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Computer Controlled Thermal Conduction & Computer Controlled Thermal Radiation

1. INTRODUCTION

The transfer of energy in the form of heat occurs in many chemical and other types of processes. In the simplest of terms, the discipline of heat transfer is concerned with only two things: temperature, and the flow of heat. Temperature represents the amount of thermal energy available, whereas heat flow represents the movement of thermal energy from place to place. Heat transfer occurs because of a temperature difference as driving force and heat flows from the high temperature region to the low temperature region¹.

Heat transfer may occur by any one or more of the three basic mechanisms of heat transfer: conduction, convection, or radiation.

Conduction is the transfer of energy from the more energetic particles of a substance to the adjacent less energetic ones as a result of interactions between the particles. Conduction can take place in solids, liquids or gases. In gases and liquids, conduction is due to the collisions and diffusion of the molecules during their random motion. In solids, it is due to the combination of vibrations of molecules in a lattice and energy transport by free electrons. A cold canned drink in a warm room, for example, eventually warms up to the room temperature as a result of heat transfer from the room to the drink through the aluminum by conduction. The rate of heat conduction through a medium depends on the geometry of the medium, its thickness and the material of the medium, as well as the temperature difference across the medium.

Convection is the mode of energy transfer between a solid surface and the adjacent liquid or gas that is in motion and it involves the combined effects of conduction and fluid motion. The faster the fluid motion, the greater the convection heat transfers. In the absence of any bulk fluid motion, the heat transfer between a solid surface and the adjacent fluid is pure conduction. Convection is called forced convection if the fluid forced to flow over the

surface by external means such as fan, pump, or the wind. In contrast, convection is called natural (or free) convection if the fluid motion is caused by buoyancy forces that are induced by density differences due to the variation of temperature in fluid¹.

Radiation heat transfer is energy transport due to emission of electromagnetic waves or photons from a surface or volume. The radiation does not require a heat transfer medium, and can occur in a vacuum. The heat transfer by radiation is proportional to the fourth power of the absolute material temperature. Examples of radiation include the transfer of heat from the sun to the earth, and from a quartz lamp to a cool object that requires warming².

2. THEORY

2.1. Steady-State Heat Conduction

A special case of general energy balance is heat balance. At unsteady state, heat balance can be written by Eq. (1).

$$\begin{array}{ccccccc} \text{Rate of} & & \text{Rate of generation} & & \text{Rate of} & & \text{Rate of accumulation} \\ \text{heat in} & + & \text{of heat} & = & \text{heat out} & + & \text{of heat} \end{array} \quad (1)$$

The heat balance in the x direction for control volume in Fig. 1 by using Eq. (1) with the cross sectional area being $A \text{ m}^2$, where \dot{q} is the rate of heat generated per unit volume.

$$q_{x|x} + \dot{q} (\Delta x \cdot A) = q_{x|x+\Delta x} + \rho \cdot C_p \cdot \frac{\partial T}{\partial t} (\Delta x \cdot A) \quad (2)$$

Assuming no heat generation and the rate of transfer occurs only by conduction, and also assuming steady-state heat transfer where the rate of accumulation is zero Eq. (.2) becomes

$$q_{x|x} = q_{x|x+\Delta x} \quad (3)$$

Eq. (3) means, the rate of heat input by conduction=the rate of heat output by conduction; or q_x is a constant with time for steady state heat transfer.

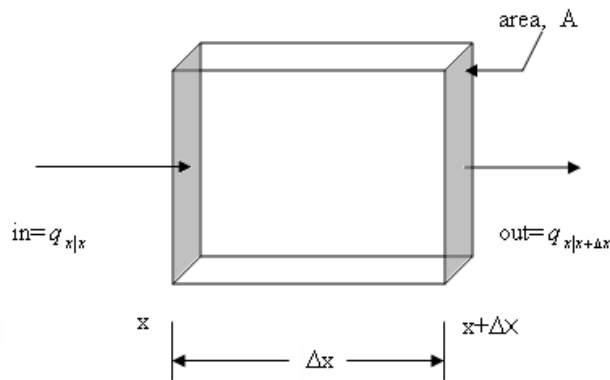


Fig.1. Unsteady state balance for heat transfer in control volume

2.2. Fourier's Law of Heat Conduction

The general molecular transport equation, all three main types of rate transfer processes (momentum transfer, heat transfer and mass transfer) are characterized by the same general type of equation. This basic equation is as follows:

$$\text{rate of a transfer process} = \frac{\text{driving force}}{\text{resistance}} \quad (4)$$

Eq. (4) states what we know intuitively: that in order to transfer a property such as heat or mass, we need a deriving force to overcome a resistance.

When a temperature gradient exists in a body, experience has shown that there is an energy transfer from the high-temperature region to the low-temperature region. We say that the energy is transferred by conduction and that the heat-transfer rate per unit area is proportional to the normal temperature gradient (Fig.2):

$$\frac{q_x}{A} \approx \frac{\partial T}{\partial x} \quad (5)$$

When the proportionality constant is inserted,

$$\frac{q_x}{A} = -k \cdot \frac{\partial T}{\partial x} \quad (6)$$

where;

q_x = heat transfer rate in the x direction, W

A = cross sectional area normal to the direction of heat, m²

T= temperature, K

x= distance, m

The quantity of q_x/A is called the heat flux in W/ m². The quantity dT/dx is the temperature gradient in the direction of heat flow. The positive constant k (W/m.K) is called the thermal conductivity of the material and the minus sign in Eq. (6) is required because if the heat flow is positive in a given direction, the temperature decreases in this direction. Eq. (6) is called Fourier's Law of heat conduction¹.

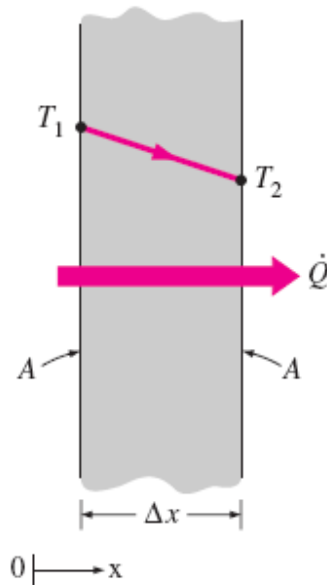


Fig.2. Heat conduction through a wall of thickness x and area A .

Fourier's Law can be integrated for the case of steady state heat transfer through a flat wall of constant cross-sectional area A , where the inside temperature at point 1 is T_1 and T_2 at point 2 a distance of $x_2 - x_1$ m away. Rearranging Eq. (6),

$$\frac{q_x}{A} \int_{x_1}^{x_2} dx = -k \int_{T_1}^{T_2} dT \quad (7)$$

Integrating, assuming that k is constant and does not vary with temperature and dropping the subscript x on q_x for convenience, Eq. (8) is obtained¹,

$$\frac{q_x}{A} = \frac{k}{x_2 - x_1} (T_1 - T_2) \quad (8)$$

2.3. Thermal Conductivity

The thermal conductivity of a material can be defined as the rate of heat transfer through a unit thickness of the material per unit area per unit temperature difference. The thermal conductivity of a material is a measure of the ability of the material to conduct heat. A high value for thermal conductivity indicates that the material is a good heat conductor, and a low value indicates that the material is a poor heat conductor or *insulator*. The thermal conductivities of some common materials at room temperature are given in Table 1. The thermal conductivity of pure copper at room temperature is $k = 401 \text{ W/m} \cdot ^\circ\text{C}$, which indicates that a 1-m-thick copper wall will conduct heat at a rate of 401 W per m^2 area per $^\circ\text{C}$ temperature difference across the wall. Note that materials such as copper and silver that are good electric conductors are also good heat conductors, and have high values of thermal conductivity. Materials such as rubber, wood, and styrofoam are poor conductors of heat and have low conductivity values².

Table 1. The thermal conductivities of some materials at room temperature²

Material	$k, \text{W/m} \cdot ^\circ\text{C}^*$
Diamond	2300
Silver	429
Copper	401
Gold	317
Aluminum	237
Iron	80.2
Mercury (l)	8.54
Glass	0.78
Brick	0.72
Water (l)	0.613
Human skin	0.37
Wood (oak)	0.17
Helium (g)	0.152
Soft rubber	0.13
Glass fiber	0.043
Air (g)	0.026
Urethane, rigid foam	0.026

*Multiply by 0.5778 to convert to $\text{Btu/h} \cdot \text{ft} \cdot ^\circ\text{F}$.

The thermal conductivities of materials vary over a wide range, as shown in Fig. 3. The thermal conductivities of gases such as air vary by a factor of 10^4 from those of pure metals such as copper. Note that pure crystals and metals have the highest thermal conductivities, and gases and insulating materials the lowest².

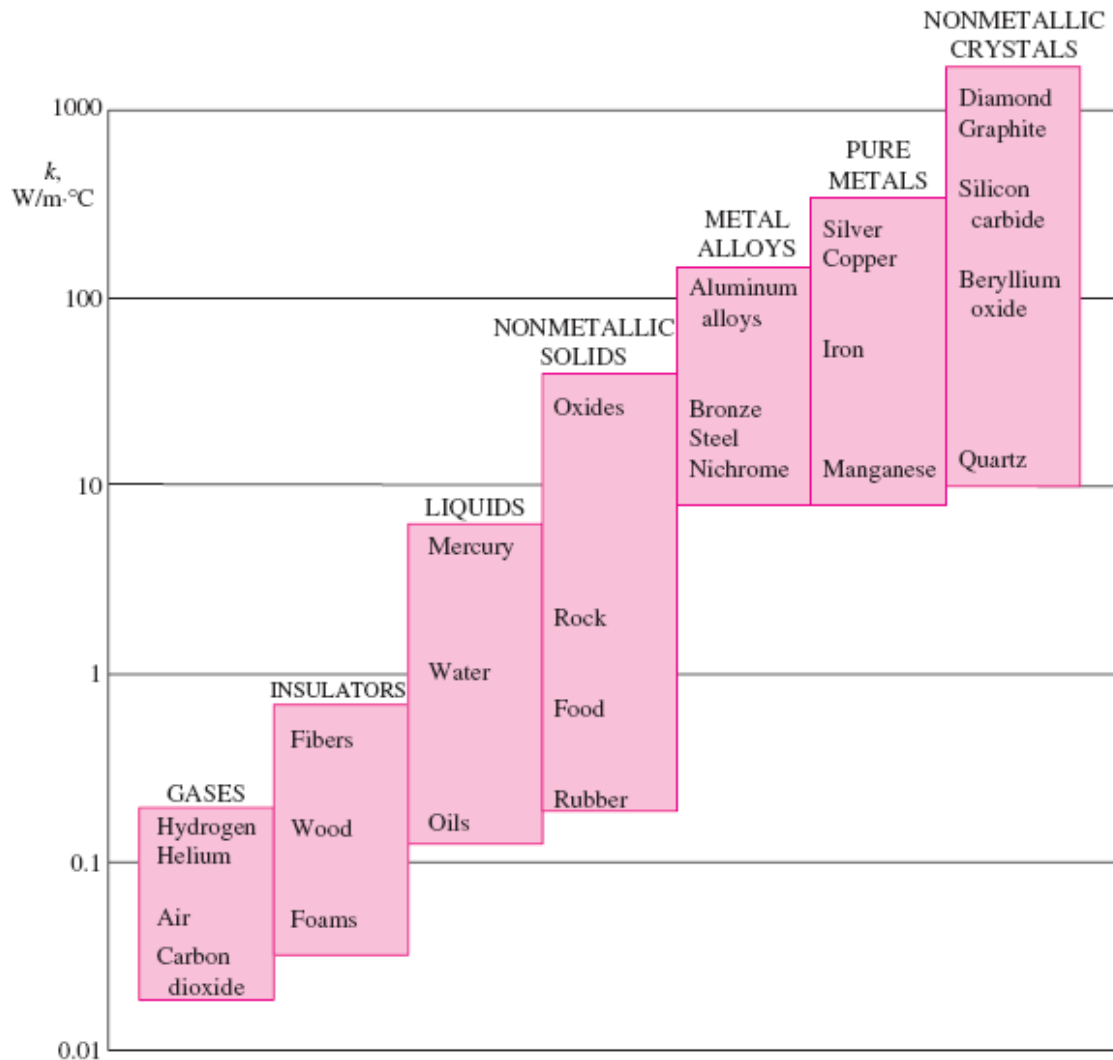


Fig.3. The range of thermal conductivity of various materials at room temperature. ²

The thermal conductivity of a material is found to depend on the chemical composition of the substances, of which it is composed, the phase, (i.e., gas, liquid or solid) in which it exists, its crystalline structure if a solid, the temperature and pressure to which it is subjected, and whether or not it is a homogeneous material. It is a property that depends only on the material and not on the geometric configuration.

Many factors are known to influence the thermal conductivity of metals, such as chemical composition, atomic structure, phase changes, grain size, temperature, pressure and deformation. The factor with the greatest influence is the temperature. The thermal

conductivities of pure metals decrease with increasing temperature, whereas the conductivities of non-metals increase; alloys show intermediate behavior.

In general, thermal conductivities of liquids are relatively insensitive to pressure, particularly at pressures not too close to the critical pressure. For this reason the temperature variation is usually the only influence which is taken into account. Most liquid, exhibit a decreasing thermal conductivity with temperature, although water is as usual a notable exception. The thermal conductivity of liquids varies moderately with temperature and often can be expressed as linear variation,

$$k=a+bT \quad (9)$$

where a and b are empirical constants.

The thermal conductivities of gases increase approximately as the square root of the absolute temperature and is independent of pressure up to a few atmospheres. At very low pressures (vacuum), however, the thermal conductivity approaches zero^{2,4}.

2.4. Conduction Heat Transfer

2.4.1. Conduction Through A Flat Slab or Wall

Fourier's equation will be used to obtain equations for one-dimensional steady state conduction of heat through some simple geometries. For a flat slab or wall where the cross-sectional area A and k in Eq. (6) are constant, Eq. (8) is obtained, which we rewrite as

$$\frac{q_x}{A} = \frac{k}{x_2 - x_1} (T_1 - T_2) = \frac{k}{\Delta x} (T_1 - T_2) \quad (10)$$

This is shown in Fig. 4, where $\Delta x=x_1-x_2$. Equation (10) indicates that that if T is substituted for T_2 and x for x_2 , the temperature varies linearly with distance as shown in Fig. 3-b¹.

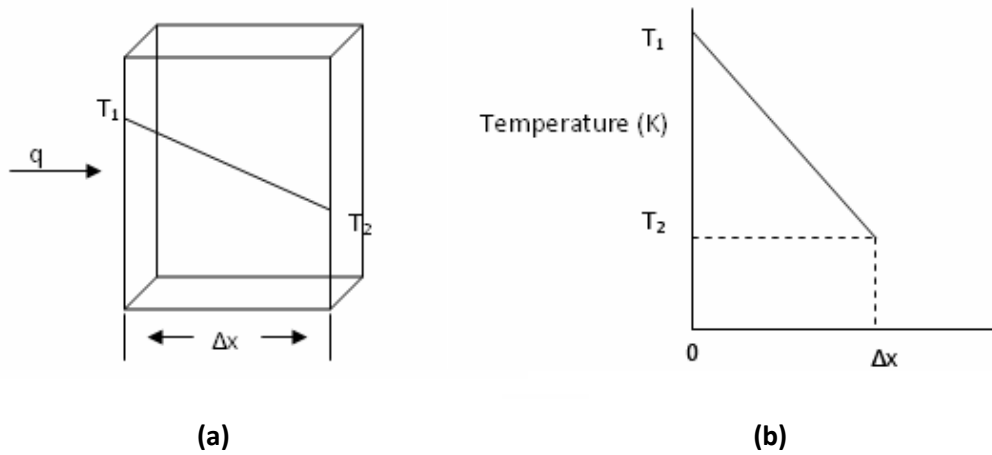


Fig. 4. Heat conduction in a flat wall: (a) geometry of wall, (b) temperature plot

2.4.2. Conduction Through Plane Walls In Series

The case where there is a multilayer wall of more than one material is shown in Fig.5. The temperature profiles in the three materials A, B, C are indicated. Since the heat flow q must be the same in each layer, we can write Fourier's equation for each layer as:

$$q = \frac{k_A A}{\Delta x_A} (T_1 - T_2) = \frac{k_B A}{\Delta x_B} (T_2 - T_3) = \frac{k_C A}{\Delta x_C} (T_3 - T_4) \quad (11)$$

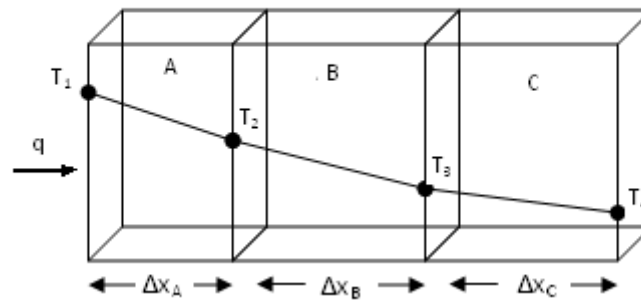


Fig. 5. Heat flow through a multilayer wall

Solving each equation for ΔT

$$T_1 - T_2 = q \frac{\Delta x_A}{k_A A} \quad T_2 - T_3 = q \frac{\Delta x_B}{k_B A} \quad T_3 - T_4 = q \frac{\Delta x_C}{k_C A} \quad (12)$$

Adding the equations for T_1-T_2 , T_2-T_3 , and T_3-T_4 , the internal temperatures T_2 and T_3 drop out and final rearranged equation is

$$\frac{q}{A} = \frac{T_1 - T_4}{\frac{\Delta x_A}{k_A A} + \frac{\Delta x_B}{k_B A} + \frac{\Delta x_C}{k_C A}} = \frac{T_1 - T_4}{R_A + R_B + R_C} \quad (13)$$

where the resistance $R_A = \Delta x_A / k_A A$, and so on. Hence, the final equation is in terms of the overall temperature drop T_1-T_4 and the total resistance, $R_A+R_B+R_C$ ¹.

2.4.3. Conduction Through a Hollow Cylinder

Heat conduction through a hollow cylinder is another case of one dimensional conduction. Consider a cylindrical surface in Fig. 6 with an inside radius of r_1 (m), where the temperature is T_1 (K) and outside radius of r_2 (m) having a temperature of T_2 (K) and a length of L (m). Heat is flowing radially from the inside surface to the outside. Rewriting Fourier's Law with distance dr instead of dx ,

$$\frac{q}{A} = -k \frac{dT}{dr} \quad (14)$$

The cross sectional area normal to the heat flow is

$$A_r = 2\pi r L \quad (15)$$

Substituting Eq.(15) into Eq.(14),rearranging and integrating,

$$\frac{q_x}{2\pi L} \int_{r_1}^{r_2} \frac{dr}{r} = -k \int_{T_1}^{T_2} dT \quad (16)$$

$$q = k \frac{2\pi L}{\ln(r_2 / r_1)} (T_1 - T_2) \quad (17)$$

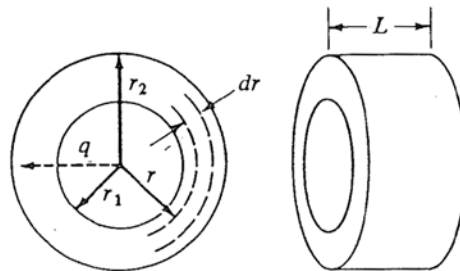


Fig. 6. Heat conduction in a cylinder

2.5. Heat Conduction in Two or More Independent Variables

Numerous experimental techniques have been devised for the measurement of the thermal conductivity of solids. Both steady-state and unsteady-state methods are used. However, an accurate determination of thermal conductivity demands a careful analysis of the experimental technique involved. To demonstrate some of the heat transfer problems, a simple unsteady-state procedure will be used to determine the thermal conductivity of solids and the results and method will be analyzed in some details³.

Under steady-state conditions without any heat generation, the heat balance on a cylindrical shell element is given by Eq. (18). It is assumed that the conduction occurs only in the radial direction³.

By using Eq.(18),Eq. (24) is obtained by steps shown in Eqs. [19-23] for constant physical properties of a short lengthened cylinder.

The boundary conditions which are necessary for the solution of this differential balance is given by Eq. (25)^{3,5}.

$$q_{r|r} - q_{r|r+\Delta r} = \rho \cdot C_p \cdot \frac{\partial T}{\partial t} (\Delta r \cdot A) \quad (18)$$

Dividing by $A \cdot \Delta r$, and letting Δr approaches zero,

$$-\frac{1}{A} \left(\frac{q_{r|r+\Delta r} - q_{r|r}}{\Delta r} \right) = \rho \cdot C_p \cdot \frac{\partial T}{\partial t} \quad (19)$$

$$-\frac{1}{r} \frac{\partial(rq)}{\partial r} = \rho C_p \frac{\partial T}{\partial t} \quad (20)$$

$$-\frac{1}{r} \frac{\partial}{\partial r} \left[r \left(-k \frac{\partial T}{\partial r} \right) \right] = \rho C_p \frac{\partial T}{\partial t} \quad (21)$$

$$\frac{k}{r} \frac{\partial}{\partial r} \left[r \left(\frac{\partial T}{\partial r} \right) \right] = \rho C_p \frac{\partial T}{\partial t} \quad (22)$$

$$\frac{k}{\rho C_p} \frac{1}{r} \frac{\partial}{\partial r} \left[r \left(\frac{\partial T}{\partial r} \right) \right] = \frac{\partial T}{\partial t} \quad (23)$$

Substituting diffusivity coefficient α into the equation which is $\alpha = \frac{k}{\rho C_p}$ and rearranging the equation;

$$\frac{\partial T}{\partial t} = \alpha \frac{1}{r} \frac{\partial}{\partial r} \left[r \frac{\partial T}{\partial r} \right] \quad (24)$$

where;

$$\alpha : \text{thermal diffusivity, m}^2/\text{s} \quad \left(\alpha = \frac{k}{\rho C_p} \right)$$

C_p : heat capacity, kJ/kg K

k : thermal conductivity, W/mK

ρ : density, kg/m³

r : radial distance, m

T : temperature, K

t : time, s

Note that Newton's law of cooling is a necessary boundary condition in this analysis. The boundary conditions which are necessary for the solution of this differential balance are;

$$\begin{aligned} \text{at } t \leq 0; \quad T &= T_0 & \text{for } 0 \leq r \leq R \\ \text{at } r=0; \quad \frac{\partial T}{\partial r} &= 0 & \text{for all } t \\ \text{at } r=R; \quad -k \frac{\partial T}{\partial r} &= h(T - T_\infty) & \text{for } t > 0 \end{aligned} \quad (25)$$

where;

h : heat transfer coefficient, W/m²K

R : radius of cylinder, m

T_0 : initial uniform cylinder temperature, K

T_∞ : bath temperature, K

Solution of the differential energy balance:

$$\frac{T_\infty - T}{T_\infty - T_0} = 2 \sum_{n=1}^{\infty} \frac{\exp[-\beta_n(\alpha t/R^2)](Rh/k)J_0[\beta_n(r/R)]}{[\beta_n^2 + (Rh/k)^2]J_0(\beta_n)} \quad (26)$$

where;

$\pm\beta_n$ = roots of $\beta J_1(\beta) = (rh/k)J_0(\beta)$

$J_n(x)$ = n^{th} order Bessel function of x

Solution to differential balance is given by Eq.(26). This solution is composed of four dimensionless groups [Eq. 28-31] and the solution can be represented in functional form as given by Eq. (27)

$$\frac{T_\infty - T}{T_\infty - T_0} = f\left[\left(\alpha t/R^2\right), (k/hr), (r/R)\right] \quad (27)$$

- $\frac{T_\infty - T}{T_\infty - T_0}$ = dimensionless temperature (28)

- $\alpha t/R^2$ = dimensionless time or Fourier number (29)

Fourier number is the measure of rate of heat conduction compared with the rate of heat storage in a given volume element. Therefore larger the Fourier number, the deeper the penetration of heat into a solid over a given time.

- $\frac{hL_c}{k}$ = Biot number(Bi) (30)

The Biot number compares the relative values of internal conduction resistance and surface convective resistance to heat transfer.

where:

h :heat transfer coefficient, W/m^2K

L_c : characteristic length of body, $x_l=V/A$

k :thermal conductivity of body, $W/m K$

The physical significance of Biot number can be fairly understood by imagining the heat flow from a hot metal sphere immersed in a pool to the surroundings fluid. The heat flow experiences two resistances: the first by the solid metal and the second by the fluid present near the surface of the sphere. The thermal resistance of the fluid exceeds that thermal resistance offered by the metal sphere, so the Biot number is less than one. Values of the Biot number smaller than 0.1 imply that the heat conduction inside the body is much faster than the heat conduction away from its surface, and temperature gradients are negligible inside of it. This can indicate the applicability of certain methods of solving transient heat transfer problems.

- r/R =dimensionless coordinate (31)

The numerical solution of Eq. (26) has been carried out and is usually presented in graphical form as functions of dimensionless groups. Chart for this solution is presented in Fig. 11. only for the condition at the see center of cylinder (see in Appendix).

2.7. Thermal Radiation

In heat transfer studies, it is interested in thermal radiation, which is the form of radiation emitted by bodies because of their temperature. All bodies at a temperature above absolute zero emit thermal radiation.

Radiative heat transfer or thermal radiation is the science of transferring energy in the form of electromagnetic waves similar to X rays, light waves, gamma rays, and so on, different only in wavelength. The electromagnetic spectrum is subdivided into a number of wavelength

ranges, as shown in Fig. 7. Thermal radiation is defined as the portion of the spectrum between the wavelengths 1×10^{-7} m and 1×10^{-4} m.

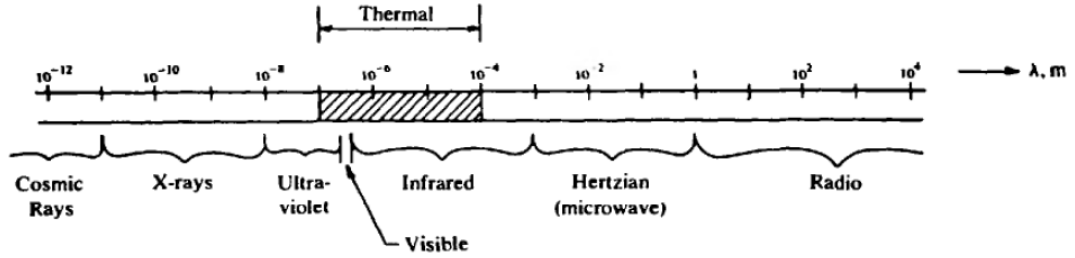


Fig. 7. Electromagnetic spectrum.

All materials continuously emit and absorb electromagnetic waves, or photons, by changing their internal energy on a molecular level. Strength of emission and absorption of radiative energy depend on the temperature of the material, as well as on the wavelength λ , frequency ν , or wave number η , that characterizes the electromagnetic waves, given as Eq.(32)^{1,2}

$$\lambda = \frac{c}{\nu} = \frac{1}{\eta} \quad (32)$$

where wavelength is usually measured in μm ($=10^{-6}$ m), while frequency is measured in hertz=cycles/s), and wave numbers are given in cm^{-1} . Electromagnetic waves or photons (which include what is perceived as “light”) travel at the speed of light, c . The speed of light depends on the medium through which the wave travels and is related to that in vacuum, c_0 , through the relation

$$c = \frac{c_0}{n} \quad c_0 = 2.998 \times 10^8 \text{ m/s} \quad (33)$$

where n is known as the refractive index of the medium. By definition, the refractive index of vacuum is $n \equiv 1$. For most gases the refractive index is very close to unity, and the c in Eq.(33) can be replaced by c_0 . Each wave or photon carries with it an amount of energy E is given Eq(34)

$$E = h\nu \quad h = 6.626 \times 10^{-34} \text{ J.s} \quad (34)$$

where h is known as Planck's constant. The frequency of light does not change when light penetrates from one medium to another because the energy of the photon must be conserved. On the other hand, the wavelength does change, depending on the values of the refractive index for the two media.

In an elementary sense, the mechanism of radiant heat transfer is composed of three distinct steps or phases:

1. The thermal energy of a hot source, such as the wall of a furnace at T_1 , is converted into energy in the form of electromagnetic radiation waves.
2. These waves travel through the intervening space in straight lines and strike a cold object at T_2 , such as a furnace tube containing water to be heated.
3. The electromagnetic waves that strike the body are absorbed by the body and converted back to thermal energy or heat.

Unlike heat conduction or convection, electromagnetic waves do not require a medium for their propagation. Therefore, because of their ability to travel across vacuum, thermal radiation becomes the dominant mode of heat transfer in low pressure (vacuum) and outer-space applications. Another distinguishing characteristic between conduction, convection and thermal radiation is their temperature dependence. While conductive and convective fluxes are more or less linearly dependent on temperature differences, radiative heat fluxes tend to be proportional to differences in the fourth power of temperature. For this reason, radiation tends to become the dominant mode of heat transfer in high-temperature applications, such as combustion (fires, furnaces, rocket nozzles), nuclear reactions, and others^{1,2}.

2.8. Absorptivity, Reflectivity and Transmittivity

Whenever radiant energy is incident upon any surface, part of the radiation is absorbed, part is reflected, and part is transmitted through the receiving body as shown in Fig.8. Thus

$$\alpha + \rho + \tau = 1 \quad (35)$$

α = fraction of incident radiation absorbed = absorptivity

ρ = fraction of incident radiation reflected = reflectivity

τ = fraction of incident radiation transmitted = transmissivity

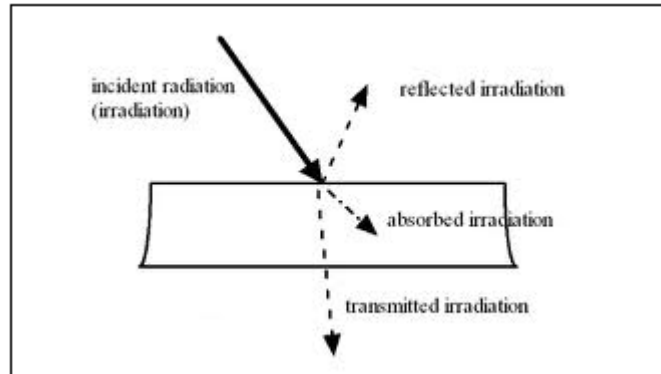


Fig.8 Definition of total radiation properties.

For most cases in process engineering, bodies are opaque to transmission, so this will be neglected. Hence for opaque bodies,

$$\alpha + \rho = 1 \quad (35)$$

A shiny silvered surface is the best reflector, whereas a dull black surface is the best absorber. Surfaces which appear black in daylight do not reflect any light so they absorbed all the light which falls on them. Such surfaces are also good absorbers of thermal radiation. Therefore, a good absorber of thermal radiation is dull black, whereas a poor absorber is shiny. This explains why you feel warmer if you wear dark coloured dresses ^{1,4}.

2.9. Radiation Behavior of Surfaces

When radiation is incident on a homogeneous body, some of the radiation is reflected and the remainder penetrates into the body. The radiation may then be absorbed as it travels through the medium. If the material thickness required to substantially absorb the radiation is large compared with the thickness dimension of the body, then most of the radiation will be transmitted entirely through the body and will emerge with its nature unchanged. If on the other hand, the material is a strong internal absorber, the radiation that is not reflected from the body will be converted into internal energy within a thin layer adjacent to the surface.

A distinction must be made between the ability of a material to let radiation pass through its surface and its ability to internally absorb the radiation after it has passed into the body. For

example, a highly polished metal will generally reflect all but a small portion of the incident radiation, but the radiation passing into the body will be strongly absorbed and converted into internal energy within a very short distance within the material. Thus the metal has strong internal absorption ability, although it is a poor absorber for the incident beam because most of the incident beam is reflected. Nonmetals may exhibit the opposite tendency. Nonmetals may allow a substantial portion of incident beam to pass into the material (small surface reflection), but a larger thickness is usually required than in the case of a metal to absorb the radiation internally and convert it into internal energy. A glass window readily allows radiation to pass through its surface but is a poor absorber for visible radiation, hence that radiation is transmitted. When all the radiation that passes into the body is absorbed internally, the body is opaque.

To be a good absorber for incident energy, a body must have a low surface reflectivity and internal absorption sufficiently high to prevent the radiation passing through. If metals in the form of very fine particles are deposited on a subsurface, the result is a surface of low reflectivity. This effect combined with the high internal absorption of the metal, causes this type of coating to be a good absorber. This is the basis for formation of the metallic blacks, such as platinum or gold black. These materials can have less than 1% reflection for the solar spectrum and provide complete internal absorption within a few micrometers of thickness. A blackbody must have zero surface reflection and complete internal absorption^{1,4}.

A blackbody is defined as an ideal body that allows all the incident radiation to pass into it (no reflected energy) and internally absorbs all the incident radiation (no transmitted energy). This is true of radiation for all wavelengths and for all angles of incidence. Hence the blackbody is a perfect absorber for all incident radiation^{1,4}.

The concept of blackbody is basic to the study of radiative energy transfer. As a perfect absorber, it serves as a standard with which real absorbers can be compared. The blackbody also emits the maximum radiant energy and hence serves as an ideal standard of comparison with a real body emitting radiation^{1,4}.

Only a few materials, such as carbon black, platinum black, gold black, carborundum and some specially formulated black paints on absorbing substrates approach the blackbody in their ability to absorb radiant energy.

2.10. Stefan-Boltzmann Law

The maximum rate of radiation that can be emitted from a surface at an absolute temperature T_s (in K or R) is given by the Stefan-Boltzmann law (Eq.37) as:

$$Q_{\text{emit,max}} = \sigma A_s T_s^4 \quad (\text{W}) \quad (37)$$

where $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$ or $0.1714 \times 10^{-8} \text{ Btu/h.ft}^2 \cdot \text{R}^4$ is the Stefan-Boltzmann constant. According of Stefan-Boltzmann's law, the energy emitted by a blackbody per unit area and unit time is proportional to the power "four" of the absolute temperature of the body. The idealized surface that emits radiation at this maximum rate is called blackbody and the radiation emitted by a blackbody is called blackbody radiation (Fig.9).

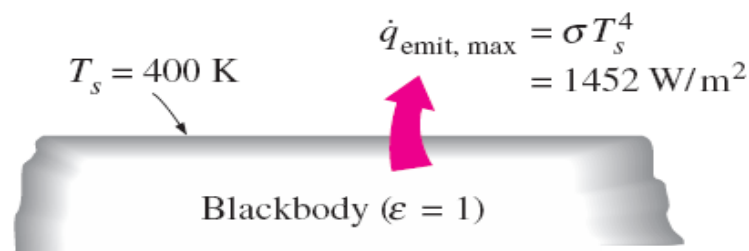


Fig.9. Blackbody radiation represents the maximum amount of radiation that can be emitted from a surface at a specified temperature.

The radiation emitted by all real surfaces is less than the radiation emitted by a blackbody at the same temperature, and is expressed in Eq. (38):

$$Q_{\text{emit}} = \epsilon \sigma A_s T_s^4 \quad (\text{W}) \quad (38)$$

where ϵ is the emissivity of the surface. The property emissivity, whose value is in the range $0 \leq \epsilon \leq 1$, is a measure of how closely a surface approximates a blackbody for which $\epsilon = 1$. The emissivities of some surfaces are given in Table 2.

The emissivity specifies how well a real body radiates energy as compared with a blackbody. The emissivity depends on factors such as body temperature, wavelength of the emitted

energy and angle of emission. The emissivity is usually measured experimentally in a direction normal to the surface and as a function of wavelength. In calculating the entire energy loss by a body, an emissivity is needed that includes all directions and wavelengths.

Table 2. Emissivities of some materials at 300 K

Material	Emissivity
Aluminum foil	0.07
Anodized aluminum	0.82
Polished copper	0.03
Polished gold	0.03
Polished silver	0.02
Polished stainless steel	0.17
Black paint	0.98
White paint	0.90
White paper	0.92–0.97
Asphalt pavement	0.85–0.93
Red brick	0.93–0.96
Human skin	0.95
Wood	0.82–0.92
Soil	0.93–0.96
Water	0.96
Vegetation	0.92–0.96

Another important radiation property of a surface is its absorptivity α , which is the fraction of the radiation energy incident on a surface that is absorbed by the surface. Like emissivity, its value is in the range $0 \leq \alpha \leq 1$. A blackbody absorbs the entire radiation that incident on it. That is, a blackbody is perfect absorber ($\alpha = 1$), as it is a perfect emitter.

The absorptivity is defined as the fraction of the energy incident on a body that is absorbed by the body. The incident radiation is the result of the radiative conditions at the source of the incident energy. The spectral distribution of the incident radiation is independent of the temperature or physical nature of the absorbing surface (unless radiation emitted from the surface is partially reflected back to the surface). Compared with emissivity, additional complexities are introduced into the absorptivity because the directional and spectral characteristics of the incident radiation must be accounted for.

2.11. Kirchhoff's Law

In general, both ε and α of a surface depend on the temperature and the wavelength of the radiation. Kirchhoff's law of radiation states that the emissivity and absorptivity of a surface at a given temperature and wavelength are equal. In many practical applications, the surface temperature and the temperature of the source of incident radiation are of the same order of magnitude and the average absorptivity of a surface is taken to be equal to its average emissivity. The rate at which a surface absorbs radiation determined from Fig.10 is given by Eq. (39).

$$Q_{\text{absorbed}} = \alpha Q_{\text{incident}} \quad (\text{W}) \quad (39)$$

where Q_{incident} is the rate at which radiation is incident on the surface and α is the absorptivity of the surface. For opaque (nontransparent) surfaces, the portion of incident radiation not absorbed by the surface is reflected back.

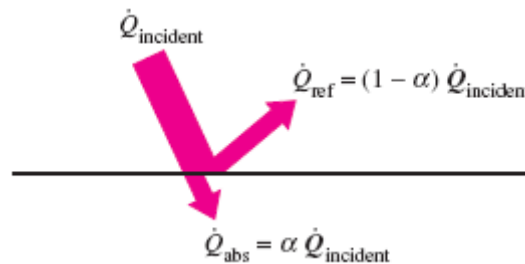


Fig. 10. The absorption of radiation incident on an opaque surface of absorptivity α .

The difference between the rates of radiation emitted by the surface and the radiation absorbed is the net radiation heat transfer. If the rate of radiation absorbed is greater than the rate of radiation emission, the surface is said to be gaining energy by radiation. Otherwise, the surface is said to be losing energy by radiation. In general, the determination of the net rate of heat transfer by radiation between two surfaces is a complicated matter since it depends on the properties of the surfaces, their orientation relative to each other and the interaction of the medium between two surfaces with radiation.

When a surface of emissivity ε and the surface area A_s at an absolute temperature T_s is completely enclosed by a much larger (or black) surface at absolute temperature T_{surr} separated by a gas (such as air) that does not intervene with radiation, the net rate of radiation heat transfer between these two surfaces shown in Fig.10 is given by Eq (40):

$$Q_{\text{rad}} = \epsilon \sigma A_s (T_s^4 - T_{\text{surr}}^4) \quad (\text{W}) \quad (40)$$

In this special case, the emissivity and the surface area of the surrounding surface do not have any effect on the net radiation heat transfer.

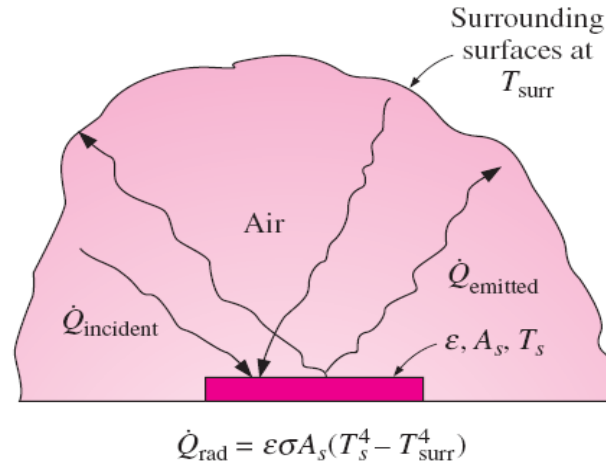


Fig.11. Radiation heat transfer between a surface and surfaces surrounding it.

3. EXPERIMENTAL SET-UP AND EXPERIMENTAL PROCEDURE

3. 1. Equipment Description Of Computer Controlled Conduction

3.1.1 Introduction

The thermal conduction is the modality of heat conduction that takes place in a material by the temperature conduction on it. A solid is chosen for the pure conduction demonstration, since liquids, as well as the gases, show excessive convective heat transference.

In a practical situation, the heat conduction takes place in three dimensions, what is complex and, to analyze it, a wide computation work is necessary. At a basic level, the demonstration of the lineal and radial conduction can be studied using the TCCC equipment ⁵.

3.1.2 Description

The heat conduction unit, TCCC, developed by EDIBON consists on two modules electrically heated and set on a support frame for tests. One of the modules is provided of a cylindrical metal bar for the realization of a series of experiments of lineal transmission of heat. While the other one consists on a metallic disk that allows studying the heat radial transmission. Both models are provided with a series of takings for the connection of a series of temperature sensors included with the equipment. To maintain the gradient of constant temperature, on a lateral of the models, a cooling system has been inserted by circulating water.

The instrumentation provided with the equipment allows making the measuring of the temperature and the electric power given to the heater element. For the control of the given energy it has a control circuit that allows the variation from 0 to 100% of the maximum resistance power.

The equipment has been especially designed to demonstrate the heat transmission principle for conduction without the knowledge necessity of the main principles of convection or radiation transmission.

The lineal experimental group is given with interchangeable samples of different materials, different diameters and different insulating materials that allow demonstrating the area effects, the conductivity and the combinations in series in the heat transmission process.

It is viable with the TCCC equipment the experimental calculation of the thermal conductivity of the different materials ⁵.

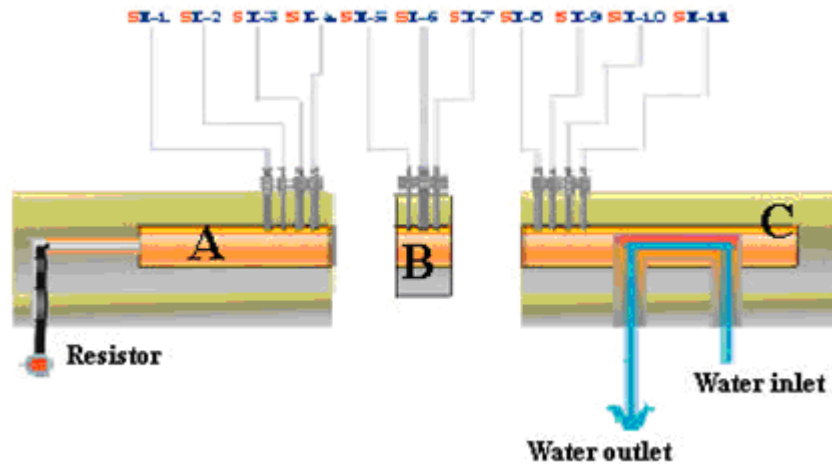
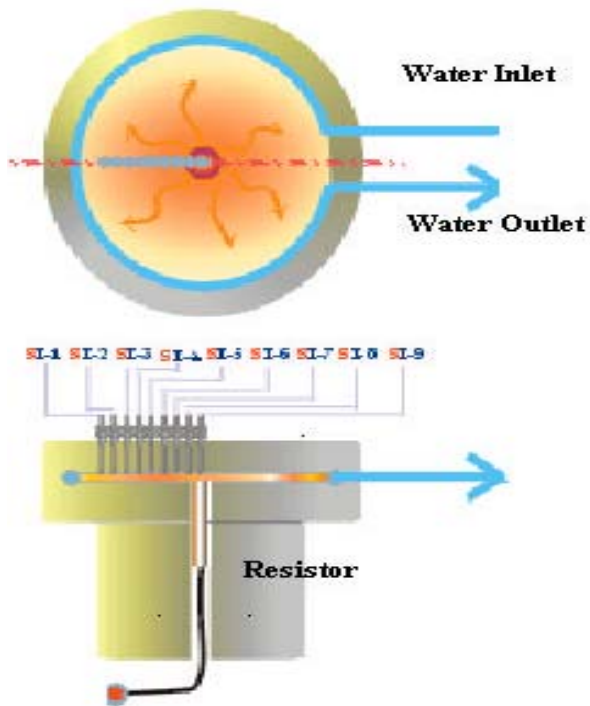


Fig. 12. Linear element

The linear element that is shown Fig. 2.12. in consists on three differentiated parts: A is the region where the contact resistance is located. It has 4 temperature takings and an insulating cover that avoids the radial transmission of heat favoring the linear transmission. The element B, shown in the same figure, corresponds to the interchangeable elements: a brass cylinder of 25mm, stainless steel cylinder of 25 mm and a brass cylinder of 10 mm.

In the piece C, an efficient refrigeration system has been incorporated by water circulation that guarantees a constant heat gradient in the system.

As it can be seen in Fig. 12. , the linear element has 11 temperature takings that will allow us to obtain a perfect temperature profile in each pattern. All the temperature sensors are interchangeable between the linear pattern and the radial pattern. In the same way, the change in the element B implies the change of the sensors associated to this element. The connection and disconnection of the sensors has been carried out in an easy and comfortable way.



In the radial pattern that is shown in Fig. 13., it consists of a disk of 110 mm of diameter with a refrigeration system in its end for water circulation. In this model, there are 6 temperature takings placed along its radius. The contact resistance is placed exactly in its central point surrounded by an effective insulating material

Fig. 13. Radial Element

3.2. Experimental Procedures of Computer Controlled Conduction

3.2.1. Conduction in a simple bar

3.2.1.1. Objectives

This practice objective is the experimental demonstration of Fourier's law.

3.2.1.2. Required Material

- SACED-TCCC Software (Supplied with the equipment)
- TCCC equipment
- Accessory: Conductor cylindrical bar plus interchangeable brass accessory of 25 mm of diameter.

3.2.1.3. Experimental Procedure

For proper practice development, follow these steps:

1. Connect the SACED program TCCC.
2. Verify that all the temperature sensors and that the heating resistance have been connected and also that the accessory is in line with the fix onduction cylinders.
3. Make a water flow to circulate through the cooling system.
4. Fix a power for the heating resistance of 8W with the power controller. You can know the power consumption in the resistance by viewing the wattmeter measurement SW-1. You should wait until ST-1 reaches a stable value, and then, start the experiment.
5. Wait until the system stays constant and no temperature variations are produced in the sensors.
6. Repeat the previous steps for heating powers of 8, 10, 12 and 14 W.
7. Complete Table 1 at data sheet.
8. Represent, for each power, the thermal evolution along the conductive metallic bar.
9. Verify Fourier's law, knowing that the distance between the temperature sensors is 10mm.

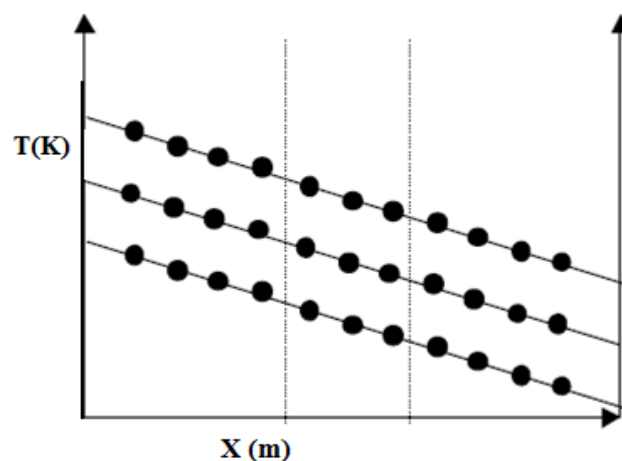


Fig. 14. Temperature versus distance diagram

3.2.2. Determination of the thermal conductivity, k, of the stainless steel and Conduction through a compound bar

3.2.2.1. Objectives

The objective of this experiment is to observe the temperature profile of a compound metallic bar. By means of this experiment we want to demonstrate that the heat transferred by the compound bar is defined by the heat transferred by the less conductive element (thermally speaking).

3.2.2.2. Required material

For the realization of this experiment we will need:

- TCCC equipment.
- SACED System for data acquisition.
- Accessory of lineal conduction.
- Stainless steel sample of 25 mm of diameter.

3.2.2.3. Experimental Procedure

For proper practice development, follow these steps:

1. Make water circulate through the refrigeration system.
2. Connect the computer, execute the program SACED and switch on the Interface.
3. Select a heating power of 8W.
4. Wait until the system reaches a stationary balance.
5. Complete the Table 2 at data sheet.
6. Verify that the following equation is completed:

$$\frac{q}{A} = \frac{t_1 - t_2}{\Delta x_{12} / k_{12}} \text{ Heating region (Brass 70 Cu, 30 Zn, } k=111 \text{ W/m}^\circ\text{C)}$$

$$\frac{q}{A} = \frac{t_4 - t_3}{\Delta x_{34} / k_{34}} \text{ Cooling region (Brass 70Cu, 30 Zn, } k=111 \text{ W/m}^\circ\text{C)}$$

NOTE: Use the experimental value given in the tables or the one experimentally obtained in the previous practice.

7. Represent the power in function of the temperature profile obtained.

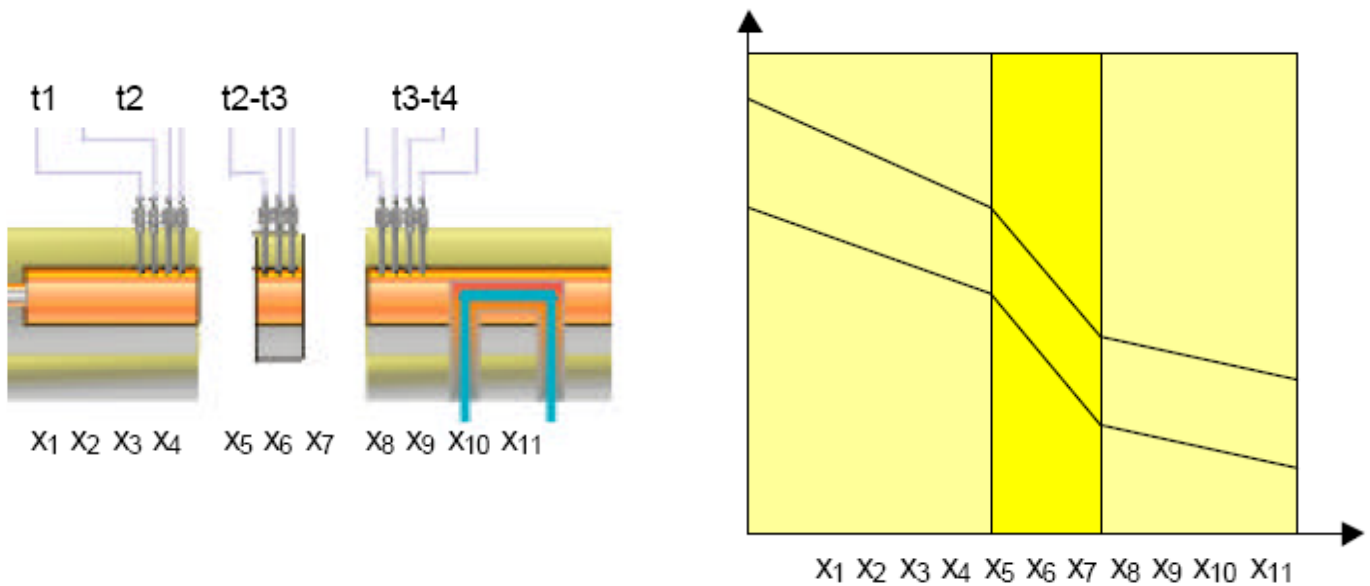


Fig. 15. Conduction in a simple bar

3.2.3. Effect of the crossing sectional area

3.2.3.1. Objectives

The objective of this experiment is to observe how the area, or section, of the transmitter element affects in the heat transmission. In this case, we will use the intermediate piece of brass (the same material that the heating and cooling elements) however, its section has been reduced from 25 mm to 10 mm of diameter.

3.2.3.2. Required material

- TCCC equipment, lineal conduction unit.
- SACED system.
- Interchangeable brass accessory of 10mm of diameter.

3.2.3.3. Experimental Procedure

1. Make water circulate through the refrigeration system.
2. Connect the computer, execute the program SACED and switch on the Interface.
3. Select a heating power of 8W.
4. Wait until the system reaches a stationary balance.
5. Complete Table 3 at data sheet.
6. Represent in a table the values obtained along the cylinder for each value of heating power.
7. Extrapolate the thermal gradient obtained for the heating and cooling elements to obtain the thermal gradient in the intermediate section of area 10mm.
8. Verify if the following equality is filled for each one of the modules:

$$A_A \left(\frac{\Delta T}{\Delta x} \right)_A = A_B \left(\frac{\Delta T}{\Delta x} \right)_B = A_C \left(\frac{\Delta T}{\Delta x} \right)_C$$

9. Establish the proportionality between the gradient of the coolers and the heater in comparison with the one of the sample and verify that their proportionality is equivalent to the proportionality of its areas.
10. That is to say:

$$\frac{A_B}{A_C} = \frac{\left(\frac{\Delta T}{\Delta x} \right)_C}{\left(\frac{\Delta T}{\Delta x} \right)_B}$$

3.2.4. Radial conduction

3.2.4.1. Objectives

The objective of this experiment is to demonstrate the heat distribution that takes place in a plane surface.

3.2.4.2. Required material

For the realization of the experiment the following elements are required:

- System of acquisition and control, SACED.
- TCCC equipment, “circular disk” accessory.

3.2.4.3. Experimental Procedure

1. Make water circulate through the refrigeration system.
2. Connect the computer, execute the program SACED TCCC2.EDB and switch on the Interface.
3. Select a heating power of 10 W.
4. Wait until the system reaches a stationary balance.
5. Complete Table 4 at data sheet.
6. Represent in a table the values obtained along the disk for each heating power value.
7. The behavior of the temperature along the radius will be drawn, at the same time, temperature T_0 should be determined in the external radius R_0 (110 mm) of the disk, starting from the curve.
8. Use these data to calculate the velocity of the heat radial conduction starting from the equation comparing the result with the heat input measured by the wattmeter (Q).
9. See the differences that can exist between the input measurements and the conduction velocity calculated.

10. Represent in a log-linear graph with radius on the logarithmic axis and the temperature on the lineal axis the data obtained.

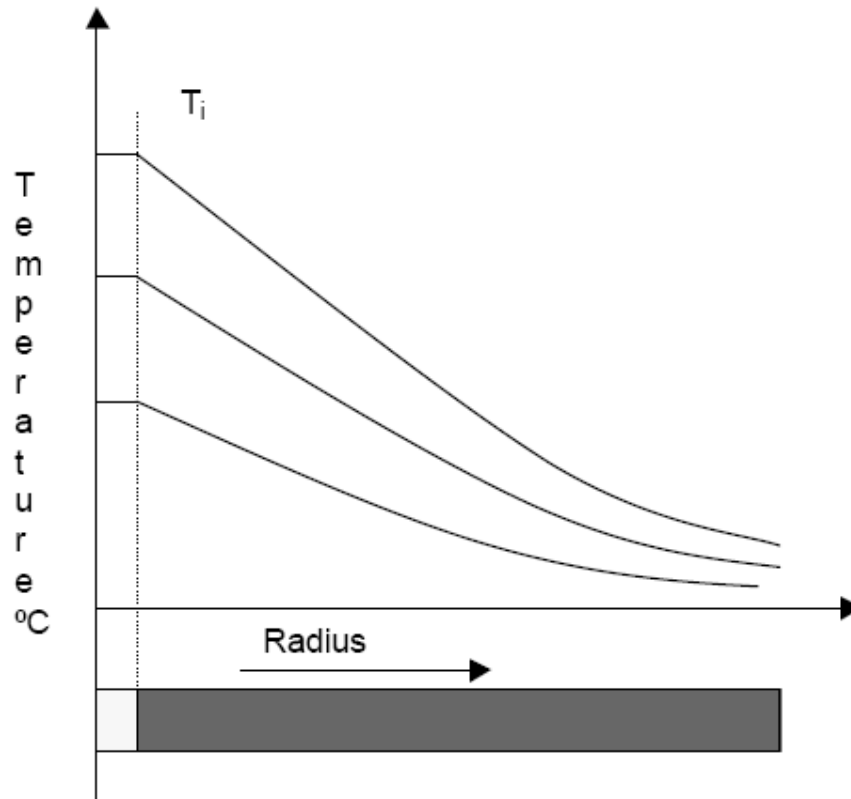


Fig. 16. Temperature versus radius diagram

3.3. Experimental Set-Up Of Computer Controlled Radiation

3.3.1. General Description

The apparatus consists metal plate with a resistance at one side and a lamp in another side. Lengthwise of the metal plate you can place the elements subminstre with the equipment.

3.3.2 Radiation Accessories

3.3.2.1. Radiometer (SR-1)

Allows to measure the intensity of the radiation. It is very important that you do not touch the radiometer surface, because is very sensitive.

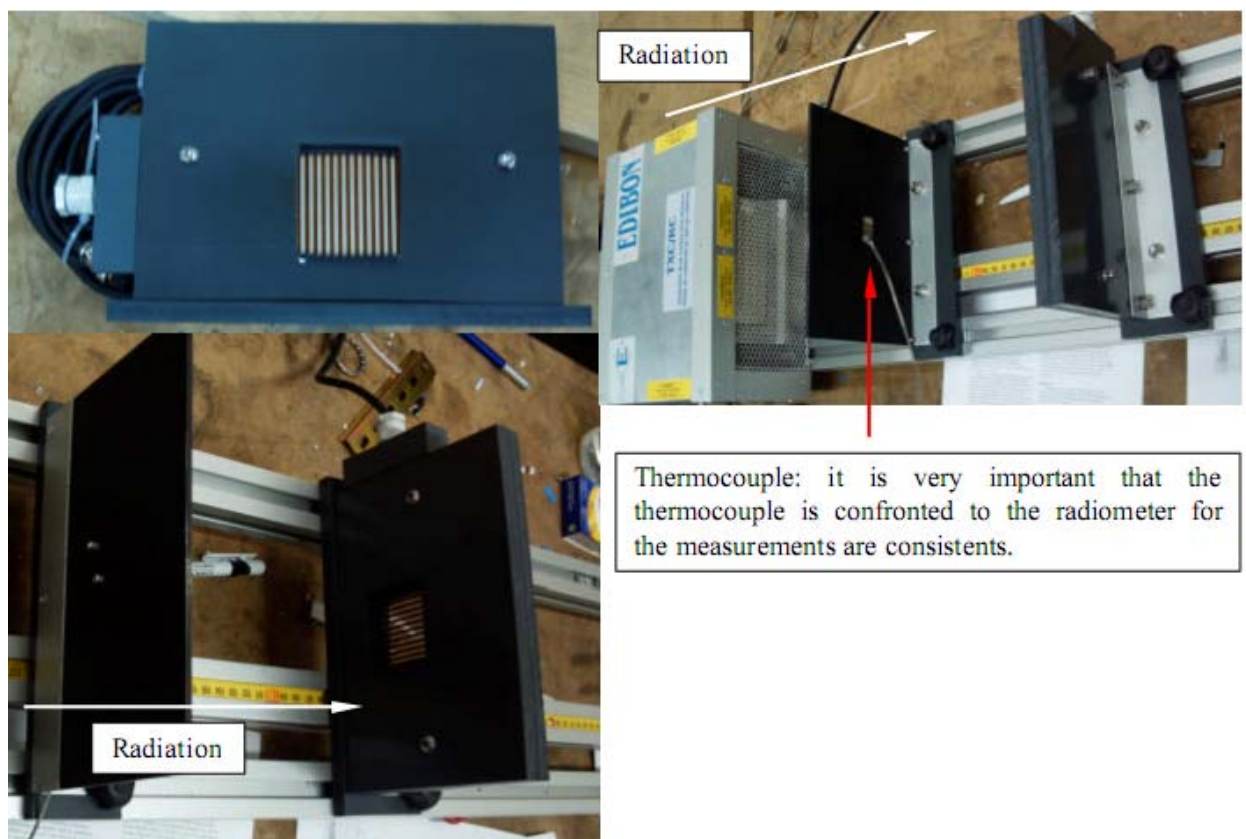


Fig. 17. Radiation heat transfer equipment.

3.3.2.2. Plane Surfaces

They are elements for studying their radiation. Each element contains a thermocouple (ST-1, ST-2, ST-3, ST-4 and ST-5).

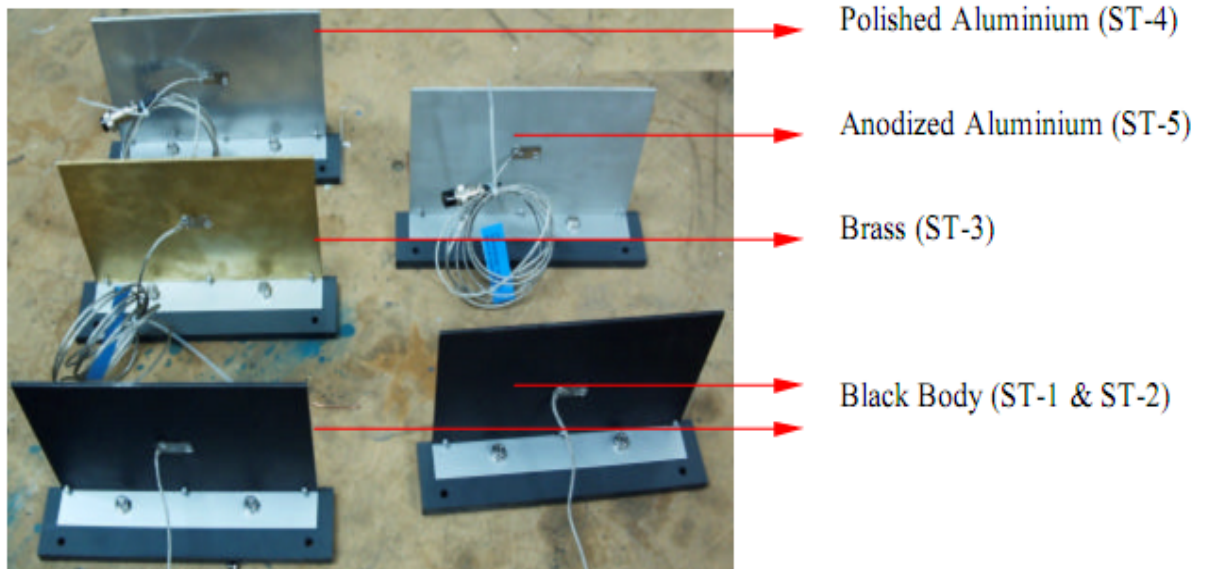


Fig. 18. Plane surfaces

3.4. Experimental Procedures of Computer Controlled Radiation

3.4.1. Inverse of the distant square law for the radiation

3.4.1.1. Objectives

This practice objective is the experimental demonstration of inverse of the distant square law for the radiation.

3.4.1.2. Required material

- SACED-TXC Software (Supplied with the equipment)
- TXC-RC equipment

3.4.1.3. Experimental Procedure

The setup of the practice is represented in the next figure,

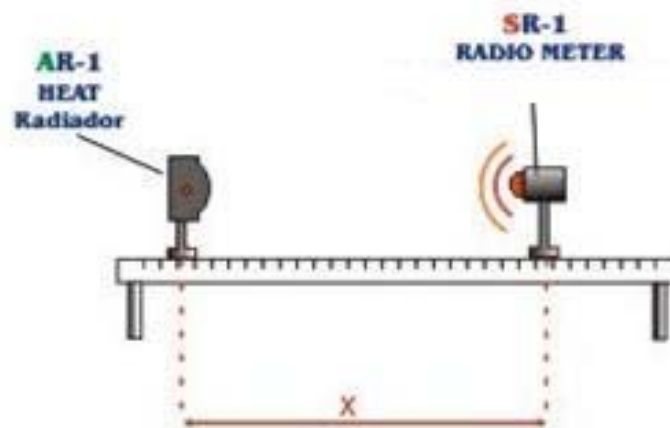


Fig. 19. Experimental set-up for inverse of the distant square law for the radiation experiment

In this case, you place the resistance (AR-1) and radiometer (SR-1) confronted. You change the distant “x” and you measure the radiation in the radiometer when the system is in stationary state.

For a proper practice development, follow these steps:

1. Connect the SACED program TXC_RC.
2. Verify that all the temperature sensors and that the heating resistances have been connected.
3. Fix a distant for the radiometer.
4. Fix a power for the heating resistance of 40 % power.

5. Wait until the system is stationary and the radiometer measure a constant value. Take the measurement in the radiometer.
6. Repeat the previous steps for another distance of the radiometer.
7. Represent, radiation versus distance.

3.4.2. Stefan-Boltzmann's law

3.4.2.1. Objectives

This practice objective is the experimental demonstration of Stefan- Boltzman's law.

3.4.2.2. Required material

For the realization of this experiment we will need:

- TXC-RC equipment.
- SACED System for data acquisition.
- Plate with the thermocouple ST-1.

3.4.2.3. Experimental procedure

The setup of the practice is represented in the next figure.

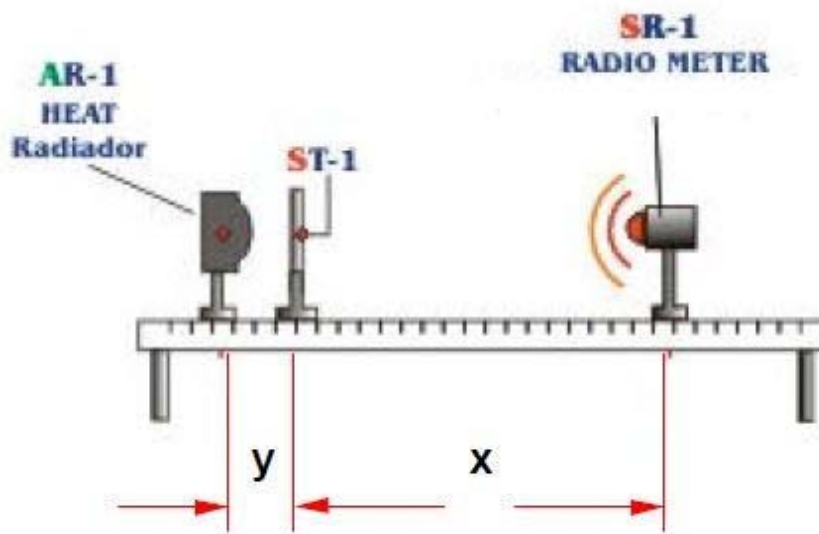


Fig.20. Experimental set-up for Stefan-Boltzmann's law experiment

In this case, the radiometer is confronted with the black plate. The resistance heats the plate and the plate irradiates energy. This energy is measured by the radiometer. You can change the temperature value in the plate (changing the distant “y” between the plate and the AR-1 or changing the power in the AR-1).

4. NOMENCLATURE

- q_x : heat transfer direction in the x direction, W
 A : cross sectional area normal to the direction of heat, m^2
 T : temperature, K
 x : distance, m
 k : thermal conductivity, W/m.K
 α : thermal diffusivity, m^2/s
 C_p : heat capacity, kJ/kg K
 k : thermal conductivity, W/mK
 ρ : density, kg/m^3
 r : radial distance, m
 T : temperature, K
 t : time, s
 h : heat transfer coefficient, W/m^2K
 R : radius of cylinder, m
 T_0 : initial uniform cylinder temperature, K
 T_∞ : bath temperature, K
 ν : kinematic viscosity, (m^2/s)
 μ : viscosity, (Pa s)
 L_c : characteristic length of body, $x_1=V/A$
 L : length of the reactor
 k_f : thermal conductivity of the fluid
 h : convective heat transfer coefficient
 λ : wavelength (μm)
 η : wave number (hertz=cycles/s)
 m : meter
 W : watt
 kg : kilogram
 J : joule

5. REFERENCES

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NAME-SURNAME:

GROUP NO:

ASSISTANT:

DATE:

DATA SHEET

Table 1. Relationship between power and temperature

Q (W)	T1	T2	T3	T4	T5	T6	T7	T8	T9	T10	T11
8											
10											
12											
14											

Table 2. Relationship between power and temperature

Q (W)	T1	T2	T3	T4	T5	T6	T7	T8	T9	T10	T11
8											
10											
12											
14											

Table 3. Relationship between power and temperature

Q (W)	T1	T2	T3	T4	T5	T6	T7	T8	T9	T10	T11
8											
10											
12											
14											

Table 4. Relationship between power and temperature

Q (W)	T1	T2	T3	T4	T5	T6
10						
15						
20						
25						

NAME-SURNAME:

GROUP NO:

ASSISTANT:

DATE:

DATA SHEET

Table 5. Experimental results for inverse of the distant square law for the radiation

R (W/m²)	X (mm)