$

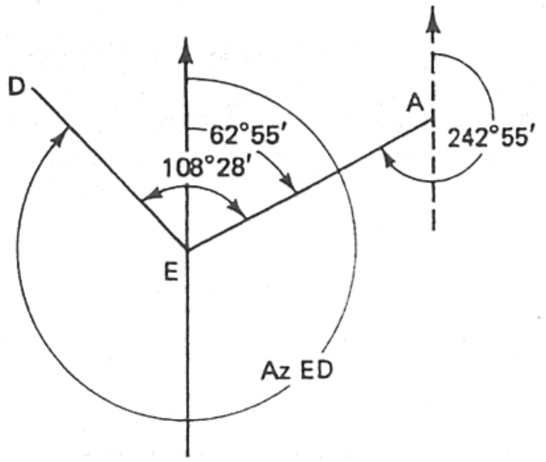


$$\begin{aligned} \text{Az } EA &= 62°55' \\ &\ + \underline{180} \\ \text{Az } AE &= 242°55' \\ + \angle A &\ \ \underline{87\ 05} \\ \text{Az } AB &= 329°60' \\ \text{Az } AB &= 330°00' \qquad \text{Check} \end{aligned}$$

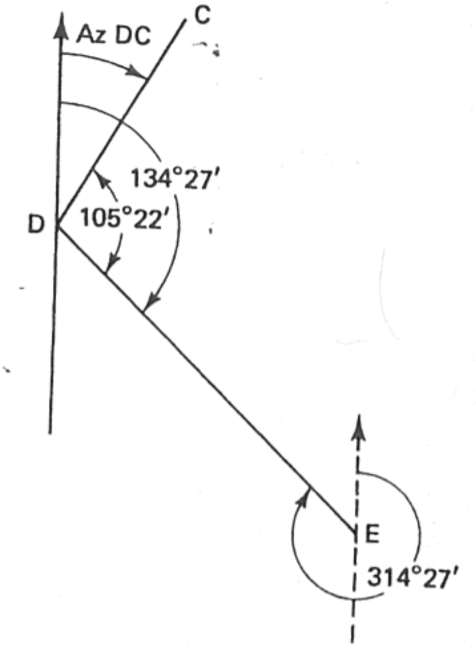
Clockwise solution



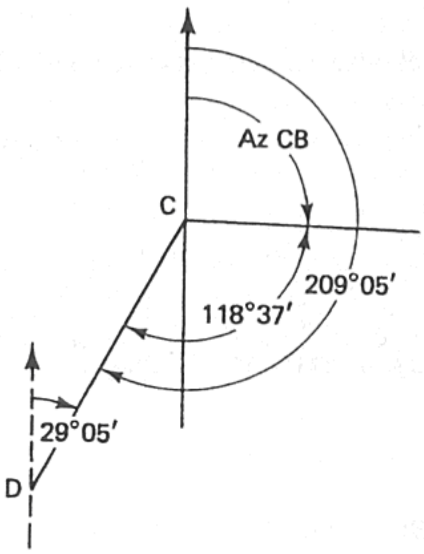
$$\begin{aligned} \text{Az } AB &= 330°00' \qquad \text{Given} \\ - \angle A &\ \ \underline{87°05'} \\ \text{Az } AE &= 242°55' \end{aligned}$$



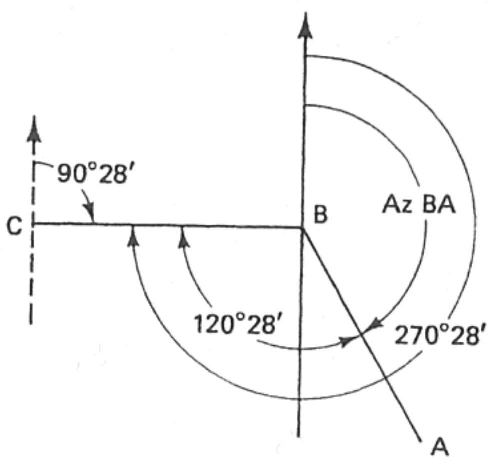
$$\begin{aligned} \text{Az } AE &= 242°55' \\ &\ - \underline{180°} \\ \text{Az } EA &= \ \ 62°55' \\ &\ + \underline{360°} \qquad \text{(to enable subtraction)} \\ \text{Az } EA &= 422°55' \\ - \angle E &\ \ \underline{108°28'} \\ \text{Az } ED &= 314°27' \end{aligned}$$



$$\begin{array}{rl} \text{Az } ED = & 314°27' \\ - & \underline{180°} \\ \text{Az } DE = & 134°27' \\ - \angle D & \underline{105°22'} \\ \text{Az } DC = & 29°05' \end{array}$$



$$\begin{array}{rl} \text{Az } DC = & 29°05' \\ + & \underline{180} \\ \text{Az } CD = & 209°05' \\ - \angle C & \underline{118°37'} \\ \text{Az } CB = & 90°28' \end{array}$$



$$\begin{array}{rl} \text{Az } CB = & 90°28' \\ + & \underline{180} \\ \text{Az } BC = & 270°28' \\ - \angle B & \underline{120°28'} \\ \text{Az } BA = & 150°00' \\ + & \underline{180°00'} \\ \text{Az } AB = & 330°00' \quad \text{Check} \end{array}$$

COUNTERCLOCKWISE SOLUTION

Course	Azimuth	Bearing
BC	270°28′	N 89°32′ W
CD	209°05′	S 29°05′ W
DE	134°27′	S 45°33′ E
EA	62°55′	N 62°55′ E
AB	330°00′	N 30°00′ W

CLOCKWISE SOLUTION

Course	Azimuth	Bearing
AE	242°55′	S 62°55′ W
ED	314°27′	N 45°33′ W
DC	29°05′	N 29°05′ E
CB	90°28′	S 89°32′ E
BA	150°00′	S 30°00′ E

Notes

1. It will be noted that for reversal of direction (i.e., clockwise versus counterclockwise) azimuths for the same side differ by 180°, whereas bearings for the same side remain numerically the same and have the letters (N/S, E/W) reversed.
2. It should be emphasized that the bearings calculated from azimuths have no built-in check. The only way that the correctness of the calculated bearings can be verified is to double-check the computation. **Constant reference to a good problem diagram will help reduce the incidence of mistakes.**

4.10 Bearing Computations

As with azimuth computations, the solution can proceed in a clockwise or counterclockwise manner. Referring to Figure 4.11, side *AB* has a given bearing of N 30°00′ W, and the bearing of either *BC* or *AE* may be computed first.

Since there is no systematic method of directly computing bearings, each bearing computation will be regarded as a separate problem; **it is essential that a neat, well-labeled diagram accompany each computation.**

The sketch of each individual bearing computation will show the appropriate interior angle together with one bearing angle. The required bearing angle should also be clearly shown.

In Figure 4.12a, the interior angle (*B*) is shown; the bearing angle (30°) for side *BA* is shown; and the required bearing angle for side *BC* is shown as a question mark. Analysis of the sketch clearly shows that the required bearing angle (?) = 180° − (120°28′ − 30°00′) = 89°32′ and that the quadrant is NW. The bearing of *BC* = N 89°32′ W.

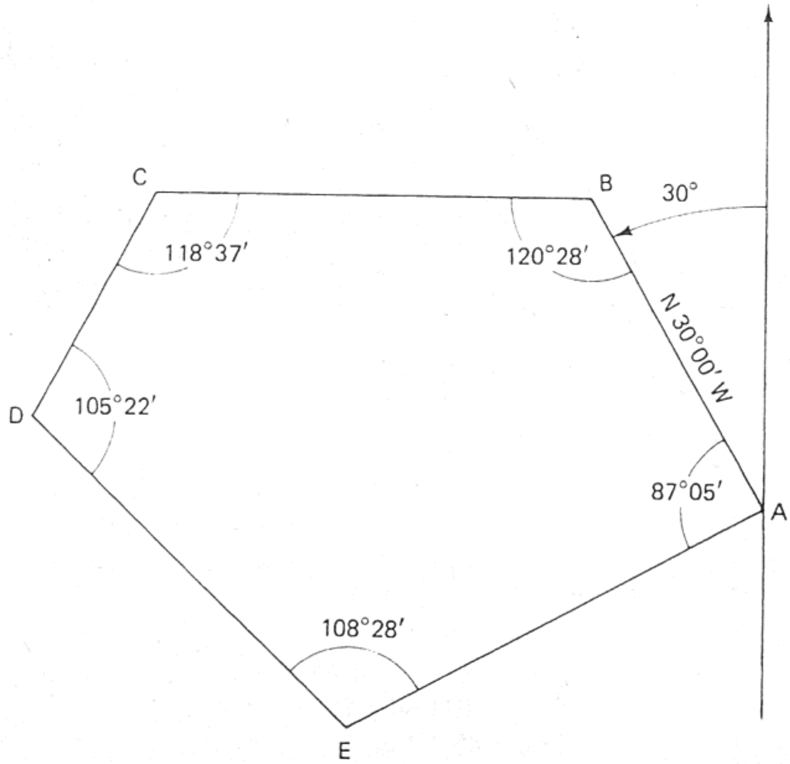


FIGURE 4.11 Sketch for bearing computations.

In Figure 4.12b the bearing for *CB* is shown as S 89°32′ E. This is the reverse of the bearing that was just calculated for side *BC*. When the meridian line is moved from *B* to *C* for this computation, it necessitates the reversal of direction. Analysis of the completely labeled sketch clearly shows that the required bearing angle for *CD* (?) = (118°37′ − 89°32′) = 29°05′ and that the direction is SW. The bearing of *CD* is S 29°05′ W.

Analysis of Figure 4.12c clearly shows that the bearing angle of line *DE* (?) = 180° − (105°22′ + 29°05′) = 45°33′ and that the direction of *DE* is SE. The bearing of *DE* is S 45°33′ E.

Analysis of Figure 4.12d clearly shows that the bearing angle of line *EA* (?) is (108°28′ − 45°33′) = 62°55′ and that the direction is NE. The bearing of *EA* is N 62°55′ E.

The problem's original data included the bearing of *AB* as being N 30°00′ W. The bearing of *AB* will now be computed using the interior angle at *A* and the bearing just computed for the previous course (*EA*). The bearing angle of *AB* = 180° − (62°55′ + 87°05′) = 30°00′ and the direction is NW. The bearing of *AB* is N 30°00′ W (see Figure 4.12e). This last computation serves as a check on all our computations.

4.11 Comments on Bearings and Azimuths

It has been shown that both bearings and azimuths may be used to give the direction of a line. North American tradition in this regard favors the use of bearings over azimuths; most legal plans (plats) show directions in bearings.

In the previous sections, it was shown that bearings can be derived from computed azimuths (Section 4.9), or bearings can be computed directly from the given data (Section 4.10). The **advantage** of computing bearings directly from the given data is that the final computation (of the given bearing) provides a check on all the problem computations, ensuring (normally)

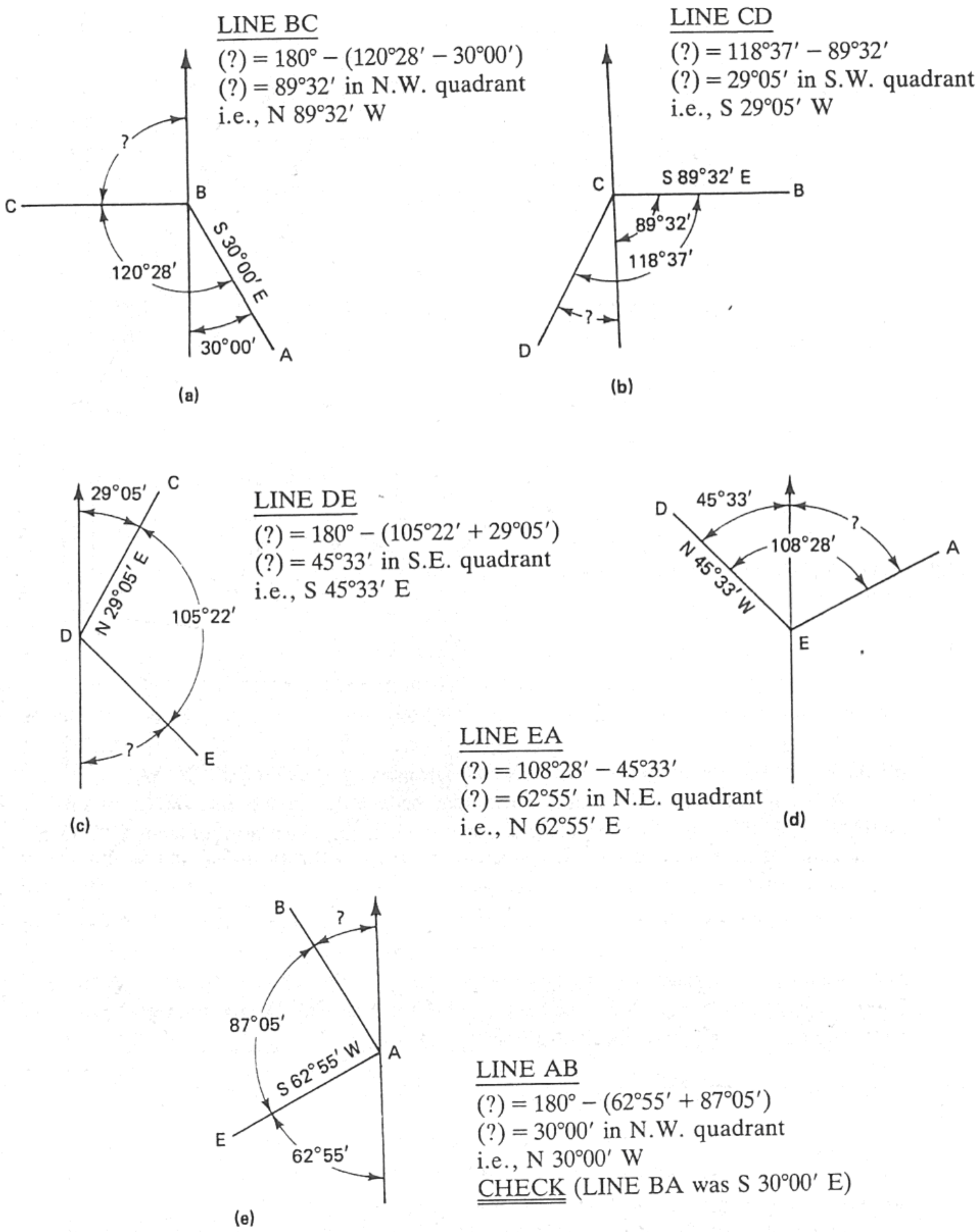


FIGURE 4.12 Sketches for each bearing calculation for the discussion of Section 4.10.

the correctness of all the computed bearings. In contrast, if bearings are to be derived from computed azimuths, there is no intrinsic check on the correctness of the derived bearings.

The **disadvantage** associated with computing bearings directly from the given data is that there is no systematic approach to the overall solution. Each bearing computation is unique, requiring individual analysis. It is sometimes difficult persuading people to prepare neat, well-labeled sketches for computations involving only intermediate steps in a problem; without neat, well-labeled sketches for each bearing computation, the potential for mistakes is quite large. Additionally, when mistakes do occur in the computation, the lack of a systematic approach to the solution often means that much valuable time is lost before the mistake is found and corrected.

In contrast, the computation of azimuths involves a highly systematic routine: **add (subtract) the interior angle from the back azimuth of the previous course.** If the computations are arranged as shown in Section 4.9, mistakes that may be made in the computation will be found in very short order.

With the advent of the universal use of computers and sophisticated hand-held calculators, it is expected that azimuths will be used more and more to give the direction of a line. It is easier to deal with straight numeric values rather than the alphanumeric values associated with bearings, and it is more efficient to have the algebraic sign generated by the calculator or computer rather than trying to remember if the direction was north, south, east, or west.

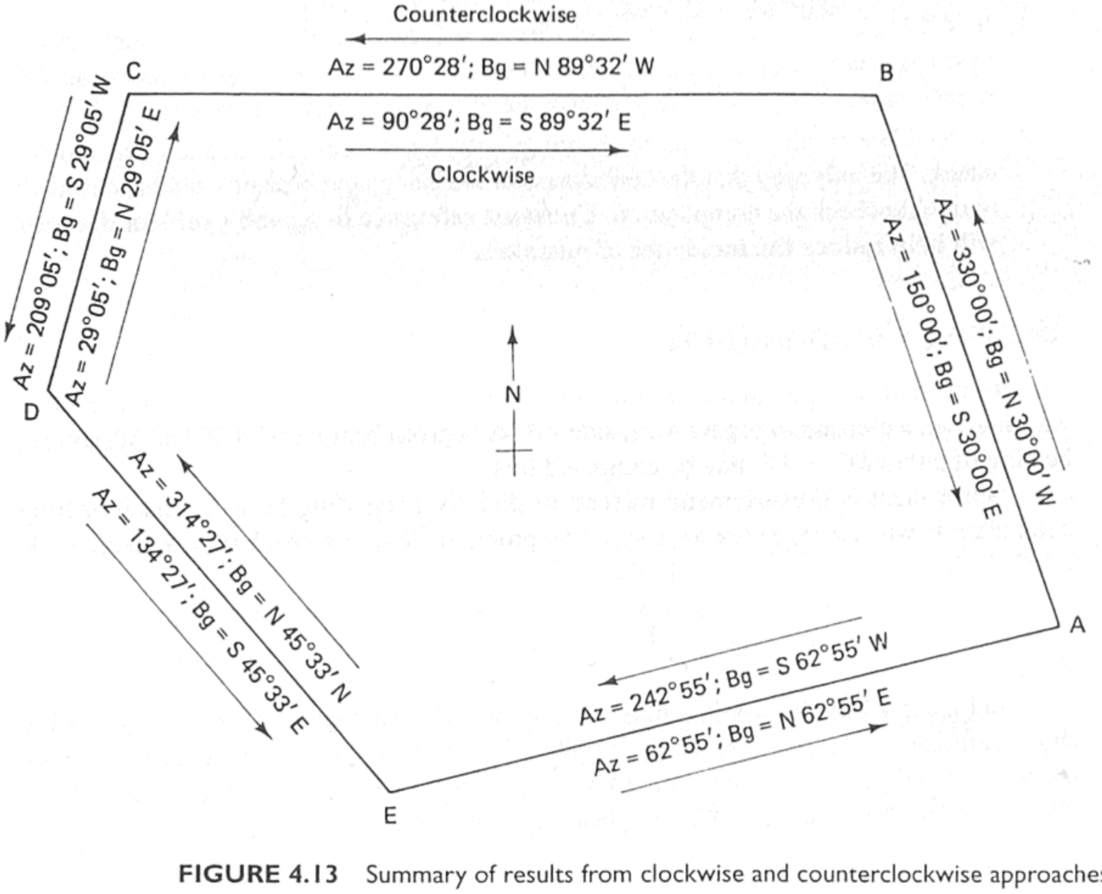


FIGURE 4.13 Summary of results from clockwise and counterclockwise approaches.

PROBLEMS:

1) A closed five-sided field traverse has the following interior angles:
A=$117^{o}31'00''$, B=$122^{o}09'30''$, C=$80^{o}10'00''$, D=$90^{o}41'30''$. Find the angle at E.

2) Convert the following azimuths to bearings.
(a) $191^{o}58'$ **(b)** $146^{o}11'$ **(c)** $167^{o}23'$ **(d)** $278^{g}59^{c}$ **(e)** $313^{g}47^{c}$ **(e)** $74^{g}86^{c}$

3) Convert the following bearings to azimuths.
(a) N $45^{o}45'$ E **(b)** S $3^{o}38'$ W **(c)** S $5^{o}21'$ E **(d)** N $88^{o}59'$ W
(e) S $13^{g}27^{c}$ W **(e)** N $64^{g}78^{c}$ W **(f)** N $37^{g}19^{c}$ E **(g)** S $92^{g}89^{c}$ E

4) Convert the angles given in questions (2) and (3) into the other angle units; degree into gon and gon into degree.

5) Convert the angles given in questions (2) and (3) into decimal expressions.

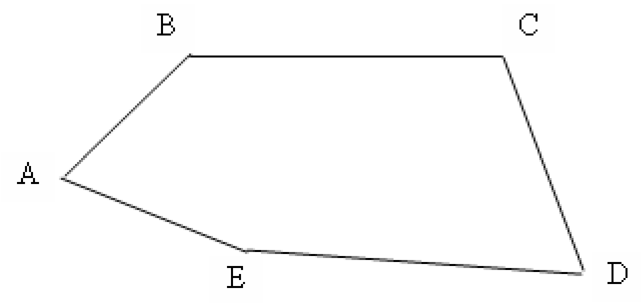
6) Convert the azimuths given in problem (2) to reverse (back) azimuths.

7) Convert the bearings given in problem (3) to reverse (back) bearings.

8) An open traverse that runs from A through H has the following deflection angles: B=$8^{o}13'$R, C=$2^{o}21'$R, D=$14^{o}41'$R, E=$21^{o}08'$L, F=$6^{o}32'$L, G=$1^{o}15'$R. If the bearing of AB is N $33^{o}58'$ E, compute the bearings of the remaining sides.

9) Closed traverse ABCD has the following bearings: AB = N $61^{o}27'$ E, BC = S $48^{o}31'$ E, CD = S $16^{o}20'$ W, DA = N $63^{o}41'$ W. Compute the interior angles and provide a geometric check on your work.

Use the following sketch and interior angles for Problems (10) through (13).



Interior angles
A = $63^{o}47'00''$
B = $140^{o}28'50''$
C = $101^{o}30'20''$
D = $72^{o}48'10''$
E = $161^{o}25'40''$
$537^{o}178'120''$ (Total)
= $540^{o}00'00''$ (closed)

10) If the bearing of AB is N $41^{o}23'20''$ E, compute the bearings of the remaining sides. Provide the solution proceeding clockwise and the other solution proceeding counterclockwise.

11) If the azimuth of AB is $41^{o}23'20''$, compute the azimuths of the remaining sides. Provide two solutions: one solution proceeding clockwise and the other proceeding counterclockwise.

12) If the bearing of AB is N $47^{o}41'$ E, compute the bearings of the remaining sides proceeding in a clockwise direction.

13) If the azimuth of AB is $47^{o}41'$, compute the azimuths of the remaining sides proceeding in a counterclockwise direction.

14) Latitudes and departures of courses of a quadrilateral are given. Determine the area within the traverse by the DMD method. Check by DPDs.

Course	AB	BC	CD	DA
Latitudes (m)	N424.02	S282.91	S938.19	N797.08
Departures (m)	E212.98	W601.51	W664.03	E1052.56

15) The latitudes and departures for a closed-polygon traverse ABCDEFGA given. Calculate:

a) the latitude and departure of line GA, b) the length of line GA, and

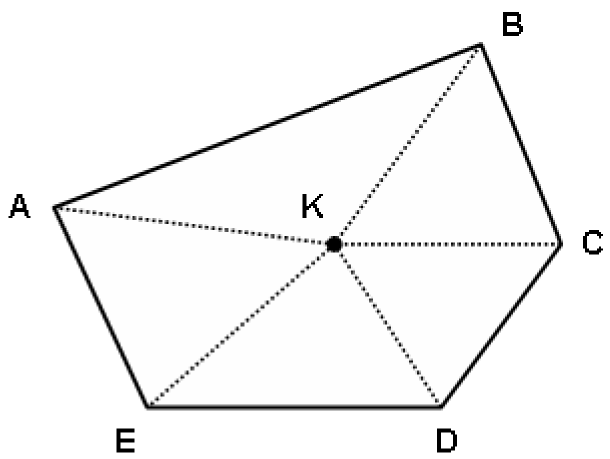
c) the area within the traverse in hectares by both DMDs and coordinates.

Course	AB	BC	CD	DE	EF	FG	GA
Latitudes (m)	N100	S900	S200	S700	N1000	N400	??
Departures (m)	E600	W600	E700	W800	W200	W600	??

16) A theodolite was set up at control station K that is within the limits of a five-sided property. Coordinates of control station K are 1,990.000N and 2,033.000E. Azimuth angles and distances to the five property corners were determined as follows.

Compute the coordinates of the property corners A, B, C, D and E.

Direction	Azimuth	Horizontal Distance (m)
KA	$286^o51'30''$	34.482
KB	$37^o35'28''$	31.892
KC	$90^o27'56''$	38.286
KD	$166^o26'49''$	30.916
KE	$247^o28'43''$	32.585



BASIC PROBLEMS OF TRAVERSE COMPUTATIONS

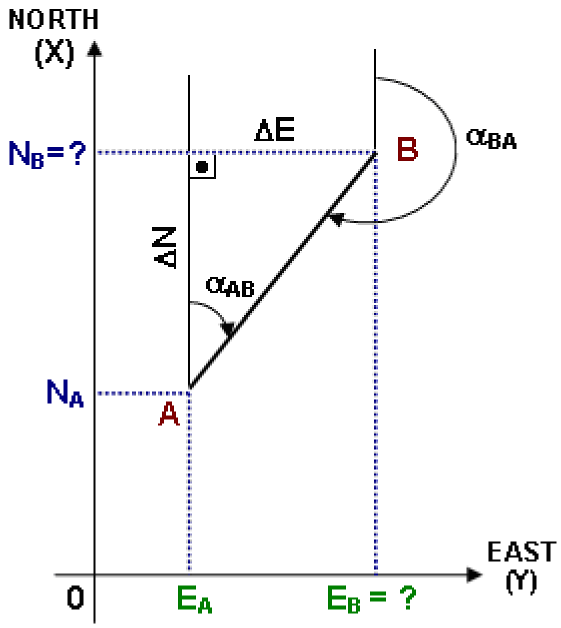
In order to carry out coordinate computations, azimuth computations, distance computations, etc. we need to know 4 basic problems explained below. If one understands these problems he/she can solve any traverse problem. Any problem will be a part or combination of these basic problems.

Basic Problem 1. The coordinate calculation (direct problem)

The coordinates of point A (N_A, E_A), the azimuth α_{AB} from A to B, i.e. the angle measured from the north direction and the distance AB from A to B are given. The coordinates of point B are asked. Thus:

GIVEN $\Rightarrow N_A, E_A, \alpha_{AB}$, AB $\alpha_{AB} = (AB)$ X ≈ North   Y ≈ East

ASKED $\Rightarrow N_B, E_B$



On the figure;

$\Delta E \approx \Delta Y$ (Departure)

$\Delta N \approx \Delta X$ (Latitude)

$\alpha_{AB} \approx$ Azimuth from A to B

$\alpha_{BA} \approx$ Azimuth from B to A ($=\alpha_{AB} + 200^g$)

AB : Distance between the points A and B

To solve this problem, it is seen from the figure that:

$$\mathbf{N_B\ (=X_B) = N_A + \Delta N = N_A + AB*\cos \alpha_{AB}}$$

$$\mathbf{E_B\ (=Y_B) = E_A + \Delta E = E_A + AB*\sin \alpha_{AB}}$$

Example :

GIVEN $\Rightarrow$ $N_A = 32647.87$ m $E_A = 24125.14$ m

$\alpha_{AB} = 140^g.7025$ AB = 348.74 m

ASKED $\Rightarrow$ N_B, E_B

$N_B = N_A + AB*\cos \alpha_{AB} = 32647.87 + 348.74 * \cos 140^g.7025$

$N_B = 32647.87 + (-208.09) =$ **32439.78 m**

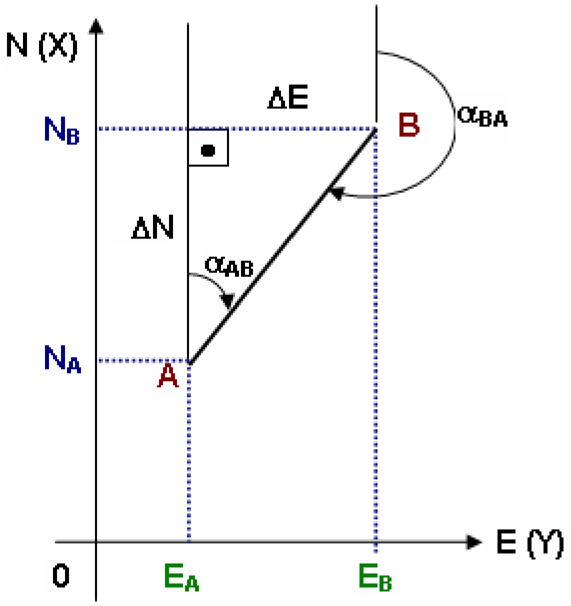
$E_B = E_A + AB*\sin \alpha_{AB} = 24125.14 + 348.74 * \sin 140^g.7025$

$E_B = 24125.14 + 279.86 =$ **24405.00 m**

<u>Basic Problem 2</u>. Azimuth and distance calculation from coordinates (inverse problem)

The coordiates of point A (N_A, E_A) and point B (N_B, E_B) are given. The computation of the azimuth α_{AB} from A to B or α_{BA} from B to A, and the distance AB from A to B are asked. Thus:

GIVEN $\Rightarrow N_A, E_A, N_B, E_B$ **ASKED** $\Rightarrow \alpha_{AB}$ (or α_{BA}), AB



From the figure;

$$\tan \alpha_{AB} = \Delta E / \Delta N = \frac{E_B - E_A}{N_B - N_A} \quad \text{and}$$

$$\alpha_{BA} = \alpha_{AB} \pm 200^g$$

$$AB = \sqrt{(\Delta E^2 + \Delta N^2)}$$

$$= \sqrt{(E_B - E_A)^2 + (N_B - N_A)^2} \quad \text{or}$$

$$AB = \frac{N_B - N_A}{\cos \alpha_{AB}} = \frac{E_B - E_A}{\sin \alpha_{AB}}$$

<u>Example :</u>

GIVEN $\Rightarrow$ $N_A = 20680.42$ m $E_A = 30540.71$ m
$N_B = 20791.51$ m $E_B = 30310.25$ m

ASKED $\Rightarrow$ α_{AB} (or α_{BA}), AB

$$\tan \alpha_{AB} = \Delta E / \Delta N = \frac{E_B - E_A}{N_B - N_A} = \frac{30310.25 - 30540.71}{20791.51 - 20680.42} = \frac{-230.46}{111.09}$$

$\tan \alpha_{AB} = -2.0745 \Rightarrow \alpha_{AB} = -71^g.4048 \Rightarrow$ $\boldsymbol{\alpha_{AB}} = -71^g.4048 + 400^g = \mathbf{328^g.5952}$ (*)

$\boldsymbol{\alpha_{BA}} = 328^g.5952 - 200^g = \mathbf{128^g.5952}$

$$AB = \sqrt{(30310.25 - 30540.71)^2 + (20791.51 - 20680.42)^2} = \sqrt{(-230.46)^2 + (111.09)^2}$$

$$\mathbf{AB} = \sqrt{65452.7997} = \mathbf{255.84\ m}$$

also $AB = \Delta N / \cos\alpha_{AB} = 111.09 / \cos 328^g.5952 = 255.84$ m

$AB = \Delta E / \sin\alpha_{AB} = -230.46 / \sin 328^g.5952 = 255.84$ m

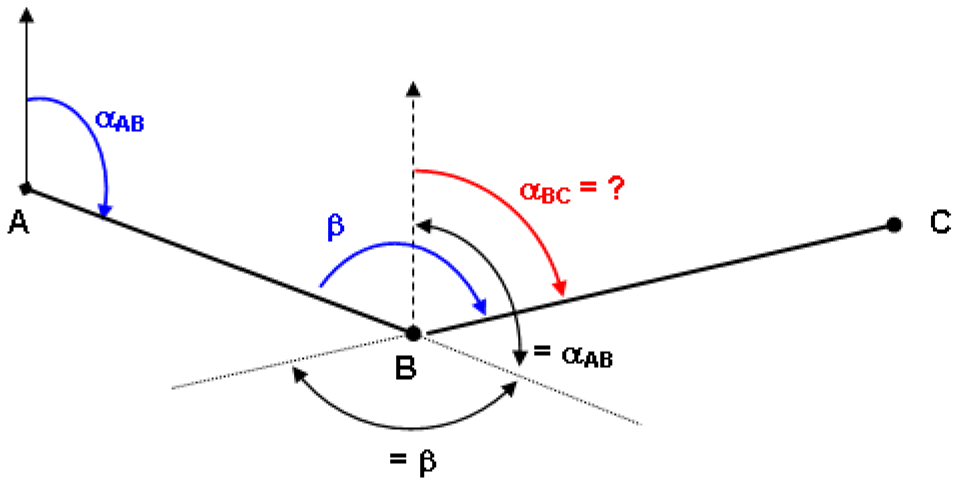
(*) NOTE : Determined azimuth (α_{AB}) must be corrected according to its quadrant as follows:

Quadrant	ΔE(Dep.)	ΔN(Lat.)	α_{AB} (determined)	α_{AB} (corrected)
1. (NE)	+	+	α_{AB}(det)	$\alpha_{AB} = \alpha_{AB}$(det)
2. (SE)	+	-	α_{AB}(det)	$\alpha_{AB} = \alpha_{AB}$(det) + 200^g
3. (SW)	-	-	α_{AB}(det)	$\alpha_{AB} = \alpha_{AB}$(det) + 200^g
4. (NW)	-	+	α_{AB}(det)	$\alpha_{AB} = \alpha_{AB}$(det) + 400^g

Basic Problem 3. **The azimuth calculation of other points**

The azimuth α_{AB} in the previous station and the angle to the right (breaking angle) at our station (point B) are given. The azimuth at our station (point B) is asked.

GIVEN $\Rightarrow \alpha_{AB}, \beta$ **ASKED** $\Rightarrow \alpha_{BC}$

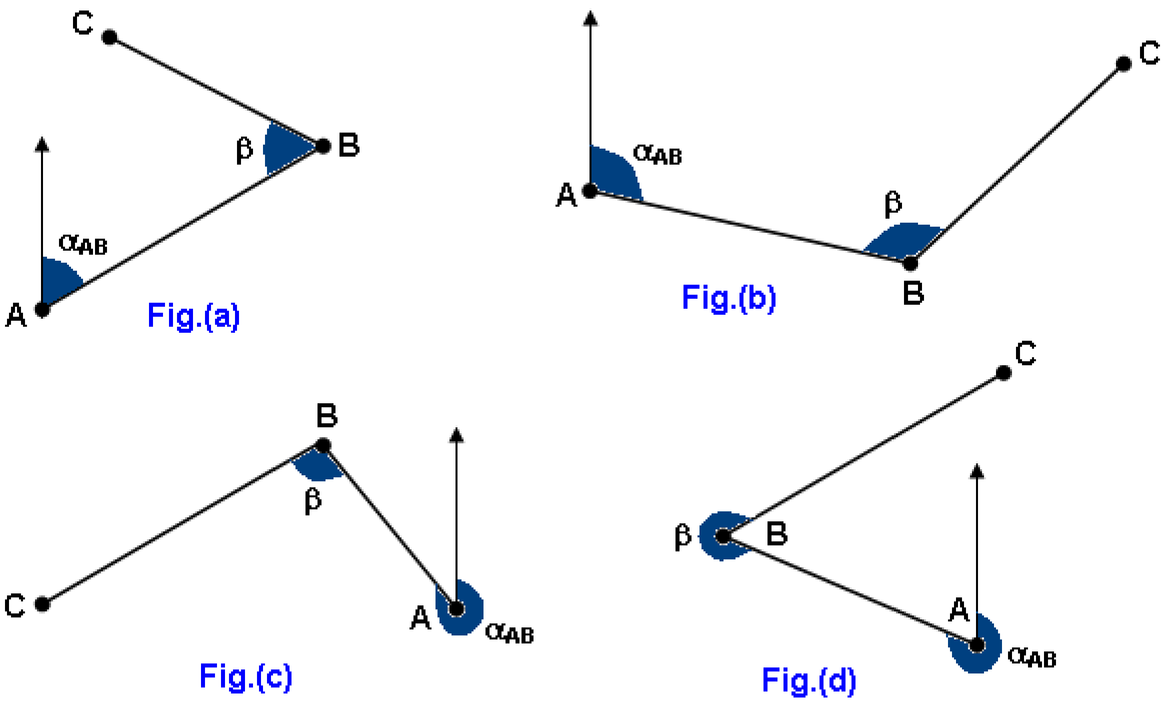


From the figure above

$$\alpha_{BC} = \alpha_{AB} + \beta - 200^g$$

To make a decision on both the amount and sign (+ or -) of this number, one should look at the result of ($\alpha_{AB} + \beta$) as follows;

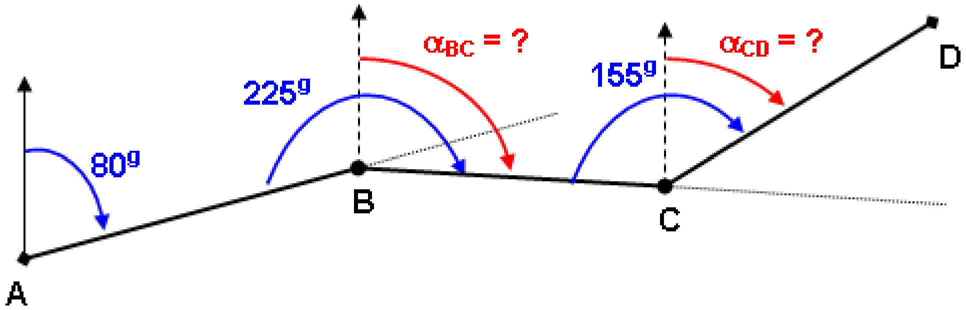
Figure	check ($\alpha_{AB} + \beta$)
a	if $0^g \leq (\alpha_{AB} + \beta) < 200^g$ then $\alpha_{BC} = \alpha_{AB} + \beta + 200^g$
b	if $200^g \leq (\alpha_{AB} + \beta) < 400^g$ then $\alpha_{BC} = \alpha_{AB} + \beta - 200^g$
c	if $400^g \leq (\alpha_{AB} + \beta) < 600^g$ then $\alpha_{BC} = \alpha_{AB} + \beta - 200^g$
d	if $600^g \leq (\alpha_{AB} + \beta) < 800^g$ then $\alpha_{BC} = \alpha_{AB} + \beta - 600^g$



Example :

GIVEN ⇒ $\alpha_{AB} = 80^g$ $\beta_1 = 225^g$ $\beta_2 = 155^g$

ASKED ⇒ α_{BC} and α_{CD}



From the figure;

at point B $\alpha_{AB} + \beta_1 = 80^g + 225^g = 305^g$ (between 200^g and 400^g)

then $\boldsymbol{\alpha_{BC}} = \alpha_{AB} + \beta_1 - 200^g = 305^g - 200^g = \mathbf{105^g}$

at point C $\alpha_{BC} + \beta_2 = 105^g + 155^g = 260^g$ (between 200^g and 400^g)

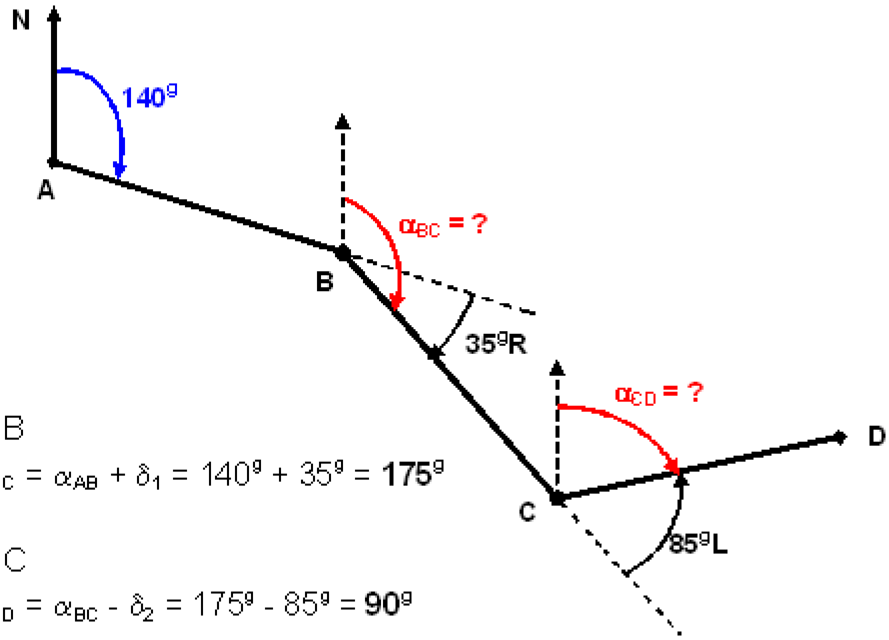
then $\boldsymbol{\alpha_{CD}} = \alpha_{BC} + \beta_2 - 200^g = 260^g - 200^g = \mathbf{60^g}$

Example :

If deflection angles (δ) are given instead of angles to the right (breaking angles) then

GIVEN ⇒ $\alpha_{AB} = 140^g$ $\delta_1 = 35^g R$ $\delta_2 = 85^g L$

ASKED ⇒ α_{BC} and α_{CD}



At point B

$\alpha_{BC} = \alpha_{AB} + \delta_1 = 140^g + 35^g = \mathbf{175^g}$

At point C

$\alpha_{CD} = \alpha_{BC} - \delta_2 = 175^g - 85^g = \mathbf{90^g}$

Basic Problem 4. The angle calculation between two directions

The coordinates of previous, present and the next stations are given. The angle between the line to the previous station and that of the next station is asked.

GIVEN $\Rightarrow N_A, E_A, N_B, E_B, N_C, E_C$ **ASKED** $\Rightarrow \beta$ (Angle to the right)

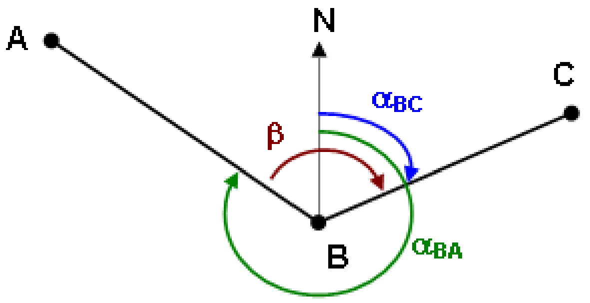


Fig.(a)

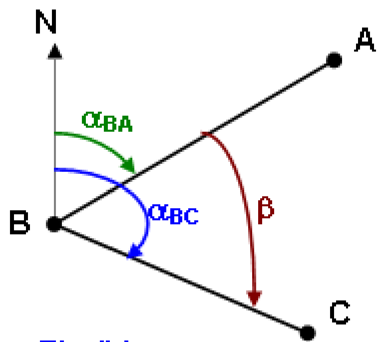


Fig.(b)

From Figure (a) above we can write;

$$\beta = 400^g - \alpha_{BA} + \alpha_{BC} = \alpha_{BC} - \alpha_{BA} + 400^g$$

and from Figure (b) we can write;

$$\beta = \alpha_{BC} - \alpha_{BA}$$

and as a general formula we can write;

$$\beta = \alpha_{BC} - \alpha_{BA} + k*400^g \quad \text{where } k=0 \text{ if } \alpha_{BC} > \alpha_{BA}$$
$$k=1 \text{ if } \alpha_{BC} < \alpha_{BA}$$

Example :

GIVEN $\Rightarrow$ **ASKED** $\Rightarrow \beta$ (Angle to the right)

Point	North (X)	East (Y)
A	4068.48 m	3808.18 m
B	3827.50 m	3821.40 m
C	4000.00 m	3500.00 m

$$\tan \alpha_{BC} = \Delta E / \Delta N = \frac{E_C - E_B}{N_C - N_B} = \frac{3500.00 - 3821.40}{4000.00 - 3827.50} = \frac{-321.4}{172.5}$$

$\tan \alpha_{BC} = -1.8632 \Rightarrow \alpha_{BC} = -68^g.6410$

If $\Delta E(-)$ and $\Delta N(+) \Rightarrow \quad \alpha_{BC} = -68^g.6410 + 400^g = \mathbf{331^g.3590}$

$$\tan \alpha_{BA} = \Delta E / \Delta N = \frac{E_A - E_B}{N_A - N_B} = \frac{3808.18 - 3821.40}{4068.48 - 3827.50} = \frac{-13.22}{240.98}$$

$\tan \alpha_{BA} = -0.0549 \Rightarrow \alpha_{BA} = -3^g.4890$

If $\Delta E(-)$ and $\Delta N(+) \Rightarrow \quad \alpha_{BA} = -3^g.4890 + 400^g = \mathbf{396^g.5110}$

if $\alpha_{BC} < \alpha_{BA}$ then k=1, therefore

$$\beta = \alpha_{BC} - \alpha_{BA} + 400^g = 331^g.3590 - 396^g.5110 + 400^g = 334^g.8480$$

AREA COMPUTATION

1. Introduction

There are a number of important reasons for determining areas of tracts of land.

- to include the acreage of a parcel of land in the deed describing the property,
- to determine the acreage of fields, lakes, and so on,
- to determine the area to be surfaced, paved, seeded, or sodded,
- (a special application) to determine end areas for earthwork volume calculation.

In plane surveying, acreage is considered to be the orthogonal projection of the area onto horizontal surface. The most common unit of area for lots is the *square foot (ft^2);* for large tracts it is the acre (1 acre=43560 ft^2). In the metric system, area is given in square meters or *hectares* (1 hectare=10000 m^2=2471 acres).

2. Methods of Measuring Area

Both field and map measurements are used to determine area. *Field measurement* methods are the more accurate, and include:

(i) – division of the tract into simple figures (triangles, rectangles, and trapezoids),
(ii) – offsets from a straight line,
(iii) – double meridian distances, and
(iv) – coordinates.

The typical system of getting field data for procedure (iii) and (iv) is traversing, with the property corners serving as hubs of a closed-polygon figure.

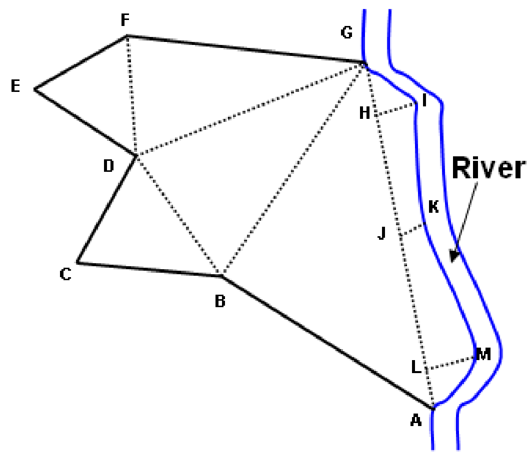
Methods of determining area from *map measurements* include:

(i) – dividing the area into triangles,
(ii) – counting coordinate squares, and
(iii) – running a planimeter over the enclosing lines.

3. Area by Division into Triangles

A tract can be divided into simple geometric figures such as the triangles and trapezoids shown in the figure.

The tract, three directions (north, west and south) are defined with straight courses and east direction is surrounded by a river- curved boundary, is divided into triangles and trapezoids.



Triangles :
ABG, BCD, BDG, DEF, DFG, GHI and ALM.
Trapezoids :
HIKJ and JKML.

The area of a triangle whose sides are known can be computed by the formula

$$Area = \sqrt{s(s-a)(s-b)(s-c)}$$ where a, b and c are sides of the triangle and

$$s=\frac{1}{2}(a+b+c)$$

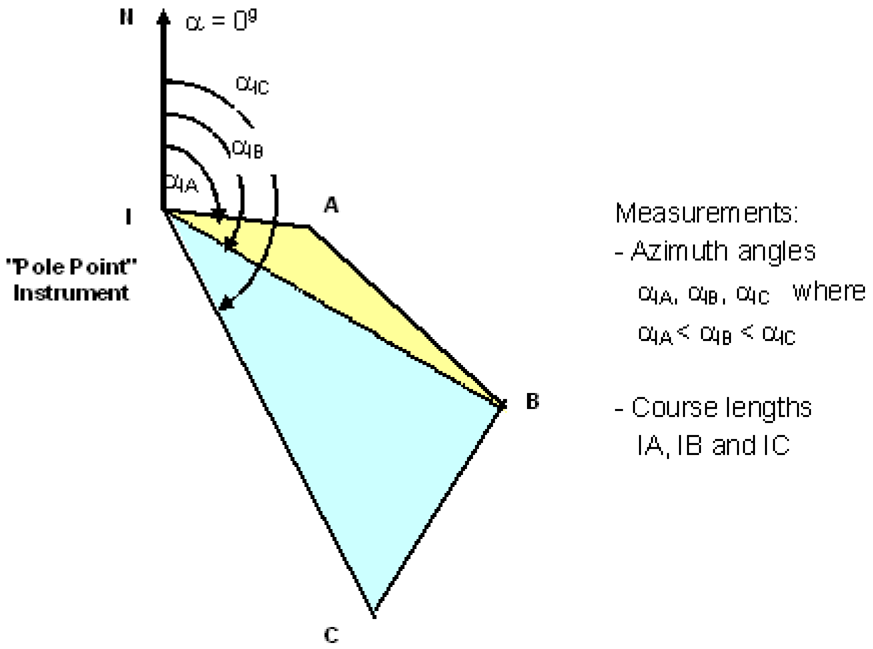
Another formula for area of a triangle is

$$Area = \frac{1}{2}(a.b.\sin C)$$ where C is the angle between sides a and b.

The area of a parcel is the sum of the areas of all geometric figures. The use of equation to compute area of a triangle will depend on available equipment. If an EDMI is available, lengths of triangle sides are readily measured and the first equation will be selected. If the 2nd equation is used, both angles and distances must be measured.

Horizontal angle measuring instrument can be positioned in two ways;

i) At a corner point of the tract : In this situation instrument (Pole point) is positioned at a suitable corner of polygon. First the instrument is directed towards the north and then the angle is adjusted to zero. Afterwards the instrument is turned in clockwise direction until it is directed to the first point and the amount of angle is read from the scale of the instrument. This is the azimuth of the first course. Also distance of the point should be measured. The procedure is reapeated for the other courses.



Once the azimuth angles and the course lengths of the tract are measured at the field, its area can be computed by computing the areas of individual triangles. Two sides and the angle betwwen these sides are known, therefore;

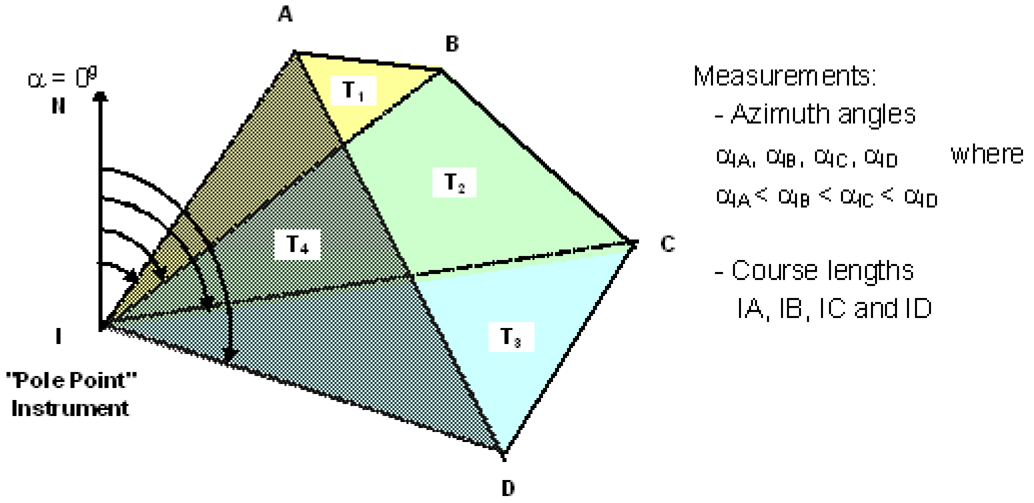
Area of tract IABC (four sided) = Area of triangle IAB + Area of triangle IBC

Area of triangle IAB = $\frac{1}{2} IA * IB * \sin(\alpha_{IB} - \alpha_{IA})$

Area of triangle IBC = $\frac{1}{2} IB * IC * \sin(\alpha_{IC} - \alpha_{IB})$

Area of tract IABC = $\frac{1}{2} IA * IB * \sin(\alpha_{IB} - \alpha_{IA}) + \frac{1}{2} IB * IC * \sin(\alpha_{IC} - \alpha_{IB})$

i) At a point out of the tract : In this situation instrument (Pole point) is positioned suitably outside the tract. The procedure is same as the first way. Again the azimuth angles and the course lengths of the tract are measured.



Once the azimuth angles and the course lengths of the tract are measured at the field, its area can be computed by computing the areas of individual triangles (T_1, T_2, T_3 and T_4). The importance has to be given on the signs of the areas. That is some triangle areas must be considered positive while the others are negative. From the figure;

Area of tract ABCD (four sided) = Area of T_1 + Area of T_2 + Area of T_3 - Area of T_4

Area of T_1 (IAB) = $\frac{1}{2} IA * IB * \sin(\alpha_{IB} - \alpha_{IA})$ *positive*

Area of T_2 (IBC) = $\frac{1}{2} IB * IC * \sin(\alpha_{IC} - \alpha_{IB})$ *positive*

Area of T_3 (ICD) = $\frac{1}{2} IC * ID * \sin(\alpha_{ID} - \alpha_{IC})$ *positive*

Area of T_4 (IAD) = $\frac{1}{2} IA * ID * \sin(\alpha_{ID} - \alpha_{IA})$ *negative*

$$\text{Area of tract ABCD} = \frac{1}{2} IA * IB * \sin(\alpha_{IB} - \alpha_{IA}) + \frac{1}{2} IB * IC * \sin(\alpha_{IC} - \alpha_{IB}) + \frac{1}{2} IC * ID * \sin(\alpha_{ID} - \alpha_{IC}) - \frac{1}{2} IA * ID * \sin(\alpha_{ID} - \alpha_{IA})$$

4. Area by Offsets from Straight Lines

Irregular tracts can be reduced to a series of trapezoids by right-angle offsets from points at regular intervals along a measured straight (reference) line, as shown in figure. Area is found by the formula

Area = $b(\frac{h_0}{2} + h_1 + h_2 + \ldots\ldots + h_{n-1} + \frac{h_n}{2})$ where b is the length of common interval between offsets and h_0, h_1, ..., h_{n-1}, h_n are the offsets.

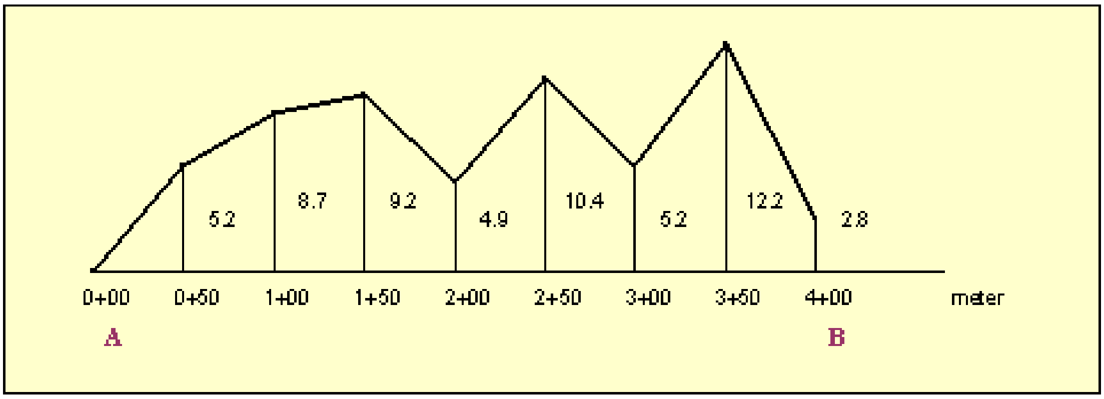


Figure 1. Area by offsets.

When we compute the area of the tract shown in Figure 1, we obtain;

Area = 50 (0 + 5.2 + 8.7 + 9.2 + 4.9 + 10.4 + 5.2 + 12.2 + 2.8/2) = 2860 m^2

For irregularly curved boundaries like that in following figure, the spacing of offsets along the reference line should be selected so that connecting straight lines on a curved boundary at offset points accyrately define the curves.

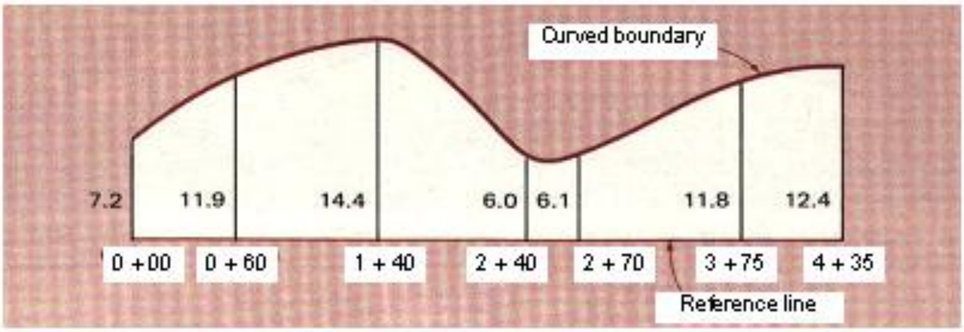


Figure 2. Area by offsets for a tract with curved boundary.

When we compute the area of the tract shown in Figure 2, that the intervals are not regular, we obtain;

$$\text{Area} = \frac{1}{2}[60(7.2+11.9)+80(11.9+14.4)+100(14.4+6.0)+30(6.0+6.1)$$
$$+105(6.1+11.8)+60(11.8+12.4)] = 4492 \text{ m}^2$$

5. Area by Double-Meridian Distance Method

The area within a closed figure can be computed by the double-meridian distance method when latitudes and departures of boundary lines (courses) are known. *The meridian distance of a traverse course is the perpendicular distance from the center point of the course to the reference meridian.* To ease the problem of signs, a reference meridian is placed through the most westerly traverse station.

In Figure 3 the meridian distances of courses AB, BC, CD, DE and EA are MM', PP', QQ', RR' and TT', respectively.

To express PP' in terms of convenient distances, MF and BG are drawn perpendicular to PP'. Then

PP' = P'F + FG + GP

= meridian distance of AB + ½ departure of AB + ½ departure of BC

Thus, the meridian distance of any course of a traverse equals the meridian distance of the preceding course, plus one-half the departure of the preceding course, plus one-half the departure of the course itself. It is simpler to employ full departures of courses. Therefore, *double meridian distances* (DMDs) equal to twice the meridian distances are used, and a single division by 2 is made at the end of the computation.

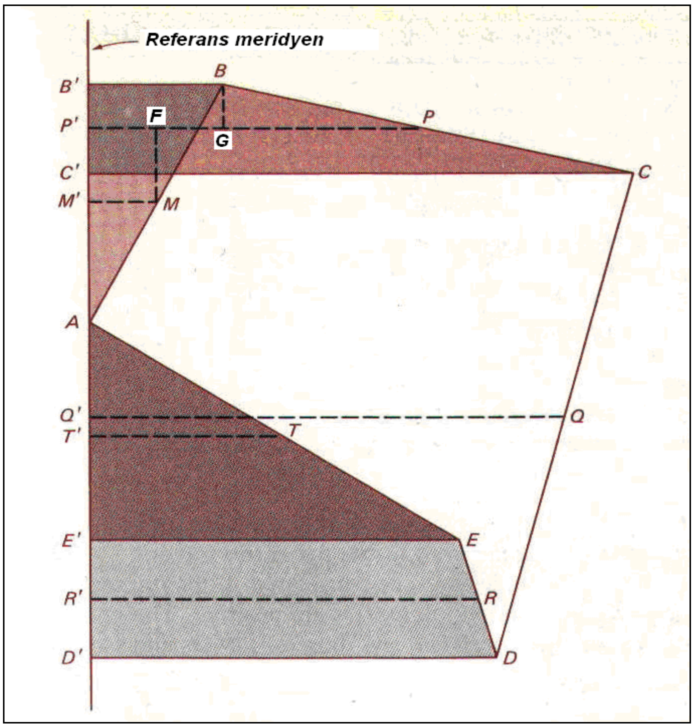


Figure 3. Meridian distances and traverse area computation
By DMD (Double Meridian Distance) method.

Based on the considerations described, the following general rule can be applied in calculating DMDs: *The DMD for any traverse course is equal to the DMD of the preceding course, plus the departure of the preceding course, plus the departure of the course itself.* Signs of the departures, east plus and west minus, must be considered. When the reference meridian is taken through the most westerly station of a closed traverse and calculations of the DMDs started with a course through that station, *the DMD of the first course is its departure.* Applying these rules, for the tarverse in Figure 3,

DMD of AB = departure of AB
DMD of BC = DMD of AB + departure of AB + departure of BC
DMD of CD = DMD of BC + departure of BC - departure of CD
DMD of DE = DMD of CD - departure of CD - departure of DE
DMD of EA = DMD of DE - departure of DE - departure of EA

A check on all computations is obtained if the DMD of the last course, after computing around the traverse, is also equal to its departure but has the opposite sign.

The area enclosed by traverse ABCDEA may be expressed in terms of trapezoid areas (shown with different color shading) as:

Area = B'BCC' + C'CDD' – (AB'B + DD'E'E + AEE') .... (1)

The area of each figure equals the meridian distance of a course times its balanced latitude. The DMD of a course multiplied by its latitude equals double the area. *Algebraic summation* of all double areas gives *twice the area* inside the entire traverse.

Signs of the products of DMDs and latitudes must be considered. If the reference line is passed through the most westerly station, all DMDs are positive. The products of DMDs and north latitudes are therefore plus, and those of DMDs and south latitudes are minus.

EXAMPLE 1:

Coordinates of traverse given in Figure 3 above are tabulated below. By using the values compute;

- DMDs of all courses,
- The area within the traverse, using the DMDs
- The area by DPDs.

	COORDINATES (m)	
Sta.	Northing	Easting
A	10,000.00	10,000.00
B	10,255.96	10,125.66
C	10,102.44	10,716.31
D	9,408.37	10,523.62
E	9,611.34	10,517.55

Departure of AB = Easting of B – Easting of A

Latitude of AB = Northing of B – Northing of A

Dep.of AB = 10125.66-10000.00 = + 125.66 (≡ E125.66) ...m

Lat.of AB = 10255.96-10000.00 = + 255.96 (≡ N255.96) ...m

Dep.of BC = 10716.31-10125.66 = + 590.65 (≡ E590.65) ...m

Lat.of BC = 10102.44-10255.96 = - 153.52 (≡ S153.52) ...m

Dep.of CD = 10523.62-10716.31 = - 192.69 (≡ W192.69) ...m

Lat.of CD = 9408.37-10102.44 = - 694.07 (≡ S694.07) ...m

Dep.of DE = 10517.55-10523.62 = - 6.07 (≡ W6.07) ...m

Lat.of DE = 9611.34-9408.37 = + 202.97 (≡ N202.97) ...m

Dep.of EA = 10000.00-10517.55 = - 517.55 (≡ W517.55) ...m

Lat.of EA = 10000.00-9611.34 = + 388.66 (≡ N388.66) ...m

TABLE 1. Computation of Double Meridian Distances

Departure of AB	= + 125.66	= ***DMD of AB***	
Departure of AB	= + 125.66		
Departure of BC	= + 590.65		
	+ 841.97	= ***DMD of BC***	
Departure of BC	= + 590.65		
Departure of CD	= - 192.69		
	+ 1239.93	= ***DMD of CD***	
Departure of CD	= - 192.69		
Departure of DE	= - 6.07		
	+ 1041.17	= ***DMD of DE***	
Departure of DE	= - 6.07		
Departure of EA	= - 517.55		
	+ 517.55	= ***DMD of EA***	Check

TABLE 2. Computation of area by DMDs and DPDs.

				Double Areas			Double Areas	
Course	Latitude	Departure	DMD	+	-	DPD	+	-
AB	N 255.96	E 125.66	+ 125.66	32,164		+ 255.96	32,164	
BC	S 153.52	E 590.65	+ 841.97		129,259	+ 358.40	211,689	
CD	S 694.07	W 192.69	+ 1239.93		860,598	- 489.19	94,262	
DE	N 202.97	W 6.07	+ 1041.17	211,326		- 980.29	5,950	
EA	N 388.66	W 517.55	+ 517.55	201,151		- 388.66	201,151	
Total	0.00	0.00		444,641	989,857		545,216	0.00

ΣLatitude = ΣNorth - ΣSouth = 847.59 – 847.59 = 0.00
ΣDeparture = ΣEast - ΣWest = 716.31 – 716.31 = 0.00
Double area of trapezoid = DMD of course * Latitude of course
(*note*: N latitude is "+", S latitude is "-" then double areas are "+" or "-" accordingly)

DPD of AB = Latitude of AB = + 255.96
DPD of BC = DPD of AB + Lat.of AB + Lat.of BC = + 255.96 + 255.96 - 153.52 = + 358.40
DPD of CD = DPD of BC + Lat.of BC + Lat.of CD = + 358.40 - 153.52 - 694.07 = - 489.19
DPD of DE = DPD of CD + Lat.of CD + Lat.of DE = - 489.19 - 694.07 + 202.97 = - 980.29
DPD of EA = DPD of DE + Lat.of DE + Lat.of EA = - 980.29 + 202.97 + 388.66 = - 388.66

Computations for area are generally arranged as in Table 2. Sums of positive and negative double areas are obtained, and the absolute value of the smaller subtracted from that of the larger. The result is divided by 2 to get the area of the tarverse.

Double area = Absolute(-989857) - Absolute (444641) = 545216 m^2
Area of traverse ABCDEA = 545216 ÷ 2 = 272608 m^2 = 27.2608 hectares

If the total of minus double areas is larger than the plus value, it signifies only that DMDs were computed by going around the traverse in a clockwise direction. If the route had been from A through E, D, C and B and back to A, the total plus double area would have been the greater.

As a check, the area can be computed by *double parallel distances (DPDs). The DPD for any traverse course is equal to the DPD of the preceding course, plus the latitude ot the preceding course, plus the latitude of the course itself.* In the DPD method, double areas of trapezoids are computed by multiplying the DPD of each course by its departure. Algebraic summation of all trapezoids areas, and division of the absolute value by 2, gives the area. The last three columns in Table 2 show the area computation by DPDs for the traverse.

In modern surveying and engineering offices, area calculations are seldom done by hand; rather, they are programmed for electronic computer solution. If an area is computed by hand, however, it should be checked by using different methods, or by two persons who employ the same system.

6. Area by Coordinates

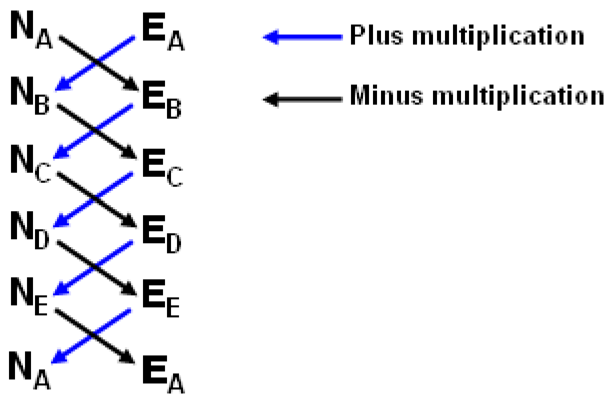
The procedure of computing areas by coordinates also can be readily developed by reference to Figure 3. Since double the meridian distances M'M and P'P in coordinate terms are $(E_B + E_A)$, and $(E_C + E_B)$, and the latitudes of lines AB and BC are $(N_B - N_A)$ and $(N_C - N_B)$, respectively, then based on the summation of trapezoidal areas, the following equation for double area can be written:

$$2 * (\text{area}) = (E_C + E_B)(N_C - N_B) + (E_D + E_C)(N_D - N_C) + (E_E + E_D)(N_E - N_D) + (E_A + E_E)(N_A - N_E) + (E_B + E_A)(N_B - N_A) \qquad \text{....... (2)}$$

Equation (2) is equivalent to the trapezoid-area formula [Eq.(1)], except that the first two products are negative because $(N_C - N_B)$ and $(N_D - N_C)$ are negative, and the last three products are positive. Thus, the double area resulting from Eq.(2) is negative but of no consequence because the absolute value is adopted. Expanding and rewriting, Eq.(2) is simplified to

$$2 * (\text{area}) = E_A N_B + E_B N_C + E_C N_D + E_D N_E + E_E N_A - E_B N_A - E_C N_B - E_D N_C - E_E N_D - E_A N_E \qquad \text{....... (3)}$$

Equation (3) can be reduced to an easily remembered form by listing the E and N coordinates of each point in succession in two columns as shown in Eq.(4), *with coordinates of the starting point repeated at the end.* The products noted by diagonal arrows are ascertained, with solid arrows considered minus and dashed ones plus. The algebraic summation of all products is computed and its absolute value divided by 2 to get area.



.... (4)

The procedure indicated in Eq.(4) is applicable to calculating any size traverse. If the computations are being made by calculator, some surveyors assume E=0 for the most westerly point and N=0 for the most southerly station. Magnitudes of coordinates and products are thereby reduced and the amount of work lessened, since four products will equal zero (if $E_A = 0$ and $N_D = 0$).

Another handy formula, easily derived similar to Eq.(2), for calculating areas within closed polygon traverses is

$$2 * (\text{area}) = E_A(N_E - N_B) + E_B(N_A - N_C) + E_C(N_B - N_D) + E_D(N_C - N_E) + E_E(N_D - N_A) \qquad \text{....... (5)}$$

Equations 2 through 5 are all readily programmed for solution by electronic computer.

EXAMPLE 2:

Figure 4 illustrates the same traverse used for Example 1, but coordinates of points reduced so that E_A =0.00 (A is the most westerly station) and N_D =0.00 (D is the most southerly station). Thus, all coordinates are positive. Compute the traverse area by the coordinate method.

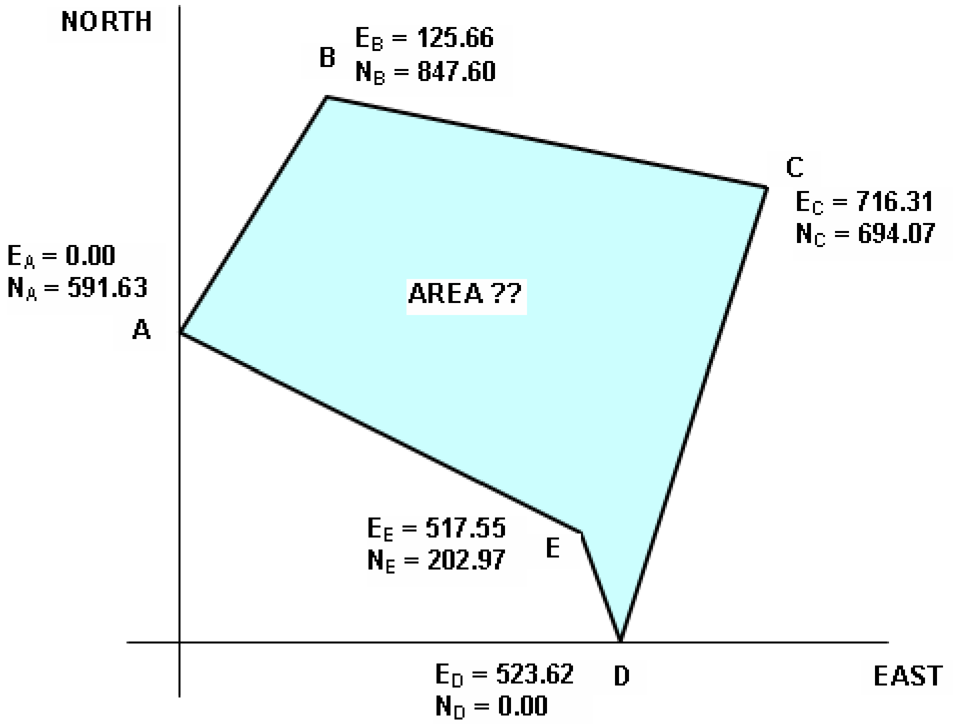


Figure 4. Area by coordinates.

TABLE 3. Computation of area by coordinates.

	ORIGINAL COORDINATES (m)		REDUCED COORDINATES (m)		DOUBLE AREA (m^2)	
Pnt.	Northing	Easting	Northing	Easting	Minus	Plus
A	10,000.00	10,000.00	591.63	0.00		
B	10,255.96	10,125.66	847.59	125.66	74,344	0
C	10,102.44	10,716.31	694.07	716.31	607,137	87,216
D	9,408.37	10,523.62	0.00	523.62	363,429	0
E	9,611.34	10,517.55	202.97	517.55	0	106,279
A	10,000.00	10,000.00	591.63	0.00	0	306,198
				TOTAL	- 1,044,910	+ 499,693

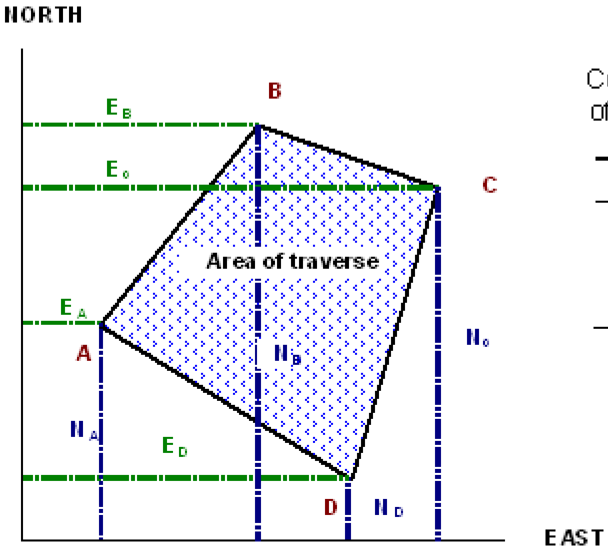
By using Eq.(3)

$$2 *(\text{area}) = E_AN_B + E_BN_C + E_CN_D + E_DN_E + E_EN_A - E_BN_A - E_CN_B - E_DN_C - E_EN_D - E_AN_E$$
$$= E_BN_C + E_DN_E + E_EN_A - E_BN_A - E_CN_B - E_DN_C$$
$$= \text{Absolute } (1044910) – \text{Absolute } (499693) = 545217 \text{ m}^2$$

Area of traverse ABCDEA = 545217 ÷ 2 = 272608 m^2 = 27.2608 hectares

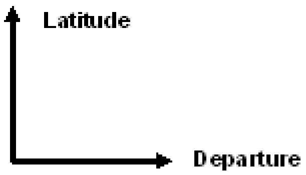
EXAMPLE 3:

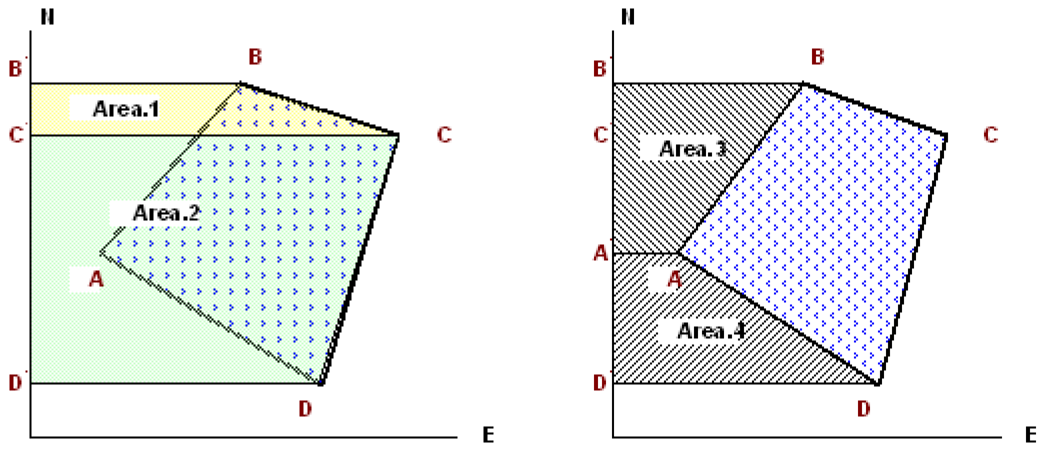
The area of the traverse given in the following figure is computed by using the areas of trapezoids.



Coordinate values of the corner points of traverse (N_i and E_i)

Point	North (m)	East (m)
A	336	123
B	648	370
C	552	653
D	96	512



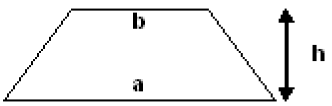


Definitions:

Area 1 : Area of quadrilateral defined by the corner points B'BCC'
Area 2 : Area of quadrilateral defined by the corner points C'CDD'
Area 3 : Area of quadrilateral defined by the corner points B'BAA'
Area 4 : Area of quadrilateral defined by the corner points A'ADD'

Area of Traverse (ABCD) = Area.1 + Area.2 – (Area.3 + Area.4)

Be careful : All quadrilaterals in these figures are trapezoid (trapezium-Br.E.) in shape. Therefore the areas will be calculated accordingly. Thus:



Area of trapezoid = h [(a+b)/2]

Now we can compute the area of traverse ABCD by using the trapezoids (By double area method) :

2*Area.1 = $(E_C+E_B)(N_B-N_C)$ = (653+370)(648-552) = **98208 m^2 (Positive)**
2*Area.2 = $(E_C+E_D)(N_C-N_D)$ = (653+512)(552-96) = **531240 m^2 (Positive)**
2*Area.3 = $(E_A+E_B)(N_B-N_A)$ = (123+370)(648-336) = **153816 m^2 (Negative)**
2*Area.4 = $(E_A+E_D)(N_A-N_D)$ = (123+512)(336-96) = **152400 m^2 (Negative)**

If **Area of Traverse (ABCD) = Area.1 + Area.2 – (Area.3 + Area.4)**

Then **2*(ABCD) = 98208 + 531240 – (153816 + 152400) = 323232 m^2**

Area of Traverse (ABCD) = 323232 ÷2 = 161616 m^2

If we apply the method used in Table 3, we can similarly fill the following table (Point A is most westerly and Point D is most southerly)

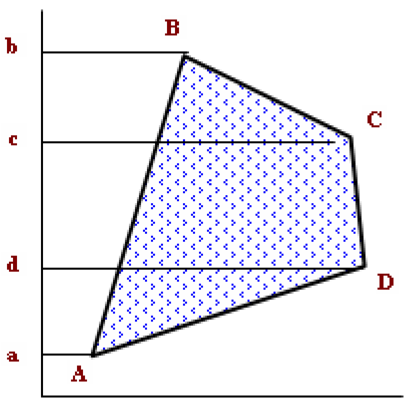
TABLE 4. Computation of area by coordinates (Example 3).

	ORIGINAL COORDINATES (m)		BALANCED COORDINATES (m)		DOUBLE AREA (m^2)	
Pnt.	Northing	Easting	Northing	Easting	Minus	Plus
A	336	123	240	0		
B	648	370	552	247	59,280	0
C	552	653	456	530	292,560	112,632
D	96	512	0	389	177,384	0
A	336	123	240	0	0	93,360
				TOTAL	- 529,224	+ 205,992

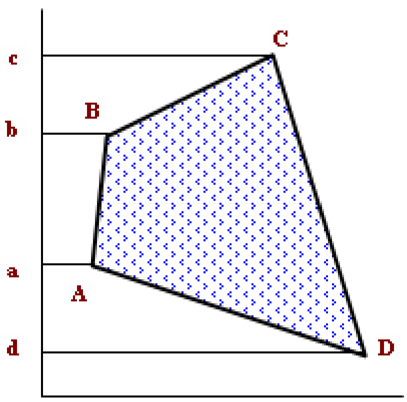
2*Area = Absolute (529224) – Absolute (205992) = 323232 m^2

Area of traverse ABCDA = 323232 ÷ 2 = 161616 m^2 = 16.1616 hectares

Be careful : To decide on the sign of the area of a trapezoid is important. Therefore the figure must be drawn carefully according to given coordinate values. For example, if the above traverse was as follows then the effect of trapezoids would be such;



This traverse 3 Positive and 1 Negative area
+ areas : bBCc, cCDd, dDAa
- area : bBAa
Area of ABCD = (bBCc+cCDd+dDAa) - bBAa



This traverse 3 Negative and 1 Positive areas
+ area : cCDd
- areas : cCBb, bBAa, aADd
Area of ABCD = cCDd - (cCBb+bBAa+aADd)

7. Area of Parcels with Circular Boundaries

The area of a tract that has a circular curve for one boundary, as in Figure 5, can be found by dividing the figure into two parts: polygon ABCDEGFA and sector EGF. The radius R=EG=FG and either central angle θ =EGF, or length EF must be known to calculate sector area EGF=$\pi R^2(\theta^o/360^o)$. To obtain the tract's total area, the sector area is added to area ABCDEGFA, found by the DMD or coordinate method.

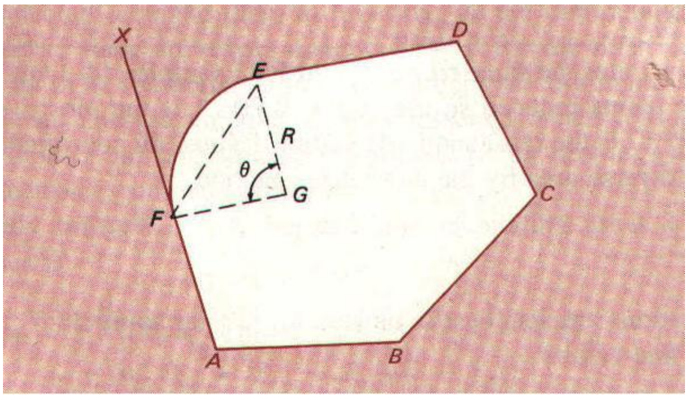


Figure 5. Area with circular curve as part of boundary.

8. Area from a Map by Triangles

A traverse can be plotted to scale, divided into triangles, the sides measured, areas of triangles is determined. This method is not accurate as computations using field measurements due to inaccuracies in scaling, but the procedure is useful for checking.

9. Area by Coordinate Squares

To find the area within a plotted traverse by coordinate squares, the map is marked off in squares of unit area. The number of complete unit squares in the traverse is counted and sum of all partial units estimated. Traverse area is the product of the total number of unit squares times the area covered by each.

10. Measurement of Area by Planimeter

A planimeter mechanically integrates area and records the answer on a drum and disk as a tracing point is moved over an outline of the figure to be measured.

There are mechanical and electronic planimeters. The major parts of the mechanical type are a scale bar, graduated drum and disk, vernier, tracing point and guard, and anchor arm, weight and point.

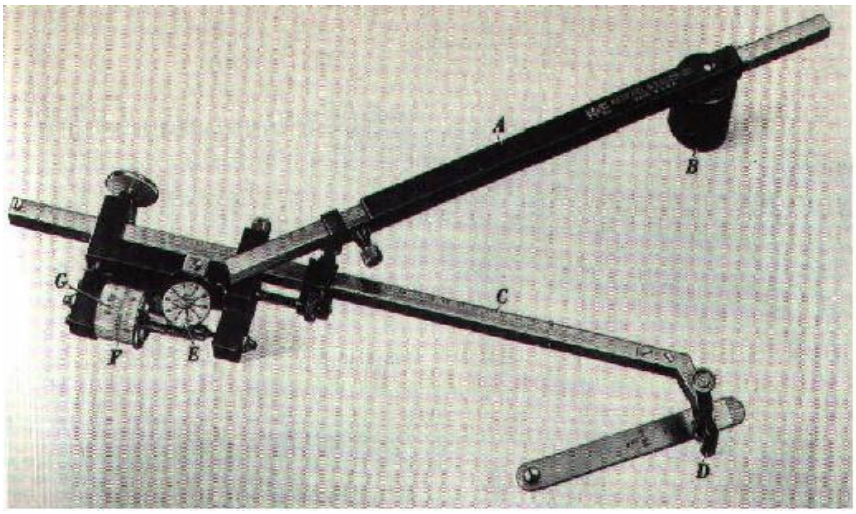
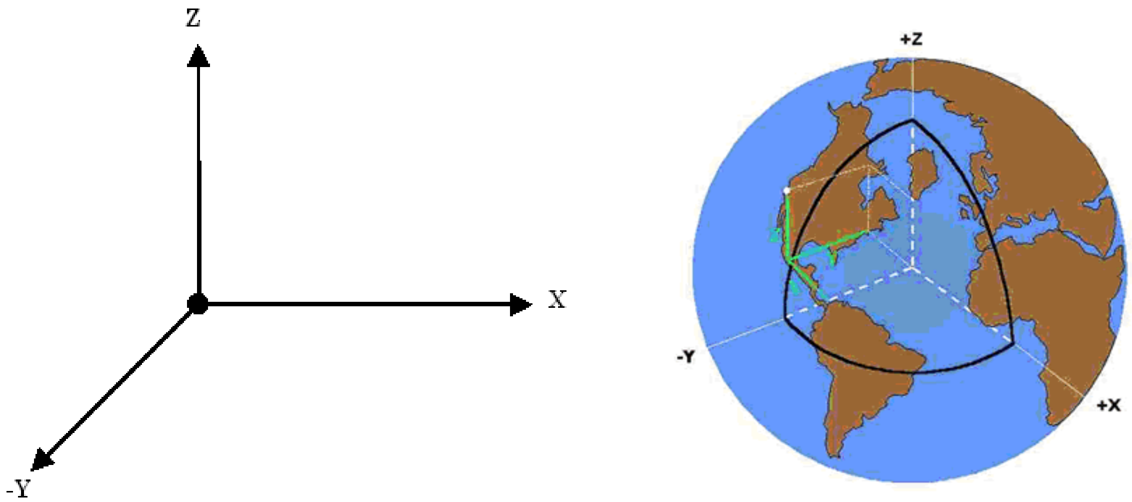


Figure 6. Mechanical planimeter: A,anchor arm; B,anchor point; C,scale bar; D,tracing point; E,disk; F,drum; G,vernier.

LEVELING (ELEVATION MEASUREMENT)

A point in space is fixed by the cartesian coordinates (X,Y,Z) or by its horizontal (plane) coordinates (x,y) and vertical coordinate (H), i.e., elevation.

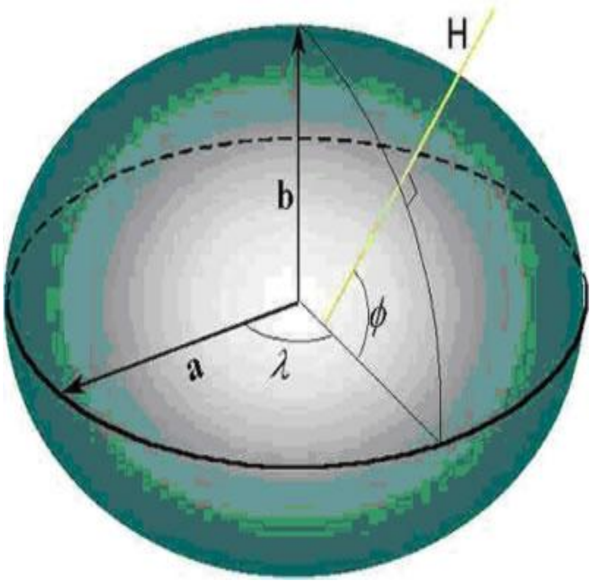
1. 3D Reference System
If the earth is a 3 dimensional body, then we should have a 3 dimensional coordinate system right? Adding a 3rd axis (z), which intersects the plane formed by the x and y axis at a right angle, and locating our point of original (the point where all three axes intersect) at the centre of mass of the Earth it would make our system 'geocentric' and would allow us to assigned a 3D, Cartesian coordinate value (x, y and z) to any point on, above, or below the surface of the earth.



3D Reference System

Once we've located our point of origin at the earth's centre of mass, we need to line up, or orient, the axes of our reference system.
If we line up the Z-axis parallel to the Earth's axis of rotation (from North pole to South) and if we line up the x/y axes so that positive X-axis intersects the Greenwich meridian (longitude 0°) we've completely defined (with the exception of a few really technical details) our 3D spatial reference system.
Officially, this system is known as the Conventional Terrestrial Reference System (CTRS), sometimes just called the Conventional Terrestrial System (CTS).

2. Geodetic Datum
Well, we talked about the earth's curved surface, so now let's add it (the earth's surface) to our 3D coordinate system.
The shape of the earth is quite complicated. The study of it's shape, size and gravity field is an entire field of study (**Geodesy**) and is covered in another module, but, to recap, from various studies, we know that the easiest way to represent the shape of the earth mathematically is by using an ellipsoid.
An ellipsoid is simply an ellipse rotated about its minor axis (b).
If we locate the centre of our ellipsoid to coincide with the centre of our 3D Cartesian coordinate system (also the centre of the mass of the earth) we have now defined what's referred to as a 'geodetic datum'.



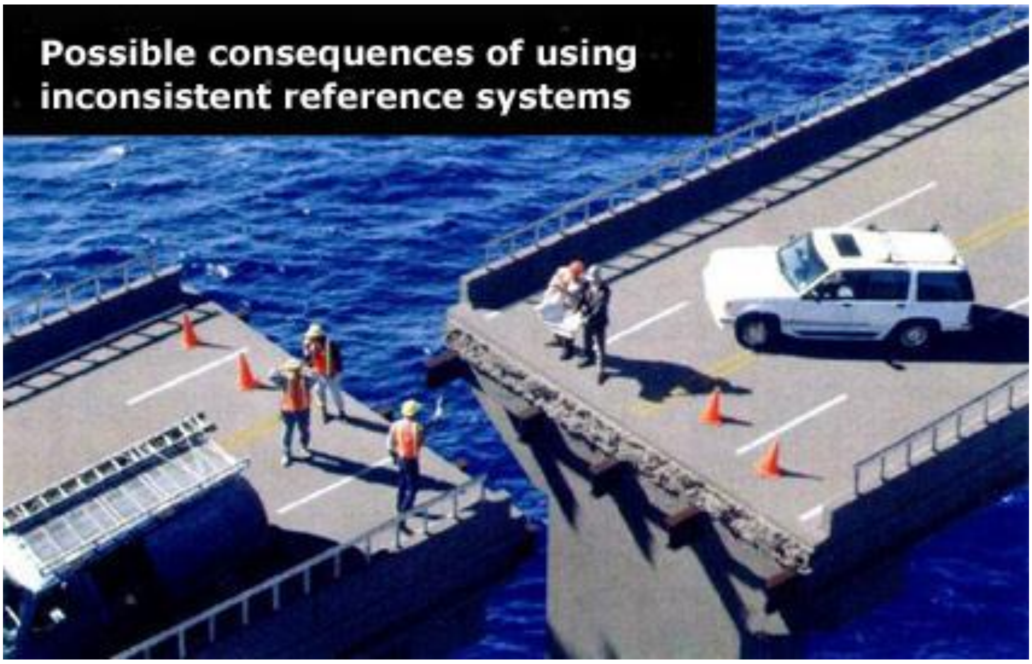
Very specialized procedures are used to accurately position a few carefully selected points. The comprehensive and highly precise observations are only made at a few points, as this type of work is very time consuming making it extremely expensive to establish and maintain points.

These points become the fiducial or datum points to which all other surveys may be connected and referred, making them the very backbone of our spatial reference frame.

We achieve our goal of fully realizing the spatial reference system once other surveys are connected and **control networks** are established so that we can extend the known positions so that all of our positioning needs can be met by referring surveys to this single common spatial reference system.

Now, we can make use of all our theory and hard work. Anyone that needs to position or locate features for mapping, engineering, construction, navigation, and GIS development will be able to do so easily and even share or exchange data and it will all be related to the same spatial reference system.

Provided of course that we all use the same spatial reference system.



3. Survey Classifications

As mentioned, there are many different types of surveys. Generally speaking, surveys will either take into account the true shape of the earth (Geodetic surveys) or treat the earth as a flat surface (Plane surveys). Additionally, surveys are conducted for the purpose of positioning features on the ground (Horizontal surveys), determining the elevation or heights of features (Vertical surveys) or a combination of both. These classifications are described in the next two sections.

3.1. Horizontal vs. Vertical Surveys

Survey measurements can further be divided into two more categories, horizontal and vertical.

Horizontal surveys are used when we are interested in where objects are geographically located (latitude and longitude) and not in their elevation.

Vertical surveys determine the heights, or elevations of objects. These two types of surveys may be, and often are, combined.

Horizontal and vertical surveys may be further classified as relative, or absolute.

Relative surveys relate one object 'relative' to another. For example, the height of your desk may be about 1 metre above the floor. That is a relative height as, relative to the floor, your desk is 1 metre higher.

Now, if we wanted to know the height of your desk above the vertical reference surface used in mapping, called 'mean sea level', the height or elevation above mean sea level would be a precise or "absolute" value.

3.2. Elevations

The height of a feature above mean sea level is called an elevation. We use the elevation of features in our everyday lives for example to build roads and in all types of construction. Also, since water flows downhill, we need to know where the water will flow, for instance when draining water away from your house or for sewer systems and water mains. Well, you get the idea.

Elevations are measured by one of two methods, either by using a survey technique called "**levelling**" or by using a **Global Positioning System (GPS)** receiver.

3.3. Vertical Survey Methods

The purpose of a vertical survey is to establish an elevation relative to a reference surface.

The most common reference surface - called "**mean sea level**" - is the one used for our national topographic maps.

Small scale, local surveys, such as **construction surveys**, may not need a height above mean sea level, only the differences in heights between points.

The most common methods to determine the difference in heights or to establish elevations are differential and trigonometric levelling.

In order to establish elevation of points, or to determine differences in elevation between points, and to control grades in construction surveys we use leveling.

There are three common types of leveling:

1. Differential leveling
2. Stadia (Three-wire) leveling
3. Trigonometric leveling

4. Differential Leveling (Spirit leveling, Geometric leveling)

This is the most common method of determining elevation differences. Let us consider 2 ground points A and B) as given in Figure 1. Suppose the height of A is known and the height of B is asked, then we follow the following procedure: After setting up the level instrument halfway between A and B, rod (or staff) readings are taken on both points. The rod reading at A is called backsight reading (BS) and that of at B is called foresight reading (FS). Then we can compute H_B as follows

$$\Delta H = H_B - H_A = BS - FS$$
$$H_B = H_A + BS - FS$$

In understanding differential leveling the following set of definitions are important to know.

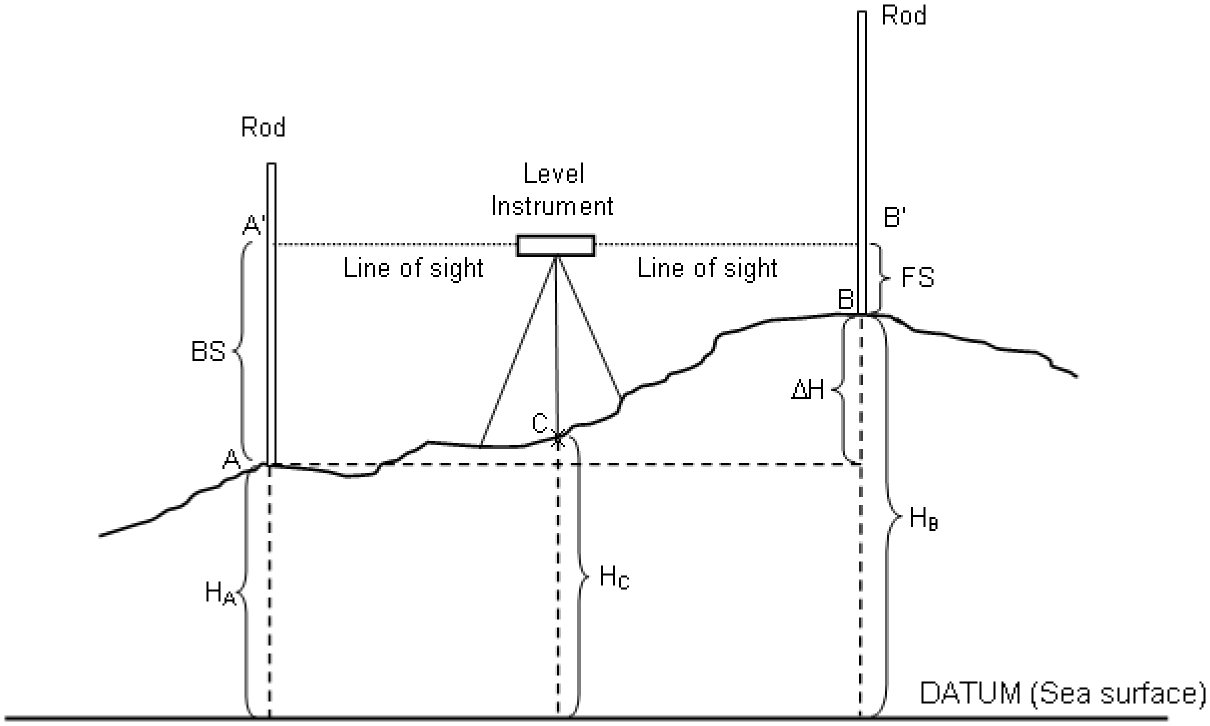


Figure 1. Differential Leveling

A backsight (BS) : The vertical distance between the line of sight and a point of known or assumed elevation, or the reading taken on such a point.

A foresight (FS) : The vertical distance between the line of sight and a point whose elevation is desired, or the reading taken on such a point.

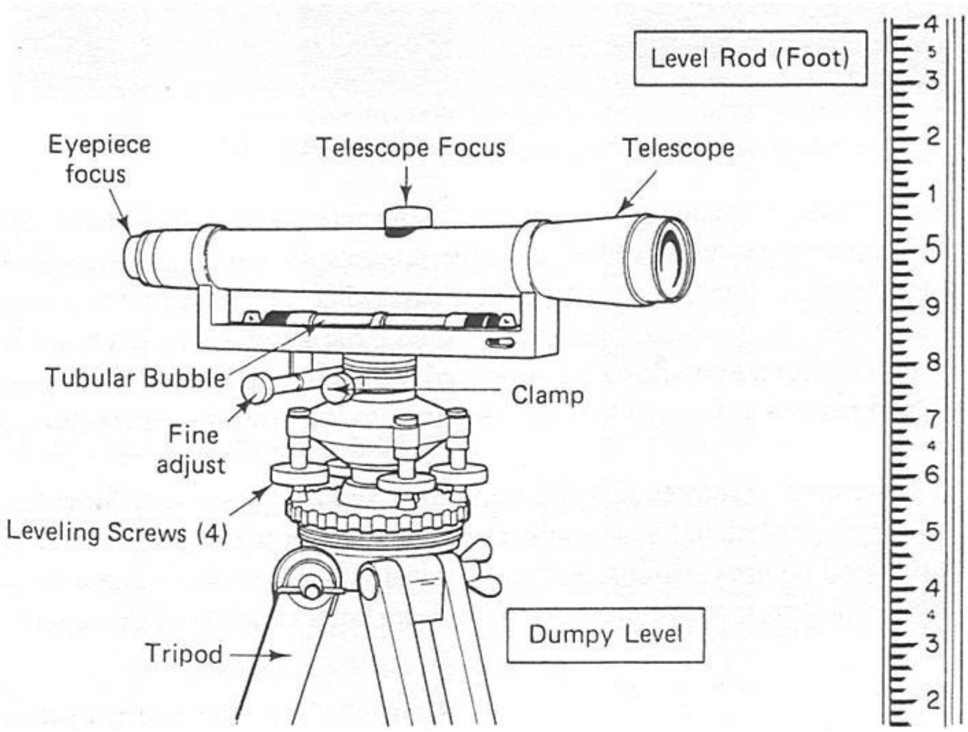
A benchmark (BM) : A reference point with known elevation. Benchmarks are established using precise leveling techniques and instrumentation. Benchmarks are bronze disks or plugs set into (usually) vertical waII faces. It is important that the benchmark be placed in a structure that has substantial footings (at least below minimum frost-depth penetration). Benchmark elevations and locations are published by federal, state or provincial, and municipal agencies and are available to surveyors for a nominal fee.

A temporary benchmark (*TBM*) : is a semipermanent point of known elevation. TBMs can be flange bolts on fire hydrants, nails in the roots of trees, top corners of concrete culvert headwalls, and so on.

A turning point (TP) : A point whose elevation has been determined by a FS and whose purpose has been served after a BS has been taken on it. We use turning points if the distance between known and unknown point is too large to have readings on rods.

An intermediate foresight (IS) or (IFS) : A secondary point having connection only to a benchmark (BM) or to a turning point (TP). It is a rod reading taken at any other point wherethe elevation is required.

Height of instrument (*HI*) : is the elevation of the line of sight through the level (i.e., elevation of BM + BS = HI). .



To accomplish differential leveling follow the following steps:

1) Set up the level instrument half way (approximately) between BS and FS points.
2) Focus the eyepiece on the reticle and perform the following steps for BS measurement.
3) Point the telescope towards the rod
4) Focus the objective.
5) Read and record the reading.

After the last reading the rodman moves to the turning point (for FS readings), then the operator goes through steps (3) thru (5) as above. The rodman now stays at the TP and the operator moves the instrument for a new set up on the other side of TP in the direction of levels. Then the steps above are repeated.

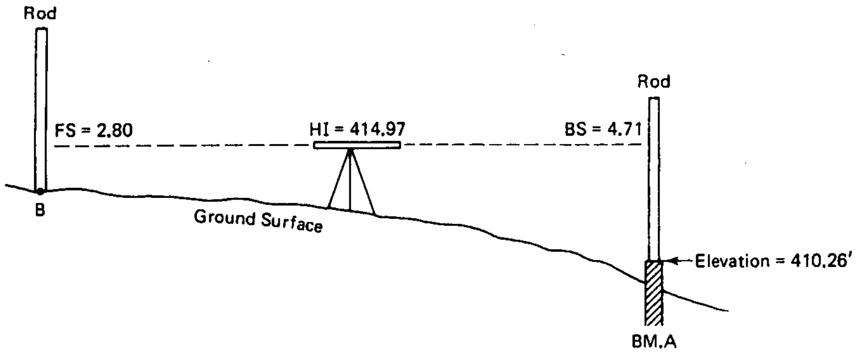


FIGURE 3.16 Leveling procedure: one setup.

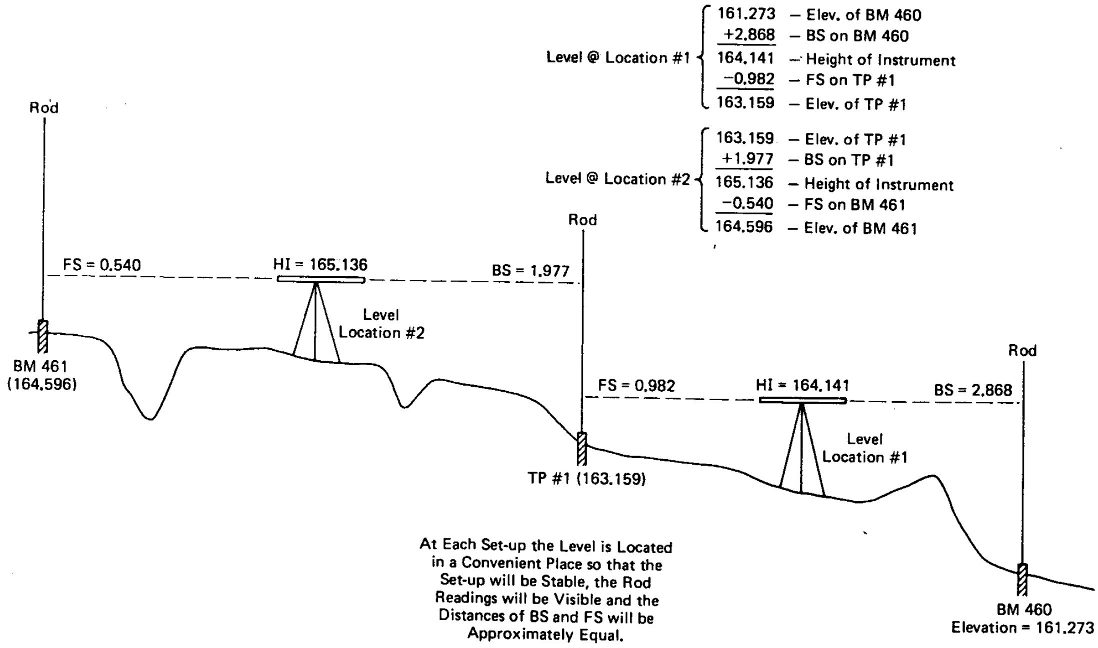
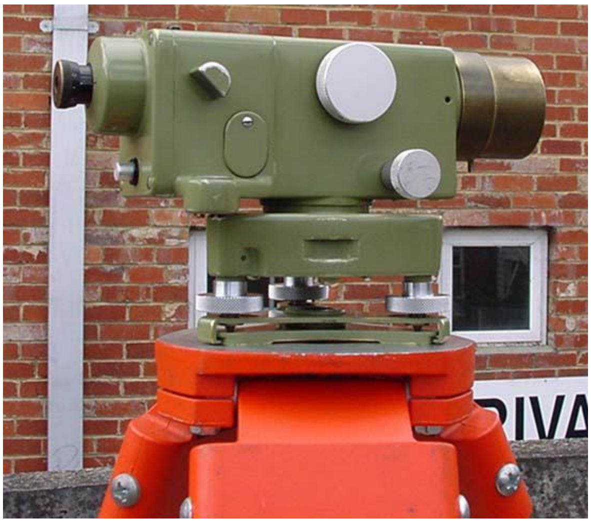


FIGURE 3.17 Leveling procedure: more than one setup.



Precise Level – 1970's

Automatic Level

Precise Level and invar staff

Automatic Level and Staff

Automatic Level

Let us give an example for this general differential leveling. Suppose BM_1 and BM_2 are given with their elevations of the turning points T_1 and T_2, and the intermediate points I_1 and I_2, which are located between T_1 and T_2 as shown in Figure 2.

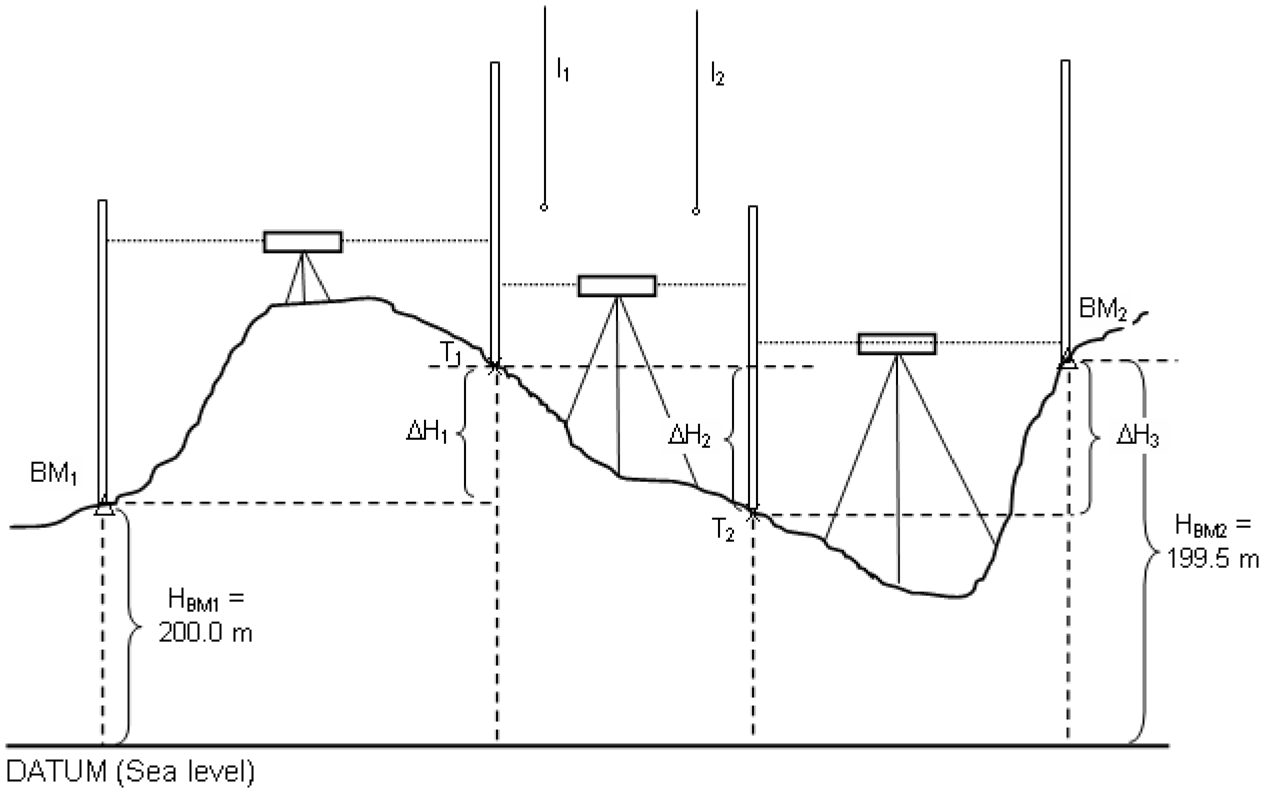


Figure 2. General differential leveling.

Horizontal distances:	Elevations:
BM_1-T_1 = 81.0 m	H_{BM1} = 200 m
T_1-T_2 = 62.0 m	
T_2-BM_2 = 99.0 m	H_{BM2} = 199.5 m

We are given the following elevations

H_{BM1} = 200 m, H_{BM2} = 199.5 m

We read the following BS and FS measurements

	BM_1	T_1	T_2	BM_2	I_1	I_2
BS	1.222	0.111	0.550	--	--	--
FS	--	0.221	2.151	0.111	1.000	0.955

From the figıre above we can write

$H_{T1} = H_{BM1} + \Delta H_1 = H_{BM1} + BS_{BM1} – FS_{T1}$

$H_{T2} = H_{T1} - \Delta H_2 = H_{T1} + BS_{T1} – FS_{T2} = H_{BM1} + BS_{BM1} + BS_{T1} - FS_{T1} – FS_{T2}$

$H_{BM2} = H_{T2} + \Delta H_3 = H_{T2} + BS_{T2} – FS_{BM2}$

$= H_{BM1} + (BS_{BM1} + BS_{T1} + BS_{T2}) – (FS_{T1} + FS_{T2} + FS_{BM2})$

$\mathbf{H_{BM2} = H_{BM1} + \Sigma BS – \Sigma FS}$

From the last equation we can derive the following general formula

CLOSING ELEVATION = STARTING ELEVATION + (ΣBS – ΣFS)

However due to imperfect human being and instruments and physical conditions above equation would not be satisfied, but rather there would be a small error of closure, say V_H, expressed as follows

V_H = (Closing elevation – Starting elevation) - (ΣBS – ΣFS)

This error is distributed to every height difference, i.e., ΔH_i, proportional to the distances between those points.

Let us consider Figure 2 and the related observations and perform the computations given in the following table.

Station	Distance (m)	BS	IS	FS	ΔH=BS-FS	Elevation	Adjusted elevation
BM_1		1.222	--	--	--	200.00	200.00
	81.0				(+0.033)		
T_1		0.111	--	0.221	1.001	201.001	201.034
I_1		--	1.000	--	-0.889	200.112	200.145
	62.0						
I_2		--	0.955	--	-0.844	200.157	200.190
					(+0.026)		
T_2		0.550	--	2.151	-2.040	198.961	199.020
	99.0				(+0.041)		
BM_2		--	--	0.111	0.439	199.400	199.500
	242.0	1.883		2.483	-0.600		-0.500
	(ΣS)	(ΣBS)		(ΣFS)	⇧ ($\Delta H_i = \Sigma BS – \Sigma FS$)		(H_{BM2}-H_{BM1})

We see from the table above that the actual elevation difference is

$\Delta H = H_{BM2} - H_{BM1} = -0.500$

But from the observations we get

$\Delta H' = \Sigma H_i = \Sigma BS – \Sigma FS = -0.600$

Hence from equation we find the closure error

$V_H = (H_{BM2} - H_{BM1}) - (\Sigma BS - \Sigma FS) = -0.500 - (-0.600) = 0.100$ m

The V_H error above is distributed proportional to the length of each side corresponding to every ΔH_i, i.e.

$V_{H1} = (S_1 / \Sigma S)* V_H = (81/242)*0.100 = 0.033$ m

$V_{H2} = (S_2 / \Sigma S)* V_H = (62/242)*0.100 = 0.026$ m

$V_{H3} = (S_3 / \Sigma S)* V_H = (99/242)*0.100 = 0.041$ m

Then these errors are added to the corresponding elevation difference, i.e.

$(\Delta H_{adjusted})_i = (\Delta H_{measured})_i + V_{Hi}$

Tus the adjustment of differentiel leveling is completed.

The magnitude of teh maximum acceptable error closure V_H changes by depending on the purpose. For practical field works we use

$V_{max} = \pm 20*(\Sigma S)^{1/2}$ .... mm where ΣS is the total distance (in km) covered between the benchmarks. In order to minimize this error we should read the measurement rod accurately, level the instrument properly, etc.

5. Stadia (Three- wire) Leveling

Leveling can be perfonned by using the stadia cross hairs found on most levels (see Figure 3). Each backsight (BS) and foresight (FS) is recorded by reading the stadia hairs in addition to the horizontal cross hair. The three readings thus obtained are averaged to obtain the desired value.

The stadia hairs (wires) are positioned an equal distance above and below the main cross hair and are spaced to give 1.00 ft (m) of interval for each 100 ft (m) of horizontal distance that the rod is away from the level.

The recording of three readings at each sighting enables the survey or to perform a relatively precise survey while using ordinary levels. Readings to the closest thousandth of a foot (mm) are estimated and recorded. The leveling rod used for this type of work should be calibrated to ensure its integrity.

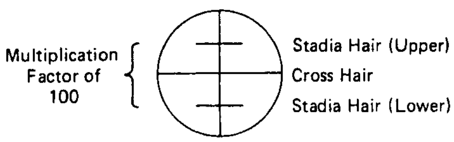


FIGURE 3 Reticle cross hairs. i.e. 100 X Stadia Hair Interval = Distance

B.M. LEVELING – 3 WIRE
B.M. #17 to B.M. 201
(RETURN RUN ON P.48)

JONES – NOTES
SMITH – X
BROWN – ROD
GREEN – ROD

Job ROD #19, INST. #L.33 8 °C CLOUDY
Date MAR 3 2000 Page 47

STA.	B.S.	DIST.	F.S.	DIST.	ELEV.	DESCRIPTION
BM #17					186.2830	BRONZE PLATE SET IN WALL --- ETC.
	0.825		1.775			
	0.725	10.0	1.673	10.2	+ 0.7253	
	0.626	9.9	1.572	10.1	187.0083	
	2.176	19.9	5.020	20.3	− 1.6733	
	$\boxed{+0.7253}$		$\boxed{-1.6733}$			
T.P. #1					185.3350	N. LUG TOP FLANGE FIRE HYD. N/S
	0.698		1.750			MAIN ST. OPP. CIVIC #181.
	0.571	12.7	1.620	13.0	+ 0.5710	
	0.444	12.7	1.490	13.0	185.9060	
	1.713	25.4	4.860	26.0	− 1.6200	
	$\boxed{+0.5710}$		$\boxed{-1.6200}$			
T.P. #2					184.2860	N. LUG TOP FLANGE FIRE HYD. N/S
	1.199		2.509			MAIN ST. OPP. CIVIC #163.
	1.118	8.1	2.427	8.2	+ 1.1180	
	1.037	8.1	2.343	8.4	185.4040	
	3.354	16.2	7.279	16.6	− 2.4263	
	$\boxed{+1.1180}$		$\boxed{-2.4263}$			
BM. 201					182.9777	BRONZE PLATE SET IN ESTLY FACE
						OF RETAINING WALL --- ETC.
Σ	+2.4143	61.5m	−5.7196	62.9m		

ARITHMETIC CHECK: 186.283 + 2.4143 − 5.7196 = 182.9777 ✓

FIGURE 3.31 Survey notes for three-wire leveling.

6. Trigonometric Leveling

The difference in elevation between A and B (see Figure 3.32) can be determined if the vertical angle (α) or Zenith angle ($90°-\alpha$) and the slope distance (S) are measured.

$V = S \sin \alpha$ or $V = S \cos (90°-\alpha)$

Elevation at ins. + hi $\pm$ V - RR = Elevation at rod

Note. The hi in this case is not the elevation of the line of sight as it is in differential leveling, but instead hi here refers to the distance from point A up to the optical center of the theodolite measured with a steel tape or rod.

Trigonometric leveling can be used where it is not feasible to use a level. The slope distance can be determined using a steel tape, stadia methods, or EDM methods. The angle is normally measured by use of a theodolite, but for lower-order surveys a clinometer could be used.

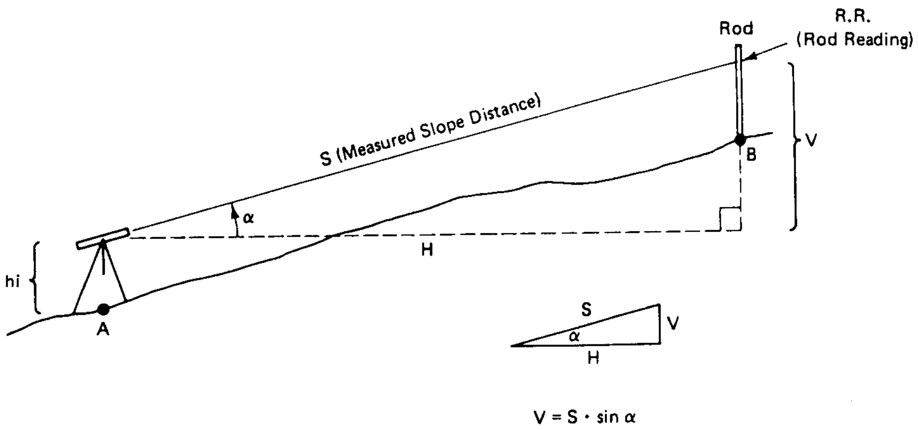


FIGURE 3.32 Trigonometric leveling.

EXAMPLE: See Figure 3.33

$V = S \sin \alpha = 82.18 \sin 30°22' = 41.54$ ft

Elev. at Ins. + hi $\pm$ V - RR = Elev. at rod

$= 361.29 + 4.72 - 41.54 - 4.00 = 320.47$

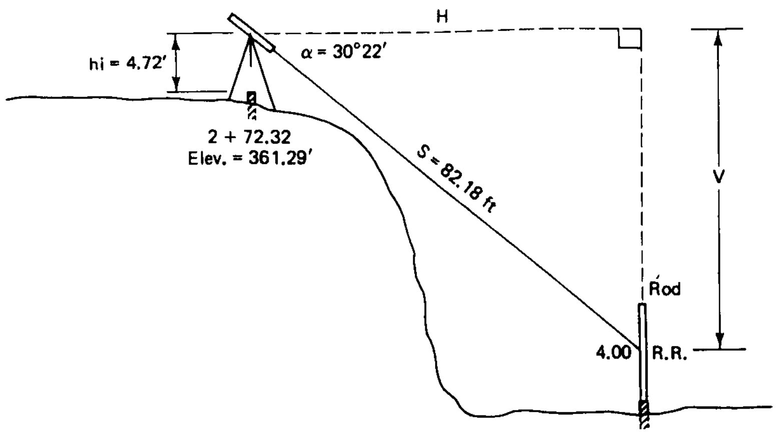


FIGURE 3.33 Example of trigonometric leveling (see Section 3.23).

7. Level Loop Adjustments

If a level survey were performed in order to establish new benchmarks, it would be desirable to suitably proportion any acceptable error throughout the length of the survey. Since the error tolerances are based on the distances surveyed, adjustments to the level loop will be based on the relevant distances, or on the number of instrument setups, whieh is a faqor direetly relııted to the distanee surveyed (see Section A.8).

EXAMPLE :

A level circuit is shown in Figure. The survey, needed for local engineering projects, commenced at BM 20, the elevations of new benchmarks 201, 202, and 203

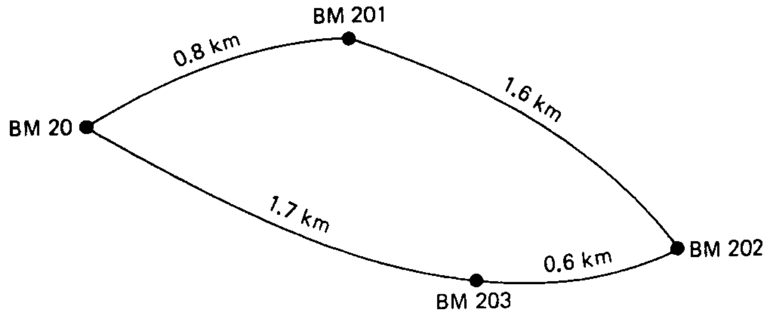


FIGURE 3.34 Level loop. Total Distance Around Loop is 4.7 km.

were determined, and then the level survey was looped baek to BM 20, the point of commencement (the survey could have terminated at any established BM).

The error in the survey was found to be -0.015 m over a total distance of 4.7 km, in this case an acceptable error. It only remains for this acceptable error to be suitably distributed over the length of the survey. The error is proportioned according to the fraction of cumulative distance over total distance, as in the following table.

BM	Loop distance, cumulative (km)	Elevation	Correction, $\frac{\text{Cumulative distance}}{\text{Total distance}} \times (-E)$	Adjusted elevation
20		186.273 (fixed)		186.273
201	0.8	184.242	$+0.8/4.7 \times 0.015 = +0.003 =$	184.245
202	2.4	182.297	$+2.4/4.7 \times 0.015 = +0.008 =$	182.305
203	3.0	184.227	$+3.0/4.7 \times 0.015 = +0.010 =$	184.237
20	4.7	186.258	$+4.7/4.7 \times 0.015 = +0.015 =$	186.273

$E = 186.258 - 186.273 = -0.015$ m

9. Suggestions for Rod Work

1. The rod should be properly extended and clamped; care should be taken to ensure that the bottom of the sole plate does not become encrusted with mud and the like, which could result in mistaken readings. If a rod target is being used, care is exercised to ensure that it is properly positioned and that it cannot slip.
2. The rod should be hel d plumb for all rod readings. Either a rod level will be used, or the rod will be gently waved to and from the instrument so that the lowest (indicating a plumb rod) reading can be determined. This practice is particularly important for all backsights and foresights.

3. The surveyor should ensure that all points used as tuming points are suitable (i.e., describable, identifiable, and capable of having the elevation determined to the closest 0.001 m).

4. Care should be taken to ensure that the rod is held in precisely the same position for the backsight as it was for the foresight for all tuming points.

5. If the rod İs being held near to, but not on, a required location, the face of the rod should be turned away from the instrument so that the instrument operator cannot take a mistaken reading. This type of mistaken reading usually occurs when the distance between the rod and the instrument is too far to allow for voice communication and sometimes even for good visual contact.

9. Suggestions for Instrument Work

1. Use a straight-leg (nonadjustable) tripod, if possible.

2. Tripod legs should be tightened so that when one leg is extended horizontally it falls slowly back to the ground under its own weight.

3. The instrnment can be comfortably carried in a vertical position resting on one shoulder; if tree branches or other obstructions threaten the safety of the instrument, it should be cradled under one arm with the instrument forward, where it can be seen.

4. When setting up the instrument, gently force the legs into the ground by applying weight on the tripod shoe spurs. On rigid surfaces (e.g., concrete) the tripod legs should be spread farther apart to increase stability.

5. When the tripod is to be set up on a sidehill, two legs should be placed downhill and the third leg placed uphill. The instrument can be set up roughly leveled by careful manipulation of the third uphill leg.

6. The location of the level setup should be wisely chosen with respect to the ability to "see" the maximum number of rod locations, particularly BS and FS locations.

7. Prior to taking rod readings, the cross hair should be sharply focused; it helps to point the instrument toward a light-colored background.

8. When the instrument operator observes apparent movement of the cross hairs on the rod (parallax), he or she should carefully check the cross-hair focus adjustment and the objective focus adjustment.

9. The instrument operator should consistently read the rod at either the top or the bottom of the cross hair.

10. Never move the level before a foresight is taken; otherwise, all work done from that HI will have to be repeated.

11. Check to ensure that the level bubble remains centered, or that the compensating device (in automatic levels) is operating.

12. Rod readings (and the line of sight) should be kept at least 0.5 m above the ground surface to help minimize refraction errors when perforrning a precise level survey.

FULL DIP, STRIKE AND APPARENT DIP

Definitions :

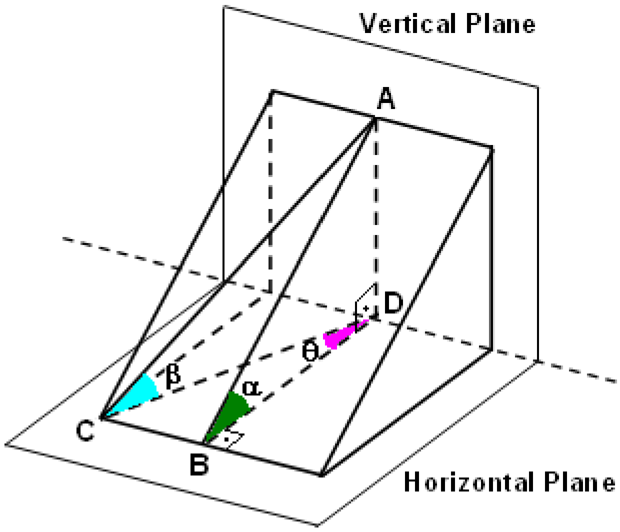
Dip : The dip of a bed is the angle of inclination of the bed below the horizontal plane.

Strike : The strike of a bed is the direction of any horizontal line in the bed. The strike and direction of full (true) dip are always at right angles to each other.

Apparent dip : If AB is direction of full dip, then the dip along any line such as AC is an apparent dip.

Let the full (true) dip α, and the apparent dip β;

If $BD \perp AD$, $CD \perp AD$ and $BC \perp BD$
Then,
ADB, BCD and ACD are right triangles. Therefore;



$\tan \alpha = AD/BD \Rightarrow AD = BD.\tan \alpha$
$\tan \beta = AD/CD \Rightarrow AD = CD.\tan \beta$

$BD.\tan \alpha = CD.\tan \beta$
$\tan \beta = (BD/CD)\tan \alpha$

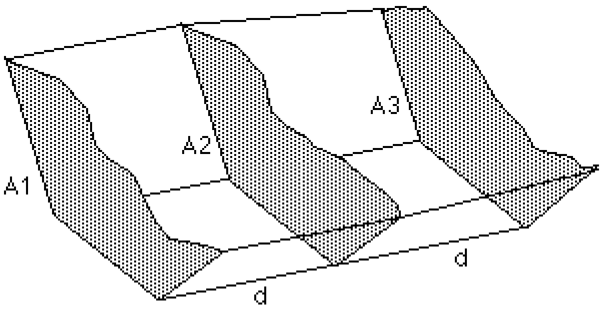
From BCD triangle,
$BD/CD = \cos \theta$
then, $\tan \beta = \tan \alpha.\cos \theta$ (tan apparent dip=tan full dip multiplied by the cos of the angle between the apparent and the full dips.)

Example : The amount and direction of full dip of a seam is $26^o33'$ due south. What is the slope of a drift driven in the seam on a bearing $S40^oW$?

$$\tan \beta = \tan \alpha.\cos \theta = \tan 26^o33'.\cos 40^o = 0.3828 \Rightarrow \beta = 20^o57'$$

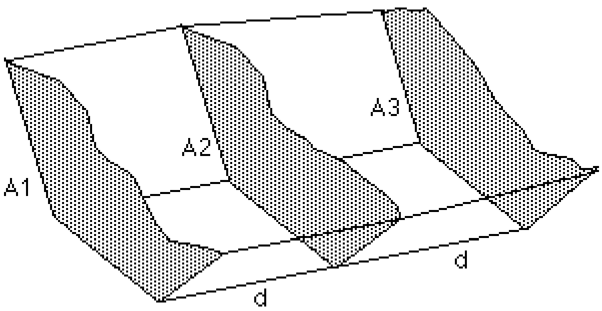
DETERMINATION OF VOLUME OF A MASS

Volumes: End Area Method



Volume = d*[First area + last area + 2S(all remaining areas)] / 2

This is the simplest method for determining volumes from cross sections. It closely follows the theory developed for the determination of areas, in this case instead of **offsets** at constant separation (resulting in areas) there are **areas** at constant separations (resulting in volumes).



In the figure it can be assumed that areas A_1, A_2 and A_3 have been determined. Therefore, if A_1 is the left end area, A_2 the right end area and d the separation between sections, the first volume is,

$$V_1 = d(A_1+A_2)/2$$

Now consider several successive cross sections situated at equal distances, d, along a fixed direction. Then,

$$V = d(A_1+A_2)/2 + d(A_2+A_3)/2 + d(A_3+A_4)/2 + \dots\dots\dots +d(A_{n-1}+A_n)/2$$

$$V = d[A_1+2A_2+2A_3+2A_4+ \dots\dots\dots +2A_{n-1}+A_n]/2$$

V = d*[First area + last area + 2 Σ(all remaining areas)] / 2

This **End Area** formula may be applied to any number of cross sections equally spaced along a straight line.

Volumes From Contours

The method used is simply the end area method or the prismoidal formula, the cross section being replaced by the areas contained within successive contours (see below). The distance between sections, or in this case contours, simply becomes the contour interval. As the contained areas are usually quite irregular they are normally determined by planimeter or by computers and digitising software. The process is laborious and is becoming less popular with the advent of digital data and digital maps.

$v_1=c(a_1+a_2)/2 \qquad v_2=c(a_2+a_3)/2 \qquad v_3=c(a_3+a_4)/2$

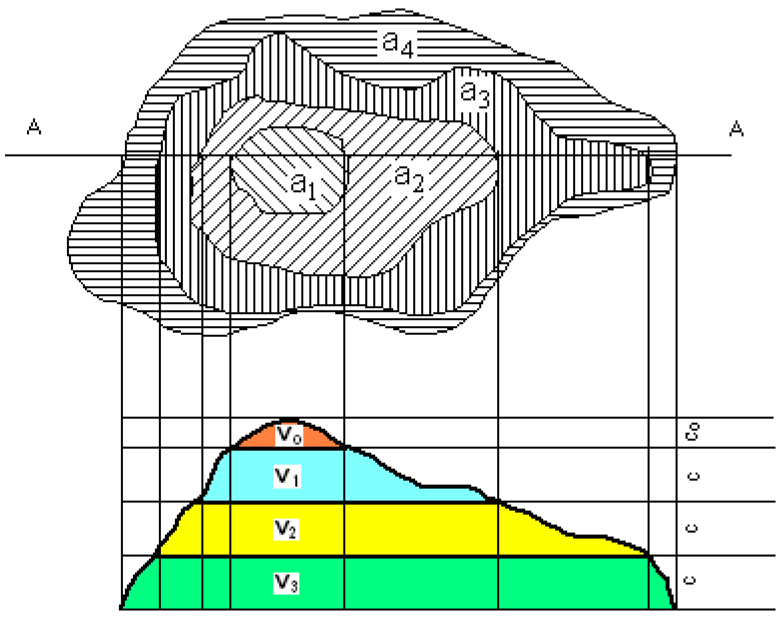
The volume of the hill shown below is (for constant intervals),

$$V=v_1+v_2+v_3 = V = \frac{c}{2}\left[(a_1 + 2a_2 + 2a_3 + a_4)\right]$$ where c is the contour interval.

The formula ignores the volume of the apex of the hill top; this could be included by making $a_o = 0$, then;

$v_o = c_o.(a_1/2)$ or by the additional use of some other suitable method.

Total volume = $v_o+v_1+v_2+v_3$



Section AA