

MAD 256 – SURVEYING

**SOLVED PROBLEMS
SORTED ACCORDING
TO THE TOPICS**

PART 1 : ERROR DETERMINATION

1.1. Side lengths of a rectangular parcel on a map is measured 5.14 and 12.85 cm with an accurate ruler. The scale of the map is 1/5000. The surveyor has to lay out this parcel onto the ground. If he uses an erroneous tape with a length of 20.01 meter, determine the lengths (sides of rectangle) to be laid out with this meter to obtain the correct sizes.

Solution :

Field lengths of rectangle are;

$$5.14 \times 5000 = 25700 \text{ cm} = 257 \text{ meter}$$

$$12.85 \times 5000 = 64250 \text{ cm} = 642.5 \text{ meter}$$

Error in one length = $20.0 - 20.01 = -1 \text{ cm}$ then Correction in one length = $-\text{error} = 1 \text{ cm}$

For short side;

$$\# \text{ of measures} = 257 / 20.01 = 12.8435$$

$$\text{Correction} = (1) \times 12.8435 = 12.8435 \text{ cm}$$

We must find the distance that, when corrected by +12.8435 cm, will give 257 m. So;

$$\text{Length to be laid out} = 25700 - 12.8435 = 25687.16 \text{ cm} = \mathbf{256.8716 \text{ meter}}$$

For long side;

$$\# \text{ of measures} = 642.5 / 20.01 = 32.1089$$

$$\text{Correction} = (1) \times 32.1089 = 32.1089 \text{ cm}$$

We must find the distance that, when corrected by +32.1089 cm, will give 642.5 m. So;

$$\text{Length to be laid out} = 64250 - 32.1089 = 64217.89 \text{ cm} = \mathbf{642.1789 \text{ meter}}$$

1.2. A horizontal distance was recorded as 342.28 m with a 30-m tape that was 30.006 m under standard conditions. If the temperature was 12°C during measurement, determine the corrected distance? [use $C_t = 0.0000116(T - 20)L$]

Solution :

$$\text{Correction per tape length} = -\text{error} = -(30 - 30.006) = +0.006 \text{ m}$$

$$\text{Tape correction} = (342.28 / 30) \times 0.006 = 0.068 \text{ m}$$

$$\text{Temperature correction } C_t = 0.0000116(12 - 20)342.28 = -0.032 \text{ m}$$

$$\text{Corrected distance} = 342.28 + 0.068 - 0.032 = 342.316 \text{ m}$$

1.3. The interior angles of a triangle are measured as 75°12'16", 38°29'45" and 66°17'38". Determine the error in this measurement and then correct the angles.

Solution :

$$\text{Total measured angle} = 75^\circ 12' 16'' + 38^\circ 29' 45'' + 66^\circ 17' 38'' = 179^\circ 59' 39''$$

$$\text{Error} = \text{measured} - \text{true} = 179^\circ 59' 39'' - 180^\circ = 179^\circ 59' 39'' - 179^\circ 59' 60'' = -21''$$

$$\text{Correction for each} = -\text{error} / 3 = -(-21'' / 3) = +7''$$

Corrected angles;

$$75^\circ 12' 16'' + 7'' = 75^\circ 12' 23''$$

$$38^\circ 29' 45'' + 7'' = 38^\circ 29' 52''$$

$$66^\circ 17' 38'' + 7'' = 66^\circ 17' 45''$$

$$\begin{array}{r} \text{sum} \quad 179^\circ 58' 120'' = 179^\circ 60' = 180^\circ \end{array}$$

PART 2 : ANGLE CONVERSION, SCALE AND SLOPE

2.1. Convert the following angles to the corresponding units.

- a) $144^\circ = ?$ grad b) $5\pi/8 = ?$ degree c) $230^\circ = ?$ radian d) $170^\circ = ?$ radian

Solution :

$$\text{In general, } \frac{D}{360} = \frac{G}{400} = \frac{M}{6400} = \frac{R}{2\pi}$$

- a) $144^\circ = ?$ grad $144 \cdot 400 / 360 = 160$ grad
 b) $5\pi/8 = ?$ degree $(5\pi/8) \cdot 360 / 2\pi = 112.5$ degree
 c) $230^\circ = ?$ radian $230 \cdot 2\pi / 360 = 1.28\pi$ or 4.014 radian
 d) $170^\circ = ?$ radian $170 \cdot 2\pi / 400 = 0.85\pi$ or 2.67 radian

- 2.2. a) Field distance between two points is 1460 m. Determine map scale if map distance between these points is measured 29,2 mm.
 b) The area of a rectangle is determined 1450 mm² on a map which has a scale of 1/5000. Determine the true area of this rectangular land in m².

Solution :

- a) $a/A = 1/M$ $M = 1460000 \text{ mm} / 29,2 \text{ mm} = 50000$ Scale = 1:50000
 b) $f/F = 1/M^2$ $F = 1450 \text{ mm}^2 \cdot 5000^2 = 1450 \cdot 25 \cdot 10^6 = 36250 \cdot 10^6 \text{ mm}^2 = 36250 \text{ m}^2$.

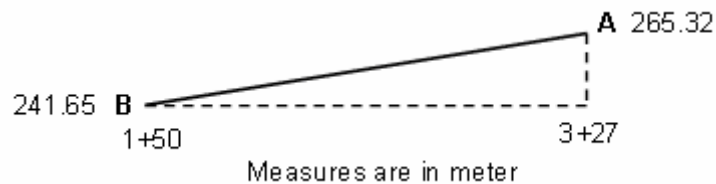
- 2.3. The side lengths of a rectangle on the plan are 5 cm and 10 cm. It represents a property with an area of 500 m². Determine the scale of the plan.

Solution :

$$f/F = 1/M^2 \rightarrow M = (F/f)^{1/2} = [5000000 \text{ cm}^2 / (5 \cdot 10 \text{ cm}^2)]^{1/2} = [100000]^{1/2} = 316$$

then, scale = 1:316

- 2.4. If the elevations of two points are known as well as the horizontal distance between them, determine the slope from point A to B.

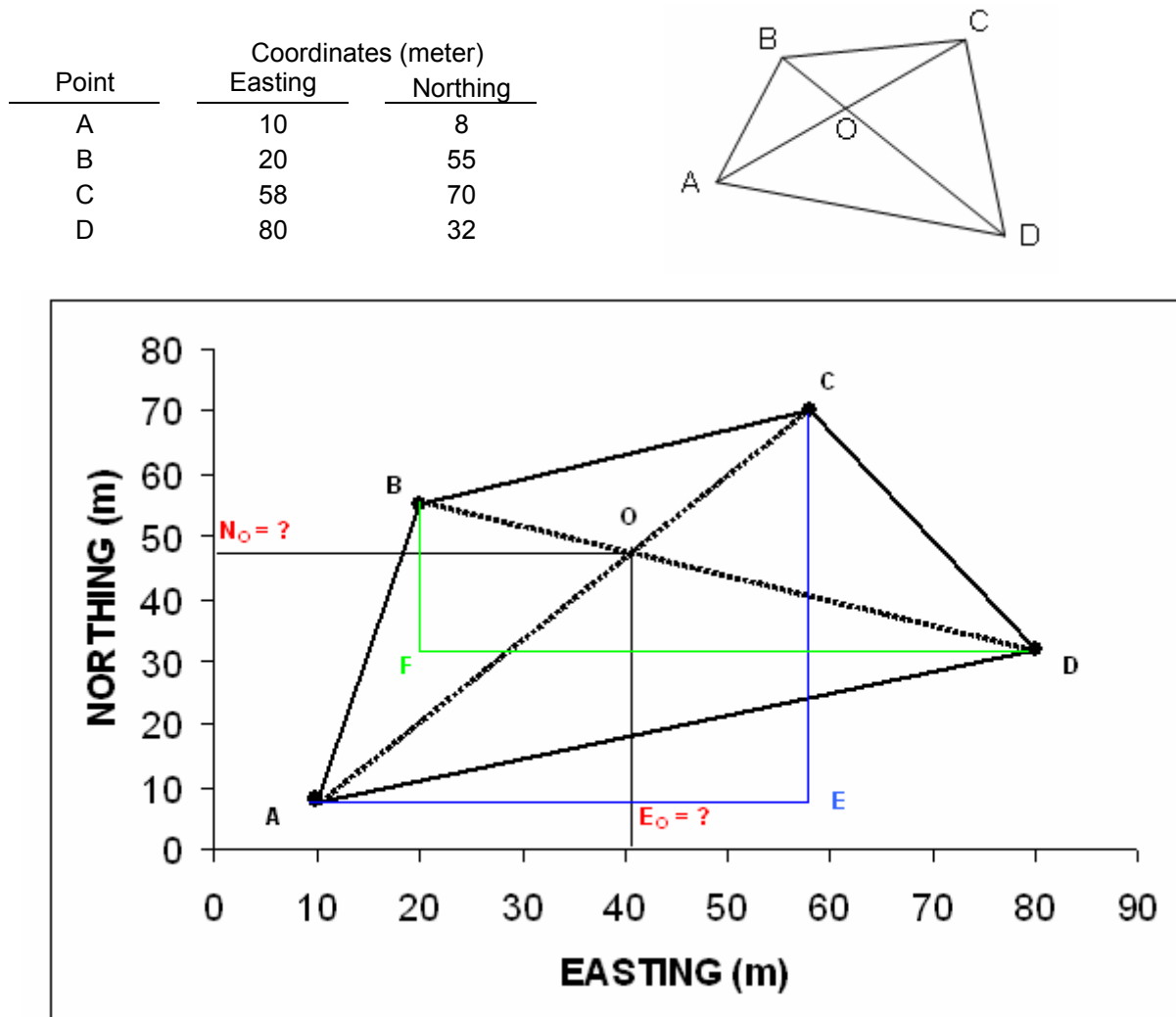


Solution :

- Elevation difference = $241.65 - 265.32 = -23.67$ m (since point B is lower)
 Distance = $327 - 150 = 177$ m
 Slope (Gradient) = $(-23.67 / 177) \cdot 100 = -13.37\%$ or $\tan^{-1}(23.67 / 177) = -7.6$ degree

PART 3 : BASIC PROBLEMS

3.1. A four sided closed traverse is defined with its coordinate values. If the opposite corner points are connected with diagonal lines, such as A with C and B with D, compute the coordinate values of the intersection point (O) of these diagonal lines.



Solution :

To determine the coordinates of point O, we can use similarity property of triangles. After drawing triangles ACE and BDF which have diagonals of traverse as their hypotenuse, we can write the following equations;

For ACE triangle;
$$\frac{(N_C - N_A)}{(N_O - N_A)} = \frac{(E_C - E_A)}{(E_O - E_A)} \quad \text{Eq.1}$$

For BDF triangle;
$$\frac{(E_D - E_O)}{(E_D - E_B)} = \frac{(N_O - N_D)}{(N_B - N_D)} \quad \text{Eq.2}$$

We have two equations and two unknowns. From eqn.2,
$$N_O = (3760 - 23 \cdot E_O) / 60$$

We substitute N_O into eqn.1 and then solve for E_O ,

$$\begin{aligned} 62 \cdot E_O - 620 &= 48 \cdot [(3760 - 23 \cdot E_O) / 60] - 384 \Rightarrow 3720 \cdot E_O - 37200 = 180480 - 1104 \cdot E_O - 23040 \\ E_O(3720 + 1104) &= 180480 + 37200 - 23040 \\ E_O &= 194640 / 4824 = 40.35 \text{ m.} \\ N_O &= (3760 - 23 \cdot 40.35) / 60 = 2831.95 / 60 = 47.20 \text{ m.} \end{aligned}$$

Point O is (40.35; 47.20)

3.2. The coordinates of the points A and B are given. Determine the azimuth of AB (in degree) and the distance between the points (in meter).

Point	Coordinates (m)	
	Easting	Northing
A	-20.50	42.30
B	-65.20	-50.10

Solution :

$$\tan \alpha = \Delta E / \Delta N = (E_B - E_A) / (N_B - N_A) = [-65.20 - (-20.50)] / [-50.10 - 42.30] = [-44.70] / [-92.40]$$

$$\tan \alpha = 0.4838 \quad \text{then} \quad \alpha = 25^\circ.816$$

If ΔE and ΔN are both negative, then Azimuth AB = $\alpha + 180 = 25^\circ.816 + 180^\circ = 205^\circ.816$

$$\text{Distance} = [(\Delta E)^2 + (\Delta N)^2]^{1/2} = [(44.70)^2 + (92.40)^2]^{1/2} = [10535.85]^{1/2} = 102.64 \text{ meter}$$

3.3. The coordinates of points A, B and C are given. Determine angle to the right at point B in degree.

Point	Coordinates (m)	
	Easting	Northing
A	30	60
B	56	8
C	22	42

Solution :

First, determine Azimuths of BA and BC

$$\text{Az. BA} = \tan^{-1}(\Delta E / \Delta N) = \tan^{-1}[(30 - 56) / (60 - 8)] = \tan^{-1}(-26 / 52)$$

$$\text{Az. BA} = -26.57^\circ$$

If ΔE is negative and ΔN is positive, then

$$\text{Az. BA} = -26.57^\circ + 360^\circ = 333.43^\circ$$

$$\text{Az. BC} = \tan^{-1}(\Delta E / \Delta N)$$

$$= \tan^{-1}[(22 - 56) / (42 - 8)] = \tan^{-1}(-34 / 34)$$

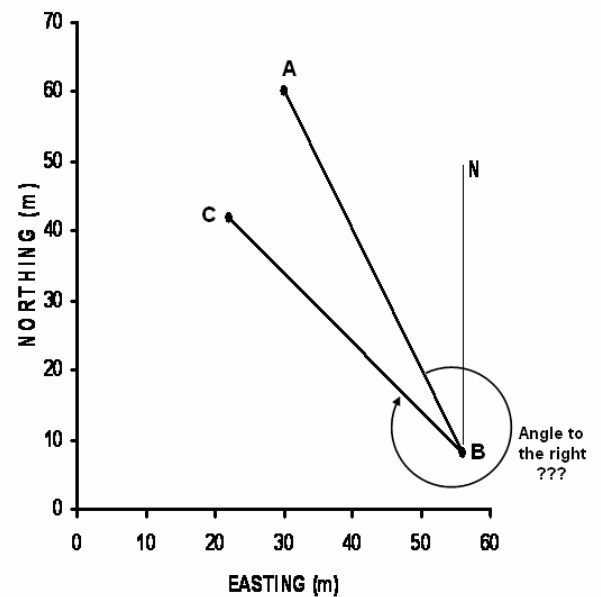
$$\text{Az. BC} = -45^\circ$$

If ΔE is negative and ΔN is positive, then

$$\text{Az. BC} = -45^\circ + 360^\circ = 315^\circ$$

$$\text{Az. BA} > \text{Az. BC}, \text{ then Angle to the right at B} = (\text{Az. BC} - \text{Az. BA}) + 360^\circ = (315^\circ - 333.43^\circ) + 360^\circ$$

$$\text{Angle to the right at B} = 341.57^\circ$$



3.4. The coordinates of three points (A, B and C) are given as follows. Determine the deflection angle (in degree unit) at point B.

Point	East (m)	North (m)
A	20	80
B	60	-30
C	-90	20

Solution :

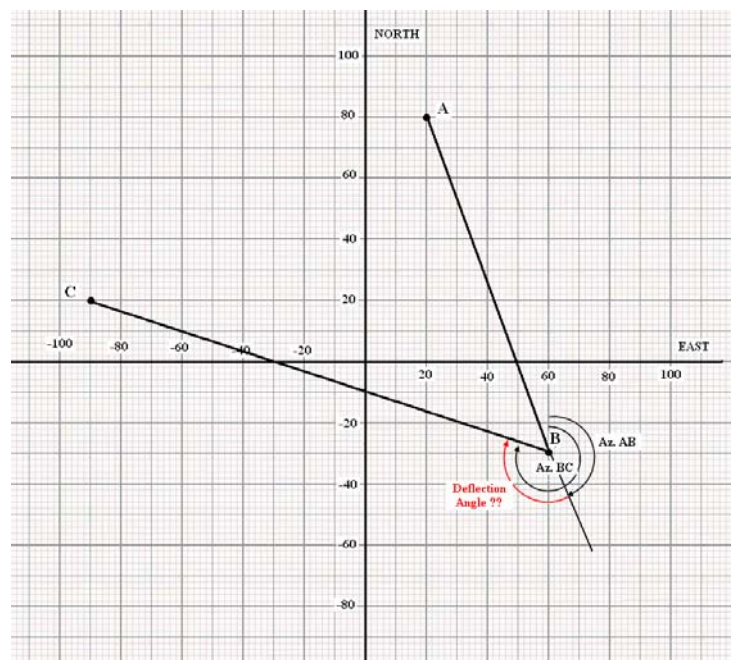
$$\text{Az. AB} = \tan^{-1}(\Delta E / \Delta N) = \tan^{-1}[(60 - 20) / (-30 - 80)] = \tan^{-1}(40 / -110) = -20^\circ \text{ (2nd Quadrant)}$$

$$\text{Az. AB} = -20^\circ + 180^\circ = 160^\circ$$

$$\text{Az. BC} = \tan^{-1}(\Delta E / \Delta N) = \tan^{-1}[(-90 - 60) / (20 + 30)] = \tan^{-1}(-150 / 50) = -71.6^\circ \text{ (4th Quadrant)}$$

$$\text{Az. BC} = -71.6^\circ + 360^\circ = 288.4^\circ$$

$$\begin{aligned} \text{Deflection Angle at B} &= (\text{Az. BC} - \text{Az. AB}) \text{ R} \\ &= (288.4^\circ - 160^\circ) \text{ R} \\ &= 128.4^\circ \text{ R} \end{aligned}$$



PART 4 : AREA DETERMINATION

4.1. Determine the area of the figure given. (Hint: Area = $[p(p-a)(p-b)(p-c)]^{1/2}$; $p=(a+b+c)/2$)

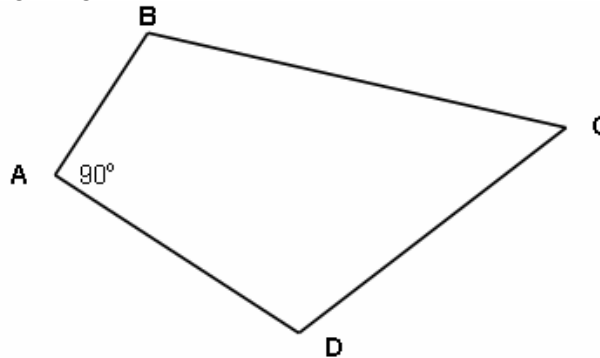
Lengths:

$$S_{AB} = 12 \text{ m.}$$

$$S_{BC} = 25 \text{ m.}$$

$$S_{CD} = 21 \text{ m.}$$

$$S_{DA} = 17 \text{ m.}$$



Solution :

If angle A is 90° , then $S_{BD} = (S_{AB}^2 + S_{DA}^2)^{1/2} = (12^2 + 17^2)^{1/2} = (433)^{1/2} = 20.81 \text{ m.}$

$$\text{Area}_{ABD} = 12 \times 17 / 2 = 102 \text{ m}^2.$$

$$p = (20.81 + 25 + 21) / 2 = 33.405 \text{ m.}$$

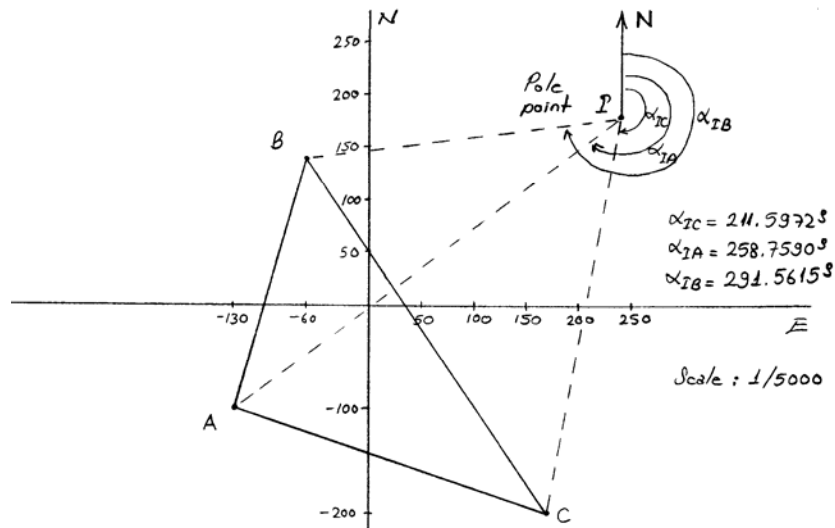
$$\text{Area}_{BCD} = [33.405(33.405 - 20.81)(33.405 - 25)(33.405 - 21)]^{1/2} = (43867.63)^{1/2} = 209.45 \text{ m}^2.$$

$$\text{Area}_{ABCD} = 102 + 209.45 = 311.45 \text{ m}^2.$$

4.2. The corner coordinates of a triangular property (ABC) and location coordinate of the pole point (I) are given as follows. If the azimuth angles are measured from the pole point, determine the area of the property in m^2 by using the equation $A = (a \cdot b \cdot \sin \alpha) / 2$.

Point	Coordinate (m)		Course	Azimuth (Gon)
	Northing	Easting		
A	-100	-130	IA	258.7590 ^g
B	140	-60	IB	291.5615 ^g
C	-200	170	IC	211.5972 ^g
I (Pole Pnt.)	180	240		

Solution :



Before the area, we need to determine the distances IB, IA and IC. These can be achieved by using the coordinates of the points. Thus:

$$IB = \sqrt{\Delta E^2 + \Delta N^2} = \sqrt{(240 - (-60))^2 + (180 - 140)^2} = \sqrt{300^2 + 40^2} = 302.655 \text{ m}$$

$$IA = \sqrt{\Delta E^2 + \Delta N^2} = \sqrt{(240 - (-130))^2 + (180 - (-100))^2} = \sqrt{370^2 + 280^2} = 464.004 \text{ m}$$

$$IC = \sqrt{\Delta E^2 + \Delta N^2} = \sqrt{(240 - 170)^2 + (180 - (-200))^2} = \sqrt{70^2 + 380^2} = 386.394 \text{ m}$$

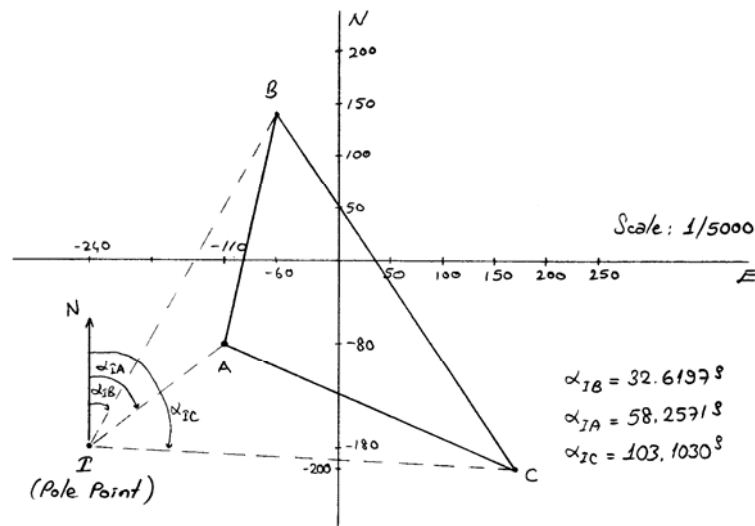
From azimuths and distances we can determine the areas of triangles by using given equation

$$\begin{aligned}
\text{Area}_{IBA} &= [\text{IB} \cdot \text{IA} \cdot \sin(\alpha_{IB} - \alpha_{IA})] / 2 = [302.655 \cdot 464.004 \cdot \sin(291.5615 - 258.7590)] / 2 \\
&= (140433.13 \cdot 0.4927) / 2 = 34599.92 \text{ m}^2 \\
\text{Area}_{IAC} &= [\text{IA} \cdot \text{IC} \cdot \sin(\alpha_{IA} - \alpha_{IC})] / 2 = [464.004 \cdot 386.394 \cdot \sin(258.7590 - 211.5972)] / 2 \\
&= (179288.36 \cdot 0.6749) / 2 = 60499.96 \text{ m}^2 \\
\text{Area}_{IBC} &= [\text{IB} \cdot \text{IC} \cdot \sin(\alpha_{IB} - \alpha_{IC})] / 2 = [302.655 \cdot 386.394 \cdot \sin(291.5615 - 211.5972)] / 2 \\
&= (116944.07 \cdot 0.9509) / 2 = 55600.07 \text{ m}^2 \\
\text{Area}_{ABC} &= \text{Area}_{IBA} + \text{Area}_{IAC} - \text{Area}_{IBC} = 34599.92 + 60499.96 - 55600.07 = 39499.81 \text{ m}^2 \\
\text{Area}_{ABC} &= \mathbf{39499.81 \text{ m}^2}
\end{aligned}$$

4.3. The corner coordinates of a triangular property (ABC) and location coordinate of pole point (I) are given as follows. If the azimuth angles are measured from the pole point, determine the area of the property in m^2 by using the equation $A = (a \cdot b \cdot \sin \alpha) / 2$.

Point	Coordinate (m)		Course	Azimuth (Gon)
	Northing	Easting		
A	-80	-110	IA	58.2571 ^g
B	140	-60	IB	32.6197 ^g
C	-200	170	IC	103.1030 ^g
I (Pole Pnt.)	-180	-240		

Solution :



Before the area, we need to determine the distances IB, IA and IC. These can be achieved by using the coordinates of the points. Thus:

$$\begin{aligned}
\text{IB} &= \sqrt{\Delta E^2 + \Delta N^2} = \sqrt{(-240 - (-60))^2 + (-180 - 140)^2} = \sqrt{180^2 + 320^2} = 367.151 \text{ m} \\
\text{IA} &= \sqrt{\Delta E^2 + \Delta N^2} = \sqrt{(-240 - (-110))^2 + (-180 - (-80))^2} = \sqrt{130^2 + 100^2} = 164.012 \text{ m} \\
\text{IC} &= \sqrt{\Delta E^2 + \Delta N^2} = \sqrt{(-240 - 170)^2 + (-180 - (-200))^2} = \sqrt{410^2 + 20^2} = 410.488 \text{ m}
\end{aligned}$$

From azimuths and distances we can determine the areas of triangles by using given equation

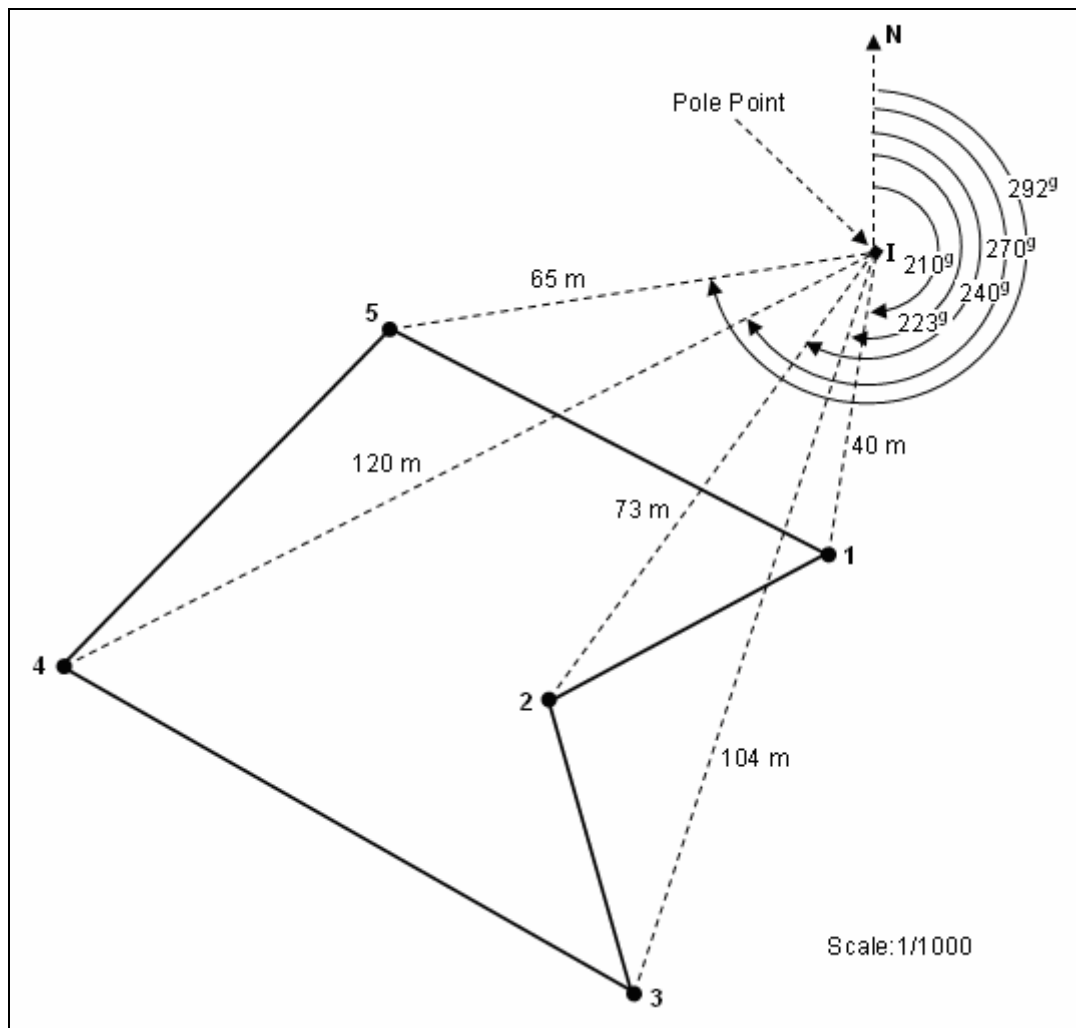
$$\begin{aligned}
\text{Area}_{IBA} &= [\text{IB} \cdot \text{IA} \cdot \sin(\alpha_{IA} - \alpha_{IB})] / 2 = [367.151 \cdot 164.012 \cdot \sin(58.2571 - 32.6197)] / 2 \\
&= (60217.17 \cdot 0.3919) / 2 = 11799.98 \text{ m}^2 \\
\text{Area}_{IAC} &= [\text{IA} \cdot \text{IC} \cdot \sin(\alpha_{IC} - \alpha_{IA})] / 2 = [164.012 \cdot 410.488 \cdot \sin(103.1030 - 58.2571)] / 2 \\
&= (67324.96 \cdot 0.6476) / 2 = 21800.00 \text{ m}^2 \\
\text{Area}_{IBC} &= [\text{IB} \cdot \text{IC} \cdot \sin(\alpha_{IC} - \alpha_{IB})] / 2 = [367.151 \cdot 410.488 \cdot \sin(103.1030 - 32.6197)] / 2 \\
&= (150711.08 \cdot 0.8944) / 2 = 67400.06 \text{ m}^2 \\
\text{Area}_{ABC} &= \text{Area}_{IBC} - (\text{Area}_{IAB} + \text{Area}_{IAC}) = 67400.06 - (11799.98 + 21800.00) = 33800.08 \text{ m}^2 \\
\text{Area}_{ABC} &= \mathbf{33800.08 \text{ m}^2}
\end{aligned}$$

4.4. Following measurements have been taken to determine the area of a parcel. The instrument is adjusted at a point out of the parcel and thus the azimuths and lengths are taken. Determine the area of the traverse (Pentagon area₁₂₃₄₅) by using triangulation method. Area of a triangle = $\frac{1}{2} \cdot s_1 \cdot s_2 \cdot \sin(\alpha_2 - \alpha_1)$.

Course	Azimuth (grad)	Length (meter)
I1	210	40
I2	240	73
I3	223	104
I4	270	120
I5	292	65

Solution :

Put the instrument at any point, e.g. = 0; 0
Assume a scale = 1/1000 to draw. 10 meter = 1 cm.



Determine the area of each triangle by using the equation, Area = $\frac{1}{2} \cdot s_1 \cdot s_2 \cdot \sin(\alpha_2 - \alpha_1)$

$$\text{Area I12} = 0.5 \cdot s_1 \cdot s_2 \cdot \sin(\alpha_2 - \alpha_1) = 0.5 \cdot 40 \cdot 73 \cdot \sin(240 - 210) = 1460 \cdot \sin 30^\circ = 662.83 \text{ m}^2$$

$$\text{Area I23} = 0.5 \cdot s_2 \cdot s_3 \cdot \sin(\alpha_2 - \alpha_3) = 0.5 \cdot 73 \cdot 104 \cdot \sin(240 - 223) = 3796 \cdot \sin 17^\circ = 1001.66 \text{ m}^2$$

$$\text{Area I34} = 0.5 \cdot s_3 \cdot s_4 \cdot \sin(\alpha_4 - \alpha_3) = 0.5 \cdot 104 \cdot 120 \cdot \sin(270 - 223) = 6240 \cdot \sin 47^\circ = 4199.60 \text{ m}^2$$

$$\text{Area I45} = 0.5 \cdot s_4 \cdot s_5 \cdot \sin(\alpha_5 - \alpha_4) = 0.5 \cdot 120 \cdot 65 \cdot \sin(292 - 270) = 3900 \cdot \sin 22^\circ = 1321.08 \text{ m}^2$$

$$\text{Area I15} = 0.5 \cdot s_1 \cdot s_5 \cdot \sin(\alpha_5 - \alpha_1) = 0.5 \cdot 40 \cdot 65 \cdot \sin(292 - 210) = 1300 \cdot \sin 82^\circ = 1248.38 \text{ m}^2$$

$$\text{Area of pentagon} = S_{I12} + S_{I34} + S_{I45} - S_{I23} - S_{I15} = 662.83 + 4199.60 + 1321.08 - 1001.66 - 1248.38$$

$$\text{Area of pentagon} = 6183.51 - 2250.04$$

$$\text{Area of pentagon} = 3933.47 \text{ m}^2$$

4.5. Latitudes and departures of the courses of a five-sided closed traverse are given as follows. Determine its area by using DMD (Double Meridian Distance) method.

Course	Latitude (m)	Departure (m)
AB	80 N	30 W
BC	100 N	80 E
CD	130 S	90 E
DE	30 N	60 W

Solution :

First of all, latitude and departure of the missing course (EA) should be determined to satisfy

$\Sigma \text{Latitude} = 0$ and $\Sigma \text{Departure} = 0$. Then:

$$80 + 100 - 130 + 30 + \text{Lat}_{EA} = 0 \rightarrow \text{Lat}_{EA} = 80 \text{ S} \quad \text{and} \quad -30 + 80 + 90 - 60 + \text{Dep}_{EA} = 0 \rightarrow \text{Dep}_{EA} = 80 \text{ W}$$

If we draw the reference meridian line passes through the most westerly point (which is W), then:

$$\text{DMD of course BC} = \text{Departure of itself} = 80$$

$$\text{DMD of course CD} = \text{DMD of BC} + \text{Dep. of BC} + \text{Dep. of CD} = 80 + 80 + 90 = 250$$

$$\text{DMD of course DE} = \text{DMD of CD} + \text{Dep. of CD} + \text{Dep. of DE} = 250 + 90 - 60 = 280$$

$$\text{DMD of course EA} = \text{DMD of DE} + \text{Dep. of DE} + \text{Dep. of EA} = 280 - 60 - 80 = 140$$

$$\text{DMD of course AB} = \text{DMD of EA} + \text{Dep. of EA} + \text{Dep. of AB} = 140 - 80 - 30 = 30 (= \text{dep. of AB})$$

Now we can determine double areas by using $\text{DMD} \times \text{Latitude}$;

$$\text{For course BC, Double area} = 80 \times 100 = + 8000$$

$$\text{For course CD, Double area} = 250 \times (-130) = - 32500$$

$$\text{For course DE, Double area} = 280 \times 30 = + 8400$$

$$\text{For course EA, Double area} = 140 \times (-80) = - 11200$$

$$\text{For course AB, Double area} = 30 \times (-80) = + 2400$$

$$\text{Sum of double areas} = \text{abs}(-24900) = 24900 \text{ m}^2, \text{ then}$$

$$\text{Area of traverse ABCDE} = 24900/2 = \mathbf{12450 \text{ m}^2}$$

4.6. Latitudes and departures of the courses of a five-sided closed traverse are given as follows. Determine its area by using DMD (Double Meridian Distance) method.

Course	Latitude (m)	Departure (m)
AB	50 N	150 W
BC	150 N	300 E
CD	250 S	50 E
DE	110 N	130 W

Solution :

First of all, latitude and departure of the missing course (EA) should be determined to satisfy

$\Sigma \text{Latitude} = 0$ and $\Sigma \text{Departure} = 0$. Then:

$$50 + 150 - 250 + 110 + \text{Lat}_{EA} = 0 \rightarrow \text{Lat}_{EA} = 60 \text{ S} \quad \text{and} \quad -150 + 300 + 50 - 130 + \text{Dep}_{EA} = 0 \rightarrow \text{Dep}_{EA} = 70 \text{ W}$$

If we draw the reference meridian line passes through the most westerly point (which is W), then:

$$\text{DMD of course BC} = \text{Departure of itself} = 300$$

$$\text{DMD of course CD} = \text{DMD of BC} + \text{Dep. of BC} + \text{Dep. of CD} = 300 + 300 + 50 = 650$$

$$\text{DMD of course DE} = \text{DMD of CD} + \text{Dep. of CD} + \text{Dep. of DE} = 650 + 50 - 130 = 570$$

$$\text{DMD of course EA} = \text{DMD of DE} + \text{Dep. of DE} + \text{Dep. of EA} = 570 - 130 - 70 = 370$$

$$\text{DMD of course AB} = \text{DMD of EA} + \text{Dep. of EA} + \text{Dep. of AB} = 370 - 70 - 150 = 150 (= \text{dep. of AB})$$

Now we can determine double areas by using $\text{DMD} \times \text{Latitude}$;

$$\text{For course BC, Double area} = 300 \times 150 = + 45000$$

$$\text{For course CD, Double area} = 650 \times (-250) = - 162500$$

$$\text{For course DE, Double area} = 570 \times 110 = + 62700$$

$$\text{For course EA, Double area} = 370 \times (-60) = - 22200$$

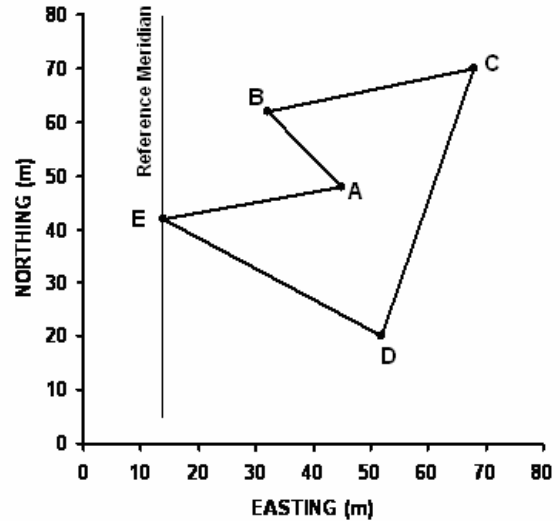
$$\text{For course AB, Double area} = 150 \times 50 = + 7500$$

$$\text{Sum of double areas} = \text{abs}(-69500) = 69500 \text{ m}^2, \text{ then}$$

$$\text{Area of traverse ABCDE} = 69500/2 = \mathbf{34750 \text{ m}^2}$$

4.7. The coordinates of a five sided polygon are given. Draw the polygon with a scale of 1/500 and determine its area (m²) by using DMD method.

	COORDINATES (m)	
Pnt	Easting	Northing
A	45	48
B	32	62
C	68	70
D	52	20
E	14	42



Point E has the lowest Easting value that shows the most westerly point. Therefore reference meridian passes through E. Now, we determine departure and latitude values of the sides.

Departure of AB = $E_B - E_A$	Latitude of AB = $N_B - N_A$
Dep. of AB = $32 - 45 = -13$ (\equiv W13) ...m	Lat. of AB = $62 - 48 = +14$ (\equiv N14) ...m
Dep. of BC = $68 - 32 = +36$ (\equiv E36) ...m	Lat. of BC = $70 - 62 = +8$ (\equiv N8) ...m
Dep. of CD = $52 - 68 = -16$ (\equiv W16) ...m	Lat. of CD = $20 - 70 = -50$ (\equiv S50) ...m
Dep. of DE = $14 - 52 = -38$ (\equiv W38) ...m	Lat. of DE = $42 - 20 = +22$ (\equiv N22) ...m
Dep. of EA = $45 - 14 = +31$ (\equiv E31) ...m	Lat. of EA = $48 - 20 = +28$ (\equiv N28) ...m
Total departure = 0	Total latitude = 0 (checking done)

Departure of EA	= + 31	= DMD of EA
Departure of EA	= + 31	
Departure of AB	= - 13	
	+ 49	= DMD of AB
Departure of AB	= - 13	
Departure of BC	= + 36	
	+ 72	= DMD of BC
Departure of BC	= + 36	
Departure of CD	= - 16	
	+ 92	= DMD of CD
Departure of CD	= - 16	
Departure of DE	= - 38	
	+ 38	= DMD of DE Checking done

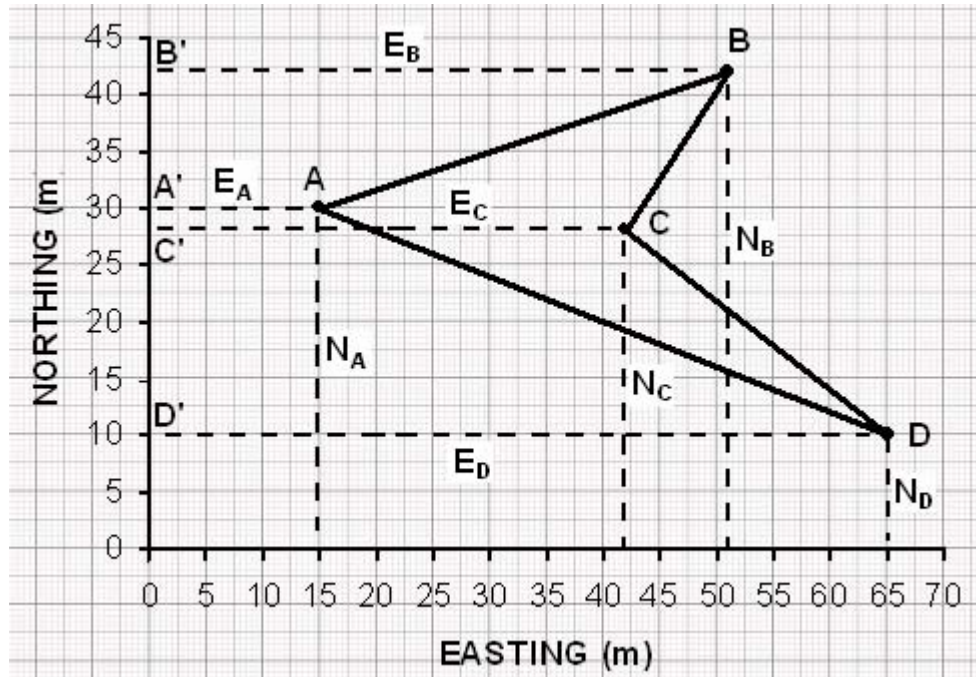
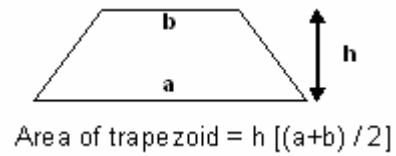
				Double Areas	
Course	Latitude	Departure	DMD	+	-
AB	N 14	W 13	+ 49	686	
BC	N 8	E 36	+ 72	576	
CD	S 50	W 16	+ 92		4600
DE	N 22	W 38	+ 38	836	
EA	N 6	E 31	+ 31	186	
Total	0.00	0.00		2284	4600

Double area = Absolute(-4600+2284) = 2316 m²

Area of the field = 2316/2 = 1158 m²

4.8. The coordinates of a four sided area are given as follows. Draw the shape by using a scale of 1/500 and determine its area (m²) by using trapezoidal area method.

Point	Coordinates (m)	
	Easting	Northing
A	15	30
B	51	42
C	42	28
D	65	10



Scale : 1/500

Solution :

$$\begin{aligned}
 \text{Area ABCD} &= \text{Area B'BCC'} + \text{Area C'CDD'} - \text{Area B'BAA'} - \text{Area A'ADD'} \\
 &= \frac{1}{2} [(E_B + E_C)(N_B - N_C) + (E_C + E_D)(N_C - N_D) - (E_B + E_A)(N_B - N_A) - (E_A + E_D)(N_A - N_D)] \\
 &= \frac{1}{2} [(51 + 42)(42 - 28) + (42 + 65)(28 - 10) - (51 + 15)(42 - 30) - (15 + 65)(30 - 10)] \\
 &= \frac{1}{2} [(93 \times 14) + (107 \times 18) - (66 \times 12) - (80 \times 20)] = \frac{1}{2} (1302 + 1926 - 792 - 1600)
 \end{aligned}$$

$$\text{Area ABCD} = 418 \text{ m}^2$$

4.9. Determine the area (m²) of the following parcel by using tarpezoid method.

Point	Coordinates (m)	
	Easting	Northing
A	-80	20
B	100	220
C	80	-30
D	200	-150
E	-20	-210

Solution :

For easier calculation, shift axes to Point A (most westerly point) and to Point E (ost southerly point).

Point	Coordinates (m)		Shifted Coordinates (m)	
	Easting	Northing	Easting	Northing
A	-80	20	0	230
B	100	220	180	430
C	80	-30	160	180
D	200	-150	280	60
E	-20	-210	60	0

Area of ABCDE is composition of trapezoidal areas.

$$\text{Area(ABCDE)} = \text{Area(B'BCC')} + \text{Area(C'CDD')} + \text{Area(D'DEE')} - \text{Area(B'BAA')} - \text{Area(A'AEE')}$$

Area of trapezoid = (Sum of Eastings/2)(Difference of Northings)

$$\text{Area(B'BCC')} = [(180+160)/2] \times (430-180) = 170 \times 250 = 42500 \text{ m}^2$$

$$\text{Area(C'CDD')} = [(160+280)/2] \times (180-60) = 220 \times 120 = 26400 \text{ m}^2$$

$$\text{Area(D'DEE')} = [(280+60)/2] \times (60-0) = 170 \times 60 = 10200 \text{ m}^2$$

$$\text{Area(B'BAA')} = [(180+0)/2] \times (430-230) = 90 \times 200 = 18000 \text{ m}^2$$

$$\text{Area(A'AEE')} = [(60+0)/2] \times (230-0) = 30 \times 230 = 6900 \text{ m}^2$$

$$\text{Area(ABCDE)} = 42500 + 26400 + 10200 - 18000 - 6900 = 79100 - 24900 = 54200 \text{ m}^2$$

Correlated according to coordinates method

	Easting	Northing	Plus	Minus
A	0	230	0	41400
B	180	430	32400	68800
C	160	180	9600	50400
D	280	60	0	3600
E	60	0	13800	0
A	0	230	55800	164200
			Area =	54200 m ²

PART 5 : AZIMUTH, BEARING, etc.

5.1. Convert the following azimuth angles into bearings (all in degree).

- a) 218 degree b) 340 grad c) $5\pi/8$ d) 130 degree

Solution :

a) Azimuth = 218° (between 180° and 270°) then Bearing = S ($218-180$) W = **S38°W**

b) Azimuth = 340^g (between 300^g and 400^g) $340^g = (340/400)*360 = 306^\circ$
then Bearing = N ($360-306$) W = **N54°W**

c) Azimuth = $5\pi/8 = [(5\pi/8)/(\pi)]*180 = 112.5^\circ$ (between 90° and 180°)
then Bearing = S ($180-112.5$) E = **S67.5°E**

d) Azimuth = 130° (between 90° and 180°) then Bearing = S ($180-130$) E = **S50°E**

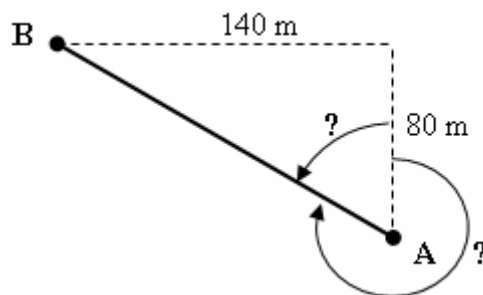
5.2. Latitude and departure of line AB are +80 m and -140 m respectively. Determine azimuth and bearing angles of AB in degrees.

Solution :

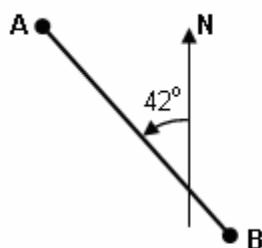
$$\alpha_{AB} = \tan^{-1}(\Delta E/\Delta N) = \tan^{-1}(-140/80) \rightarrow \alpha_{AB} = -60.25^\circ$$

If ΔE is negative and ΔN is positive, then Azimuth AB = $-60.25^\circ + 360^\circ = 299.75^\circ$

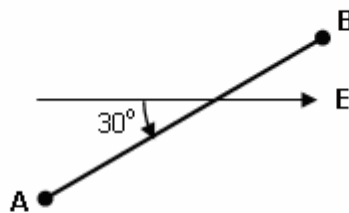
Azimuth AB = 299.75°
Bearing AB = N 60.25° W



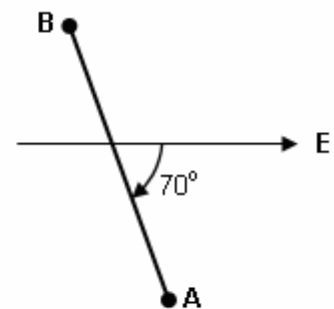
5.3. Determine the bearing and the azimuth angles of the lines AB in each of the following figures.
(Show your drawings and calculations)



(a)



(b)



(c)

Solution :

a) Azimuth AB = $180^\circ - 42^\circ = 138^\circ$

b) Azimuth AB = $90^\circ - 30^\circ = 60^\circ$

c) Azimuth AB = $360^\circ - (90^\circ - 70^\circ) = 340^\circ$

Bearing (SE quadrant) = S ($180^\circ - 138^\circ$) E = S 42° E

Bearing (NE quadrant) = N 60° E

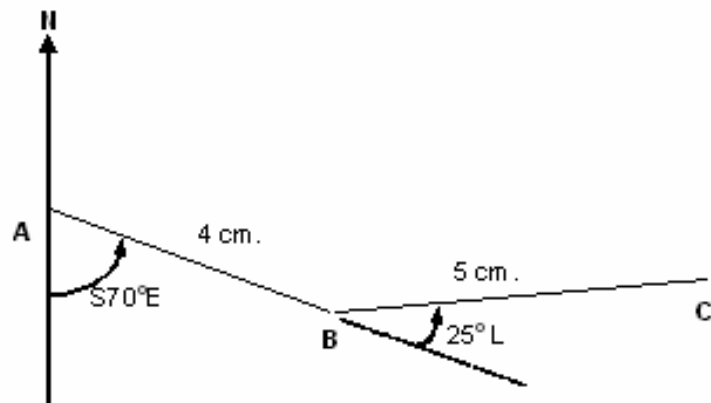
Bearing (NW quadrant) = N ($360^\circ - 340^\circ$) W = N 20° W

5.4. Draw the following lines on a milimetric paper by using the scale 1:1.

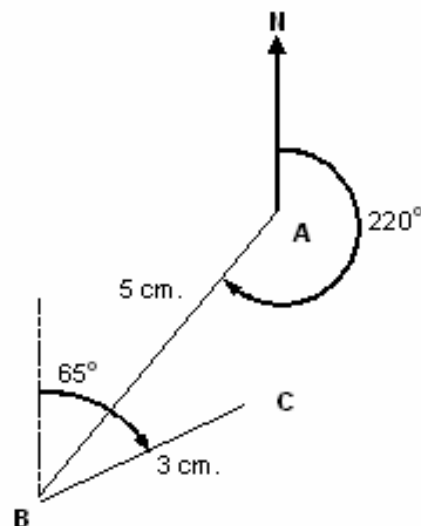
- | | | |
|---|----------|----------|
| a) Bearing of AB= S70°E, Deflection angle at point B = 25°L | AB=4 cm. | BC=5 cm. |
| b) Azimuth of AB= 220°, Azimuth of BC = 65° | AB=5 cm. | BC=3 cm. |
| c) Angle to left of AB= 60°, Bearing of BC = N35°E | AB=3 cm. | BC=6 cm. |
| d) Azimuth of AB= 250°, Deflection angle at point B = 40°L | AB=6 cm. | BC=4 cm. |

Solution :

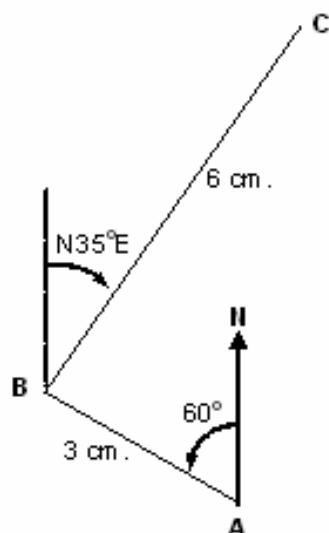
a)



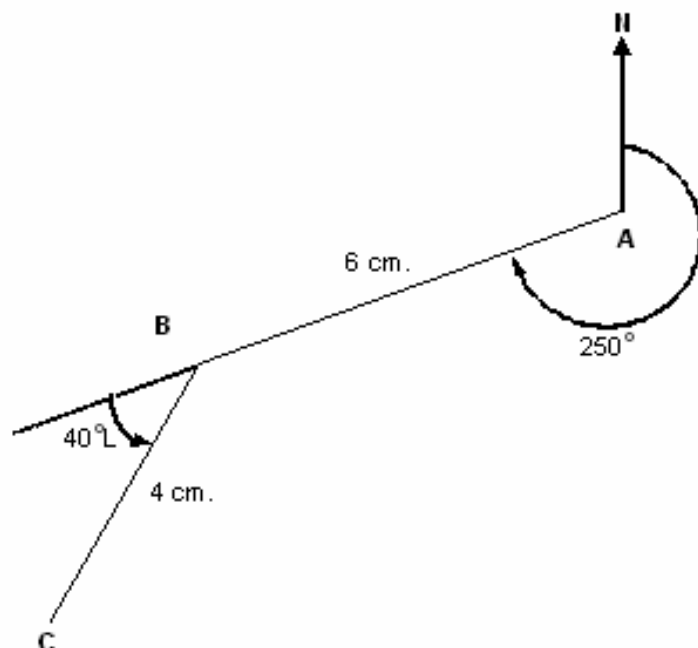
b)



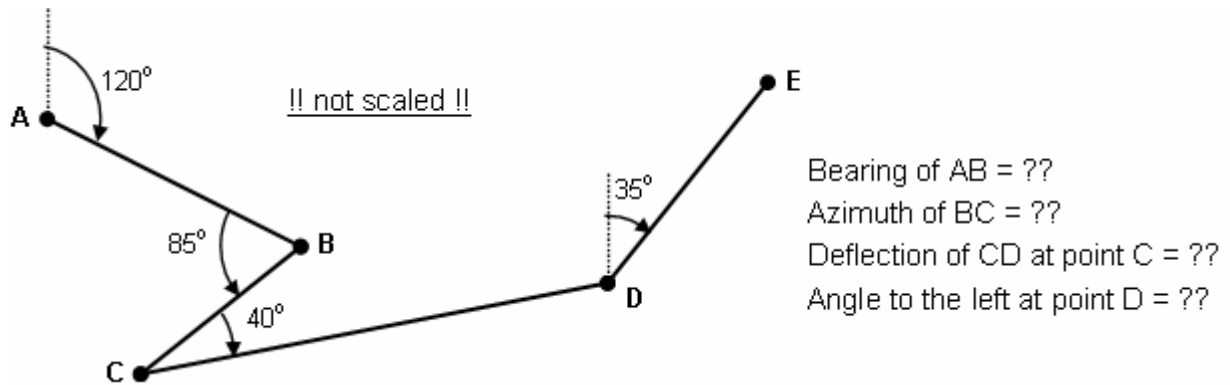
c)



d)



5.5. Determine the following angles according to information given on the figure (Follow the letters).



Solution :

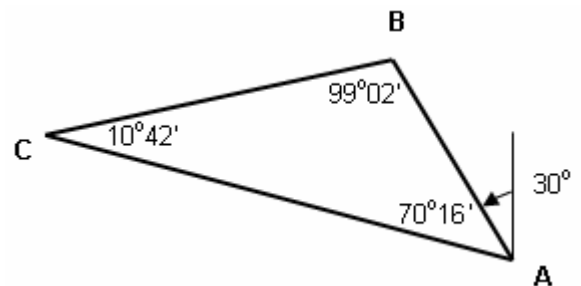
- i) Bearing of AB = S (180-120) W = **S60°E**
- ii) Azimuth AB = 120° Azimuth BA (reverse) = 120+180 = 300°
 Azimuth of BC = (300-85) = **215°**
- iii) Deflection of CD at point C = (180-40) = **140°L**
- iv) Angle to the left at point D
 Azimuth of CB = Az.BC – 180° = 35°
 Azimuth of CD = Az.CB + 40° = 75°
 Azimuth of DC = Az.CD + 180° = 255°
 Angle to the left at point D = (255-35) = **220°**

5.6. Interior angles of a triangle are obtained as follows. If bearing of side AB is N30°W, determine azimuths and bearings of each side (move counterclockwise direction) after correcting the angles.

- Interior angle at point A : 70°12'
 Interior angle at point B : 98°58'
 Interior angle at point C : 10°38'

Solution :

Sum of interior angles (A+B+C) = 178°108' = 179°48'
 Error = 179°48'-179°60' = -12'
 Correction (equal) = 12/3 = 4' for each angle
 Corrected angle A = 70°12' + 4' = 70°16'
 Corrected angle B = 98°58' + 4' = 99°02'
 Corrected angle C = 10°38' + 4' = 10°42'

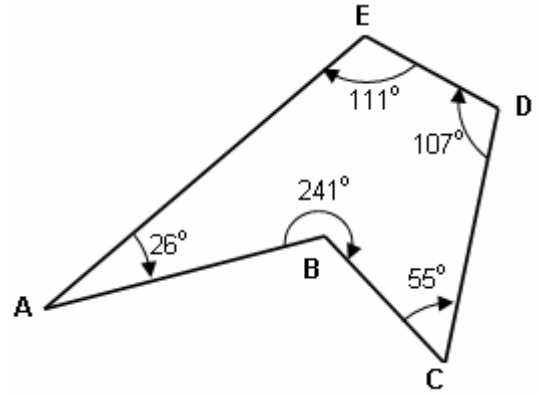


Bearing AB = N30°W (given)
 Azimuth AB = 360°-30° = **330°**
 Azimuth BA = 330°-180° = 150°
 Azimuth BC = 150°+99°02' = **249°02'**
 Bearing BC = S(249°02'-180°)W = **S69°02'W**
 Azimuth CB = 69°02'
 Azimuth CA = 69°02'+10°42' = **79°44'**
 Bearing CA = **N79°44'E**
 Azimuth AC = 79°44'+180° = 259°44'
 Azimuth AB = 259°44'+70°16' = 330° (check)

5.7. A five sided figure is given below. If the azimuth of AB is 74° , determine the bearing angles of all sides (AB, BC, CD, DE, EA) and deflection angles at the points (B, C, D, E).

Solution :

Bearing AB (NE quadrant) = N 74° E
 Deflection at B = $241 - 180 = 61^\circ$ R
 Azimuth BA (reverse) = $180 + 74 = 254^\circ$
 Azimuth BC = $254 + 241 - 360 = 135^\circ$
 Bearing BC (SE quadrant) = $180 - 135 = \text{S } 45^\circ \text{ E}$
 Deflection at C = $180 - 55 = 125^\circ$ L
 Azimuth CB (reverse) = $180 + 135 = 315^\circ$
 Azimuth CD = $315 + 55 - 360 = 10^\circ$
 Bearing CD (NE quadrant) = N 10° E
 Deflection at D = $180 - 107 = 73^\circ$ L
 Azimuth DC (reverse) = $180 + 10 = 190^\circ$
 Azimuth DE = $190 + 107 = 297^\circ$
 Bearing DE (NW quadrant) = $360 - 297 = \text{N } 63^\circ \text{ W}$
 Deflection at E = $180 - 111 = 69^\circ$ L
 Azimuth ED (reverse) = $297 - 180 = 117^\circ$
 Azimuth EA = $117 + 111 = 228^\circ$
 Bearing EA (SW quadrant) = $228 - 180 = \text{S } 48^\circ \text{ W}$
 Check \rightarrow Azimuth AE (reverse) = $228 - 180 = 48^\circ$ is equal to $74 - 26 = 48^\circ$

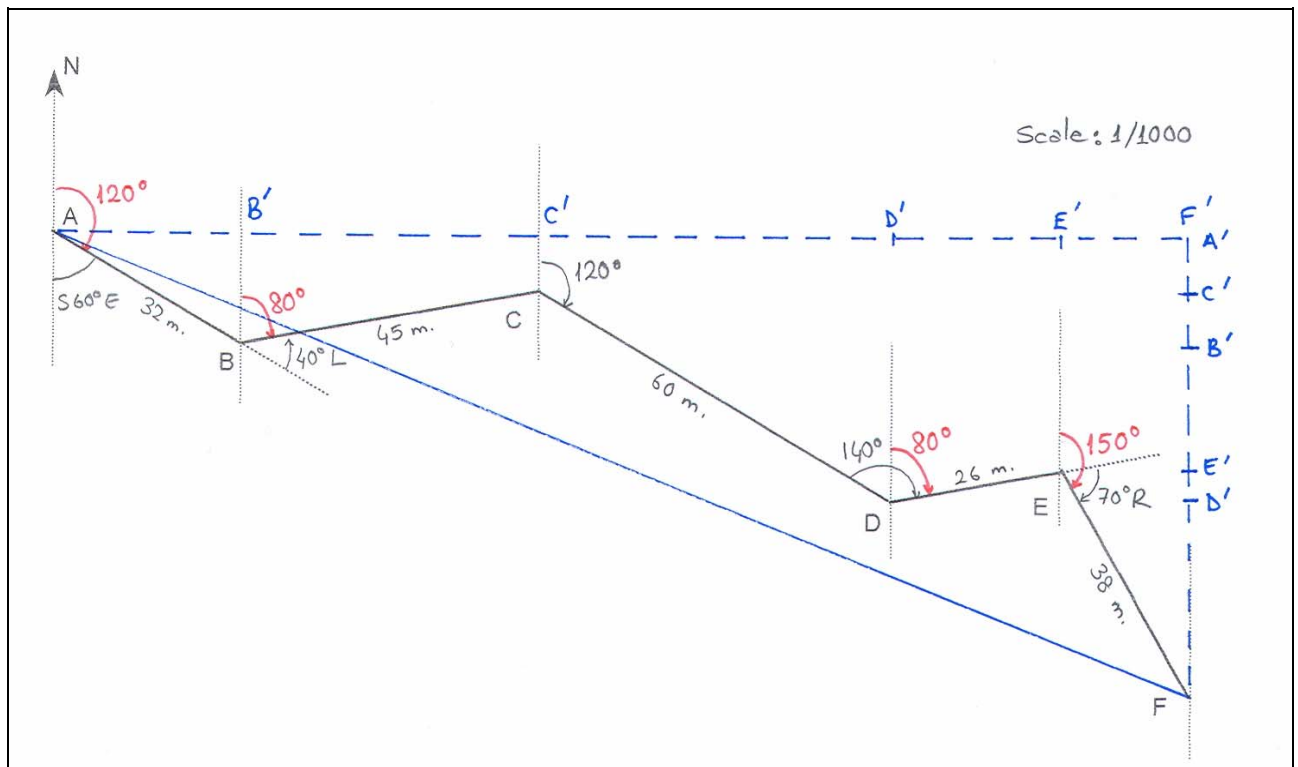


PART 6 : OPEN TRAVERSE

6.1. (a) Draw an open traverse starting from point A and ends at point F, according to information given below. Use a scale of 1/1000 and show your calculations in details.

Bearing of course AB	= S 60° E	Deflection angle at point B	= 40°L
Azimuth of course CD	= 120°	Angle to the right at point D	= 140°
Deflection angle at point E	= 70°R	Length of course AB	= 32 meter
Length of course BC	= 45 meter	Length of course CD	= 60 meter
Length of course DE	= 26 meter	Length of course EF	= 38 meter

Solution :



(b) Determine bearing and azimuth angles of the courses BC, DE and EF of the traverse given in (a). Write your calculations clearly.

Solution :

Course	Azimuth	Bearing
BC	$120^\circ - 40^\circ = 80^\circ$	= azimuth → = N 80° E
DE	$120^\circ + 140^\circ - 180^\circ = 80^\circ$	= azimuth → = N 80° E
EF	$80^\circ + 70^\circ = 150^\circ$	2nd Qua. = $180^\circ - \text{azimuth}$ → = S 30° E

(c) Determine the distance between the points A and F of the open traverse given in (a).

Solution :

$$\text{Distance AF} = \sqrt{(AF')^2 + (FF')^2} \quad \text{from the figure}$$

In E-W direction $AF' = AB' + B'C' + C'D' + D'E' + E'F'$

In N-S direction $F'F = A'B' - B'C' + C'D' - D'E' + E'F'$

$$AF' = 32 \cdot \cos 30^\circ + 45 \cdot \cos 10^\circ + 60 \cdot \cos 30^\circ + 26 \cdot \cos 10^\circ + 38 \cdot \cos 60^\circ$$

$$= 27.713 + 44.316 + 51.962 + 25.605 + 19.000 = 168.596 \text{ m.}$$

$$F'F = 32 \cdot \sin 30^\circ - 45 \cdot \sin 10^\circ + 60 \cdot \sin 30^\circ - 26 \cdot \sin 10^\circ + 38 \cdot \sin 60^\circ$$

$$= 16.000 - 7.814 + 30.000 + 4.515 + 32.909 = 66.580 \text{ m.}$$

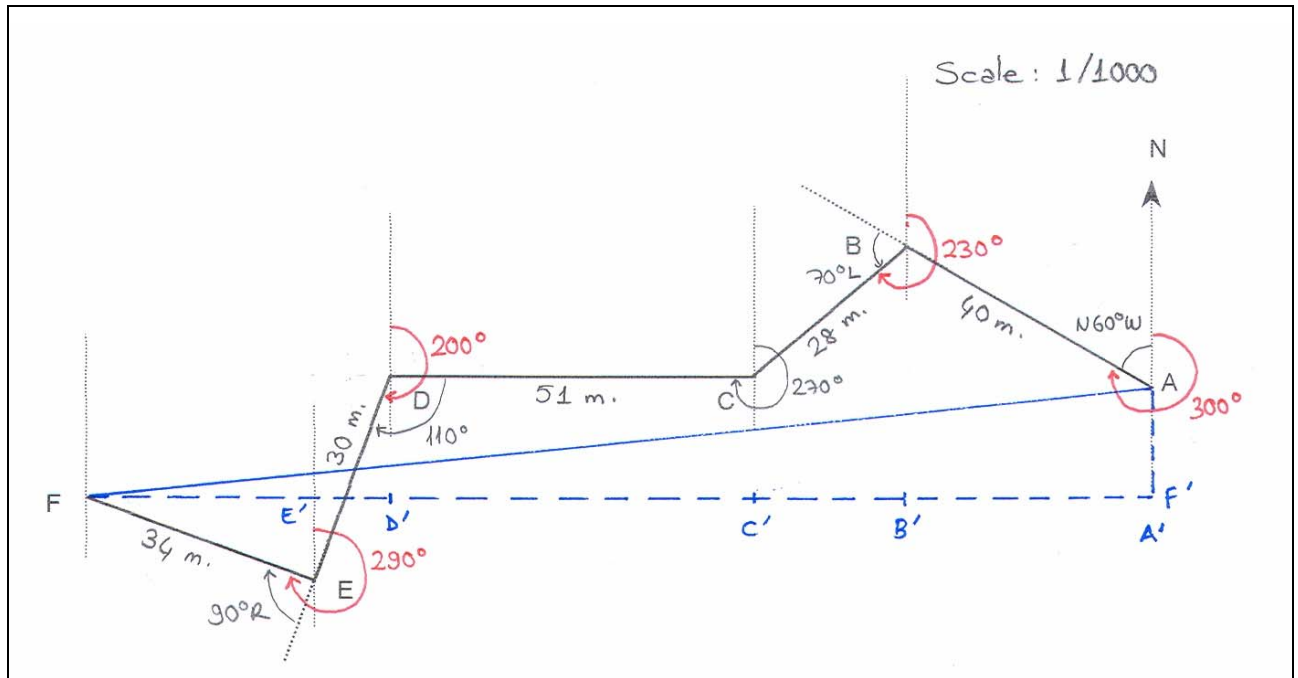
$$\text{Distance AF} = \sqrt{(168.596)^2 + (66.580)^2} = \sqrt{32857.508} = 181.266 \text{ m}$$

Distance AF = 181.266 m

6.2. (a) Draw an open traverse starting from point A and ends at point F, according to information given below. Use a scale of 1/1000 and show your calculations in details.

Bearing of course AB	= N 60° W	Deflection angle at point B	= 70°L
Azimuth of course CD	= 270°	Angle to the right at point D	= 110°
Deflection angle at point E	= 90°R	Length of course AB	= 40 meter
Length of course BC	= 28 meter	Length of course CD	= 51 meter
Length of course DE	= 30 meter	Length of course EF	= 34 meter

Solution :



(b) Determine bearing and azimuth angles of the courses BC, DE and EF of the traverse given in (a). Write your calculations clearly.

Solution :

Course	Azimuth	Bearing
BC	$300^\circ - 70^\circ = 230^\circ$	3rd Qua. = azimuth-180° → = S 50° W
DE	$270^\circ + 110^\circ - 180^\circ = 200^\circ$	3rd Qua. = azimuth-180° → = S 20° W
EF	$200^\circ + 90^\circ = 290^\circ$	4th Qua. = 360°-azimuth → = N 70° W

(c) Determine the distance between the points A and F of the open traverse given in (a).

Solution :

$$\text{Distance AF} = \sqrt{(FA')^2 + (AF')^2} \quad \text{from the figure}$$

In E-W direction $FA' = A'B' + B'C' + C'D' + D'E' + E'F'$

In N-S direction $AF' = -A'B' + B'C' + C'D' + D'E' - E'F'$

$$FA' = 40 \cos 30^\circ + 28 \cos 40^\circ + 51 \cos 0^\circ + 30 \cos 70^\circ + 34 \cos 20^\circ$$

$$= 34.641 + 21.449 + 51.000 + 10.261 + 31.950 = 149.301 \text{ m.}$$

$$AF' = -40 \sin 30^\circ + 28 \sin 40^\circ + 51 \sin 0^\circ + 30 \sin 70^\circ - 34 \sin 20^\circ$$

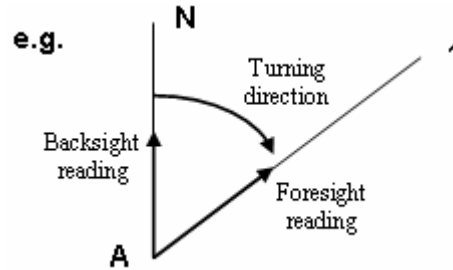
$$= -20.000 + 17.998 + 0.0 + 28.191 - 11.629 = 14.560 \text{ m.}$$

$$\text{Distance AF} = \sqrt{(149.301)^2 + (14.560)^2} = \sqrt{22502.782} = 150.010 \text{ m}$$

Distance AF = 150.010 m

6.3. Courses of an open traverse with known coordinates of starting and ending points (A and B) are measured. Forward readings are taken after backward ones by turning the instrument in clockwise direction. The readings are given as:

Station	Backsight reading	Foresight reading
A	20 ^g	75 ^g
1	120 ^g	340 ^g
2	50 ^g	380 ^g
3	370 ^g	30 ^g
4	280 ^g	130 ^g



Coordinates: $E_A = 170.00$ m. $N_A = 125.00$ m. $E_B = 232.60$ m. $N_B = 121.00$ m.

Lengths: $S_{A1} = 17$ m. $S_{12} = 15$ m. $S_{23} = 28$ m. $S_{34} = 22$ m. $S_{4B} = 20$ m.

Determine :

- Angle to the right, azimuth angle, computed latitude-departure and coordinates,
- Linear misclose at point B,
- Accuracy of the measurement,
- Amount of adjustments (dE&dN) in departure and latitude according to compass rule,
- Adjusted departure and latitude, and then adjusted coordinates.

Put your results in the table.

Solution :

Pnt	Course	Length (m)	Angle to the right (gon)	Azimuth (gon)	Computed		Computed		Adjustments		Adjusted		Adjusted	
					Departure (m)	Latitude (m)	Easting (m)	Northing (m)	dE (m)	dN (m)	Departure (m)	Latitude (m)	Easting (m)	Northing (m)
A			55				170.00	125.00					170.00	125.00
	A1	17.00		55	12.93	11.04			-0.03	0.05	12.90	11.09		
1			220				182.93	136.04					182.90	136.09
	12	15.00		75	13.86	5.74			-0.03	0.05	13.83	5.79		
2			330				196.79	141.78					196.73	141.88
	23	28.00		205	-2.20	-27.91			-0.06	0.08	-2.26	-27.83		
3			60				194.59	113.87					194.47	114.05
	34	22.00		65	18.76	11.49			-0.04	0.07	18.72	11.56		
4			250				213.35	125.36					213.19	125.61
	4B	20.00		115	19.45	-4.67			-0.04	0.06	19.41	-4.61		
B							232.80	120.69					232.60	121.00
TOTAL		102.00							-0.20	0.31				

Given values

- a) Angle to the right at point A = Foresight Angle – Backsight Angle = $75 - 20 = 55^g$
 Angle to the right at point 1 = Foresight Angle – Backsight Angle = $340 - 120 = 220^g$
 Angle to the right at point 2 = Foresight Angle – Backsight Angle = $380 - 50 = 330^g$
 Angle to the right at point 3 = Foresight Angle – Backsight Angle = $(30 - 370) + 400 = 60^g$ (since BS>FS)
 Angle to the right at point 4 = Foresight Angle – Backsight Angle = $(130 - 280) + 400 = 250^g$ (since BS>FS)
 Azimuth of course A1 = Angle to the right at point A = 55^g
 Azimuth of course 12 = (Az.A1+Angle to the right at point 1) – $200^g = (55 + 220) - 200 = 75^g$
 Azimuth of course 23 = (Az.12+Angle to the right at point 2) – $200^g = (75 + 330) - 200 = 205^g$
 Azimuth of course 34 = (Az.23+Angle to the right at point 3) – $200^g = (205 + 60) - 200 = 65^g$
 Azimuth of course 4B = (Az.34+Angle to the right at point 4) – $200^g = (65 + 250) - 200 = 115^g$
 Dep.A1 = $S_{A1} \cdot \sin(Az.A1) = 17 \cdot \sin(55^g) = 12.93$ m. Lat.A1 = $S_{A1} \cdot \cos(Az.A1) = 17 \cdot \cos(55^g) = 11.04$ m.
 Dep.12 = $S_{12} \cdot \sin(Az.12) = 15 \cdot \sin(75^g) = 13.86$ m. Lat.12 = $S_{12} \cdot \cos(Az.12) = 15 \cdot \cos(75^g) = 5.74$ m.
 Dep.23 = $S_{23} \cdot \sin(Az.23) = 28 \cdot \sin(205^g) = -2.20$ m. Lat.23 = $S_{23} \cdot \cos(Az.23) = 28 \cdot \cos(205^g) = -27.91$ m.
 Dep.34 = $S_{34} \cdot \sin(Az.34) = 22 \cdot \sin(65^g) = 18.76$ m. Lat.34 = $S_{34} \cdot \cos(Az.34) = 22 \cdot \cos(65^g) = 11.49$ m.
 Dep.4B = $S_{4B} \cdot \sin(Az.4B) = 20 \cdot \sin(115^g) = 19.45$ m. Lat.4B = $S_{4B} \cdot \cos(Az.4B) = 20 \cdot \cos(115^g) = -4.67$ m.

$$\begin{aligned}
 E_1 &= E_A + \text{Dep.} A1 = 170 + 12.93 = 182.93 \text{ m.} \\
 E_2 &= E_1 + \text{Dep.} 12 = 182.93 + 13.86 = 196.79 \text{ m.} \\
 E_3 &= E_2 + \text{Dep.} 23 = 196.79 - 2.20 = 194.59 \text{ m.} \\
 E_4 &= E_3 + \text{Dep.} 34 = 194.59 + 18.76 = 213.35 \text{ m.} \\
 E_B &= E_4 + \text{Dep.} 4B = 213.35 + 19.45 = 232.80 \text{ m.}
 \end{aligned}$$

$$\begin{aligned}
 N_1 &= N_A + \text{Lat.} A1 = 125 + 11.04 = 136.04 \text{ m.} \\
 N_2 &= N_1 + \text{Lat.} 12 = 136.04 + 5.74 = 141.78 \text{ m.} \\
 N_3 &= N_2 + \text{Lat.} 23 = 141.78 - 27.91 = 113.87 \text{ m.} \\
 N_4 &= N_3 + \text{Lat.} 34 = 113.87 + 11.49 = 125.36 \text{ m.} \\
 N_B &= N_4 + \text{Lat.} 4B = 125.36 - 4.67 = 120.69 \text{ m.}
 \end{aligned}$$

b) Linear misclose at B = $(\text{Misclose in Dep.}^2 + \text{Misclose in Lat.}^2)^{1/2}$

Misclose in Dep. = Comp. Dep. of B – Given Dep. of B = 232.80 - 232.60 = 0.20 m.

Misclose in Lat. = Comp. Lat. of B – Given Lat. of B = 120.69 - 121.00 = -0.31 m.

Linear misclose at B = $(0.20^2 + 0.31^2)^{1/2} = 0.37 \text{ m.}$

c) Accuracy = $1 / (\text{Total course length} / \text{Linear Misclose}) = 1 / (102 / 0.37) = 1 / 275$

d) $dE_{A1} = -\text{Dep. Misc.} * (S_{A1} / \Sigma S) = -0.20 * (17 / 102) = -0.03 \text{ m.}$

$dN_{A1} = \text{Lat. Misc.} * (S_{A1} / \Sigma S) = 0.31 * (17 / 102) = 0.05 \text{ m.}$

$dE_{12} = -\text{Dep. Misc.} * (S_{12} / \Sigma S) = -0.20 * (15 / 102) = -0.03 \text{ m.}$

$dN_{12} = \text{Lat. Misc.} * (S_{12} / \Sigma S) = 0.31 * (15 / 102) = 0.05 \text{ m.}$

$dE_{23} = -\text{Dep. Misc.} * (S_{23} / \Sigma S) = -0.20 * (28 / 102) = -0.06 \text{ m.}$

$dN_{23} = \text{Lat. Misc.} * (S_{23} / \Sigma S) = 0.31 * (28 / 102) = 0.08 \text{ m.}$

$dE_{34} = -\text{Dep. Misc.} * (S_{34} / \Sigma S) = -0.20 * (22 / 102) = -0.04 \text{ m.}$

$dN_{34} = \text{Lat. Misc.} * (S_{34} / \Sigma S) = 0.31 * (22 / 102) = 0.07 \text{ m.}$

$dE_{4B} = -\text{Dep. Misc.} * (S_{4B} / \Sigma S) = -0.20 * (20 / 102) = -0.04 \text{ m.}$

$dN_{4B} = \text{Lat. Misc.} * (S_{4B} / \Sigma S) = 0.31 * (20 / 102) = 0.06 \text{ m.}$

e) Adj. Dep. A1 = Comp. Dep. A1 + $dE_{A1} = 12.93 - 0.03 = 12.90 \text{ m.}$

Adj. $E_1 = 170.00 + 12.90 = 182.90 \text{ m.}$

Adj. Lat. A1 = Comp. Lat. A1 + $dN_{A1} = 11.04 + 0.05 = 11.09 \text{ m.}$

Adj. $N_1 = 125.00 + 11.09 = 136.09 \text{ m.}$

Adj. Dep. 12 = Comp. Dep. 12 + $dE_{12} = 13.86 - 0.03 = 13.83 \text{ m.}$

Adj. $E_2 = 182.90 + 13.83 = 196.73 \text{ m.}$

Adj. Lat. 12 = Comp. Lat. 12 + $dN_{12} = 5.74 + 0.05 = 5.79 \text{ m.}$

Adj. $N_2 = 136.09 + 5.79 = 141.88 \text{ m.}$

Adj. Dep. 23 = Comp. Dep. 23 + $dE_{23} = -2.20 - 0.06 = -2.26 \text{ m.}$

Adj. $E_3 = 196.73 - 2.26 = 194.47 \text{ m.}$

Adj. Lat. 23 = Comp. Lat. 23 + $dN_{23} = -27.91 + 0.08 = -27.83 \text{ m.}$

Adj. $N_3 = 141.88 - 27.83 = 114.05 \text{ m.}$

Adj. Dep. 34 = Comp. Dep. 34 + $dE_{34} = 18.76 - 0.04 = 18.72 \text{ m.}$

Adj. $E_4 = 194.47 + 18.72 = 213.19 \text{ m.}$

Adj. Lat. 34 = Comp. Lat. 34 + $dN_{34} = 11.49 + 0.07 = 11.56 \text{ m.}$

Adj. $N_4 = 114.05 + 11.56 = 125.61 \text{ m.}$

Adj. Dep. 4B = Comp. Dep. 4B + $dE_{4B} = 19.45 - 0.04 = 19.41 \text{ m.}$

Adj. $E_B = 213.19 + 19.41 = 232.60 \text{ m.}$

Adj. Lat. 4B = Comp. Lat. 4B + $dN_{4B} = -4.67 + 0.06 = -4.61 \text{ m.}$

Adj. $N_B = 125.61 - 4.61 = 121.00 \text{ m.}$

6.4. Following measurements (Azimuth angles and lengths) have been taken from an open traverse measurement. By using given data, determine the distance and bearing angle of the course AE. Prepare a table for the results in a logical order.

Course	Azimuth (grad)	Length (meter)
AB	250	78
BC	75	152
CD	170	90
DE	380	126

Solution :

Assume point A = 0; 0

Assume a scale = 1/2000 to draw.

20 meter = 1 cm.

Departure of a side

$\Delta E = \text{Length} * \sin(\text{Azimuth})$

Latitude of a side

$\Delta N = \text{Length} * \cos(\text{Azimuth})$

Easting B = Easting A + ΔE

and so on

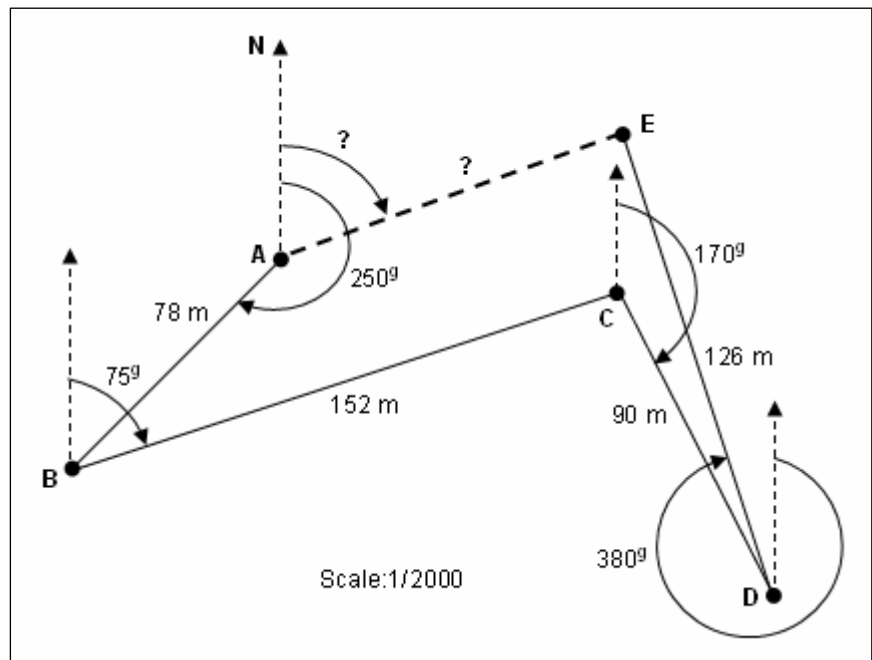
Northing B = Northing A + ΔN and so on

$\Delta E_{AB} = 78 * \sin 250^\circ = -55.15 \text{ m}$

$\Delta N_{AB} = 78 * \cos 250^\circ = -55.15 \text{ m}$

$E_B = 0.00 + (-55.15) = -55.15 \text{ m}$

$N_B = 0.00 + (-55.15) = -55.15 \text{ m}$



$\Sigma \Delta E = -55.15 + 140.43 + 40.86 - 38.94 = 87.20 \text{ m}$ (Easting of point E)

$\Sigma \Delta N = -55.15 + 58.17 - 80.19 + 119.83 = 42.66 \text{ m}$ (Northing of point E)

$\alpha_{AE} = \tan^{-1}[\Delta E_{AE} / \Delta N_{AE}] = \tan^{-1}[87.20 / 42.66] = \tan^{-1}(2.044) = 71.04^\circ$

Distance AE = $[\Delta E_{AE}^2 + \Delta N_{AE}^2]^{1/2} = [87.20^2 + 42.66^2]^{1/2} = (9423.7)^{1/2}$

\Rightarrow Bearing AE = N 71.04° E

\Rightarrow Distance AE = 97.08 meter

Pnt	Side	Length (m)	Azimuth (grad)	ΔE (m)	ΔN (m)	Easting (m)	Northing (m)
A						0.00	0.00
	AB	78	250	-55.15	-55.15		
B						-55.15	-55.15
	BC	152	75	140.43	58.17		
C						85.28	3.02
	CD	90	170	40.86	-80.19		
D						126.14	-77.17
	DA	126	380	-38.94	119.83		
E						87.20	42.66

- 6.5. Angles and distances of an open traverse are given as follows. Also the coordinates of point A as 200E and 200N, point D as 348.01E and 387.87N, and the azimuth of side AB as 12.00^g are given. Compute;
- Coordinate misclose (ΔE and ΔN) at point D,
 - Linear misclosure, its azimuth and accuracy of measurement,
 - Adjusted coordinates by applying compass rule.

Solution :

Pnt	Side	Length (m)	Angle to the right (gon)	Azimuth (gon)	Departure (m)	Latitude (m)	Computed Easting (m)	Computed Northing (m)	Adjusted Departure (m)	Adjusted Latitude (m)	Adjusted Easting (m)	Adjusted Northing (m)
A							200.00	200.00			200.00	200.00
	AB	87.00		12.00^g	16.30	85.46			16.39	85.29		
B			286^g				216.30	285.46			216.39	285.29
	BC	58.00		98.00^g	57.97	1.82			58.03	1.70		
C			142^g				274.27	287.28			274.42	286.99
	CD	125.00		40.00^g	73.47	101.13			73.59	100.88		
D							347.74	388.41			348.01	387.87
		270.00			147.74	188.41			148.01	187.87		

- a) Azimuth AB= 12^g (given)
Azimuth BC= $12^g+286^g-200^g=98^g$
Azimuth CD= $98^g+142^g-200^g=40^g$
Departure AB= $87 \cdot \sin 12^g = 16.30$ m. Latitude AB= $87 \cdot \cos 12^g = 85.46$ m.
Easting AB= $200+16.30=216.30$ m Northing AB= $200+85.46=285.46$ m
Departure BC= $58 \cdot \sin 98^g = 57.97$ m. Latitude BC= $58 \cdot \cos 98^g = 1.82$ m.
Easting BC= $216.30+57.97=274.27$ m Northing BC= $285.46+1.82=287.28$ m
Departure CD= $125 \cdot \sin 40^g = 73.47$ m. Latitude CD= $125 \cdot \cos 40^g = 101.13$ m.
Easting AB= $274.27+73.47=347.74$ m Northing AB= $287.28+101.13=388.41$ m
Error in departure (ΔE) = $347.74-348.01=-0.27$ m
Error in latitude (ΔN) = $388.41-387.87=0.54$ m
- b) Linear misclose = $[(\Delta E)^2 + (\Delta N)^2]^{1/2} = [(0.27)^2 + (0.54)^2]^{1/2} = [0.365]^{1/2} = 0.60$ m
Its azimuth = $\tan^{-1}[(\Delta E)/(\Delta N)] = \tan^{-1}[(-0.27)/(0.54)] = \tan^{-1}[-0.5] = -29.5^g$
 ΔE is negative and ΔN is positive, therefore Azimuth = $-29.5^g + 200 = 170.50^g$
Accuracy = $1/(270/0.6) = 1/450$
- c) Correction in departure = $0.27/270 = 0.001$ m/m Correction in latitude = $-0.54/270 = -0.002$ m/m
Adjusted departure AB = $16.30 + 0.001 \cdot 87 = 16.39$ m Adjusted latitude AB = $85.46 - 0.002 \cdot 87 = 85.29$ m
Adjusted departure BC = $57.97 + 0.001 \cdot 58 = 58.03$ m Adjusted latitude BC = $1.82 - 0.002 \cdot 58 = 1.70$ m
Adjusted departure CD = $73.47 + 0.001 \cdot 125 = 73.59$ m Adjusted latitude CD = $101.13 - 0.002 \cdot 125 = 100.88$ m

PART 7 : CLOSED TRAVERSE

7.1. Angles and distances of a four sided closed traverse are measured in clockwise direction as given. Also the coordinate of point A as 100E and 100N, and the azimuth of side AB as 12.1883^g are given. Compute;

- Misclose of angles,
- Adjusted angles,
- Azimuth of the sides,
- Linear misclosure, its azimuth and accuracy of measurement,
- Adjusted coordinates.

Solution :

Pnt	Side	Length (m)	Angle to the right (gon)	Adjusted angle to the right (gon)	Azimuth (gon)	Departure (m)	Latitude (m)	Computed Easting (m)	Computed Northing (m)	Adjusted Departure (m)	Adjusted Latitude (m)	Adjusted Easting (m)	Adjusted Northing (m)
A			277.8904^g	277.8935^g				100.00	100.00			100.00	100.00
	AB	87.30			12.1883^g	16.61	85.70			16.63	85.69		
B			289.7284^g	289.7315^g				116.61	185.70			116.63	185.69
	BC	58.45			12.1883^g	58.42	-1.76			58.43	-1.77		
C			299.1775^g	299.1806^g				175.03	183.94			175.06	183.92
	CD	127.48			101.9198^g	-2.20	-127.46			-2.18	-127.48		
D			333.1913^g	333.1944^g				172.83	56.48			172.88	56.44
	DA	84.92			201.1004^g	-72.89	43.57			-72.88	43.56		
A								99.94	100.05			100.00	100.00
TOTAL		358.15	1199.9876^g	1200.00^g	334.2948^g	-0.06	0.05	-0.06	0.05	0.00	0.00	0.00	0.00

Coloured cells denote the given values.

a) If traverse is worked in clockwise direction, then angle to the right is exterior angle at related point. Therefore sum of these angles should satisfy $200^g(n+2)$. Therefore;
 $200^g(4+2) = 1200^g$ Then Angular misclose = $1199.9876^g - 1200^g = 0.0124^g$

b) Correction for each angle = $0.0124^g / 4 = 0.0031^g$ (equally distributed)
 Adjusted angle = $277.8904^g + 0.0031^g = 277.8935^g$, and the similar calculations for the others.

c) Azimuth of any point = Previous azimuth + Angle to the right – 200^g
 Az. BC = $12.1883^g + 289.7315^g - 200^g = 101.9198^g$
 Az. CD = $101.9198^g + 299.1806^g - 200^g = 201.1004^g$
 Az. DA = $201.1004^g + 333.1944^g - 200^g = 334.2948^g$
 Az. AB = $334.2948^g + 277.8935^g - 200^g = 412.1883^g = 12.1883^g$ (to check)

d) Departure = Side length * sin (Az.) Latitude = Side length * cos (Az.)
 For AB, Dep.= $87.30 \times \sin(12.1883^g) = 16.61$ m. Lat.= $87.30 \times \cos(12.1883^g) = 85.70$ m.
 For BC, Dep.= $58.45 \times \sin(101.9198^g) = 58.42$ m. Lat.= $58.45 \times \cos(101.9198^g) = -1.76$ m.
 For CD, Dep.= $127.48 \times \sin(201.1004^g) = -2.20$ m. Lat.= $127.48 \times \cos(201.1004^g) = -127.46$ m.
 For DA, Dep.= $84.92 \times \sin(334.2948^g) = -72.89$ m. Lat.= $84.92 \times \cos(334.2948^g) = 43.57$ m.
 Σ Departure = -0.06 m. Σ Latitude = 0.05 m. (Error in closure)
 Linear misclose = $[(-0.06)^2 + (0.05)^2]^{1/2} = 0.078$ meter.
 Azimuth = $\tan^{-1}(\Delta\text{Dep.}/\Delta\text{Lat.}) = -0.06/0.05 = -55.7716^g$ (if $\Delta\text{Dep.}$ "-" and $\Delta\text{Lat.}$ "+", add 400^g),
 then Azimuth = $-55.7716^g + 400.0000^g = 344.2284^g$
 Accuracy = $1 : (358.15/0.078) = 1:4591 = 1:4600$

e) Correction according to compass rule (Bodwitch Method)

For side AB; Correction for dep. = $\Delta\text{Dep.} \times \text{length}/\text{total perimeter} = 0.06 \times 87.30/358.15 \cong 0.02$ m.
 Correction for lat. = $\Delta\text{Lat.} \times \text{length}/\text{total perimeter} = 0.05 \times 87.30/358.15 \cong -0.01$ m.
 For side BC; Correction for dep. = $\Delta\text{Dep.} \times \text{length}/\text{total perimeter} = 0.06 \times 58.45/358.15 \cong 0.01$ m.
 Correction for lat. = $\Delta\text{Lat.} \times \text{length}/\text{total perimeter} = 0.05 \times 58.45/358.15 \cong -0.01$ m.
 For side CD; Correction for dep. = $\Delta\text{Dep.} \times \text{length}/\text{total perimeter} = 0.06 \times 127.48/358.15 \cong 0.02$ m.
 Correction for lat. = $\Delta\text{Lat.} \times \text{length}/\text{total perimeter} = 0.05 \times 127.48/358.15 \cong -0.02$ m.
 For side DA; Correction for dep. = $\Delta\text{Dep.} \times \text{length}/\text{total perimeter} = 0.06 \times 84.92/358.15 \cong 0.01$ m.
 Correction for lat. = $\Delta\text{Lat.} \times \text{length}/\text{total perimeter} = 0.05 \times 84.92/358.15 \cong -0.01$ m.

- 7.2. Angles and distances of a four sided closed traverse are measured in counterclockwise direction as given. Also the coordinates of point A as $E_A=100$ and $N_A=120$, and the bearing of side AB as $S62^\circ E$ are given. Compute;
- Misclose of angles,
 - Adjusted angles,
 - Azimuth of the sides,
 - Linear misclosure, its azimuth and accuracy of measurement,
 - Adjusted coordinates.

Solution :

Pnt	Side	Length (m)	Angle to the right ($^\circ$)	Adjusted angle to the right ($^\circ$)	Azimuth ($^\circ$)	Departure (m)	Latitude (m)	Computed Easting (m)	Computed Northing (m)	Adjusted Departure (m)	Adjusted Latitude (m)	Adjusted Easting (m)	Adjusted Northing (m)
A			93	92				100.00	120.00			100.00	120.00
	AB	40			118	35.32	-18.78			35.34	-18.87		
B			98	97				135.32	101.22			135.34	101.13
	BC	90			35	51.62	73.72			51.66	73.52		
C			43	42				186.94	174.94			187.00	174.65
	CD	70			257	-68.21	-15.75			-68.17	-15.91		
D			130	129				118.73	159.19			118.83	158.74
	DA	43			206	-18.85	-38.65			-18.83	-38.74		
A								99.88	120.54			100.00	120.00
TOTAL		243	364	360		-0.12	0.54			0.00	0.00		

These values are given

- If traverse is worked in counter-clockwise direction, then angle to the right is interior angle at related point. Therefore sum of these angles should satisfy $180^\circ(n-2)$. Thus;
 $180^\circ(4-2) = 360^\circ$ Then Angular misclose = $364^\circ - 360^\circ = 4^\circ$
- Correction for each angle = $-4^\circ / 4 = -1^\circ$ (equally distributed)
Adjusted angle to the right = $93^\circ - 1^\circ = 92^\circ$, and the similar calculations for the others.
- Azimuth of AB = $180^\circ - \text{Bearing of AB} = 180^\circ - 62^\circ = 118^\circ$
Azimuth of any point = Previous azimuth + Angle to the right $\pm 180^\circ$
Az. BC = $118^\circ + 97^\circ - 180^\circ = 35^\circ$ Az. CD = $35^\circ + 43^\circ + 180^\circ = 257^\circ$
Az. DA = $257^\circ + 129^\circ - 180^\circ = 206^\circ$ Az. AB = $206^\circ + 92^\circ - 180^\circ = 118^\circ$ (to check)
- Departure = Side length * sin (Az.) Latitude = Side length * cos (Az.)
For AB, Dep.= $40 \times \sin 118 = 35.32$ m. Lat.= $40 \times \cos 118 = -18.78$ m.
For BC, Dep.= $90 \times \sin 35 = 51.62$ m. Lat.= $90 \times \cos 35 = 73.72$ m.
For CD, Dep.= $70 \times \sin 257 = -68.21$ m. Lat.= $70 \times \cos 257 = -15.75$ m.
For DA, Dep.= $43 \times \sin 206 = -18.85$ m. Lat.= $43 \times \cos 206 = -38.65$ m.
Computed Easting = Computed Easting of Previous Point + Departure = $100 + 35.32 = 135.32$ m.
Computed Northing = Computed Northing of Previous Point + Latitude = $120 + (-18.87) = 101.22$ m.
 $\Sigma \text{Departure} = -0.12$ m. $\Sigma \text{Latitude} = 0.54$ m. (Error in closure)
Linear misclose = $[(-0.12)^2 + (0.54)^2]^{1/2} = 0.553$ meter.
Azimuth = $\tan^{-1} (\Delta \text{Dep.}) / (\Delta \text{Lat.}) = \tan^{-1} (-0.12/0.54) = -12.53^\circ$ (if ΔDep "-" and ΔLat "+", add 400°),
then Azimuth = $-12.53^\circ + 360^\circ = 347.47^\circ$
Accuracy = $1 : (243/0.553) = 1:459 = 1:460$
- Correction according to compass rule (Bodwitch Method)
For side AB; Correction for dep. = $\Delta \text{Dep.} \times \text{length} / \text{total perimeter} = 0.12 \times 40 / 243 = 0.02$ m.
Correction for lat. = $\Delta \text{Lat.} \times \text{length} / \text{total perimeter} = -0.54 \times 40 / 243 = -0.09$ m.
For side BC; Correction for dep. = $\Delta \text{Dep.} \times \text{length} / \text{total perimeter} = 0.12 \times 90 / 243 = 0.04$ m.
Correction for lat. = $\Delta \text{Lat.} \times \text{length} / \text{total perimeter} = -0.54 \times 90 / 243 = -0.20$ m.
For side CD; Correction for dep. = $\Delta \text{Dep.} \times \text{length} / \text{total perimeter} = 0.12 \times 70 / 243 = 0.04$ m.
Correction for lat. = $\Delta \text{Lat.} \times \text{length} / \text{total perimeter} = -0.54 \times 70 / 243 = -0.16$ m.
For side DA; Correction for dep. = $\Delta \text{Dep.} \times \text{length} / \text{total perimeter} = 0.12 \times 43 / 243 = 0.02$ m.
Correction for lat. = $\Delta \text{Lat.} \times \text{length} / \text{total perimeter} = -0.54 \times 43 / 243 = -0.09$ m.
Adjusted Departure = Departure + Correction for Departure = $35.32 + 0.02 = 35.34$ m.
Adjusted Latitude = Latitude + Correction for Latitude = $-18.78 + (-0.09) = -18.87$ m.
Adjusted Easting = Adjusted Easting of Previous Point + Adjusted Departure = $100 + 35.34 = 135.34$ m.
Adjusted Northing = Adjusted Northing of Previous Point + Adjusted Latitude = $120 + (-18.87) = 101.13$ m.

7.3. A four sided closed traverse was measured and the following readings are obtained. The instrument is adjusted at a point, to find the interior angles, first foresight angle and then backsight angle were measured by turning the instrument in clockwise direction.

- Calculate angular misclose and then adjust the interior angles.
- Compute azimuth angles according to adjusted interior angles.
- Compute the coordinates of each point starting with given coordinates of point A and find linear misclose at point A.
- Determine accuracy of the measurement.
- Adjust departure (ΔE) and latitude (ΔN) for each side according to compass rule.
- Determine corrected coordinates of the points.

Azimuth of AB = 60°

Northing of A : 120 m.

Easting of A : -84 m.

Point	Foresight (grad)	Backsight (grad)	
A	362	65	Distance AB = 40 meter
B	218	327	Distance BC = 90 meter
C	103	151	Distance CD = 70 meter
D	372	116	Distance DA = 43 meter

Solution :

a) First we find the interior angles. In general;

Interior angle = Backsight angle - Foresight angle (!Add 400° if negative)

Interior angle at point A = $(65 - 362) + 400 = 103^\circ$

Interior angle at point B = $(327 - 218) = 109^\circ$

Interior angle at point C = $(151 - 103) = 48^\circ$

Interior angle at point D = $(116 - 372) + 400 = 144^\circ$

SUM OF THE INTERIOR ANGLES = $103 + 109 + 48 + 144 = 404^\circ$

Sum of these angles should satisfy $200^\circ(n-2)$. Therefore;

$200^\circ(4-2) = 400^\circ$ Then **Angular misclose = $404^\circ - 400^\circ = 4^\circ$**

Correction for each interior angle = $-4^\circ/4 = -1^\circ$ (equally distributed)

Adjusted interior angle at point A = $103^\circ - 1^\circ = 102^\circ$, and the similar calculations for the others.

b) Azimuth of AB = 60°

Azimuth of a side = Previous azimuth $\pm 200^\circ$ - Interior angle

Az. BC = $60^\circ + 200^\circ - 108^\circ = 152^\circ$

Az. CD = $152^\circ + 200^\circ - 47^\circ = 305^\circ$

Az. DA = $305^\circ + 200^\circ - 143^\circ = 362^\circ$

Az. AB = $362^\circ - 200^\circ - 102^\circ = 60^\circ$ **(to check)**

c) Departure = Side length * sin (Az.)

Latitude = Side length * cos (Az.)

For AB, Dep. = $40 \sin 60 = 32.361$ m.

Lat. = $40 \cos 60 = 23.511$ m.

For BC, Dep. = $90 \sin 152 = 61.609$ m.

Lat. = $90 \cos 152 = -65.607$ m.

For CD, Dep. = $70 \sin 305 = -69.784$ m.

Lat. = $70 \cos 305 = 5.492$ m.

For DA, Dep. = $43 \sin 362 = -24.170$ m.

Lat. = $43 \cos 362 = 35.564$ m.

Σ Departure = 0.016 m.

Σ Latitude = -1.039 m. (Error in closure)

Linear misclose = $[(0.016)^2 + (1.039)^2]^{1/2} = 1.039$ meter

Computed Easting = Computed Easting of Previous Point + Departure = $-84 + 32.361 = -51.639$ m.

Computed Northing = Computed Northing of Previous Point + Latitude = $120 + 23.511 = 143.511$ m.

d) **Accuracy = 1 : (243/1.039) = 1:234**

e) Correction according to compass rule (Bodwitch Method)

For side AB; Corr. for dep. = $\Delta \text{Dep.} \cdot \text{length} / \text{total perimeter} = 40 \cdot (-0.016 / 243) = -0.003$ m.

Corr. for lat. = $\Delta \text{Lat.} \cdot \text{length} / \text{total perimeter} = 40 \cdot (1.039 / 243) = 0.171$ m.

For side BC; Corr. for dep. = $\Delta \text{Dep.} \cdot \text{length} / \text{total perimeter} = 90 \cdot (-0.016 / 243) = -0.006$ m.

Corr. for lat. = $\Delta \text{Lat.} \cdot \text{length} / \text{total perimeter} = 90 \cdot (1.039 / 243) = 0.385$ m.

For side CD; Corr. for dep. = $\Delta \text{Dep.} \cdot \text{length} / \text{total perimeter} = 70 \cdot (-0.016 / 243) = -0.005$ m.

Corr. for lat. = $\Delta \text{Lat.} \cdot \text{length} / \text{total perimeter} = 70 \cdot (1.039 / 243) = 0.299$ m.

For side DA; Corr. for dep. = $\Delta \text{Dep.} \cdot \text{length} / \text{total perimeter} = 43 \cdot (-0.016 / 243) = -0.003$ m.

Corr. for lat. = $\Delta \text{Lat.} \cdot \text{length} / \text{total perimeter} = 43 \cdot (1.039 / 243) = 0.184$ m.

Adjusted Departure = Departure + Correction for Departure = $32.361 - 0.003 = 32.358$ m.

Adjusted Latitude = Latitude + Correction for Latitude = $23.511 + 0.171 = 23.682$ m.

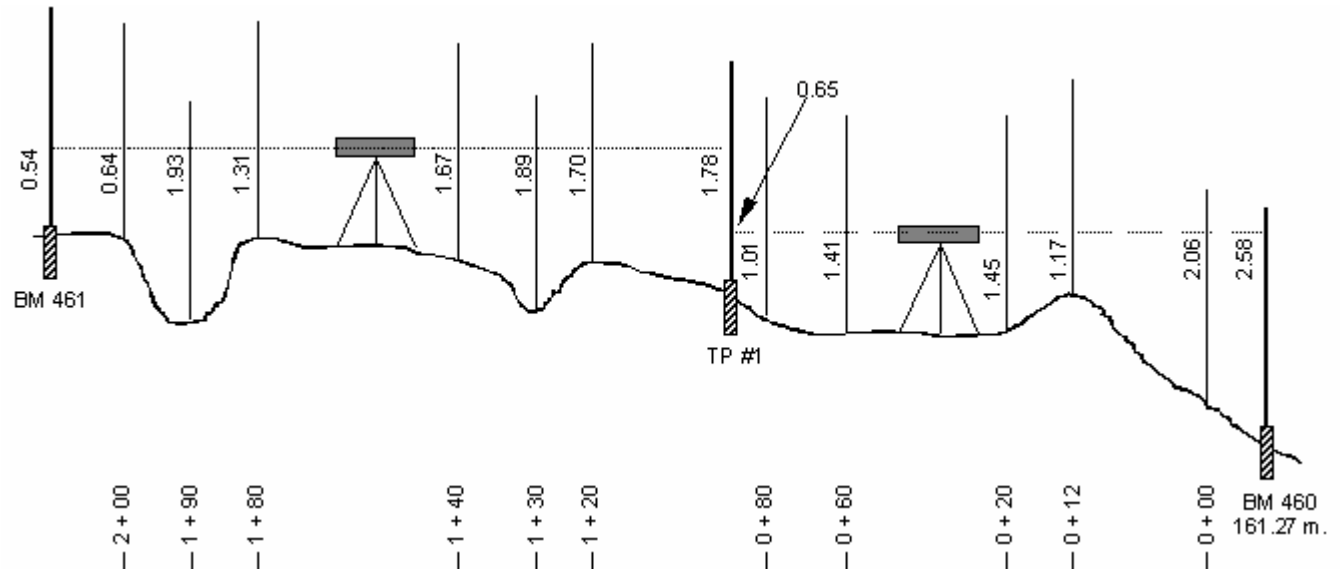
f) Adjusted E = Adjusted E of Previous Point+Adjusted Departure = -84.000+32.358= -51.642 m.
Adjusted N = Adjusted N of Previous Point+Adjusted Latitude = 120+23.682 = 143.682 m.

Pnt	Side	Length (m)	Fore sight (grad)	Back sight (grad)	Inter. angle (grad)	Adj. Inter. angle (grad)	Adjust. Azimuth (grad)	COMPUTED				ADJUSTED			
								Depar- ture (m)	Latitude (m)	Easting (m)	Northing (m)	Depar- ture (m)	Latitude (m)	Easting (m)	Northing (m)
A			362	65	103	102				-84,000	120,000			-84,000	120,000
	AB	40					60	32,361	23,511			32,358	23,682		
B			218	327	109	108				-51,639	143,511			-51,642	143,682
	BC	90					152	61,609	-65,607			61,603	-65,222		
C			103	151	48	47				9,970	77,904			9,961	78,460
	CD	70					305	-69,784	5,492			-69,789	5,792		
D			372	116	144	143				-59,814	83,396			-59,828	84,252
	DA	43					362	-24,170	35,564			-24,172	35,748		
A										-83,984	118,961			-84,000	120,000
SUM		243			404	400		0.016	-1.039			0.000	0.000		

PART 8 : LEVELING

8.1. A profile leveling measurement is conducted starting from station BM460 (with elevation 161.27 meter) and ending at station BM461 (with elevation 164.49 meter) as illustrated in the following figure. TP #1 is 100 m away from BM460 and 140 m away from BM461. First, prepare a set of notes (Station, BS, IS, FS) similar to the table given, and then compute HI, ΔH (change in elevation) and elevation of each station. Lastly, check the elevation of BM461 and compute adjusted elevations if you recognise any error in elevation of BM461 at the end of computation. (Warning: All numbers are in meter).

NOTE: Do your calculations on a different page and write the results in the table.



Station	BS (m)	HI (m)	IS (m)	FS (m)	$\Delta H = BS - FS$ (m)	Computed elevation (m)	Corrected ΔH (m)	Adjusted elevation (m)
BM 460	2.58	163.85				161.27		161.27
0 + 00			2.06		+0.52	161.79	+0.54	161.81
0 + 12			1.17		+1.41	162.68	+1.43	162.70
0 + 20			1.45		+1.13	162.40	+1.15	162.42
0 + 60			1.41		+1.17	162.44	+1.19	162.46
0 + 80			1.01		+1.57	162.84	+1.59	162.86
TP #1	1.78	164.98		0.65	+1.93	163.20	+1.95	163.22
1 + 20			1.70		+0.08	163.28	+0.11	163.33
1 + 30			1.89		-0.11	163.09	-0.08	163.14
1 + 40			1.67		+0.11	163.31	+0.14	163.36
1 + 80			1.31		+0.47	163.67	+0.50	163.72
1 + 90			1.93		-0.15	163.05	-0.12	163.10
2 + 00			0.64		+1.14	164.34	+1.17	164.39
BM 461				0.54	+1.24	164.44	+1.27	164.49

BS: Backsight; HI: Height of instrument; IS: Inner sight; FS: Foresight

Given values

Computed values

Solution :

HI at first position of instrument = $161.27 + 2.58 = 163.85$ m.

HI at second position of instrument = $163.20 + 1.78 = 164.98$ m.

Compute (BS-IS) or (BS-FS) to find change in elevation (ΔH) between two preceding stations.

e.g., at station 0+00 $\rightarrow \Delta H = BS - IS = 2.58 - 2.06 = 0.52$ m

Look for $\Sigma BS - \Sigma FS = (\text{Elevation of BM461} - \text{Elevation of BM460})$ must be satisfied for computation.

$$\Sigma BS - \Sigma FS = (2.58 + 1.78) - (0.65 + 0.54) = 4.36 - 1.19 = 3.17 \text{ m.}$$

$$\Delta H = 164.44 - 161.27 = 3.17 \text{ m.}$$

The elevation of BM461 was given as 164.49 m. Therefore, closure error = $164.44 - 164.49 = -0.05 \text{ m.}$

Correction according to distances; Total distance = $100 + 140 = 240 \text{ m.}$

$$\text{Correction for the first instrument position (BM460-TP\#1)} = (0.05/240) \times 100 = 0.021 \cong 0.02 \text{ m.}$$

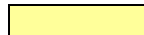
$$\text{Correction for the second instrument position (BM461-TP\#1)} = (0.05/240) \times 140 = 0.029 \cong 0.03 \text{ m.}$$

$$\text{Corrected } \Delta H = \text{Computed } \Delta H + \text{correction}$$

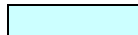
8.2. Reduce the accompanying set of profile notes (HI, ΔH , Elevation) and perform the arithmetic check.
(Show details of your calculations, write the results in the table)

Station	BS (m)	HI (m)	IS (m)	FS (m)	ΔH (m)	Elevation (m)
BM 20	3.27	303.05				299.78
TP #1	4.21	303.88		2.60	0.67	300.45
0 + 00			3.30		0.91	301.36
0 + 50			2.60		0.70	302.06
0 + 80			3.71		-1.11	300.95
1 + 00			1.10		2.61	303.56
TP #2	2.33	304.55		1.66	-0.56	303.00
1 + 50			3.80		-1.47	301.53
2 + 00			2.97		0.83	302.36
BM 21				3.88	-0.91	301.45
TOTAL	9.81			8.14	1.67	

BS: Backsight; HI: Height of instrument; IS: Inner sight; FS: Foresight



Given values



Computed values

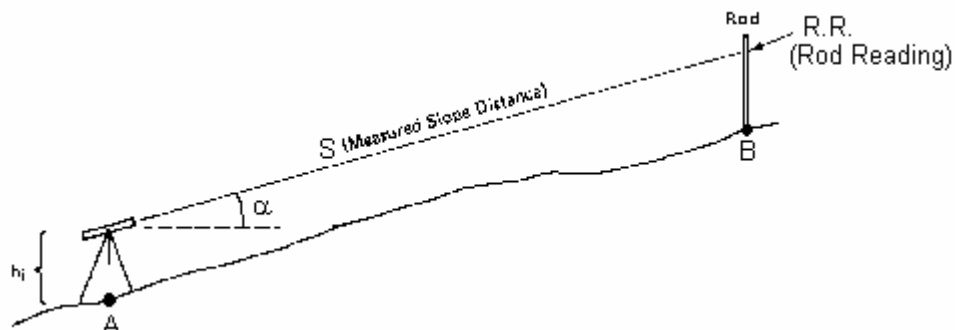
Arithmetic check $\rightarrow \Sigma BS - \Sigma FS = 9.81 - 8.14 = 1.67 \text{ m.}$ is equal to $\Sigma \Delta H = 1.67 \text{ m.}$

$$\text{Also BM21-BM20} = 301.45 - 299.78 = 1.67 \text{ m.}$$

8.3. If the following data are given for trigonometric leveling measurement as seen in the figure, determine the elevation at point B.

Elevation of point A = 423.45 m.

$$S = 82 \text{ m.} \quad h_i = 1.42 \text{ m.} \quad RR = 4.56 \text{ m.} \quad \alpha = 24^\circ$$



Solution :

$$V = S \sin 24^\circ = 82 \sin 24^\circ = 33.35 \text{ m.}$$

$$\text{Ele. Point B} = \text{Ele. Point A} + h_i + V - RR = 423.45 + 1.42 + 33.35 - 4.56 = 453.66 \text{ m.}$$

8.4. Elevations of the bench marks BM20 and BM21 are given as 299.780 m and 299.630 m respectively. Determine the elevations of the points and check it for point BM21. If the given elevation is not obtained, make necessary corrections according to compass rule and make necessary arithmetic checks. (Show details of your calculations, write the results in the table).

Point	Hori. Dist. (m)	BS (m)	FS (m)	ΔH (m)	Elevation (m)	Corrected ΔH (m)	Corrected Elevation (m)
BM 20		1.780			299.780		299.780
	50.00			-0.350		-0.384	
A		0.570	2.130		299.430		299.396
	68.00			-0.130		-0.176	
B		1.670	0.700		299.300		299.220
	34.00			0.920		0.897	
C		1.090	0.750		300.220		300.117
	82.00			-1.190		-1.246	
D		1.730	2.280		299.030		298.871
	76.00			0.810		0.759	
BM 21			0.920		299.840		299.630
TOTAL	310.00	6.840	6.780	0.060		-0.150	

Solution :

Arithmetic check $\rightarrow \Sigma BS - \Sigma FS = 6.84 - 6.78 = 0.06$ m. is equal to $\Sigma \Delta H = 0.06$ m.

Also $BM21 - BM20 = 299.84 - 299.78 = 0.06$ m.

Elevation of BM21 is given as 299.63 m. Therefore, error is;

Error in BM21 = $299.84 - 299.63 = 0.21$ m.

Correlation = $-0.21/310 = -0.00068$ m for per meter of horizontal distance. Therefore

At line BM20&A, correlation = $-0.00068 \times 50 = -0.034$ m

At line A&B, correlation = $-0.00068 \times 68 = -0.046$ m

At line B&C, correlation = $-0.00068 \times 34 = -0.023$ m

At line C&D, correlation = $-0.00068 \times 82 = -0.056$ m

At line D&BM21, correlation = $-0.00068 \times 76 = -0.051$ m

TOTAL = -0.210 m (equals to total error)

Corrected $\Delta H = \Delta H + \text{correlation} = -0.350 + (-0.034) = -0.384$ m. and so on for the others.

Total Corrected $\Delta H = -0.150$ m. which is equal to

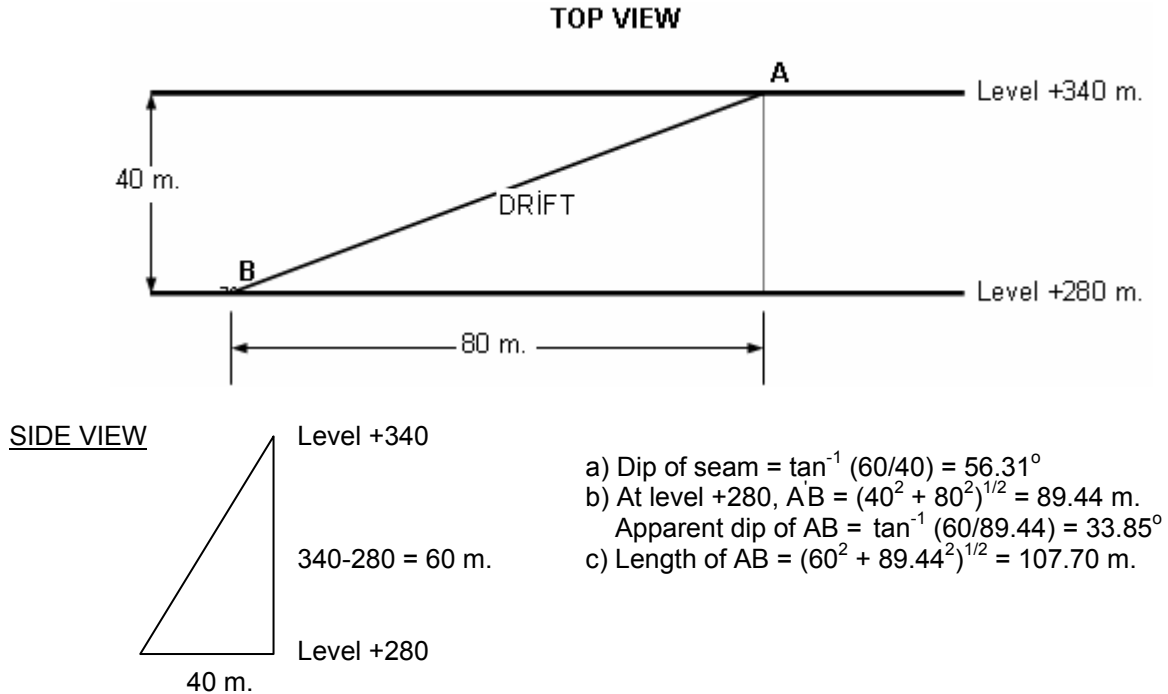
Corr. El. BM21 - El. BM21 = $299.630 - 299.720 = -0.150$ m.

After finding corrected ΔH values, corrected elevations are determined accordingly and the elevation of point BM21 is obtained as given (299.630)

PART 9 : DIP, VOLUME, CONTOUR

9.1. As seen from the figure (top view), two levels driven in an inclined coal seam are connected at points A and B with an inclined drift. Determine:

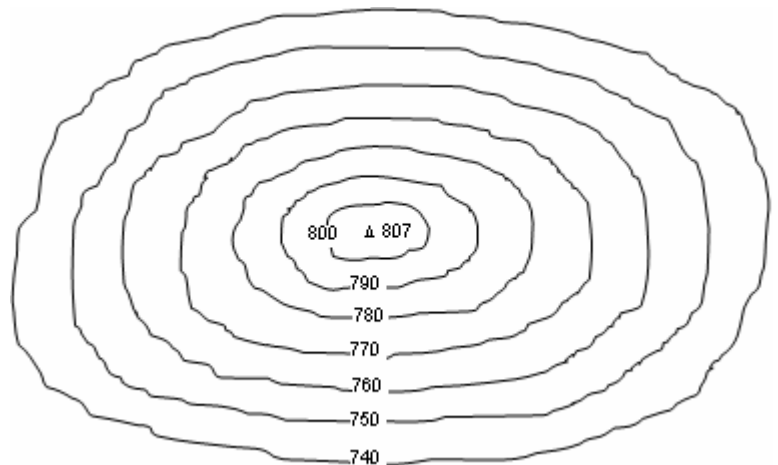
- Dip of the coal seam,
- Apparent dip of the drift AB,
- True length of the drift AB.



9.2. A coal deposit is lying horizontally between the elevations 748 and 752 m. with a regular thickness. Compute:

- Volume of coal in m^3 ,
- Volume of overburden above coal seam in m^3 .

Contour	It's area (m^2)
800 m.	620
790 m.	1700
780 m.	3800
770 m.	7900
760 m.	16000
752 m.	37000
750 m.	40000
748 m.	43000
740 m.	85000



Coal thickness = $752 - 748 = 4$ m.

Coal volume = $[(\text{Area } 752 + \text{Area } 748) / 2] \times \text{Thickness} = (\text{Area } 750 / 2) \times 4 = 40000 \times 4 = \mathbf{160\,000\,m^3}$.

Overburden volume = $[(\text{Area } 752 + \text{Area } 760) / 2] \times 8 \text{ m} + [(\text{Area } 760 + \text{Area } 770) / 2] \times 10 \text{ m} + [(\text{Area } 770 + \text{Area } 780) / 2] \times 10 \text{ m} + [(\text{Area } 780 + \text{Area } 790) / 2] \times 10 \text{ m} + [(\text{Area } 790 + \text{Area } 800) / 2] \times 10 \text{ m} + [(\text{Area } 800 + \text{Area } 807) / 2] \times 7 \text{ m}$
 $= [(37000 + 16000) / 2] \times 8 \text{ m} + [(16000 + 7900) / 2] \times 10 \text{ m} + [(7900 + 3800) / 2] \times 10 \text{ m} + [(3800 + 1700) / 2] \times 10 \text{ m} + [(1700 + 620) / 2] \times 10 \text{ m} + [(620 + 0) / 2] \times 7 \text{ m}$
 $= 212000 + 119500 + 58500 + 27500 + 11600 + 2170$

Overburden volume = $\mathbf{431270\,m^3}$.

9.3. A four sided closed traverse is measured to obtain contour map of the field. Elevation of the corner points and side lengths are given. Draw elevation contour lines with 1 m intervals (such as 101, 102, 103, etc.). A scale of 1/200 is used.

NOTE: Do your calculations on a different page and draw your lines on this figure.

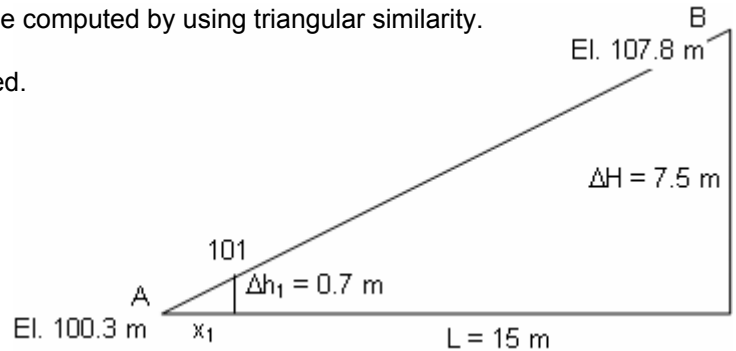
Side Lengths (meter)	Point Elevation (meter)
AB = 15	A = 100.3
BC = 23	B = 107.8
CD = 21.7	C = 96.3
DA = 27	D = 109.3

Horizontal distances of the contour lines from a point can be computed by using triangular similarity.

For line AB: (from figure)

$7.5/15 = \Delta h_1/x_1$ unknown x_1 can be easily computed.

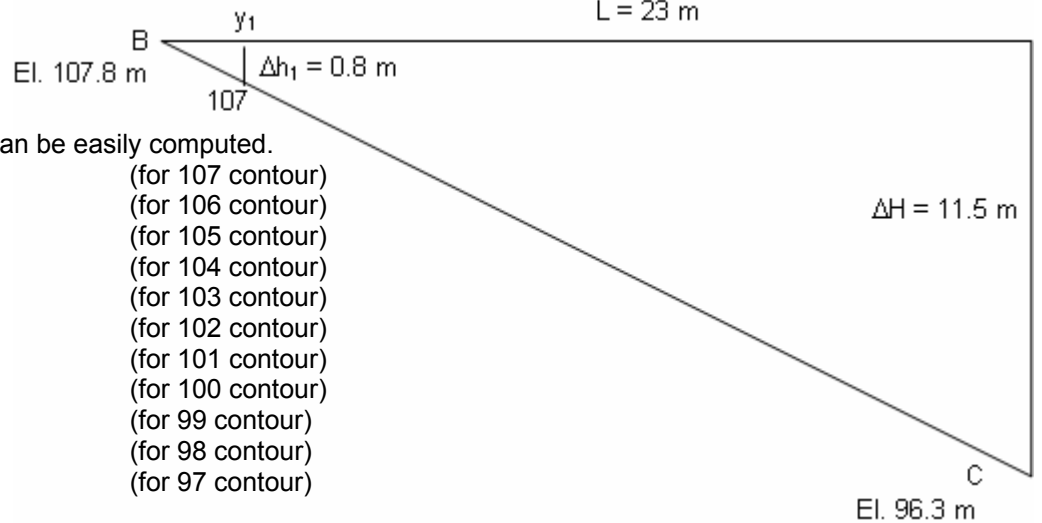
- $x_1 = 0.7 \times (15/7.5) = 1.4$ m. (for 101 contour)
- $x_2 = 1.7 \times (15/7.5) = 3.4$ m. (for 102 contour)
- $x_3 = 2.7 \times (15/7.5) = 5.4$ m. (for 103 contour)
- $x_4 = 3.7 \times (15/7.5) = 7.4$ m. (for 104 contour)
- $x_5 = 4.7 \times (15/7.5) = 9.4$ m. (for 105 contour)
- $x_7 = 5.7 \times (15/7.5) = 11.4$ m. (for 106 contour)
- $x_8 = 6.7 \times (15/7.5) = 13.4$ m. (for 107 contour)



For line BC: (from figure)

$11.5/23 = \Delta h_1/y_1$ unknown y_1 can be easily computed.

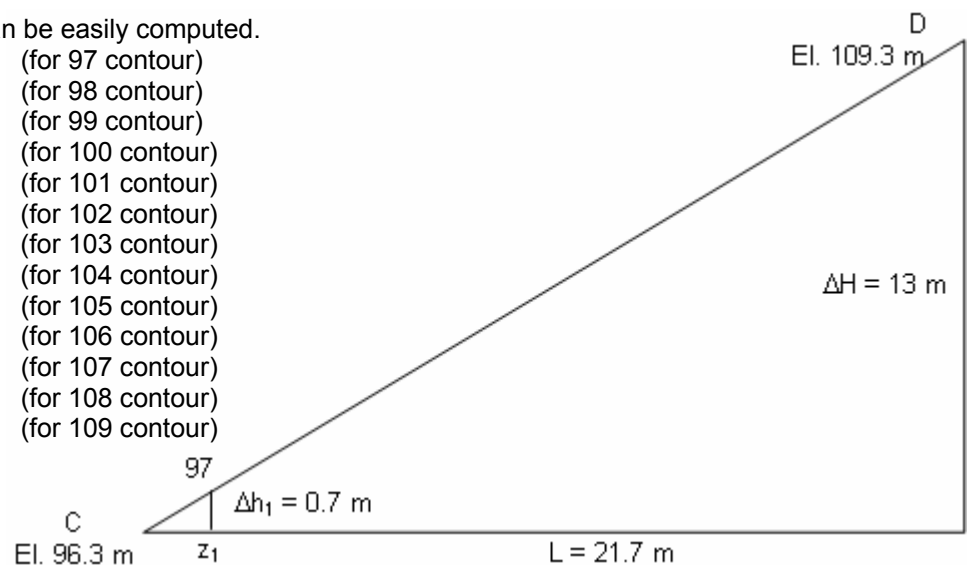
- $y_1 = 0.8 \times (23/11.5) = 1.6$ m. (for 107 contour)
- $y_2 = 1.8 \times (23/11.5) = 3.6$ m. (for 106 contour)
- $y_3 = 2.8 \times (23/11.5) = 5.6$ m. (for 105 contour)
- $y_4 = 3.8 \times (23/11.5) = 7.6$ m. (for 104 contour)
- $y_5 = 4.8 \times (23/11.5) = 9.6$ m. (for 103 contour)
- $y_6 = 5.8 \times (23/11.5) = 11.6$ m. (for 102 contour)
- $y_7 = 6.8 \times (23/11.5) = 13.6$ m. (for 101 contour)
- $y_8 = 7.8 \times (23/11.5) = 15.6$ m. (for 100 contour)
- $y_9 = 8.8 \times (23/11.5) = 17.6$ m. (for 99 contour)
- $y_{10} = 9.8 \times (23/11.5) = 19.6$ m. (for 98 contour)
- $y_{11} = 10.8 \times (23/11.5) = 21.6$ m. (for 97 contour)



For line CD: (from figure)

$13/21.7 = \Delta h_1/z_1$ unknown z_1 can be easily computed.

- $z_1 = 0.7 \times (21.7/13) = 1.2$ m. (for 97 contour)
- $z_2 = 1.7 \times (21.7/13) = 2.8$ m. (for 98 contour)
- $z_3 = 2.7 \times (21.7/13) = 4.5$ m. (for 99 contour)
- $z_4 = 3.7 \times (21.7/13) = 6.2$ m. (for 100 contour)
- $z_5 = 4.7 \times (21.7/13) = 7.8$ m. (for 101 contour)
- $z_7 = 5.7 \times (21.7/13) = 9.5$ m. (for 102 contour)
- $z_8 = 6.7 \times (21.7/13) = 11.2$ m. (for 103 contour)
- $z_9 = 7.7 \times (21.7/13) = 12.9$ m. (for 104 contour)
- $z_{10} = 8.7 \times (21.7/13) = 14.5$ m. (for 105 contour)
- $z_{11} = 9.7 \times (21.7/13) = 16.2$ m. (for 106 contour)
- $z_{12} = 10.7 \times (21.7/13) = 17.9$ m. (for 107 contour)
- $z_{13} = 11.7 \times (21.7/13) = 19.5$ m. (for 108 contour)
- $z_{14} = 12.7 \times (21.7/13) = 21.2$ m. (for 109 contour)



For line AD: (from figure)

$9/27 = \Delta h_1/k_1$ unknown x_1 can be easily computed.

$k_1 = 0.7 \cdot (27/9) = 2.1$ m. (for 101 contour)

$k_2 = 1.7 \cdot (27/9) = 5.1$ m. (for 102 contour)

$k_3 = 2.7 \cdot (27/9) = 8.1$ m. (for 103 contour)

$k_4 = 3.7 \cdot (27/9) = 11.1$ m. (for 104 contour)

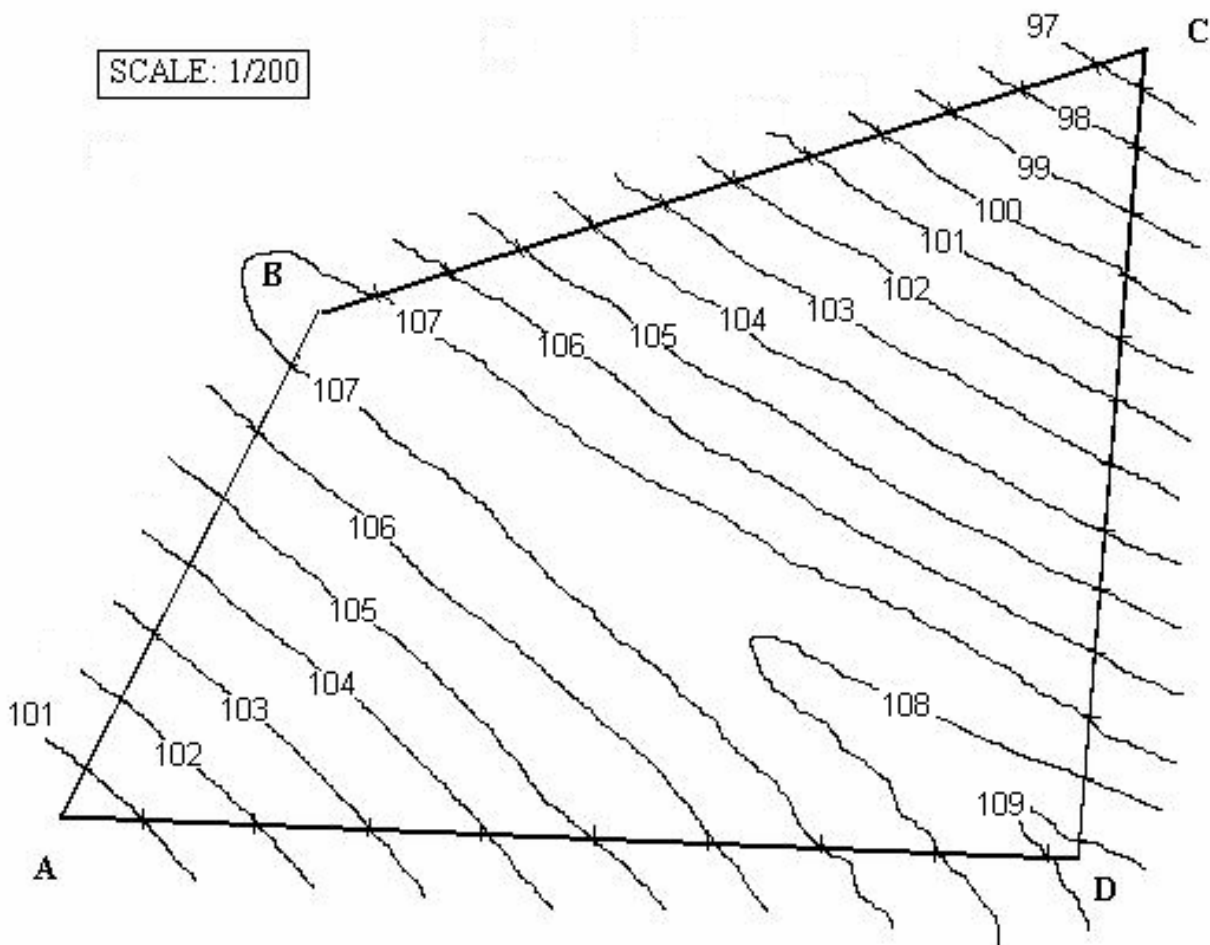
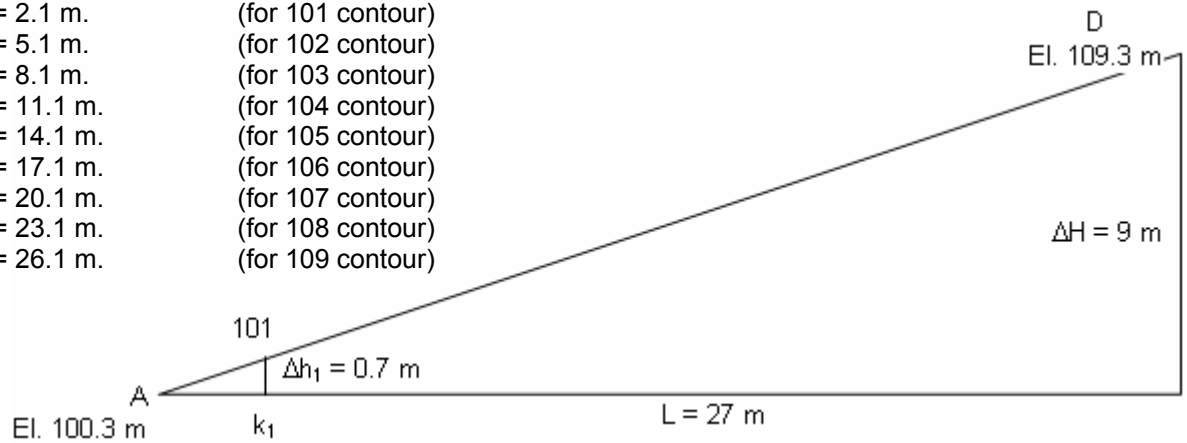
$k_5 = 4.7 \cdot (27/9) = 14.1$ m. (for 105 contour)

$k_7 = 5.7 \cdot (27/9) = 17.1$ m. (for 106 contour)

$k_8 = 6.7 \cdot (27/9) = 20.1$ m. (for 107 contour)

$k_9 = 7.7 \cdot (27/9) = 23.1$ m. (for 108 contour)

$k_8 = 8.7 \cdot (27/9) = 26.1$ m. (for 109 contour)

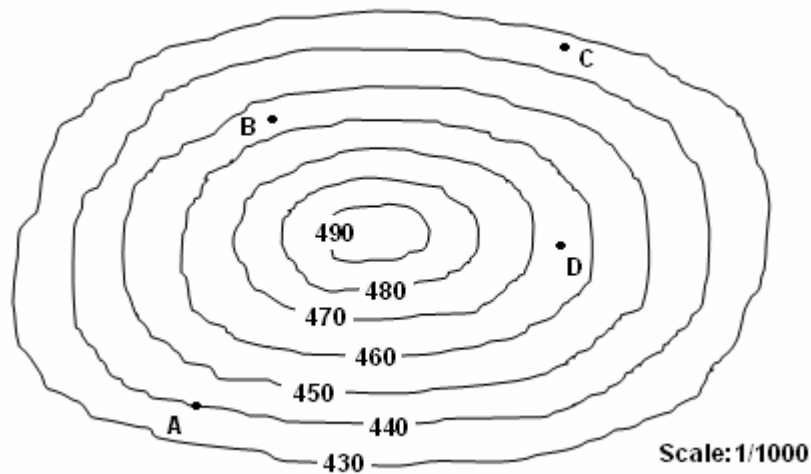


9.4. Following topographic map is obtained after leveling measurements. Determine;

a) Inclination of the courses (vertical angle in degree) AB, BC, CD and DA in given direction.

b) Area of the course denoted by the points A, B, C and D (Area on horizontal plane).

Note: Obtain some values from the figure by using ruler and the scale when necessary.



- a) From the figure, El.A=440 m., El.B=455 m., El.C=433 m., El.D=465 m.,
 Distances measured by ruler and then converted into true length by scale;
 Distance AB=3.9 cm*1000=3900 cm=39 m.
 Distance BC=4 cm*1000=4000 cm=40 m.
 Distance CD=2.6 cm*1000=2600 cm=26 m.
 Distance DA=5.3 cm*1000=5300 cm=53 m.
 Distance AC=6.8 cm*1000=6800 cm=68 m.
 Distance BD=4.2 cm*1000=4200 cm=42 m.

Inclination of line AB= $\tan^{-1}(\Delta H/\text{Distance}) = \tan^{-1}[(455-440)/39] = 21$ degree (upwards)

Inclination of line BC= $\tan^{-1}(\Delta H/\text{Distance}) = \tan^{-1}[(433-455)/40] = -29$ degree (downwards)

Inclination of line CD= $\tan^{-1}(\Delta H/\text{Distance}) = \tan^{-1}[(465-433)/26] = 51$ degree (upwards)

Inclination of line DA= $\tan^{-1}(\Delta H/\text{Distance}) = \tan^{-1}[(440-465)/53] = -25$ degree (downwards)

b) The area can be determined by using the equation if the lengths of three sides are known. The quadrant is divided into two triangles by drawing the diagonal either BD or AC. Then;

Area ABC triangle; $p = (39+40+68)/2 = 73.5$ m

$$\begin{aligned} \text{Area} &= [73.5(73.5-39)(73.5-40)(73.5-68)]^{1/2} \\ &= [73.5 \cdot 34.5 \cdot 33.5 \cdot 5.5]^{1/2} = [467212]^{1/2} = 683.53 \text{ m}^2 \end{aligned}$$

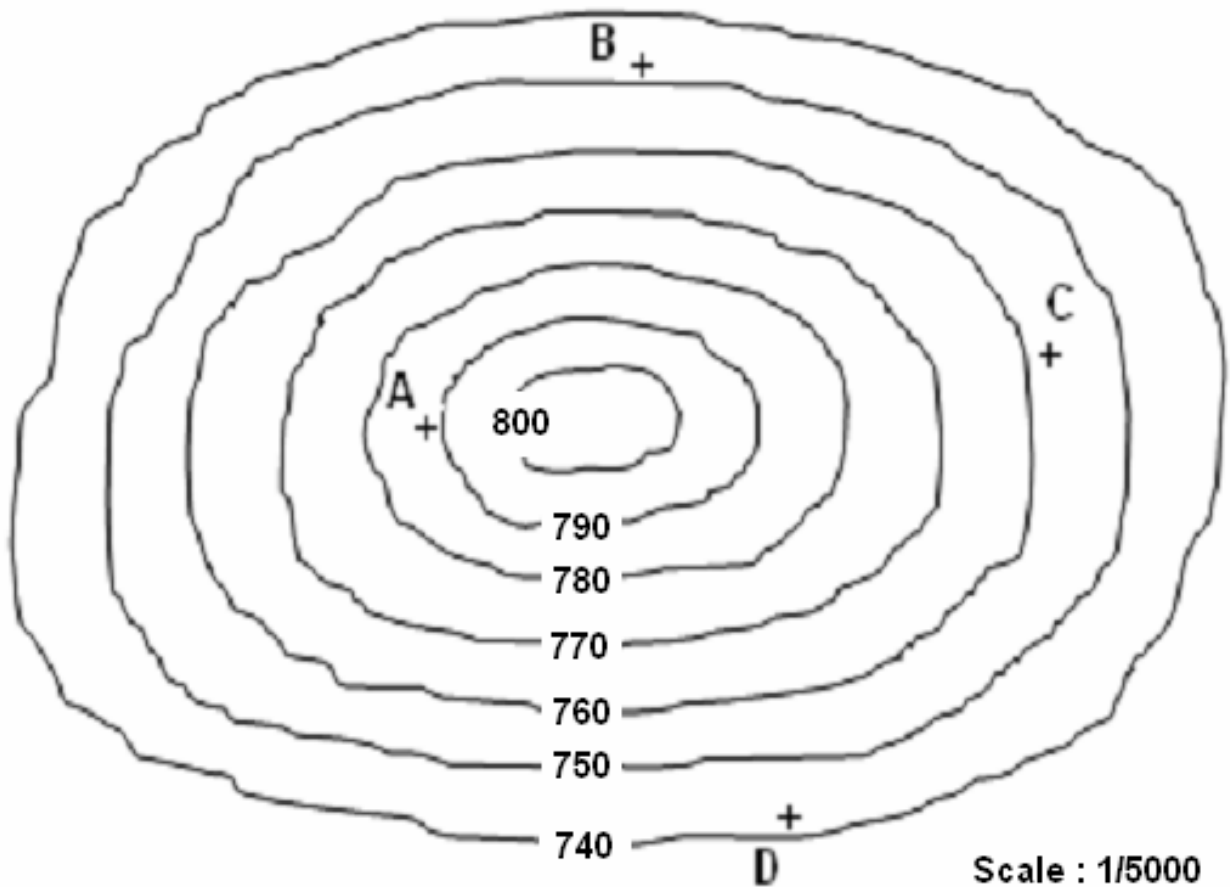
Area ACD triangle; $p = (68+26+53)/2 = 73.5$ m

$$\begin{aligned} \text{Area} &= [73.5(73.5-68)(73.5-26)(73.5-53)]^{1/2} \\ &= [73.5 \cdot 5.5 \cdot 47.5 \cdot 20.5]^{1/2} = [393638]^{1/2} = 627.41 \text{ m}^2 \end{aligned}$$

Area of ABCD = 683.53+627.41 = **1310.94 m²**

9.5. Following figure is taken from a contour map. Determine,

- Elevations of the points A, B, C and D by using interpolation method (Show your calculations and drawings clearly).
- The area of parcel defined by the points A, B, C and D by using triangulation method (Area is asked on the horizontal plane).



a) According to interpolation method;

$$\text{El. of A} = 780 + (8/10) \times 10 = 788 \text{ m.}$$

$$\text{El. of B} = 740 + (6.5/9) \times 10 = 747.2 \text{ m.}$$

$$\text{El. of C} = 750 + (9.5/12.5) \times 10 = 757.6 \text{ m.}$$

$$\text{El. of D} = 740 + (2.5/10) \times 10 = 742.5 \text{ m.}$$

b) Distances between the points can be measured from the figure and then converted to true distance by using the scale. Then;

$$\text{Area ABCD} = \text{Area ABC} + \text{Area ACD} \quad (\text{Triangles})$$

$$\text{For triangle ABC} \quad p = (280 + 330 + 415) / 2 = 512.5 \text{ m}$$

$$\text{Area ABC} = [512.5(512.5 - 280)(512.5 - 330)(512.5 - 415)]^{1/2}$$

$$\text{Area ABC} = [512.5 \times 232.5 \times 182.5 \times 97.5]^{1/2}$$

$$\text{Area ABC} = [2120236523]^{1/2}$$

$$\text{Area ABC} = 46046 \text{ m}^2$$

$$\text{For triangle ACD} \quad p = (415 + 350 + 352.5) / 2 = 558.75 \text{ m}$$

$$\text{Area ACD} = [558.75(558.75 - 415)(558.75 - 350)(558.75 - 352.5)]^{1/2}$$

$$\text{Area ACD} = [558.75 \times 143.75 \times 208.75 \times 206.25]^{1/2}$$

$$\text{Area ACD} = [3458165955]^{1/2}$$

$$\text{Area ACD} = 58806 \text{ m}^2$$

Side	Map distance (mm)	Multiply 1 mm = 5 m	True distance (m)
AB	56	5	280.0
BC	66	5	330.0
CD	70	5	350.0
AD	70.5	5	352.5
AC	83	5	415.0
BD	101.5	5	507.5

$$\text{Area ABCD} = 46046 + 58806 = 104852 \text{ m}^2$$

9.6. Four points are adjusted as seen in the figure to obtain contour map of the field. Elevations of the points and the side lengths are given. Draw elevation contour lines with 1 m intervals (such as 121, 122, 123, etc.). A scale of 1/500 is used.

Side Lengths (meter) →	AC = 30	BC = 25	CD = 40	
Elevations (meter) →	A = 120.3	B = 121.5	C = 128.0	D = 118.7

