STATISTICS
INTRODUCTION TO PROBABILITY

Instructor:

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Source: Kaplan, Robert M. <u>Basic Statistics for the Behavioral Sciences</u>, Allyn and Bacon, Inc., Boston, 1987. SENTENCES IN THIS POWER POINT PRESENTATION ARE USUALLY BORROWED FROM KAPLAN'S BOOK.

INFERENTIAL STATISTICS

In statistics, there is a distinction between a population and a sample. We sample to project conclusions about population. These projected estimations are subject to error. Inferential statistics are used to make educated people that take the chances of making errors into consideration.

INTRODUCTION TO PROBABILITY

Gambles in Everyday Life

Life is a serious of gambles. Although we are sometimes aware of it, nearly all decisions require an assessment of probabilities.

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Gambles in Everyday Life

A probability of 0 means the event is certain not to occur. A probability of 1.0 means the event will occur with certainity. For example, the sun will rise tomorrow.

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Gambles in Everyday Life

For most decisions, it is necessary to use some estimate of probability that is between 0 and 1.0. In other words, many decisions are bets against uncertainity.

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Inferential statistics are used to make inferences or general statements about a population based on a sample from that population. The major concern is whether the sample mean is equivalent. Inferential statistics are used to estimate the degree of correspondence between these two.

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Basic Terms

Random Experiment: is the experiment that the results are determined by chance.

Set: is a collection of things or objects that are clearly defined by some rule.

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Basic Terms

Element: is any member within a set.

Empty Set: is a set with no elements.

Union: is all elements that are in two different set at the same time or in any of two sets. Union is symbolized as (A U B)

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Basic Terms

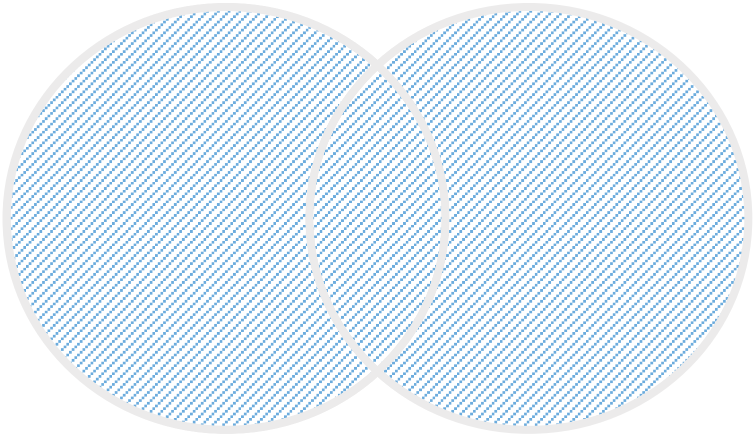
Intersection: is the subset of all elements that are commonly in two different sets at the same time. It is symbolized as ($A \cap B$)

Mutually Exclusive Events: Two events are mutually exclusive if they share no common elements. e.g. genders are mutually exclusive.

Complement: is made up of all other elements in the set outside of the subset.

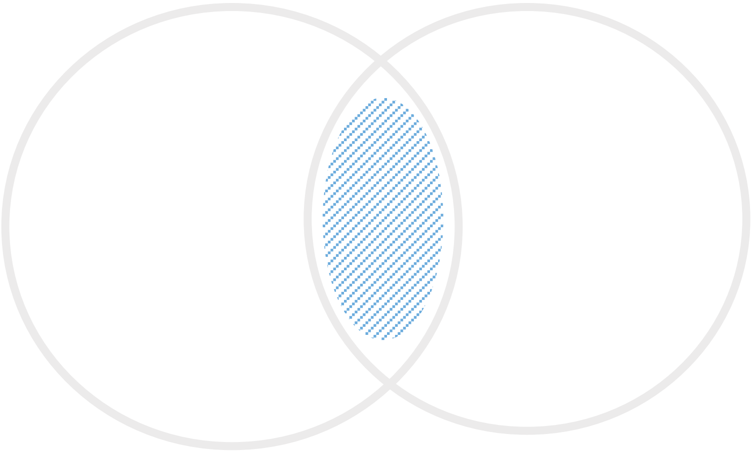
UNION

$P \cup Q$

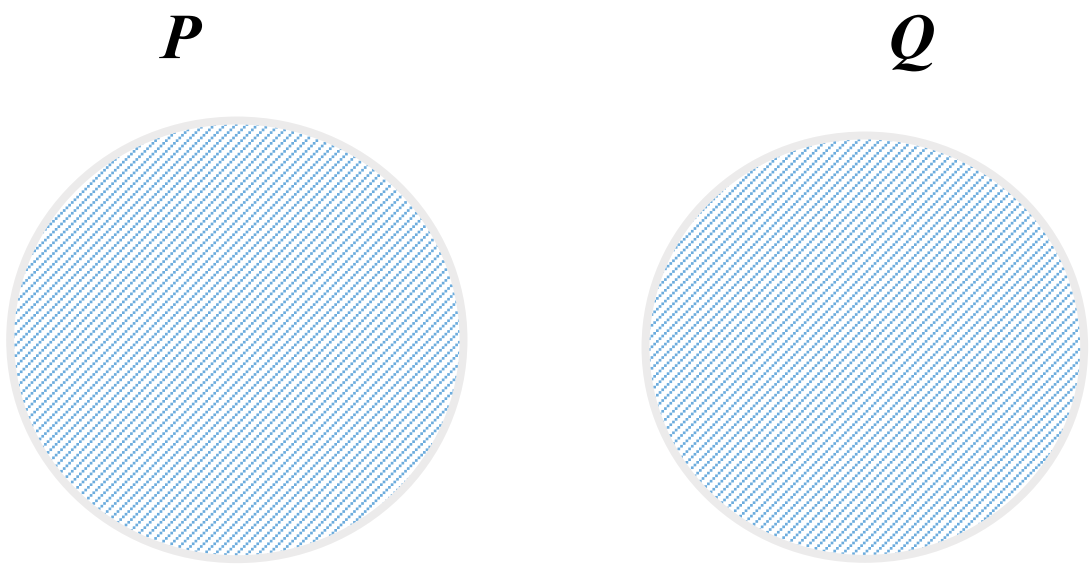


INTERSECTION

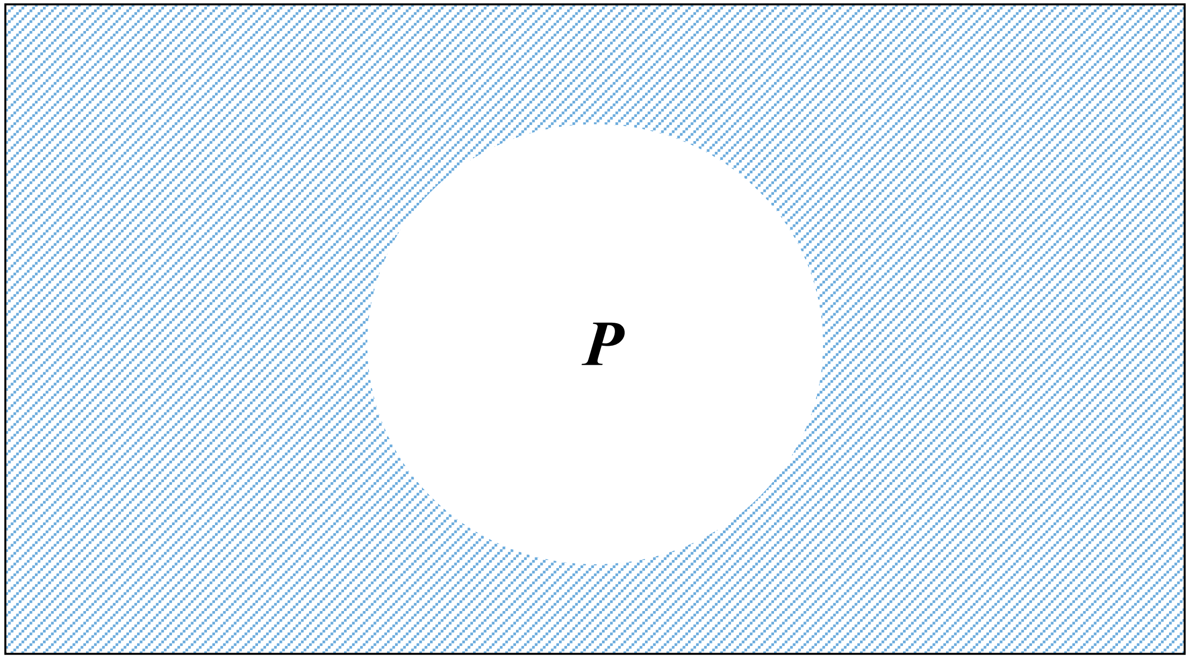
$$P \cap Q$$



MUTUALLY EXCLUSIVE EVENTS



COMPLEMENT



INTRODUCTION TO PROBABILITY

Basic Probability for Independent Events

Probability is the study of odds and chances. To calculate the probability of an outcome in an independent event, it is necessary to know all possible outcomes at first. For example, if you flip a coin, there are two possible outcome: namely HEAD and TAIL. Either of outcome will occur with a certainity. If we try to calculate the probability of an event in more than one independent event, we have to know about the number of all possible alternative outcomes. For example, if you flip a coin two times, you may have outcomes like HH, HT, TH, TT. There are four possible alternative outcomes.

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Basic Probability for Independent Events

Calculation of all possible alternatives can be formulated as such:

(X^a) Number of Possible Outcomes$^{\text{Number of Independent Events}}$

Example: For three independent coin tosses, it is 2^3
and the number of alternative outcomes is, thus, 8.

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Additive and Multiplicative Rules

Many problems in statistics and probability require us to combine two independent probability estimates. For many times it is difficult to determine how to combine independent probabilities to make a joint statement. We use additive rule and multiplicative rule. According to additive rule, we add the two probabilities together to calculate the probability of the occurance of either of events. That is to say, probability of A OR B. In summary, the additive rule expresses the probability of UNION.

Example: For a coin toss, the probability of getting either a head or a tail. ½ + ½ = 1.00

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Additive and Multiplicative Rules

Example: What is the probability of drawing an 8 OR a King from a standard 52-card deck?

$1/13 + 1/13 = 2/13 = 0.15$

Example: What is the probability of drawing an 8 OR getting a five in rolling of a die?

$1/13 + 1/6 = 19/78 = 0.24$

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Additive and Multiplicative Rules

When calculating the probability of joint occurence of events in totally different events, we use the multiplication rule. According to the multiplication rule, we multiply the independent probabilities together.

Example: For the chances of obtaining both a head in a coin toss AND a six in the roll of a die, the probability of getting the result :½ x 1/6 = 1/12 = 0.08

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Permutations and Combinations

Permutation is the list of joint occurence of all possible outcomes for independent events in a specific order.

For example, there are four aces: spades ♠, clubs ♣, hearts ♥, and diamonds ♦. What is the probability of drawing a ♥ followed by a ♦? We can list all possible outcomes for this joint occurance as such:

♠♣ ♠♥ ♠♦ ♣♠ ♣♥ ♣♦ ♥♣ ♥♠ ♥♦ ♦♣ ♦♠ ♦♥

SC SH SD CS CH CD HC HS HD DC DS DH

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Permutations and Combinations

The probability of drawing a ♥ followed by a ♦ is

¼ x 1/3 = 1/12 = 0.08. It means 8 percent.

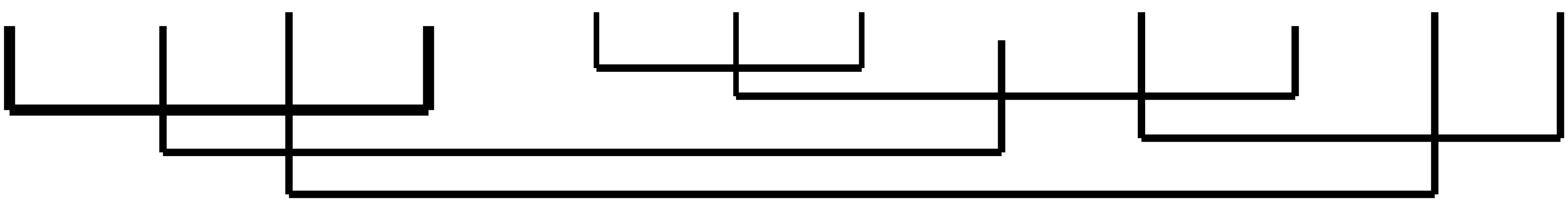
Permutation is getting the joint occurance in a specific order.

Factorials

When we ignore getting the joint occurance in a specific order (i.e. either of events may be first or second) the number of possible alternative outcomes changes. For example, the number of possible outcomes for the probability of drawing a ♥ AND a ♦ without specific order is 6.

♠♣ ♠♥ ♠♦ ♣♠ ♣♥ ♣♦ ♥♣ ♥♠ ♥♦ ♦♣ ♦♠ ♦♥

SC SH SD CS CH CD HC HS HD DC DS DH



Factorials

Thus, the probability of drawing a ♥ AND a ♦ is 1/6 = 0.167. That means 16.7 percent. This approach, which does not consider the order, is called combinations.

Factorials

There are formulations to find permutations and combinations. To understand these formulas, we must review the concept of factorial. The factorial for a number is the product of the integers from 1 to the number. The factorial is signified by an exclamation point. For example, the factorial of 6 is expressed as

$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

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Permutations

The formula for complex permutations is

N = Number of objects.   P = Permutation
M = Number of objects taken at a time.

$$ {}_{N}P_{M} = \frac{N!}{(N-M)!} $$

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Permutations

Remember that the permutation for the probability of drawing a ♥ followed by a ♦ is 12. We can test it with formula:

N = 4 M = 2

$${}_4P_2 = \frac{4!}{(4-2)!} = \frac{4x3x2x1}{2x1} = \frac{24}{2} = 12$$

$${}_NP_M = \frac{N!}{(N-M)!}$$

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Combinations
The formula for complex combinations is

N = Number of objects. C = Combination
M = Number of objects taken at a time.

$$ {}_{N}C_{M} = \frac{N!}{M!(N-M)!} $$

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Combinations

Remember that the permutation for the probability of drawing a ♥ followed and a ♦ is 12. We can test it with formula:

N = 4 M = 2

$$ {}_{4}C_{2} = \frac{4!}{2!(4-2)!} = \frac{4x3x2x1}{2x(2x1)} = \frac{24}{4} = 6 $$

$$ {}_{N}C_{M} = \frac{N!}{M!(N-M)!} $$

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Winning ticket names the first, second and third place horses (There are 8 horses).

$$prob(w) = \frac{1}{336} = 0.0030$$

$$prob(w) = \frac{1}{{}_{N}P_{M}}$$

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Probability of winning ticket names the horses finishing in the top three.

$$prob(w) = \frac{1}{56} = 0.0179$$

$$prob(w) = \frac{1}{{}_{N}C_{M}}$$